Linear vs angular

Since we know that
\[ s = r \theta \]
we can relate our "linear world" to our "angular world":
\[ \frac{ds}{dt} = r \frac{d\theta}{dt} \] (use \( l \) since we care about speed)

\[ v = r \omega \] linear speed of rotating object

\[ \Rightarrow \text{Farther a point is from axis of rotation, the larger its speed!} \]

\[ a_{\text{tan}} = \frac{dv}{dt} = r \frac{d\omega}{dt} = r \dot{\omega} \] linear acceleration of point on rotating object

\[ a_{\text{cp}} = \frac{v^2}{r} = \omega^2 r \]

\[ \Rightarrow \text{Always use radians!} \]
Since \( s = r \theta \) only works in radians.
If the runner can run 2 m/s, what is the angular velocity of the person hanging from the merry-go-round?

What is the linear velocity of the person sitting in the middle?

\[ \omega = \frac{v}{r} = \frac{2 \text{ m/s}}{1 \text{ m}} = 2 \text{ rad/s} \]

\[ v = rw \]

\[ = 0.25 \text{ m} \cdot 2 \text{ rad/s} = 0.5 \text{ m/s} \]

If my foot is measured to be traveling at 3.0 m/s, how fast is the bike moving?

\[
\begin{align*}
\omega_{\text{pedal}} &= \omega_{\text{front}} = \frac{v_{\text{foot}}}{\text{pedal}} \\
\omega_{\text{chain}} &= \omega_{\text{front}} = \frac{v_{\text{foot}}}{\text{forward}} \\
\omega_{\text{wheel}} &= \omega_{\text{chain}} = \frac{v_{\text{foot}}}{\text{forward}}
\end{align*}
\]
Energy in rotational motion

A rotating body is moving \( \Rightarrow KE \)

Depending on \( \omega \) & moment of inertia.

\[
KE = \frac{1}{2}mv^2 = \frac{1}{2}mr^2\omega^2
\]

\( \omega \) is the same for all particles

\[
k = \frac{1}{2} \left( \sum m_i r_i^2 \right) \omega^2
\]

\[
I = \text{moment of inertia}
\]

depends on how the mass is distributed in space

\[
KE = \frac{1}{2} I \omega^2 \rightarrow \text{rotational kinetic energy of a rigid body}
\]
Easier to turn \hspace{2cm} \text{Harder to turn}

Mass is the same, but \textit{I} is not!

\textit{Discrete masses vs. continuous masses...}

\begin{align*}
\text{cylinder} & \quad m = 1.2\, \text{kg} \\
& \quad V = 1.41\, \text{m/s} \quad \text{Table 9.2}
\end{align*}

\text{a) What is } KE_{\text{trans}}? \quad I_{\text{cyl}} = \frac{1}{2} m R^2

\text{b) What is } KE_{\text{rot}}? \\
\text{c) What is } KE_{\text{tot}}?

\begin{align*}
\text{a) } KE &= \frac{1}{2} m V^2 = \frac{1}{2} (1.2\, \text{kg})(1.41\, \text{m/s})^2 = 1.19\, \text{J} \\
\text{b) } KE_{\text{rot}} &= \frac{1}{2} I \omega^2 = \frac{1}{2} \left(\frac{1}{2} m R^2\right) \left(\frac{V^2}{R^2}\right) = \frac{1}{2} \cdot \frac{1}{2} m V^2 = 0.595\, \text{J} \\
\text{c) } KE_{\text{tot}} &= 1.19 + 0.595\, \text{J} = 1.79\, \text{J}
\end{align*}
Parallel axis

A body as a moment of inertia around a defined axis \( \Rightarrow \) you can define infinitely many axes!

Parallel Axis Theorem

\[
I_P = I_{cm} + Md^2
\]

\[
I_{cm} = \frac{1}{2} ML^2
\]

\[
I_{end} = I_{cm} + M \left( \frac{d}{2} \right)^2
\]

\[
= \frac{1}{12} ML^2 + \frac{M d^2}{4}
\]

\[
= \frac{1}{12} ML^2 + \frac{3}{12} ML^2 = \frac{4}{12} ML^2 = \frac{1}{3} ML^2
\]