\[ \Delta U_{int} = \mathbf{W} \text{ friction} \]

\[ k_1 + u_1 - \Delta U_{int} = k_2 + u_2 \]

\[ \Delta k + \Delta u + \Delta U_{int} = 0 \]

\[ \rightarrow \text{Energy is conserved} \]

\[ \text{PE} \]

\[ \text{KE} + \text{PE} = \Delta U_{int} \]

\[ \text{KE} \rightarrow \text{PE} - \Delta U_{int} \]

"Momentum impulsive, + collisions"

What if we don't know anything about the forces involved when two bodies interact?

\[ \mathbf{F} = m \frac{dv}{dt} = \frac{d}{dt}(mv) \]

"Rate of change of 'mv' equals the net force"
So define, \( \vec{p} = m \vec{v} \) momentum.

Unlike energy, Vector!

\[
\begin{align*}
E_1 & = E_2 \\
\Rightarrow & \quad E_1 > E_2 \\
\Rightarrow & \quad \frac{1}{2}v
\end{align*}
\]

\[
\begin{align*}
P_i & = P_e \\
\Rightarrow & \quad P_i = P_e
\end{align*}
\]

\[
\vec{P} = P_x \hat{i} + P_y \hat{j} + P_z \hat{k} \quad P_x = m v_x, \quad P_y = m v_y, \quad P_z = m v_z
\]

\[
L = \Sigma \vec{F} = \frac{d\vec{p}}{dt}
\]

The net force acting on a system equals the time rate of change of the momentum of the system.
Why is there $\frac{1}{2}mv^2 - mv^2$? What is the difference?

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$\uparrow \quad \uparrow \quad \uparrow \quad \uparrow$

$\tau = \int_0^\infty F dt \quad \text{(constant F)}$ 

Impulse

$\sum F = \frac{p_f - p_i}{t_2 - t_1} \quad \text{(again, F const)}$

$\Rightarrow \sum F(\tau_2 - \tau_1) = p_f - p_i \Rightarrow \tau = p_f - p_i$

- The change in momentum of a system equals the impulse of the net force that acts on the system.
- Impulse-momentum theorem

$L \Rightarrow \int_{t_1}^{t_2} \sum F dt = \int_{p_i}^{p_f} \frac{dp}{dt} dt = \int_{p_i}^{p_f} dp = p_f - p_i$

If $F$ varies with time, still holds:

$\tau = p_f - p_i$

If not $\text{const } \vec{F}$

$\Rightarrow \sum F_{\text{net}} = F_{\text{avg}}(t_2 - t_1)$
In (1) and (2), \( J \) is the same since the area under the curves is equal.

So, \( J = \vec{F}_2 - \vec{F}_1 \) depends on how much time a force acts for.

\[ W_{cm} = k_2 - k_1 \] depends on over how much distance a force acts.

Which is harder to catch?

- 0.5 kg ball @ 4.0 m/s \( \vec{p} = mv = 2.0 \text{ kg m/s} \)
- 0.1 kg ball @ 20.0 m/s \( \vec{p} = 2.0 \text{ kg m/s} \)

\[ \text{but } KE_1 = \frac{1}{2} \cdot 5 \text{ kg} \cdot (4.0 \text{ m/s})^2 = 40 \text{ J} \]
\[ KE_2 = \frac{1}{2} \cdot 1 \text{ kg} \cdot (20 \text{ m/s})^2 = 20 \text{ J} \]

Since \( p_1 = p_2 \), the impulse required to stop both balls is the same.
or, if \( F \) applied to stop the ball's is the same, then \( v \) is also.

However, it will take \( 5 \) as much distance to bring the lighter ball to a stop! → Ouch!