Chapter 6

Work.

Using the principle that applying a force over a displacement means you are doing work on an object, we can analyze motion in the pressure of constant or non-constant forces.

\[ W = F \Delta s \ [Nm] \cdot [J] \]

But what if \( \Delta s \) and \( F \) aren't in the same direction?

\[ W = F \Delta s \cos \theta \]

(Since, as would have to be zero, or over force)
Therefore
\[ W = \vec{F} \cdot \Delta \vec{s} = F \Delta s \cos \theta \]

\[ \begin{array}{c}
\text{Work is a scalar!}
\end{array} \]

\[ \begin{array}{c}
\text{1) } \theta = 0, \Delta s = 1 \text{ m}
\end{array} \]

\[ \begin{array}{c}
\text{2) } \theta = 30^\circ, \Delta s = 1 \text{ m}
\end{array} \]

What is the total work done by the child?

1) \[ W = F \Delta s \cos \theta = 0.8 \text{ N} \cdot 1 \text{ m} \cdot \cos 0^\circ = 0.8 \text{ J} \]

2) \[ W = F \Delta s \cos \theta = 0.8 \text{ N} \cdot 1 \text{ m} \cdot \cos 30^\circ = 0.69 \text{ J} \]

\[ W_1 = W_1 + W_2 = 1.49 \text{ J} \]

While it's not a scalar, work can be positive, negative, or zero.
\[ W_T = W_N + W_f + W_g + W_f \]

or \[ W_T = F_r \cdot a \cdot \cos \theta \]

\[ F_r \cos \theta_1, F_r \cos \theta_2, \ldots \]

Clearly, work is related to a displacement as the result of a net force.

We can also relate work to change in kinetic energy as a result of a net force.

\[ \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = W \]

\[ F_a s = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = W \]

\[ a \mid \frac{v_f^2 - v_i^2}{s} \]

\[ a = \text{const} \]

Apply \[ \sum F_x = ma_x = m \frac{v_f^2 - v_i^2}{s} \]

\[ \text{End} \]
The total work done on an object by a net force is equal to its change in kinetic energy.

\[ W_f = K_2 - K_1 = \Delta K \rightarrow \text{Work-energy theorem} \]

\[ \text{Same mass, } \quad \text{Same speed, different direction} \quad \text{Same kinetic energy} \]

\[ \text{Same speed, } \quad 2 \times \text{mass, } \quad 2 \times \text{Kinetic energy} \]

\[ \text{Same speed, } 2 \times \text{KE} \]

How much work does it take to move a 1000 kg Mazda 2 from rest to 27 m/s on a level road? What is the average net force acting on the car if it acts over a distance of 140 m?

\[ \text{0 (initially at rest)} \]

a) \[ W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 \]

\[ = \frac{1}{2} (1000 \text{kg} \times (27 \text{m/s})^2) = 3.6 \times 10^5 \text{ J} \]

b) \[ W = F \cdot \Delta s, \text{ since } F \cdot \Delta s \text{ are in the same direction} \]

\[ F = \frac{W}{\Delta s} = \frac{3.6 \times 10^5 \text{ J}}{140 \text{ m}} = 2.6 \times 10^3 \text{ N} \]
This example demonstrates the meaning of Kinetic Energy.

To accelerate an object of mass \( m \) up to a speed \( v \), the total work done must be equal to the change in energy from zero to \( k = \frac{1}{2}mv^2 \).

The KE of an object is equal to the total work that was done in accelerating it from rest to its current speed.

\[ W = F \Delta x_1 + F_2 \Delta x_2 + F_3 \Delta x_3 + \ldots + F_n \Delta x_n \]

\[ W = \int F \, dx \]

Work-Energy Theorem applies even for non-constant forces.
For a constant force,
\[ W = \int F \, dx \]
\[ = F_x \int_0^{\Delta x} \, dx \]
\[ = F_x (x_2 - x_1) = F_x \Delta x \]

Springs exert non-constant forces.
\[ F = kx \]  
\[ \text{unstretched} \rightarrow \quad \text{stretched} \]

\[ \text{Hooke's Law} \]

\[ k \text{ is force constant, depends on spring} \]

Work done to stretch a spring then is
\[ W = \int_{x_1}^{x_2} F \, dx = \int_{x_1}^{x_2} kx \, dx = \frac{1}{2} k(x_2^2 - x_1^2) \]
\[ = \frac{1}{2} kx_2^2 - \frac{1}{2} kx_1^2 \]
So, for us, we have talked about work done by a constant force, non-constant force along a straight line. What about in general?

\[
W = \int F \cdot \cos \theta \, dx
\]

Exert a horizontal force to push someone on a swing at a constant slow velocity. What is the total work done?
$F$ must vary with position

$F_x = F \cdot \sin \theta \rightarrow F = F \sin \theta$

$F_y = T \cos \theta \cdot \omega = 0 \rightarrow T = 0 / \cos \theta$

$\Rightarrow F = \omega T \tan \theta$

Total work done,

$EF = 0$ (equilibrium)

$\Rightarrow F_{net} = 0$

$W = \int F \cdot dl = 0$

Work done by force $F$?

$W = \int \frac{F \cdot de}{dl} = \int F \cos \theta e d\theta$  

$= \int \omega \tan \theta \cos \theta e R \, d\theta$

$= \omega R \int \tan \theta \cos \theta e \, d\theta$

$= \omega R (\cos(\theta) - \cos(\theta_0))$

$= \omega R (1 - \cos \theta_0)$

if $\theta_0 = 0, \theta_0 = 0, w = \omega \rightarrow \omega = \omega$
So far, we haven't worried about how much time has passed while doing work.

\[ W = F \cdot d = 20 \text{N} \cdot 5 \text{m} = 100 \text{ J} \] (18, 19, etc.)

Power is the rate at which work is done.

\[ P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} \]

\[ P = \frac{\Delta W}{\Delta t} = \frac{3 \text{ J}}{2 \text{ s}} = 1.5 \text{ W} \]

\[ 1 \text{ HP} = 746 \text{ W} \]

\[ 100 \text{ W} = \frac{100 \text{ J}}{1 \text{ s}} \]
Power can also be calculated in terms of velocity. If a force acts on a body while the body undergoes a displacement:

\[ P_{av} = \frac{W}{\Delta t} = \frac{F \cdot \Delta s}{\Delta t} = \text{F \cdot V}_{avg} \]

or

\[ P = \text{F \cdot V} = \text{F \cdot V} \]

ex: What is the minimum power required to push a slow moving truck if your Marsh-2 must accelerate from 13.4 m/s to 17.9 m/s in 3s?

\[ m = 17,300 \text{ kg} \]

\[ \text{V}_{i} = 13.4 \text{ m/s} \]

\[ \text{V}_{f} = 17.9 \text{ m/s} \]

\[ \text{W} = \Delta KE = \frac{1}{2} m \text{V}_{f}^2 - \frac{1}{2} m \text{V}_{i}^2 \]

\[ = \frac{1}{2} (17,300 \text{ kg})(17.9 \text{ m/s})^2 - \frac{1}{2} (17,300 \text{ kg})(13.4 \text{ m/s})^2 \]

\[ = 9,16 \times 10^7 \text{ J} \]

\[ P = \frac{\text{W}}{\Delta t} \]

\[ = \frac{9,16 \times 10^7 \text{ J}}{3 \text{ s}} \]

\[ = 3,05 \times 10^7 \text{ W} \]