Energy-aware Persistent Coverage and Intruder Interception in 3D Dynamic Environments

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Abstract—This paper considers the persistent coverage of a 2-D manifold that has been embedded in 3-D space. The manifold is subject to continual collisions by intruders that are generated with random trajectories. The trajectories of intruders are estimated online with an extended Kalman filter and their predicted impact points contribute normally distributed decay terms to the coverage map. A formal hybrid control strategy is presented that allows for power-constrained 3-D free-flyer agents to persistently monitor the domain, track and intercept intruders, and periodically deploy from and return to a single charging station on the manifold. Guarantees on intruder interception with respect to agent power lifespans are formally proven. The efficacy of the algorithm is demonstrated through simulation.

I. INTRODUCTION

The advent of inexpensive autonomous research platforms has spurred recent interest in teams of mobile sensors collaborating on complex surveillance and monitoring tasks. Coverage control problems have been particularly popular due to their numerous applications: e.g., environmental monitoring [1], battlefield surveillance [2], lawn mowing and vacuuming, search and rescue [3], and hull inspections [4], [5]. The latter application is actively supported by NASA whose work on the Mini AERCam paves the way for a future of extravehicular robotic (EVR) free flyers performing independent visual inspections of spacecraft exterior areas of interest [6]. Free flyer visual inspection is the primary motivating example for our work.

Coverage is often partitioned into three classes of problems: static, dynamic, and persistent. Static coverage problems (e.g., area coverage, k-coverage and point coverage) often explore the optimal arrangement of sensor nodes in a network and the agents tend to immobilize after this arrangement has been achieved [7]. Dynamic coverage problems involve the active exploration of a domain. Agents typically must sweep their sensors over all points of a domain until some desired level of coverage has been achieved [8]–[10]. Persistent coverage is often similar to dynamic coverage with the addition of information decay within the environment: i.e., agents are required to continually return to areas of interest in order to restore a deteriorating coverage level.

The term "persistent coverage" appears as early as [11] where agents must cover all points in a 2-D convex polygonal domain every $T^n$ time units. This was accomplished with the design of concentric polygonal trajectories with agents following closed paths in steady state. The work in [12] is similar but also introduces a linear coverage decay rate for specific points of interest. In this paper, as well as [13], controller design is akin to regulating the velocity along paths generated offline to increase observation time at select points of interest. As the decay rates are known and time invariant, optimal speed control is computed via linear programming.

Palacios-Gasós et al have published multiple works recently on persistent coverage [14]–[16] which also consider time-invariant coverage decay rates. These works use techniques from discrete optimization and linear programming to iteratively compute optimal paths. Similar techniques are used in [17] which also considers that agents must periodically return to refueling depots.

Common themes through all of these persistent coverage works are convex 2-D domains, predictable environments, and simplified sensing and dynamic models for agents. Coverage surfaces embedded in $\mathbb{R}^3$ are considered in [18]; however, this work is closer to that of [11] in that agents also follow predefined trajectories without any consideration of spatially dependent coverage decay maps.

Monitoring of stochastic environments is presented in [19], [20], outside of the strict persistent coverage formulation. In [19], the authors consider that agents must observe events at multiple points of interest and the precise arrival times of events are unknown \textit{a priori}. Arrival time statistics are used to inform a multi-objective scheduling protocol that results in fixed cyclic servicing policies. In [20], the environment contains smart intruders which actively attempt to evade a camera surveillance network. Camera motion is restricted to a single pan axis and thus the the system model is essentially that of a 1-D pursuit evasion problem.

The primary contributions are: a formal hybrid control strategy for multi-agent persistent coverage of non-planar surfaces embedded in $\mathbb{R}^3$ that does not make overly simplifying assumptions with respect to agent dynamic and sensing models, a formal guarantee on agent interception of stochastic intruders, and an energy-aware agent deployment and scheduling protocol. The primary hybrid mode builds upon the authors’ previous work in [21]. In addition, agents now operate with finite resources and are required to periodically return to a refueling station while observing stochastic events at locations and times that are not known \textit{a priori}. Unlike related works, these events may occur anywhere in...
the domain and the events themselves introduce non-linear
time varying decay terms to the coverage map.
This paper is organized as follows: Section II describes the
agents sensing and kinematic models, local coverage stra-
gy, and intruder detection model. Section III describes our
particle (e.g., intruder) interception hybrid mode. Section IV
details our energy-aware domain partitioning and scheduling
protocols. Section V presents our collision avoidance hybrid
mode with the formal hybrid automaton presented in Section
VI. Simulations and conclusions are presented in Sections
VII and VIII respectively.

II. PROBLEM FORMULATION

A. System Model

Consider a network of spherical autonomous agents in-
dexed \( i \in \{1, ..., N\} \), of radius \( r_i \), whose motion is
subject to 3-D rigid body kinematics [22]:

\[
\begin{bmatrix}
\dot{x}_i \\
\dot{y}_i \\
\dot{z}_i
\end{bmatrix} = 
\begin{bmatrix}
\cos \theta_i \cos \psi_i \\
\sin \theta_i \sin \psi_i - \cos \phi_i \sin \psi_i \\
\sin \theta_i \cos \psi_i + \cos \phi_i \cos \psi_i
\end{bmatrix}
\begin{bmatrix}
\dot{x}_i \\
\dot{y}_i \\
\dot{z}_i
\end{bmatrix} + 
\begin{bmatrix}
u_i \\
v_i \\
t_i
\end{bmatrix},
\]

where \( p_i = [x_i \ y_i \ z_i]^T \) is the position vector and \( \Omega_i = \begin{bmatrix} \dot{\phi}_i \ \dot{\theta}_i \ \dot{\psi}_i \end{bmatrix}^T \) is the vector of 3-2-1 Euler angles taken with
respect to a global Cartesian coordinate frame \( \mathcal{G} \) with origin
\( O \). The linear velocities \( [u_i \ v_i \ w_i]^T \) and angular velocities \( [q_i \ \bar{r}_i \ \bar{s}_i]^T \) are both presented in the body fixed frame \( \mathcal{B}_i \)
with origin \( p_i \). The state vector of agent \( i \) is defined as \( \eta_i = [p_i^T \ \Omega_i^T]^T \). In the sequel, the rotation matrices of (1) and
(2) shall be denoted \( R_1 \) and \( R_2 \) respectively. The agents
travel within a stationary domain, \( D \subset \mathbb{R}^3 \). Their task is to
survey a two-dimensional manifold, \( C \subset D \), known as our
surface of interest. For the purpose of this work, we assume
that the surface is an ellipsoid of revolution with semi-major
axis \( x_{C,r} \) and semi-minor axis \( x_{C,s} \) aligned with the global
coordinate axes \( \hat{x}_G \) and \( \hat{z}_G \) respectively with center at \( O \).

Each agent, \( i \), is equipped with a forward facing sensor
whose footprint shall be referred to as \( S_i \). A spherical sector
model is chosen for \( S_i \) as it is representative of the space
typically observable to a single camera lens. The center-line
\( S_i \) provides anisotropic sensing data which degrade in quality
wards of the footprint in a similar manner to that of a camera lens. Spherical sector \( S_i \) has vertex at \( p_i \) and a
centerline parallel to the \( \hat{x}_{B_i} \) axis. It is also characterized
by field of view \( 2\alpha_i > 0 \) and sensing range \( R_i > 0 \) where
\( R_i > r_i \). See [21] for a full sensing model description.

Denote the quality of information available at each point
in \( S_i \) as \( S_i(\eta_i(t), \bar{p}) \) where \( \bar{p} = [\bar{x} \ \bar{y} \ \bar{z}]^T \) is the position
of a point in \( D \) with respect to \( G \). Define the cover-
age level provided by agent \( i \) at time \( t \) as \( Q_i(t, \bar{p}) =
\int_0^t S_i(\eta_i(\tau), \bar{p}) \, d\tau \), where \( C \) is defined as \( C(\bar{p}) = \{ \bar{p} \in C \land 0 \forall \bar{p} \notin C \} \) and encodes that the accumulation of
information only occurs along our surface of interest, \( C \).

As the agents cover \( C \), a set of \( N_p \) high-speed particles
denoted \( k \in \{1, ..., N_p\} \), each of which travels
at a constant linear velocity, pass through the domain. Each particle shall have an associated map decay term,
\( \Lambda_k(\tau, \bar{p}) \), which is defined later in Section II-C. We define
the global coverage level as \( Q(t, \bar{p}) = \sum_{i=1}^N Q_i(t, \bar{p}) - \sum_{k=1}^{N_p} \int_0^t \Lambda_k(\tau, \bar{p}) \, d\tau \).

In this work, coverage refers to the accumulation of
sensing data over time. Points, \( \bar{p} \), are said to be sufficiently
covered when \( Q(t, \bar{p}) \geq C^* \). The purpose of this work is to
derive a hybrid control strategy which persistently sweeps \( S_i \)
across \( C \) while emphasizing surveillance around the predicted
impact points of intruders \( k \in \{1, ..., N_p\} \) on \( C \). This must
be done while avoiding collisions as defined in Definition 1.

Definition 1: Agent \( i \) avoids collision so long as \( ||p_i(t) -
p_j(t)|| > r_i + r_j \), \( \forall j \neq i \in \{1, ..., N\} \) and \( ||n_i|| > r_i \) where
the vector \( n_i \) has direction normal to \( C \) and length equal to the
Euclidean distance of its intersection point on \( C \) to \( p_i \).

Furthermore, agents operate with finite power resources
and are required to periodically return to a fueling station
denoted \( F \). Thus, a scheduling protocol is derived whereby
agents periodically deploy from \( F \) to cover within an
assigned partition of \( C \). These partitions are bounded by latitude
lines and are sorted by geodesic distance from \( F \) with agents
transferring to increasingly closer partitions as their power
resource dwindles requiring a return to \( F \). This partitioning
scheme also has the benefit of ensuring that the network
of agents is well distributed across \( C \) with agents nominally
assigned to intrude intruders with predicted impact points
within their own partition.

B. Local Coverage

Our previous work [21], considers the same environment of
stochastic intruders with agents operating under a single
control law. This controller, referred to as local coverage
mode in the sequel, constitutes the first of five hybrid
modes in our new automaton. Local coverage is gradient
following in nature and essentially commands agent
\( i \) to always seek to orient and translate \( S_i \) such that the volume
of uncovered space intersecting \( S_i \) is increased. The control
laws are derived by defining an error function of \( Q(t, \bar{p}) \) with
respect to its desired level \( C^* \) and then using Lyapunov-
like arguments to drive this error function towards smaller
values. They take the form:

\[
\begin{bmatrix}
u_{i}^{loc} \\
u_{i}^{loc} \\
u_{i}^{loc} \\
\gamma_{i}^{loc} \\
\bar{r}_{i}^{loc} \end{bmatrix} =
\begin{bmatrix}
k_{i,a_{11}} \frac{k_{i,a_{12}}}{\sqrt{a_{11}^2 + a_{21}^2 + a_{31}^2}} \\
\frac{k_{i,a_{13}}}{\sqrt{a_{11}^2 + a_{21}^2 + a_{31}^2}} \\
\frac{k_{i,a_{23}}}{\sqrt{a_{11}^2 + a_{21}^2 + a_{31}^2}} \\
\frac{k_{i,a_{33}}}{\sqrt{a_{11}^2 + a_{21}^2 + a_{31}^2}} \\
\bar{s}_{i} \text{sat}(\bar{r}_{i}^{loc})
\end{bmatrix} T
\]

\
\begin{bmatrix}
\bar{r}_{i}^{loc} \\
\bar{s}_{i}^{loc}(\bar{r}_{i}^{loc})
\end{bmatrix} T +
\begin{bmatrix}
\bar{p}_{i} T \\
\rho_{a,i} \cdot \hat{y}_{B_i} \\
\rho_{a,i} \cdot \hat{z}_{B_i}
\end{bmatrix},
\]

where \( \rho_{a,i} = -\ln(\frac{1}{R_i - r_i}(||n_i|| - r_i))R_i^{-1}n_i \) and
\( \rho_{a,i} = \xi R_2^{-1} \)

\[
\begin{bmatrix}
\arcsin(\hat{n}_i \cdot \hat{z}_G) - \Theta_i \\
\tan(\bar{n}_i \cdot \hat{y}_G, -\bar{n}_i \cdot \hat{x}_G - \Psi_i)
\end{bmatrix}.
\]
Note that $a_{i,\ell}(t, \hat{Q}_i(t, \hat{p}))$, $\forall \ell \{1, \ldots, 5\}$ are functions of agent $i$’s modified coverage map $\hat{Q}(t, \hat{p}) = Q(t, \hat{p})$... where $f_i \in \{0, 1\}$ is a particle assignment flag for agent $i$ defined of the local coverage laws and definitions of $a_{i,\ell}$, see Section III of [21].

$\rho_{i,i}$ is a collision avoidance term with respect to the surface of interest. It takes a value of zero when agent $i$’s normalized distance from $C$ is $\gamma R_i$ for $\gamma \in (0, 1)$ and is logarithmically repulsive and attractive from the surface when the distance is decreased or increased respectively. $\rho_{a,i}$, for $\xi \ll 1$, encodes that the agents should tend to align $\hat{x}_{B_i}$ with $-\hat{n}_i$ if the coverage terms associated with $r_i$ and $s_i$ have become sufficiently small. The physical meaning of $\rho_{a,i}$ is to direct $\mathcal{S}_i$ back onto $C$ if it has reached a configuration in which it no longer intersects $C$. See Fig. 1 for further illustration of the effects of $\rho_{i,i}$ and $\rho_{a,i}$.

$r_i$ and $s_i$ are saturation limits for the coverage angular velocity inputs to the system. $k_u, k_v$ and $k_w$ are tuning gains which are chosen to satisfy $\sqrt{k_u^2 + k_v^2 + k_w^2} \leq U_{\text{max}}$. As $\rho_{i,i}$ is normal to the surface, it can be shown that $U_{\text{max}}$ is an upper bound to agent velocity tangential to $C$.

C. Intruder Detection Model

We assume that an omnidirectional range sensor (e.g., LiDAR) is co-located with $O$ and provides measurements of each intruder’s position in spherical coordinates. Define a model for the motion of intruder $k$:

\[
\dot{\hat{q}}_k(t) = \begin{bmatrix}
0_{3 \times 3} & I_{3 \times 3} \\
0_{3 \times 3} & 0_{3 \times 3}
\end{bmatrix} \hat{q}_k(t),
\]

\[
\ddot{z}_k(t) = \begin{bmatrix}
\sqrt{\dot{x}_k^2 + \dot{y}_k^2 + \dot{z}_k^2} \\
\text{atan2}(y_k, x_k) \\
\text{arccos}\left(\frac{\hat{x}_k}{\sqrt{\hat{x}_k^2 + \hat{y}_k^2 + \hat{z}_k^2}}\right)
\end{bmatrix} + \epsilon,
\]

where $\hat{q}_k = [x_k, y_k, z_k, \dot{x}_k, \dot{y}_k, \dot{z}_k]^T$ and $\ddot{z}_k = [\hat{p}_k, \theta_k, \psi_k]^T$ are the Cartesian state and spherical coordinate measurement vectors of intruder $k$ resolved in $G$. $\rho_k$, $\theta_k$, and $\psi_k$ are the range, azimuthal angle, and polar angle of $k$ respectively. Intruders are assumed to be particles and shall interchangeably be referred to as such.

If intruder $k$ is detected at time $t_{dk}$, we initialize the state estimate at $t_{dk'} = t_{dk} + \Delta t$ where $\Delta t$ is lower bounded by the time required to record two measurements of $\hat{z}_k(t)$. The evolution of intruder state estimate $\hat{q}_k(t)$ and associated covariance $P_k(t)$ are then estimated online via extended Kalman Filter. For a full description of this model, see [21].

At any time $t$, we define our decay rate map for intruder $k$ in terms of its predicted position and covariance evolution over a horizon $T_{H,k}(t)$. As the intruders are assumed to travel at fixed velocities, the predicted values for Cartesian position $\hat{p}_k(t + \tau)$ and associated covariance $P_k(t + \tau)$ may be defined as $\hat{p}_k(t + \tau) = G(\tau) \hat{q}_k(t)$ and $P_k(t + \tau) = G(\tau) \hat{P}_k(t) G(\tau)^T$ respectively where $G(\tau) = [\mathbb{I}_{3 \times 3} \tau \mathbb{I}_{3 \times 3}]$.

We define the decay rate map associated with particle $k$ as the integral of our predicted normal distribution $\mathcal{N}(\hat{p}_k(t + \tau), P_k(t + \tau))$ through horizon $T_H$:

\[
\Lambda_k(t, \hat{p}) = \int_0^{T_{H,k}(t)} \lambda_k(\hat{p}_k(t + \tau), P_k(t + \tau)) \, d\tau. \tag{5}
\]

For $t < t_{dk'}$, define $\Lambda_k(t, \hat{p}) = 0$, $\forall \hat{p} \in \mathcal{D}$. Our formulation for (5) essentially takes a normal distribution for the position of particle $k$ at time $t$ and cumulatively sweeps it forward in time up to our horizon $T_{H,k}(t)$. The horizon is lower-bounded by an estimate of the remaining time until impact of particle $k$ on $C$. This may be computed using $\hat{z}_k(t)$ along with the surface geometry. With this design, $Q(t, \hat{p})$ decays along the predicted trajectory of $k$ with tapering omnidirectional decay rates spreading out from the predicted path. This design lends itself naturally to our local coverage formulation, which is gradient following in nature, in that the agents may follow these tapered decay paths towards the predicted impact points on our surface of interest. The parameter $\lambda_k > 0$ may be adjusted to scale how rapidly the coverage level will decay in time.

III. Particle Intercept Mode

We denote the detection and state estimation of particle $k$ as an event, $\epsilon_k$, which concludes at $t_{dk'}$. Assuming that particle $k$ is embedded within the surface upon impact, its position shall intersect $C$ at most one time. Thus, we define particle $k$’s estimated impact (collision) time as

\[
t_{ck} = \min\left\{t \in \mathbb{R}^+ \mid \left(\frac{\dot{x}_k + \dot{z}_k}{x_{k,v}}\right)^2 + \left(\frac{\dot{y}_k + \dot{z}_k}{x_{k,v}}\right)^2 + \left(\frac{\dot{z}_k}{x_{k,v}}\right)^2 = 1\right\},
\]

with estimated point of impact $\hat{p}_k(t_{ck}) = G(t_{ck} - t) \hat{q}_k(t)$.

Upon conclusion of event $\epsilon_k$, particle $k$ is assigned to a free agent $i$ with the minimum geodesic distance from the estimated point of impact. We define a new index, $i_k$, as the index of the agent assigned to intercept particle $k$. $i_k := i$ may occur on condition that $\tau_p \neq 1$ and $f_i \neq 1$ where $f_i \in \{0, 1\}$ is a particle assignment flag for agent $i$ defined
as 0 when the agent is free (i.e., not currently assigned a particle), \( i_p \), the power index of agent \( i \), shall be defined in Section IV.

When agent \( i \) has been assigned to intercept particle \( k \), \( f_i \) is set to 1 and it is said to have transitioned into particle intercept mode. In this mode, agent \( i \) follows an optimal trajectory to the point \( \bar{p}_k^i(t_{ck}) \) and remains within some distance tolerance \( \varepsilon_i \) until \( t > t_{ck} \) at which time \( f_i \) is set to 0. The optimal trajectory is referred to as a geodesic and its computation may be executed in an iterative manner. Specifically, we use Vincenty’s formulae as presented in [23]. For cases involving nearly antipodal points in which the standard inverse method does not converge, we use Vincenty’s supplemental algorithm presented in [24].

As an input, Vincenty’s algorithm requires two points, current and desired position, on the surface of an ellipsoid of revolution. The algorithm also requires the length of the semi-major and semi-minor axes of the ellipsoid and it returns a heading angle measured clockwise from North. This heading angle shall be referred to as \( \chi_i \). We now define the nominal heading unit vector \( \hat{\nu}_i \) which lies in the plane tangent to the surface at \( p_i \). It may be computed by rotating the North-pointing vector at \( p_i \) clockwise by an angle of \( \chi_i \) within the tangent plane.

As with our local coverage strategy, it is assumed that the agent shall nominally maintain a distance \( \gamma_R_i \) normal to \( C \). We define an ellipsoid of revolution, \( C' \) which is concentric with \( C \) and has the property that each semi-principal axis is \( \gamma_R_i \) longer than its associated counterpart in \( C \), i.e., \( x_{C',r} = x_{C,r} + \gamma R_i, y_{C',r} = y_{C,r} + \gamma R_i, \) and \( z_{C',r} = z_{C,r} + \gamma R_i \). Note that this is an ellipsoid of revolution we have that \( x_{C',r} = y_{C,r} \). The nominal trajectories of \( i \) shall be attractive to \( C' \). Thus, \( \chi_i \) and \( \hat{\nu}_i \) shall be calculated at any given time with respect to this \( C' \).

The position controller used to guide agent \( i \) to \( \bar{p}_k^i(t_{ck}) \) is composed of two additive terms: one which commands velocity tangential to \( C' \) along \( \hat{\nu}_i \) and one logarithmic term which commands velocity normal to \( C' \) in order to maintain the property that \( ||n_i|| \approx \gamma R_i \). The particle intercept mode position control law is:

\[
\begin{bmatrix}
\dot{u}_{pim}^i \\
\dot{\gamma}_{pim}^i \\
\dot{\nu}_{pim}^i
\end{bmatrix} = U_{max} R_1^{-1} \begin{bmatrix}
\hat{\nu}_i - \ln\left(\frac{1}{\gamma R_i - \kappa}(||n_i|| - \kappa)\right) \hat{n}_i \\
\hat{n}_i - \ln\left(\frac{1}{\gamma R_i - \kappa}(||n_i|| - \kappa)\right) \hat{n}_i
\end{bmatrix}
\]

As (6) commands the vehicle to follow the optimal length path along \( C' \) to \( \bar{p}_k^i(t_{ck}) \), we can establish a guarantee on the feasibility of particle interception. To simplify notation, define:

\[
g = \left[1 + \sum_{n=1}^{\infty} \left(\frac{2n - 1}{2^n n!}\right)^2 \left(\frac{x_{C',r} - z_{C',r}}{x_{C',r} + z_{C',r}}\right)^2 \left(\frac{2n}{2n - 1}\right)^2\right].
\]

Theorem 1: Assuming that sufficient time is provided, i.e.,

\[
t_{ck} - t_{dk'} > \frac{\pi x_{C',r} + \pi (x_{C',r} + z_{C',r})}{U_{max}}
\]

of control law (6) shall guide agent \( i \) to reach impact point \( \bar{p}_k^i(t_{ck}) \) before \( t_{ck} \).

Proof: Assume that at time \( t_{dk'} \), agent \( i \) is at position \( p_i(t_{dk'}) \) which lies on \( C' \) and must travel to point \( \bar{p}_k^i(t_{ck}) \) which is also on \( C' \). Denote the length of this path as \( S_{p_i(t_{dk'})}(t_{ck}) \). As (6) directs agent \( i \) to follow the shortest feasible path along \( C' \), we may upper bound \( S_{p_i(t_{dk'})}(t_{ck}) \) by the sum of two paths: a path of constant latitude \( S_{lat} \) followed by a path of constant longitude \( S_{long} \):

\[
S_{p_i(t_{dk'})}(t_{ck}) \leq S_{lat} + S_{long}.
\]

For two generic points on \( C' \), we have:

\[
S_{lat} \leq \frac{\pi}{2} (x_{C',r} + z_{C',r}) g.
\]

where the bound on \( S_{lat} \) is half of the circumference of our ellipsoid of revolution about its equator and the bound on \( S_{long} \) is half of the perimeter of our revolved ellipse. The infinite series expression term, denoted \( g \) in (10), is first presented in [25]. Assuming that agent \( i \) follows the geodesic path at velocity \( U_{max} \), we have that:

\[
S_{p_i(t_{dk'})}(t_{ck}) = U_{max} t_{ik},
\]

where \( t_{ik} \) is the time for agent \( i \) to travel to the impact point of particle \( k \) along the geodesic. Substituting (11) for \( S_{p_i(t_{dk'})}(t_{ck}) \) in (8) and combining with inequalities (9) and (10) yields: \( U_{max} t_{ik} \leq \frac{\pi}{2} (x_{C',r} + z_{C',r}) g \), which may be rearranged to form:

\[
t_{ik} \leq \frac{\pi x_{C',r} + \pi (x_{C',r} + z_{C',r})}{U_{max}}
\]

Agent \( i \) reaching \( \bar{p}_k^i(t_{ck}) \) before \( t_{ck} \) implies that: \( t_{ik} < t_{ck} - t_{dk'} \). From (12), it is clear that this is guaranteed so long as:

\[
\frac{\pi x_{C',r} + \pi (x_{C',r} + z_{C',r})}{U_{max}} < t_{ck} - t_{dk'}.
\]

This concludes the proof.

As agent \( i \) travels towards \( \bar{p}_k^i(t_{ck}) \) along the geodesic, it is desirable that it should point \( S_i \) towards \( C \). Therefore, the orientation controller for particle intercept mode is similar to that of Section II-B:

\[
\begin{bmatrix}
\dot{q}_i^{pim} \\
\dot{\gamma}_i^{pim} \\
\dot{\nu}_i^{pim}
\end{bmatrix} = R_2^{-1} \begin{bmatrix}
0 \\
\arcsin(\hat{\nu}_i \cdot \hat{z}_G) - \Theta_i \\
\tan^{-1}(\hat{\nu}_i \cdot \hat{y}_G, -\hat{n}_i \cdot \hat{x}_G) - \Psi_i
\end{bmatrix},
\]

which is essentially a proportional controller that tends to align \( \hat{z}_B \) with \( -\hat{n}_i \).

IV. ENERGY-AWARE SCHEDULING PROTOCOL

A. Domain Partitioning

As this is a persistent coverage protocol, which operates indefinitely, it is necessary to establish an agent deployment and scheduling protocol that realistically considers the agents’ finite power and/or propulsive resources.

Our strategy is to periodically deploy agents from a fueling station \( F \) which we assume to be located at the North pole of \( C' \), i.e., at the point \( [0 \ 0 \ z_{C',r}]^T \). Define \( T^* \) as the power lifespan of each agent in the network. With our definition
for $T^*$, it is intuitive that deployment windows from $F$ will arise every $\frac{T}{N}$ seconds.

In order to reduce redundancy between agents surveying $C$, it is desirable to partition the domain and assign agents to monitor separate regions. Specifically, partitioning the domain by latitude, rather than longitude, ensures that agents are poised to intercept particles without the need for frequent crossings of the equator which tend to be associated with larger values of $S_p(t_{\text{tk}}^*, x_{\text{tk}}^*)$ on an oblate spheroid.

Define $i_p(t) \in \{1, ..., N\}$ as the power index of agent $i$. Upon deployment from $F$, agent $i$ has power index $i_p = N$ and this index is reduced by one every $\frac{T}{N}$ seconds until $i_p = 1$, i.e., agent $i$ is the power critical agent. Thus, the power index obeys $i_p(t) = 1 + \text{mod} \left( i - 2 - \left\lfloor \frac{N}{p} \right\rfloor, N \right)$, where the first argument of our modulo operation is the divisor and the second argument is the divisor. The lower-bracketed delimiters represent the floored division operation.

This definition implies that agent $i = 1$ is deployed first.

We may now define our latitude partitions. The partitions are divided such that $N - 1$ agents are assigned equal surface areas of $C$ to explore. Each partition is characterized by an upper bound in $\tilde{z}$ denoted $\tilde{z}_{i_p - 2}$ and a lower bound $\tilde{z}_{i_p - 1}$. The surface area of our ellipsoid of revolution $C$ is defined as:

$$A_C = 2\pi x_{C,r}^2 \left(1 + \frac{1 - \frac{z_{C,r}^2}{x_{C,r}^2}}{1 - \frac{z_{C,r}^2}{x_{C,r}^2}} \right) \text{artanh} \left( \sqrt{1 - \frac{z_{C,r}^2}{x_{C,r}^2}} \right)$$

(14)

The agent with $i_p = 2$ is assigned to monitor the partition characterized by upper bound at north pole of $C$, i.e., $\tilde{z}_0 = z_{C,r}$. The lower bound $\tilde{z}_1$ may be computed by dividing (14) by $(N - 1)$, equating with the integral of ellipse cross sectional circumferences parametrized by $\tilde{z}$, and then numerically solving for $\tilde{z}_1$:

$$A_C \frac{N - 1}{N - 1} \int_{\tilde{z}_r}^{\tilde{z}_1} 2\pi \left(x_{C,r}^2 - \frac{x_{C,r}^2}{x_{C,r}^2} \right) \left(1 + \frac{z_{C,r}^2}{(z_{C,r}^2 - z_{C,r}^2 + \tilde{z}^2)} \right) d\tilde{z}$$

(15)

One may then iteratively solve for the remaining bounds for increasing values of $i_p$ up to $i_p = N - 1$:

$$A_C \frac{N - 1}{N - 1} \int_{\tilde{z}_{i_p - 1}}^{\tilde{z}_{i_p - 2}} 2\pi \left(x_{C,r}^2 - \frac{x_{C,r}^2}{x_{C,r}^2} \right) \left(1 + \frac{z_{C,r}^2}{(z_{C,r}^2 - z_{C,r}^2 + \tilde{z}^2)} \right) d\tilde{z}$$

(16)

The final computation of (16) for $i_p = N$ is not necessary as $\tilde{z}_{N-1}$ is the south pole of $C$, i.e., $\tilde{z}_{N-1} = -z_{C,r}$, although this may be shown through numerical computation as well. Our partitioning strategy for the case where $N = 4$ is presented in Fig. 2.

Our map augmentation term $M_i(t, \tilde{p})$ for agent $i$, first referenced in Section II-B, is defined in terms of power index $i_p$:

$$M_{i_p}(t, \tilde{p}) = \begin{cases} 0, & \text{if } \tilde{z}_{i_p-1} \leq \tilde{z} \leq \tilde{z}_{i_p-2}; \forall i_p \in \{2, ..., N\}; \\ 0, & \text{if } i_p = 1; \\ \bar{C}^*, & \text{otherwise}. \end{cases}$$

(17)

This map augmentation term encodes that regions of the domain outside of an agent’s partition are “uninteresting” and our local coverage controller will tend to direct the $S_i$ away from these regions when $S_i$ intersects $\tilde{z}_{i_p - 1}$ or $\tilde{z}_{i_p - 2}, \forall i | i_p \in \{2, ..., N\}$.

Note that no partition has been assigned to the agent for which $i_p = 1$. This is the power critical agent and it shall have flag $f_i := 1$ at the instant $i_p := 1$. The power critical agent cannot be assigned a new particle to intercept after $i_p := 1$ as this opens the possibility that particle assignment could occur near the end of the $\frac{T}{N}$ time window during which time the agent with $i_p = 1$ should be transitioning back to $F$ to exchange its power source. The power critical agent will instead spend the majority of this time window in local coverage mode assisting the other agents in gathering information. It can only be tasked with intercepting a particle if this assignment had occurred previously when $i_p = 2$.

In this scenario, the agent should be capable of intercepting particle $k$ and then transitioning back to $F$ so long as a bound is established on the length of our deployment scheduling window $\frac{T}{N}$.

**Theorem 2:** If agent power lifespan $T^*$ satisfies $\frac{T}{N} \geq t_{ck} - t_{dk'} + \frac{\pi}{2U_{\text{max}}} (x_{C',r} + z_{C',r}) g$, then the agent with $i_p = 1$ shall always be capable of reaching $F$ within $\frac{T}{N}$ of the time at which $i_p := 1$.

**Proof:** Consider the worst case scenario in which the agent with $i_p = 2$ is assigned to intercept particle $k$ at the instant before $i_p := 1$. It’s remaining flight time is currently $\frac{T^*}{N}$. The time required to intercept the particle is $t_{ck} - t_{dk'}$, after which our control strategy dictates that the agent will follow a geodesic trajectory to $F$. As $F$ lies at the north pole of $C$, this will be a trajectory of constant longitude which may be upper bounded by a length half the perimeter of our revolved ellipsoid: $\frac{\pi}{V} (x_{C',r} + z_{C',r}) g$ by definition. As the agent is controlled by (6) with a North-pointing $\hat{v}_j$, it will proceed along this geodesic at speed $U_{\max}$. Thus the time required to complete this trajectory is $\frac{\pi}{U_{\max}} (x_{C',r} + z_{C',r}) g$ and we may bound our deployment window: $\frac{T}{N} \geq t_{ck} -$
\( t_{dk} + \frac{\pi}{2U_{\max}} (x_{C',r} + z_{C',r}) g, \forall k \). This concludes the proof.

In summary, the appropriate design method for this surveillance system is to first ensure that the time from detection to collision of any arbitrary particle, \( t_{dk} - t_{dk'} \), as governed by the omnidirectional range sensor satisfies Theorem 1. One must subsequently ensure that power lifespan \( T^* \), for all agents, satisfies Theorem 2.

B. Partition Transfer and Return to Base

If an agent with \( i_p \in \{2, \ldots, N \} \) lies outside of its prescribed partition, and we have \( f_i = 0 \), then the agent shall enter partition transfer mode. This mode uses the same geodesic position and orientation controllers (6) and (13) with the destination position set to the point:

\[
[x_{id} \ y_{id} \ z_{id}]^T = \begin{bmatrix}
  x_{C',r} \cos(\arcsin(\frac{z_{id}}{z_{C',r}})) \\
  y_{C',r} \cos(\arcsin(\frac{z_{id}}{z_{C',r}})) \\
  z_{id} - 1 \text{if } z_{id} < z_{i_p-1}; 1 \text{if } z_{id} > z_{i_p-1}
\end{bmatrix},
\]

i.e., the closest agent along the point’s current longitude which lies on the boundary of its assigned partition.

The return to base mode is similar to partition transfer mode but is defined for the agent with \( i_p = 1 \). This mode is activated when the time since agent \( i \)'s last deployment from \( F \), denoted \( t_{i,F} \geq T^* - \frac{\pi}{2U_{\max}} (x_{C',r} + z_{C',r}) g \) as established in Theorem 2. The control strategy is the same as partition transfer mode with the desired position set to \( F \). Control laws for partition transfer mode and return to base shall be denoted with superscripts \( ptm \) and \( rtb \) respectively.

V. Collision avoidance

To encode collision avoidance, we have designed an explicit avoidance mode which may be transitioned into from local coverage, partition transfer, or particle intercept mode. This mode is triggered for agent \( i \) when we have the condition that \( \|p_i - p_j\| \leq R_i \) for \( i \neq j \). Denote \( j = i \cup j \) as the set of agents satisfying this condition. Agents in \( j \) are ranked by \( t_{j,F} \). One agent, denoted \( i_{pr} \), whose value for \( t_{j,F} \) is highest, i.e., \( i_{pr} = \arg \max_j(t_{j,F}) \) is permitted to proceed while the remaining agents transition to collision avoidance mode. Agents in \( j \backslash i_{pr} \), are controlled to follow \( \hat{n}_i \) until they have reached a height \( R_i \) above \( C' \), i.e., they ascend to and hover about points on an ellipsoid concentric with \( C' \) whose semi-major and semi-minor axes are a factor of \( R_i \) longer than those of \( C' \). They remain in this hover configuration until \( i_{pr} \) passes underneath the hovering agents along its nominal trajectory and removes itself from the deadlock. At this point, a new \( i_{pr} \) is selected and that agent will descend to \( C' \) and return to its previous hybrid state.

The avoidance position control strategy is:

\[
[w_i^{av} \ q_i^{av} \ w_i^{av}]^T = U_{\max} R^{-1}_i \hat{n}_i.
\]

As the agents shall ascent to a point at which \( R_i \) does not intersect \( C \), sensing information is not gathered in avoidance mode and thus the avoidance orientation control is simply \( [q_i^{av} \ q_i^{av} \ s_i] = [0 \ 0 \ 0] \). With an additional assumption on agent size, we may establish a collision avoidance guarantee.

**Theorem 3:** For agents \( \{i, j\} \in \mathcal{J} \), the condition that \( \min(R_i) > 2R_i + 2s_i \), implies that \( i \) shall not collide with \( j \).

**Proof:** Consider the case in which \( i \neq i_{pr} \) and \( j \neq i_{pr} \). Both agents operate in accordance with (18) and the agents follow trajectories along \( \hat{n}_i \) and \( \hat{n}_j \) respectively. Both unit vectors are normal to surface \( C \), an ellipsoid of revolution, and thus diverge from one another away from \( C \). Agents \( i \) and \( j \) shall enter avoidance mode at an instant when \( \|p_i - p_j\| \geq \min(R_i) \) and their distance shall tend to increase under (18). Thus \( \min(R_i) > s_i + s_j \) and subsequently \( \min(R_i) > 2s_i + 2s_j \) imply that they avoid collision.

Consider the case in which \( i = i_{pr} \) and thus \( j \neq i_{pr} \). In the instant that \( j \) transitions to avoidance mode we have that \( \|p_i - p_j\| \geq \min(R_j) \). Thus the distance for \( i \) to travel until collision is greater than or equal to \( \min(R_j) - s_i - s_j \). This straight line path for \( i \) is a conservative estimate as the true path is curved. Collision will be avoided if agent \( j \), whose path is normal to the surface, may cover a distance \( s_i + s_j \) before \( i \) covers \( \min(R_j) - s_i - s_j \). As \( j \) moves at speed \( U_{\max} \) and \( i \)'s tangential speed is upper bounded by \( U_{\max} \), this condition is satisfied if \( \min(R_j) - s_i - s_j > s_i + s_j \). This may equivalently be written as \( \min(R_j) > 2s_i + 2s_j \). These arguments apply to the case in which \( j = i_{pr} \) and \( i \neq i_{pr} \) as well. This concludes the proof.

VI. Hybrid Formulation

![Fig. 3. Agent i operates in accordance with this automaton. For clarity, elements of the reset map are shown explicitly.](image)

To provide a compact notation in this section, define \( \hat{f}_i = \frac{x_i^2}{(x_{C'} + s_i)^2} + \frac{y_i^2}{(y_{C'} + s_i)^2} + \frac{z_i^2}{(z_{C'} + s_i)^2} \). The coverage strategy for agent \( i \) is represented by the hybrid automaton in Fig. 3, described by the following entities [26]:

A set of discrete states: $Z_t = \{z_0, z_{11}, z_{12}, z_{31}, z_{41}\}$.

A set of continuous states: $\eta_t = \{x_t, y_t, z_t, \Phi_t, \Theta_t, \Psi_t\}$.

A vector field:
$$\begin{align*}
  f(z_i^0, \eta_t) &= \mathcal{R}_{i} \left[ \begin{array}{cccc}
  s_i^0 & v_i^0 & w_i^0 & 0 \\
  & & & 0 \\
  & & & 0
  \end{array} \right]^T, \\
  f(z_i^1, \eta_t) &= \mathcal{R}_{i} \left[ \begin{array}{cccc}
  s_i^1 & v_i^1 & w_i^1 & q_i^1 \\
  & & & 0 \\
  & & & 0
  \end{array} \right]^T, \\
  f(z_i^2, \eta_t) &= \mathcal{R}_{i} \left[ \begin{array}{cccc}
  s_i^2 & v_i^2 & w_i^2 & q_i^2 \\
  & & & 0 \\
  & & & 0
  \end{array} \right]^T, \\
  f(z_i^3, \eta_t) &= \mathcal{R}_{i} \left[ \begin{array}{cccc}
  s_i^3 & v_i^3 & w_i^3 & q_i^3 \\
  & & & 0 \\
  & & & 0
  \end{array} \right]^T, \\
  f(z_i^4, \eta_t) &= \mathcal{R}_{i} \left[ \begin{array}{cccc}
  s_i^4 & v_i^4 & w_i^4 & 0 0 0 0 \\
  & & & 0 \\
  & & & 0
  \end{array} \right]^T
  \end{align*}$$

where $\mathcal{R} = \begin{bmatrix} \mathcal{R}_1 \ 0 \\ 0 \ \mathcal{R}_2 \end{bmatrix}$.

A set of initial states: $\{\xi_0 \times \{\eta_t \in \mathbb{R}^6 | p_t = \mathcal{F} \wedge \Phi_t \in (-\pi, +\pi) \wedge \Theta_t \in \left[ -\frac{\pi}{2}, \frac{\pi}{2} \right] \wedge \Psi_t \in (-\pi, +\pi)\}$.

A domain: $\text{Dom}(\xi_0) = \{\eta_t \in \mathbb{R}^6 | f_t \geq 1 \wedge \{i_p \in \{2, ..., N\} \implies \hat{z}_{i_p-1} \leq z_t \leq \hat{z}_{i_p-2}\}\}$.

A set of edges: $E = \{\xi_0, \xi_1\} , \{\xi_0, \xi_2\} , \{\xi_0, \xi_3\} , \{\xi_4, \xi_5\} , \{\xi_4, \xi_6\} , \{\xi_4, \xi_7\} , \{\xi_4, \xi_8\} , \{\xi_4, \xi_9\}$.

A set of guard conditions:
$$\begin{align*}
  G(\xi_0, \xi_1) &= \{i_p = 1 \wedge t_{i_F} \geq T^* - \frac{\pi}{2\max_{i_F}}(x_{i_F},r + z_{i_F}),\}
  \\
  G(\xi_0, \xi_2) &= \{\exists k | i = i_k\}
  \\
  G(\xi_0, \xi_3) &= \{i_p \neq 1 \wedge \{z_t < \hat{z}_{i_p-1} \vee z_t > \hat{z}_{i_p-2}\}\}
  \\
  G(\xi_0, \xi_4) &= \{\|p_t - p_{i_F}\| \leq {R_i} \wedge i_p \neq \text{argmax}_{j} (t_{j,F})\}
  \\
  G(\xi_1, \xi_2) &= \{\|p_t - \hat{p}_k(t_{k_F})\| \leq \hat{z}_{i} \wedge t \geq t_{k_F} \wedge i_p = 1 \wedge t_{i_F} \geq T^* - \frac{\pi}{2\max_{i_F}}(x_{i_F},r + z_{i_F})\}
  \\
  G(\xi_1, \xi_3) &= \{\|p_t - \hat{p}_k(t_{k_F})\| \leq \hat{z}_{i} \wedge t \geq t_{k_F} \wedge i_p \in \{2, ..., N\} \wedge \{z_t < \hat{z}_{i_p-1} \vee z_t > \hat{z}_{i_p-2}\}\}
  \\
  G(\xi_1, \xi_4) &= \{i_p = 1 \vee \{i_p \neq 1 \wedge \hat{z}_{i_p-1} \leq z_t \leq \hat{z}_{i_p-2}\}\}
  \\
  G(\xi_2, \xi_4) &= \{\|p_t - \hat{p}_k(t_{k_F})\| > R_i \wedge \forall j \vee i_{pr} = \text{argmax}_{j} (t_{j,F})\}
  \\
  G(\xi_3, \xi_4) &= \{\|p_t - \hat{p}_k(t_{k_F})\| > R_i \wedge \forall j \vee i_{pr} = \text{argmax}_{j} (t_{j,F})\}
  \end{align*}$$

One should note that Zeno oscillations are avoided in $G(\xi_0, \xi_3)$ and $G(\xi_3, \xi_0)$ as (17) implies that the local coverage control laws will tend to pull $S_t$ inside of the assigned partition from the boundary.

VII. Simulations

A simulation was performed in MATLAB to verify the efficacy of the algorithm. Four agents are deployed to cover the surface of an ellipsoid, $C$, whose radius in the $xy$-plane is 80 and whose radius in the $z$-plane is 20. For each agent, $R_i = 10$, $\alpha_i = 1$, $\alpha_i = 30^\circ$, $k_u = 1$, $k_v = 5$, $k_w = 1$, $k_r = 0.1$, $k_s = 0.1$, $\bar{r}_i = 0.4$, $\bar{s}_i = 0.4$. Upon initialization of the simulation, $C$ was set to a fully covered level of $C^* = 20$ which would begin decaying upon detection of the first particle $k \in \{1, \ldots, 4\}$ at $t = 600$ sec. Particles, which travelled in random directions at a speed of 1 distance unit per second, were generated every 25 seconds.

Agents were able to successfully intercept nearly all particles along their geodesic trajectories while actively avoiding collision (see Fig. 4); however, agents spent the majority of their time in particle intercept mode hovering near predicted impact points due to the high rate of particle generation (see Fig. 5). This negatively impacted their ability to explore away from the site of impacts thus contributing to a slow but continued growth in the coverage error. In fact, Fig. 6 illustrates precisely that only the power critical agent tended to spend any time in local coverage mode. This always preceded its return to base. Future simulations will test longer time trials under the same particle generation rate to determine if the coverage error growth tapers off. Additional simulations will test less frequent particle generation to increase the time agents spend in local coverage mode.
Fig. 5. Agent $i = 2$ follows its geodesic trajectory to the predicted impact point of particle $\kappa$. The true trajectory of the particle is indicated in red and the estimated trajectory in green.

Fig. 6. The Normalized coverage error remains on the order of $10^{-4}$. Agent operating modes are indicated over time with the abbreviations from top to bottom referring to avoidance mode, partition transfer mode, particle intercept mode, return to base, and local coverage mode.

VIII. Conclusions

In this paper, we presented a hybrid formulation for the persistent coverage problem in an environment subject to stochastic intruders. This formulation was motivated in part by extravehicular applications of the NASA Mini AERCam. Agents operated with finite power resources and were required to periodically return to a refueling station while patrolling assigned latitude partitions along the surface of an ellipsoid. The efficacy of the algorithm was demonstrated in simulation.

REFERENCES


