Distributed coordination in multi-agent systems under local directed interactions: avoidance and aggregation

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Abstract—This paper proposes a velocity coordination protocol for the control of multiple nonholonomic agents, which aims to address two distinct scenarios in a unified manner. The former case is aggregation of multiple agents around a goal location, while the latter case is navigation of multiple agents to multiple goal locations along with collision avoidance. The proposed protocol imposes directed interactions among agents in the following sense: Each agent \( i \) makes a decision on its control inputs by choosing and interacting with a subset of its neighbor agents \( j \) only. The proposed decision making process is based on semi-cooperative coordination, which can be thought as an implicit prioritization among locally connected agents, and which results in a suitable adjustment of the linear velocities of the connected agents. The angular velocities of the agents are regulated to reference directions which are dictated by a family of vector fields introduced in earlier work of the author’s. Simulation results involving multiple unicycle agents achieving either aggregation or navigation towards goal locations along with collision avoidance are provided to demonstrate the efficacy of the proposed algorithm.

I. INTRODUCTION

Multi-Agent Systems have been a very popular research topic during the past fifteen years, motivated in part by applications which range from robotics and multi-robot systems, to networked control systems, to power systems and sensor networks, to name a few. A common ground within the diversity of the aforementioned applications can be the collaboration of multiple agents towards achieving common tasks and goals. Research efforts have now attributed various formulations and methodologies on multi-agent problems, which are often specialized based on the control objectives and the characteristics of the problems at hand. It is out of the scope of this paper to provide a detailed overview of the related results. Very briefly, special emphasis during the past ten years has been given in consensus problems (also called synchronization, rendezvous or agreement), flocking and formation control problems for multiple agents, see [1] for a recent survey.

When it comes to planning and coordination of the motion of multiple vehicles, such as autonomous cars or unmanned aerial vehicles, then navigating towards goal locations while avoiding collisions is a requirement of highest priority, see for instance [2], [3] and the references therein. In a not-that-different spirit, applications which are relevant to reconfigurable and modular robotics [4], automated self-organization, construction and transportation [5]–[7], or patrolling and protection, typically involve multiple agents (robots or autonomous vehicles) which need to come together, interact, and form various shapes in their workspace, creating thus a structure of augmented or improved capabilities. These problems are often characterized as aggregation or swarming behavior in multi-agent systems [8]–[13]. At the same time, limitations in the available sensing and communication platforms impose additional constraints to the multi-agent system. Given a pair \((i,j)\) of agents \(i\) and \(j\), agents typically make decisions on their actions based on available information, which can be either locally measured by each agent using onboard sensors, or transmitted and received across the nodes of the multi-agent system via wireless communication links. Thus, information flow between two agents can be either bidirectional (undirected) or unidirectional (directed). During the past ten years, research efforts have achieved the formalization of problems such as consensus and formation control in multi-agent networks using tools and notions from graph theory, matrix theory and Lyapunov stability theory [14]–[17]. The case of directed information exchange has recently attracted increased interest [18]–[22], motivated in part by the fact that undirected information flow is not always a realistic and practical assumption, due to bandwidth limitations in the network, anisotropic sensing of the agents etc. Extending consensus algorithms to nonlinear systems has also become popular, see for instance [23], [24].

Nevertheless, despite that consensus, flocking, and formation control algorithms achieve collision avoidance in multi-vehicle systems by carefully selecting initial conditions and controlling relative distance and heading, they are typically not used in encoding problems such as navigation to specific goal locations for each one of the agents. In that respect, we are developing coordination algorithms for the motion of multiple agents along with safety guarantees. We consider a group of multiple nonholonomic agents with unicycle kinematics, and while keeping a unified view, we propose a coordination protocol for two different coordination problems: namely (i) aggregation of multiple agents around a goal location and (ii) navigation of multiple agents to multiple goal locations.

While the problem of coordinating the motion of multiple robots towards a single (for aggregation) or multiple (for navigation) goal locations is not new, here we consider a unified view of these two problems, and furthermore we consider directed interactions among agents. By directed interaction we mean that each agent \( i \) makes a decision on its control actions by interacting with a subset of its neighbor agents \( j \neq i \). The decision is interpreted as that pairwise collision avoidance is achieved by at least one of

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the involved agents $i, j$. The control action of agent $i$ is determined using information on the states of all its neighbor agents $j$, and results in collision-free trajectories for agent $i$ with respect to (w.r.t.) the worst case neighbor agent $j$, i.e., w.r.t. the neighbor agent $j$ which is more susceptible to collision. Thus, the information flow for implementing our protocol is varying with time, in the sense that interactions are local, and the decision making on the control action of each agent $i$ is directed towards a subset of the neighbor agents $j \neq i$.

Our goal is not to just provide a new coordination methodology for multiple nonholonomic agents, but rather to highlight that the same coordination protocol for the agents’ linear velocities, along with slight modifications on the coordination of their angular velocities, achieves distinct collective behaviors: the former case is aggregation of multiple agents around a goal location, while the latter one is navigation of multiple agents towards multiple goal locations, along with collision avoidance. The primary motivation on studying these tasks relies on planning problems for complex control systems, such as those encountered in aerial and modular robotics.

The paper is organized as follows: Sections II and III present the mathematical formulation and the proposed coordination protocol, along with the analysis on the correctness and fulfillment of the associated control objectives, respectively. Section IV provides simulation results for a group of unicycle robots which perform aggregation around, and individually. Section V provides the fulfillment of the associated control objectives, respectively. Section IV provides simulation results for a group of unicycle robots which perform aggregation around, and individually.

II. Multi-Agent Coordination via Vector Fields

We consider a group of $N$ agents with unicycle kinematics. The motion of the $i$-th agent, $i \in \{1, \ldots, N\}$, is governed by the equations:

$$\begin{bmatrix} x_i \\ y_i \\ \theta_i \end{bmatrix} = \begin{bmatrix} \cos \theta_i & 0 \\ \sin \theta_i & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_i \\ \omega_i \end{bmatrix},$$

where $q_i = [x_i, y_i, \theta_i]^T$ is the state comprising the position $r_i = [x_i, y_i]^T$ and the orientation $\theta_i$ of agent $i$ w.r.t. a global cartesian coordinate frame $G$, $u_i$ is the linear velocity and $\omega_i$ is the angular velocity of agent $i$ w.r.t. the body-fixed frame $B_i$. We assume that each agent $i$:

(i) is a circular disk of radius $\rho_i$ centered at $r_i$,
(ii) has access to its state $q_i$ and velocities $u_i, \omega_i$,
(iii) can reliably exchange information with any agent $j \neq i$ which lies within its communication region

$$C_i : \{ r_i \in \mathbb{R}^2, r_j \in \mathbb{R}^2 \mid ||r_i - r_j|| \leq R_c \},$$

where $R_c$ is the communication range. In other words, a pair of agents $(i, j)$ is connected as long as the distance $d_{ij} = ||r_i - r_j|| \leq R_c$.

A. A family of vector fields for robot navigation

In our earlier work [25] we defined a class of vector fields $F_i : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ for each agent $i$ as:

$$F_i = \prod_{j=1, j \neq i}^N (1 - \sigma_{ij}) F_{gij} + \sum_{j=1, j \neq i}^N \sigma_{ij} F_{oij},$$

where:

$$F_{gix} = \frac{(x - x_{gi})^2 - (y - y_{gi})^2}{(x - x_{gi})^2 + (y - y_{gi})^2},$$
$$F_{giy} = \frac{2(x - x_{gi})(y - y_{gi})}{(x - x_{gi})^2 + (y - y_{gi})^2},$$

are the vector field components of a (normalized) attractive vector field $F_{gi} : \mathbb{R}^2 \rightarrow \mathbb{R}^2$, which is by construction vanishing at the goal location $r_{gi}$ only, $F_{oij}$ is a normalized repulsive vector field around each agent $j$, which will be defined later on, and $\sigma_{ij}$ is a bump function defined as:

$$\sigma_{ij} = \begin{cases} 1, & \text{for } d_m \leq d_{ij} \leq d_r; \\ a d_{ij}^3 + b d_{ij}^2 + c d_{ij} + d, & \text{for } d_r < d_{ij} < d_c; \\ 0, & \text{for } d_{ij} \geq d_c; \end{cases}$$

for $d_m \leq d_{ij} \leq d_r$, $a d_r^3 + b d_r^2 + c d_r + d$, for $d_r < d_{ij} < d_c$, $0$, for $d_{ij} \geq d_c$,

where: $d_{ij} = ||r_i - r_j||$ the Euclidean distance between agents $i, j$, the physical meaning of the values $d_m, d_r, d_c$ is to be highlighted later on, and the coefficients $a, b, c, d$ computed as:

$$a = -\frac{2}{(d_r - d_c)^3}, \quad b = \frac{3(d_r + d_c)}{(d_r - d_c)^3},$$
$$c = -\frac{6d_c d_r}{(d_r - d_c)^3}, \quad d = \frac{d_r^2(3d_c - d_r)}{(d_r - d_c)^3},$$

so that (4) is a $C^2$ function.

The vector field (2), for the repulsive vector field $F_{oij}$ around each agent $j \neq i$ defined as:

$$F_{oij} = \begin{cases} F_{oij}^{iA} & \text{for } p_{ij}^T r_{ji} \geq 0; \\ F_{oij}^{iB} & \text{for } p_{ij}^T r_{ji} < 0, \end{cases}$$

where $p_{ij} = [\cos \phi_{ij}, \sin \phi_{ij}]^T$, $r_{ji} = r_i - r_j$, $\phi_{ij} = \text{atan2}((y_j - y_{gi}), (x_j - x_{gi}))$,

$$F_{oij}^{iA} = \begin{bmatrix} \sin \phi_{ij} (x_i - x_j)(y_i - y_j) - \cos \phi_{ij} (y_i - y_j)^2 \\ \cos \phi_{ij} (x_i - x_j)(y_i - y_j) - \sin \phi_{ij} (x_i - x_j)^2 \end{bmatrix},$$
$$F_{oij}^{iB} = \begin{bmatrix} -\cos \phi_{ij} (x_i - x_j)^2 - \cos \phi_{ij} (y_i - y_j)^2 \\ -\sin \phi_{ij} (x_i - x_j)^2 - \sin \phi_{ij} (y_i - y_j)^2 \end{bmatrix},$$

is shown [25] under certain, mild assumptions to be a safe, almost global feedback motion plan for agent $i$ operating in an idealistic environment, where each other agent $j \neq i$ is static and serves as a circular obstacle.

Mild assumptions refer to ensuring that the repulsive flows around each pair of agents $j \neq i$ do not overlap, which is achieved if the minimum distance $d_m$ in (4) is defined as $d_m = 2(2\rho + \rho_c)$, or equivalently, as having the clearance between any pair of static agents greater than $2(\rho + \rho_c)$, where $\rho_c$ is the minimum allowable clearance between any pair of
agents, and \( \rho \) is the radius of the agents. This is physically interpreted as ensuring that the clearance between any pair of agents \( i, j \) is sufficiently large so that another agent \( k \) can safely navigate among them. For more details on the geometric construction, the reader is referred to [25].

Almost global means that the resulting integral curves of (2) converge to the goal location \( r_{gi} \), except for a set of initial conditions of measure zero; if agent \( i \) initiates on this set, then it converges to undesired singular points of (2). This result is naturally expected, yet it is noteworthy that the resulting safety and convergence properties are rendered without any parameter tuning, as often done in relevant Artificial Potential Fields (APF) methods.

Having said that, the scope of this paper is not to just provide a vector field based planning and coordination methodology, but rather to highlight that complex coordination tasks with multiple agents can be achieved by using appropriate forms of the vector field (2), and more specifically, by suitably choosing the form of the repulsive flow (5).

III. A DISTRIBUTED COORDINATION PROTOCOL BASED ON DIRECTED INTERACTIONS

We are interested in designing a palette of control strategies which achieve semi-cooperative coordination and control objectives such as: aggregation of multiple agents w.r.t. a goal location, and navigation of multiple agents towards goal locations along collision-free trajectories. We propose and study the correctness of the following coordination protocol.

**Proposition 1:** Recall that each agent has a circular communication region of radius \( R_c \) centered at \( r_i = [x_i, y_i]^T \), and is assigned a goal location \( r_{gi} \). Assume that:

- the linear velocity \( u_i \) of each agent evokes under the control strategy:

\[
 u_i = \begin{cases} 
 \max \left\{ 0, \min_{j \in \mathcal{N}_i \mid \|r_j - r_{gi}\| < \lambda} u_{ij} \right\}, & d_m \leq d_{ij} \leq R_c, \\
 u_{ic}, & R_c < d_{ij}; 
\end{cases}
\]

(6)

where: \( u_{ij} \) is the safe velocity of agent \( i \) w.r.t. an agent \( j \) lying in the communication region of \( C_i \) of agent \( i \), given as:

\[
u_{ij} = u_{ic} \left( \frac{d_{ij} - d_m}{R_c - d_m} + \frac{R_c - d_{ij}}{R_c - d_m} \right),
\]

(7)

with the terms in (7) defined as:

\[
u_{ic} = k_i \tanh(\|r_i - r_{gi}\|), \quad u_{ij} = u_{ij}^T \frac{r_{ji}^T \eta_j}{r_{ji}^T \eta_j}, \quad \eta_i = \begin{bmatrix} \cos \phi_i \\ \sin \phi_i \end{bmatrix},
\]

\[
 I_j = r_{ji}^T \eta_i, \quad r_{ji} = r_i - r_j,
\]

\[
 \phi_i = \arctan(F_{iy}, F_{ix}), \quad \text{with } F_{ix}, F_{iy} \text{ taken out of (2)},
\]

\[
d_m = 2(2\rho + \rho_c), \quad d_m < \rho_c < \rho < d_c \leq R_c,
\]

where the term \( \{ j \in \mathcal{N}_i \mid I_j < 0 \} \) denotes the set of neighbor agents \( j \) of agent \( i \) for which the term \( I_j \) defined above is negative,

- the angular velocity \( \omega_i \) of agent \( i \) evolves under the control strategy:

\[
 \omega_i = -\lambda_i (\theta_i - \phi_i) + \phi_i,
\]

(8)

where \( \lambda_i > 0 \) and \( \phi_i = \arctan(F_{iy}, F_{ix}) \), with \( F_{ix}, F_{iy} \) taken out of (2).

Define the attractive force \( F_{gi} \) for each agent \( i \) in (2) as in (3). Then the group of agents:

1) aggregates in the region of goal locations if the repulsive flows for each agent \( i \) in (2) are defined as in (5),

2) navigates along collision-free trajectories and each agent converges to its assigned goal location if the repulsive flows for each agent \( i \) in (2) are defined as:

\[
 F_{ix}^i = \frac{x_i - x_j}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}}, \quad \text{and } F_{iy}^i = \frac{y_i - y_j}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}}.
\]

(9)

**Proof:** It is straightforward to verify that under the angular velocity control law (8) the orientation \( \theta_i \) of agent \( i \) is globally exponentially stable to the orientation \( \phi_i \) of the vector field (2). The distribution of goal locations implies that at least a pair \((i, j)\) of agents \( i, j \) will be at conflict at some time \( t \). Consider agent \( i \), and the set of its neighbor agents \( \mathcal{N}_i \) defined as the set of agents \( j \neq i \) lying in its communication region \( C_i \). The proposed algorithm forces each agent \( i \) to consider the subset of its neighbor agents \( j \in \mathcal{N}_i \) towards whom agent \( i \) is moving – i.e., not all its neighbor agents, but rather only those towards whom agent \( i \) is susceptible to collision.

To see why we resort to this choice, consider the time derivative of the collision avoidance constraint

\[
c_{ij} = (x_i - x_j)^2 + (y_i - y_j)^2 - d_m^2 \geq 0
\]

evaluated at the reference angles \( \phi_i, \phi_j \) of the vector field (2) for agent \( i \) and \( j \), respectively, which reads:

\[
 \frac{dc_{ij}}{dt} = 2u_{ij} r_{ji}^T \begin{bmatrix} \cos \phi_i \\ \sin \phi_i \end{bmatrix} - 2u_j r_{ji}^T \begin{bmatrix} \cos \phi_j \\ \sin \phi_j \end{bmatrix},
\]

(10)

where \( r_{ji} \equiv r_i - r_j \) and the velocities \( u_{ij}, u_j \) are positive by construction (this is justified later on). Denote \( \eta_i \equiv [\cos \phi_i \sin \phi_i]^T \), \( \eta_j \equiv [\cos \phi_j \sin \phi_j]^T \). To proceed with the analysis, let us first provide the following definitions:

**Definition 1:** Assume that \( r_{ji}^T \eta_i \geq 0 \) and \( r_{ji}^T \eta_j \leq 0 \):

Then \( I \geq 0 \) and \( J \leq 0 \), which implies that both agents \( i, j \) contribute in satisfying the collision avoidance condition. We say that collision avoidance is “fully cooperative.”

**Definition 2:** Assume that \( r_{ji}^T \eta_i \geq 0 \) and \( r_{ji}^T \eta_j > 0 \):

Then \( I \geq 0 \) and \( J < 0 \), which implies that agent \( i \) contributes towards avoiding collision, whereas agent \( j \) does not. If \( I \geq 0 \) and \( J \geq 0 \), we say that collision avoidance is “semi-cooperative by agent \( i \),” otherwise, “collision” occurs.

**Definition 3:** Assume that \( r_{ji}^T \eta_i \leq 0 \) and \( r_{ji}^T \eta_j \leq 0 \):

Then \( I \leq 0 \) and \( J \geq 0 \), which implies that agent \( j \) contributes towards avoiding collision, whereas agent \( i \) does
not. If $I + J \geq 0$, we say that collision avoidance is “semi-cooperative by agent $j$”; otherwise, “collision” occurs.

Definition 4: Assume that $r_{ji}^T \eta_i \leq 0$ and $r_{ji}^T \eta_j \geq 0$: Then $I \leq 0$ and $J \leq 0$, which implies that “collision” occurs.

Let us now consider the following cases:

- $I \geq 0$: This physically means that agent $i$ moves away from or maintains fixed distance w.r.t., agent $j$. Collision avoidance may then be either “fully cooperative” or “semi-cooperative by agent $i$”, depending on the effect of agent $j$ in (10) via the term $J$. Collision occurs if and only if $J$ is negative enough to render the condition (10) negative. We let agent $i$ move with positive linear velocity:

$$u_{ic} = k_i \tanh(\|r_i - r_{gi}\|).$$  \hspace{1cm} (11)

The case of agent $j$ moving so that $I+J < 0$ is excluded through the coordination imposed below.

- $I < 0$: This physically means that agent $i$ moves towards agent $j$. Collision is avoided if and only if the term $J$ renders the condition (10) positive, i.e., avoiding collision is, at best, “semi-cooperative by agent $j$.” Nevertheless, agent $i$ ignores the intentions of agent $j$. Thus, a way to ensure that $I+J \geq 0$ is to have agent $i$ suitably adjust its linear velocity $u_i$. To this end we assume that agent $i$ communicates with agent $j$, acquires its linear velocity $u_j$ and orientation $\phi_j$, and moves according to:

$$u_{ij} = u_{ic} \frac{d_{ij} - d_m}{R_c - d_m} + u_{isij} \frac{R_c - d_{ij}}{R_c - d_m},$$  \hspace{1cm} (12)

where:

$$u_{isij} \leq u_j \frac{r_{ji}^T \eta_j}{r_{ji}^T \eta_i}$$  \hspace{1cm} (13)

is the safe (i.e., collision avoiding) velocity for agent $i$ w.r.t. agent $j$ dictated by the condition (10), and $d_{ij}$ is the distance between $i$ and $j$. A straightforward option is to set $u_{isij}$ satisfying the equality in (13). The velocity profile $u_{ij}$ in (12) is depicted in Fig. 1.

Under this choice, it is easy to verify that:

1) If $r_{ji}^T \eta_j > 0$, i.e., if agent $j$ is moving towards agent $i$, then $u_{isij} < 0$, which implies that the linear velocity $u_i$ in (12) decreases. Depending also on the magnitude of $u_{ic} = k_i \tanh(\|r_i - r_{gi}\|)$, the linear velocity $u_i$ may become negative. The use of the maximum function between zero and the minimum over $u_{ij}$ is in order to guarantee that agent $i$ does not move with negative linear velocity, i.e., backwards, for reasons that become evident later on.

2) If $r_{ji}^T \eta_j < 0$, i.e., if agent $j$ is moving away from agent $i$, then $u_{isij} > 0$. This implies that the linear velocity $u_i$ in (12) may increase. Nevertheless, when $d_{ij} = d_m$, the velocity $u_i$ is equal to the safe velocity $u_{isij}$, implying that collision with agent $j$ is avoided.

Denote now with $N_i$ the set of neighbor agents for agent $i$. It follows that a sufficient condition for agent $i$ to avoid collisions is to adjust its linear velocity as:

$$u_i = \min_{j \in N_i : I_j > 0} u_{isij},$$  \hspace{1cm} (14)

where $j \in N_i : I_j < 0$ denotes the neighbor agents $j$ of agent $i$ for which the term $I$ in (10) is negative; geometrically, this describes the subset of neighbor agents $j \in N_i$ which lie in front of agent $i$, i.e., the subset of neighbor agents $j$ towards whom agent $i$ is moving. Hence, the safe velocity $u_{isij}$ for agent $i$ per neighbor agent $j$ implies that the distance $d_{ij}$ between agents $i$ and $j$ is lower bounded by the distance $d_m$. Therefore, taking the minimum over all safe velocities $u_{isij}$, $j \in N_i$ implies that agent $i$ adjusts its linear velocity so that it remains collision-free w.r.t. all its neighbors $j$. The maximum function between zero and the minimum over $u_{isij}$ is to guarantee that agent $i$ does not move with negative linear velocity, i.e., backwards. This is to ensure that there will not be any collision among two agents that may happen to move backwards while participating in a collision avoiding maneuver.

We continue with the analysis by considering the two distinct scenarios of interest separately:

1) Aggregation: So far we have that each agent $i$ adjusts its linear velocity $u_i$ so that no collisions occur with any of its neighbor agents $j$. The relative motion of agent $i$ w.r.t. agent $j$ and vice versa depends on the form of the integral curves of (2), or in other words, on the form of the repulsive flows around each agent $j \neq i$. The following cases may occur:

   (i) The orientation $\phi_j$ of agent $j$ is such that agent $j$ moves away from agent $i$; then the velocity $u_j$ of agent $i$ is adjusted according to (6), so that collision is avoided and both agents keep moving towards their goal locations.

   (ii) The orientation $\phi_j$ of agent $j$ is such that agent $j$ moves towards agent $i$; then both agents exchange information on their current states $\mathbf{q}_i$, $\mathbf{q}_j$ and linear velocities $u_i$, $u_j$, and adjust their linear velocities according to (6), respectively, so that collision is avoided; this physically reads that both agents slow...
down until the inter-agent distance \( d_{ij} \) converges to \( d_m \). It is straightforward to verify that the subset \( \Omega_{ij} \) of the state space defined as \( \Omega_{ij} = \{ r_i, r_j \ | \ |r_i - r_j| = d_m \} \) is the largest invariant set for the system, since by construction the terms \( r_{ji}^T \eta_j \) and \( r_{ji}^T \eta_i \) are zero on \( \Omega_{ij} \). This property holds by construction for the repulsive flows (5), see also [25], implying that the linear velocities \( u_i \) and \( u_j \) out of (6) are zero on \( \Omega_{ij} \). Thus, system trajectories converge to \( \Omega_{ij} \), which further reads that agents converge to arbitrary locations such that their inter-agent distance is \( d_m \).

2) Avoidance: In a similar spirit to aggregation, the following cases may occur:

(i) The orientation \( \phi_j \) of agent \( j \) is such that agent \( j \) moves away from agent \( i \); then the same reasoning holds as in case 1(i).

(ii) The orientation \( \phi_j \) of agent \( j \) is such that agent \( j \) moves towards agent \( i \); then both agents exchange information on their current states \( q_i, q_j \) and linear velocities \( u_i, u_j \), and adjust their linear velocities according to (6), respectively, so that collision is avoided; this physically reads that both agents slow down to avoid collision. It is straightforward to verify that the subset \( \Omega_{ij} \) of the state space defined as \( \Omega_{ij} = \{ r_i, r_j \ | \ |r_i - r_j| = d_m \} \) is not an invariant set for the system, since by construction the terms \( r_{ji}^T \eta_j \) and \( r_{ji}^T \eta_i \) are non-zero on \( \Omega_{ij} \). This property holds by construction for the repulsive flows (9), see also [25]. This reads that the linear velocities \( u_i \) and \( u_j \) out of (6) are non-zero on \( \Omega_{ij} \), which further reads that system trajectories perform sliding-like behavior across the surface \( \Omega_{ij} \). Unless system trajectories get stuck on a possible chattering Zeno point on the sliding surface (geometrically this reads that the goal locations and the current locations of the locally connected interacting agents happen to lie on the same line), the agents converge to their goal locations while avoiding collisions.

IV. Simulation Results

The efficacy of the proposed coordination control algorithms is demonstrated via simulation results.

A. Aggregation

We initially consider a scenario involving \( N = 50 \) agents which are assigned with the task to move towards a region of interest and aggregate w.r.t. a point of interest \( r_0 \). The agents are randomly assigned their initial positions and have to move towards their goal locations \( r_{ji} \), which are randomly assigned within a circular region centered at the point of interest \( r_0 \). The control gains are also randomly selected. The evolution of the motion of the agents along with the final aggregation shape are depicted in Fig. 2. For clarity purposes, not all agents are drawn in the first 4 snapshots, as the lie far away from the goal locations.

Let us stress that the proposed algorithm does not prescribe any specific pattern on the agents’ final locations; all we know is that agents will move towards their goal locations, selected around a point of interest, and will converge to this region. Achieving specific patterns might be beneficial in having multiple agents moving through confined environments, and is thus ongoing research.

B. Avoidance

We consider \( N = 25 \) agents which are moving towards their goal locations (depicted with square markers) starting from goal positions (depicted with cross markers) while avoiding collisions, see the resulting paths in Fig. 3. The goal locations are defined sufficiently far apart so that the communication regions do not overlap when agents lie on their goal locations. The evolution of agents’ motion is depicted throughout the snapshots in Fig. 4.

V. Conclusions

This paper presented a unified view of coordination control strategies which achieve either aggregation or navigation of unicycle multi-robot systems towards goal locations, using only local information and directed interactions. The planning method is built upon vector fields serving as reference motion plans for each agent. The form of the integral curves locally around agents imposes the local coordination rule achieving either aggregation, or navigation towards goal locations. It was shown that aggregation and navigation in unicycle multi-robot systems can be achieved using the same linear velocity protocol, and slightly different angular velocity protocols. The notion of semi-cooperative coordination was also introduced as a means of collision avoidance, which can be also seen as implicit prioritization among locally connected agents.
Fig. 2. Aggregation of 50 nonholonomic agents under the proposed control strategy.
Fig. 4. Collision-free motion of 25 nonholonomic agents under the proposed control strategy.
Current work focuses on the definition of vector fields encoding input constraints, such as curvature bounds, which may be more appropriate for aircraft and car-like vehicles.

REFERENCES


