Distributed coordination protocols for aggregation and navigation in multi-agent systems under local directed interactions

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***Abstract*— This paper proposes a velocity coordination protocol for the control of multiple unicycle agents, which aims to address two distinct scenarios in a unified manner. The former case is aggregation around a goal location, while the latter case is navigation to multiple goal locations. The same linear velocity protocol is used in both cases, but slightly different angular velocity protocols are designed to achieve either aggregation or navigation, respectively. The proposed angular velocity protocols regulate the angular velocities of the agents to reference directions imposed by vector fields which are different for aggregation and navigation. The proposed linear velocity protocol imposes directed interactions among agents in the following sense: an implicit prioritization among locally connected agents is ad-hoc decided, which results in a suitable adjustment of the linear velocities of the connected agents so that each one slows down with respect to (w.r.t.) the neighbor agent who maximizes the rate of decrease of the inter-agent distance. Simulation results with multiple unicycle agents achieving either aggregation or navigation along collision-free trajectories are provided to demonstrate the efficacy of the proposed algorithm.**

I. INTRODUCTION

Multi-Agent Systems have been a very popular research topic during the past fifteen years [1]. Motion planning and coordination for multiple vehicles, such as autonomous cars or unmanned aerial vehicles, typically requires collision-free navigation of the agents towards desired destinations. In a not-that-different spirit, applications which are relevant to reconfigurable and modular robotics [2], automated self-organization, construction and transportation [3]–[5], or patrolling and protection, typically involve multiple agents (robots or autonomous vehicles) which need to come together, interact, and form various shapes, creating thus a structure of augmented or improved capabilities. These problems are often characterized as aggregation or swarming behavior in multi-agent systems [6]–[11].

However, limitations in the available sensing and communication platforms impose additional constraints to the multi-agent system. In a pair (i, j) of agents i and j, information flow between them can be either bidirectional (undirected) or unidirectional (directed). Research efforts have achieved the formalization of problems such as consensus and formation control in multi-agent networks using tools and notions from graph theory, matrix theory and Lyapunov stability theory [12]–[15]. The case of directed information exchange has recently attracted increased interest [16]–[20], motivated in part by the fact that undirected information flow is not always a realistic assumption, for instance due to bandwidth limitations in the network or anisotropic sensing of the agents. Extending consensus algorithms to nonlinear systems has also become popular, see for instance [21], [22]. Nevertheless, despite that consensus, flocking, and formation control algorithms can achieve collision avoidance in multi-vehicle systems by carefully selecting initial conditions and controlling relative distance and heading, they are typically not used in encoding navigation to *specific* goal locations for each one of the agents.

While the problems of coordinating the motion of multiple robots towards a single (for aggregation) or multiple (for navigation) goal locations are not new, here we consider a unified view of these two problems, and furthermore we consider *directed interactions* among agents. By directed interaction we mean that each agent i makes a decision on its control actions by interacting with a subset, *i.e., not necessarily all,* of its neighbor agents $j \neq i$, and more specifically, with the agents towards whom it is currently moving. The proposed coordination algorithms result in achieving pairwise collision avoidance by at least one of, i.e., not necessarily both, the involved agents i, j, imposing thus a semi-cooperative sense. More specifically, agent i is controlled so that it slows down and its trajectories remain collision-free w.r.t. the worst case neighbor agent j, that is, the agent j who maximizes the rate of decrease of the inter-agent distance.

Our goal is not to just provide a new coordination method for multiple nonholonomic agents, but rather to highlight that the same coordination protocol for the agents' linear velocities, along with slight modifications on the coordination of their angular velocities, achieves distinct collective behaviors: the former case is aggregation around a goal location, while the latter one is navigation towards multiple goal locations. The primary motivation on studying these tasks relies on planning problems for complex control systems, such as those encountered in aerial and modular robotics.

The paper is organized as follows: Sections II and III present the mathematical formulation and the proposed coordination protocol, along with the analysis on the correctness and fulfillment of the underlying tasks. Section IV provides simulation results for aggregation and navigation scenarios, while our conclusions and thoughts on future work are summarized in Section V.

II. MULTI-AGENT COORDINATION VIA VECTOR FIELDS

We consider a group of N agents with unicycle kinematics. The motion of the i-th agent, $i \in \{1, \ldots, N\}$, is governed

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by the equations:

$$\begin{bmatrix}\dot{x}_i\\ \dot{y}_i\\ \dot{\theta}_i\end{bmatrix} = \begin{bmatrix}\cos\theta_i & 0\\ \sin\theta_i & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}u_i\\ \omega_i\end{bmatrix}, \tag{1}$$

where $\boldsymbol{q}_i = \begin{bmatrix}x_i & y_i & \theta_i\end{bmatrix}^T$ is the state vector comprising the position $\boldsymbol{r}_i = \begin{bmatrix}x_i & y_i\end{bmatrix}^T$ and the orientation θ_i of agent i w.r.t. a global cartesian coordinate frame $\mathcal{G}$, u_i is the linear velocity and ω_i is the angular velocity of agent i w.r.t. the body-fixed frame $\mathcal{B}_i$. We assume that each agent i: (i) is a circular disk of radius ρ_i centered at $\boldsymbol{r}_i$, (ii) has access to its state $\boldsymbol{q}_i$ and velocities u_i, ω_i, (iii) can reliably exchange information with any agent $j \neq i$ which lies within its communication region $\mathcal{C}_i : \{\boldsymbol{r} \in \mathbb{R}^2 \mid \|\boldsymbol{r}_i - \boldsymbol{r}\| \leq R_c\}$, where R_c is the communication range. In other words, a pair of agents (i, j) is connected as long as the distance $d_{ij} = \|\boldsymbol{r}_i - \boldsymbol{r}_j\| \leq R_c$.

In our earlier work [23] we defined a class of vector fields $\mathbf{F}_i : \mathbb{R}^2 \to \mathbb{R}^2$ for each agent i as:

$$\mathbf{F}_i = \prod_{j=1, j\neq i}^{N} (1 - \sigma_{ij})\, \mathbf{F}_{gi} + \sum_{j=1, j\neq i}^{N} \sigma_{ij}\, \mathbf{F}^i_{oj}, \tag{2}$$

where:

$$\mathrm{F}_{gix} = \frac{(x - x_{gi})^2 - (y - y_{gi})^2}{(x - x_{gi})^2 + (y - y_{gi})^2}, \tag{3a}$$

$$\mathrm{F}_{giy} = \frac{2(x - x_{gi})(y - y_{gi})}{(x - x_{gi})^2 + (y - y_{gi})^2}, \tag{3b}$$

are the vector field components of a (normalized) attractive vector field $\mathbf{F}_{gi} : \mathbb{R}^2 \to \mathbb{R}^2$, which is by construction vanishing at the goal location $\boldsymbol{r}_{gi}$ only,
$\mathbf{F}^i_{oj}$ is a normalized repulsive vector field around each each agent $j \neq i$, which will be defined later on, and σ_{ij} is a bump function defined as:

$$\sigma_{ij} = \begin{cases} 1, & \text{for } d_m \leq d_{ij} \leq d_r;\\ a\, {d_{ij}}^3 + b\, {d_{ij}}^2 + c\, d_{ij} + d, & \text{for } d_r < d_{ij} < d_c;\\ 0, & \text{for } d_{ij} \geq d_c; \end{cases} \tag{4}$$

where: $d_{ij} = \|\boldsymbol{r}_i - \boldsymbol{r}_j\|$ the Euclidean distance between agents i, j, the physical meaning of the values d_m, d_r, d_c is to be highlighted later on, and the coefficients a, b, c, d computed as: $a = -\frac{2}{(d_r - d_c)^3}$, $b = \frac{3(d_r + d_c)}{(d_r - d_c)^3}$, $c = -\frac{6\, d_r d_c}{(d_r - d_c)^3}$, $d = \frac{{d_c}^2(3d_c - d_r)}{(d_r - d_c)^3}$, so that (4) is a $\mathcal{C}^2$ function.

The vector field (2), for the repulsive vector field $\mathbf{F}^i_{oj}$ around each agent $j \neq i$ defined as:

$$\mathbf{F}^i_{oj} = \begin{cases} \mathbf{F}^i_{oj|_A}, & \text{for } {\boldsymbol{p}_{ij}}^T \boldsymbol{r}_{ji} \geq 0;\\ \mathbf{F}^i_{oj|_B}, & \text{for } {\boldsymbol{p}_{ij}}^T \boldsymbol{r}_{ji} < 0, \end{cases} \tag{5}$$

where: $\boldsymbol{p}_{ij} = \begin{bmatrix}\cos\phi_{ij} & \sin\phi_{ij}\end{bmatrix}^T$, $\boldsymbol{r}_{ji} = \boldsymbol{r}_i - \boldsymbol{r}_j$, $\phi_{ij} = \operatorname{atan2}\left((y_j - y_{gi}), (x_j - x_{gi})\right)$,

$$\mathbf{F}^i_{oj|_A} = \begin{bmatrix}\sin\phi_{ij}(x_i - x_j)(y_i - y_j) - \cos\phi_{ij}(y_i - y_j)^2\\ \cos\phi_{ij}(x_i - x_j)(y_i - y_j) - \sin\phi_{ij}(x_i - x_j)^2\end{bmatrix},$$

$$\mathbf{F}^i_{oj|_B} = \begin{bmatrix}-\cos\phi_{ij}(x_i - x_j)^2 - \cos\phi_{ij}(y_i - y_j)^2\\ -\sin\phi_{ij}(x_i - x_j)^2 - \sin\phi_{ij}(y_i - y_j)^2\end{bmatrix}$$

is shown [23] under certain, mild assumptions to be a safe, almost global feedback motion plan for agent i operating in an idealistic environment, where each other agent $j \neq i$ is static and serves as a circular obstacle.

Mild assumptions refer to ensuring that the repulsive flows around each pair of agents $j \neq i$ do not overlap, which is achieved if the minimum distance d_m in (4) is defined as $d_m = 2(2\rho + \rho_\epsilon)$, or equivalently, as having the clearance between any pair of static agents greater than $2(\rho+\rho_\epsilon)$, where ρ_ϵ is the minimum allowable clearance between any pair of agents, and ρ is the radius of the agents. This is physically interpreted as ensuring that the clearance between any pair of agents i, j is sufficiently large so that another agent k can safely navigate among them. For more details on the geometric construction, the reader is referred to [23].

Almost global means that the resulting integral curves of (2) converge to the goal location $\boldsymbol{r}_{gi}$, except for a set of initial conditions of measure zero; if agent i initiates on this set, then it converges to undesired singular points of (2).[1]

The scope of this paper is rather to highlight that coordination tasks for multiple agents such as avoidance or aggregation can be achieved by using appropriate forms of the vector field (2), and more specifically, by suitably choosing the form of the repulsive flow (5).

III. A Distributed Coordination Protocol based on Directed Interactions

We are interested in designing a palette of control strategies which achieve semi-cooperative coordination and control for aggregation around a goal location, and navigation towards goal locations along collision-free trajectories. We propose and study the correctness of the following coordination protocol.

Proposition 1: Recall that each agent has a circular communication region $\mathcal{C}_i$ of radius R_c centered at $\boldsymbol{r}_i$, and is assigned a goal location $\boldsymbol{r}_{gi}$. Assume that for each agent i:

- The linear velocity u_i is given by the control strategy:

$$u_i = \begin{cases} \max\left\{0, \min\limits_{j \in \mathcal{N}_i | I_j < 0} u_{i|j}\right\}, & d_m \leq d_{ij} \leq R_c,\\ u_{ic}, & R_c < d_{ij}; \end{cases}, \tag{6}$$

  where: $u_{i|j}$ is the safe velocity of agent i w.r.t. an agent j lying in $\mathcal{C}_i$, given as:

$$u_{i|j} = u_{ic}\,\frac{d_{ij} - d_m}{R_c - d_m} + u_{is|j}\frac{R_c - d_{ij}}{R_c - d_m}, \tag{7}$$

  with the terms in (7) defined as:

$$u_{ic} = k_i \tanh(\|\boldsymbol{r}_i - \boldsymbol{r}_{gi}\|), \quad k_i > 0, \tag{8}$$

$$u_{is|j} = u_j\, \frac{{\boldsymbol{r}_{ji}}^T \boldsymbol{\eta}_j}{{\boldsymbol{r}_{ji}}^T \boldsymbol{\eta}_i}, \quad \boldsymbol{\eta}_i = \begin{bmatrix}\cos\phi_i\\ \sin\phi_i\end{bmatrix}, \quad I_j = {\boldsymbol{r}_{ji}}^T \boldsymbol{\eta}_i,$$

$$\boldsymbol{r}_{ji} = \boldsymbol{r}_i - \boldsymbol{r}_j, \quad \phi_i = \arctan\left(\mathrm{F}_{iy}, \mathrm{F}_{ix}\right),$$

[1]This result is naturally expected, yet it is noteworthy that the resulting safety and convergence properties are rendered without any parameter tuning, as often done in relevant potential functions methods.

where $\mathrm{F}_{ix}, \mathrm{F}_{iy}$ are taken out of (2), $d_m = 2(2\rho + \rho_\epsilon)$ and $d_m < d_r < d_c \leq R_c$. Note that the term $\{j \in \mathcal{N}_i \mid I_j < 0\}$ denotes the set of neighbor agents j of agent i towards whom agent i is moving.

- The angular velocity ω_i is given by the control strategy:

$$\omega_i = -\lambda_i \left(\theta_i - \phi_i\right) + \dot{\phi}_i, \tag{9}$$

where $\lambda_i > 0$, $\phi_i = \arctan\left(\mathrm{F}_{iy}, \mathrm{F}_{ix}\right)$, and $\mathrm{F}_{ix}, \mathrm{F}_{iy}$ taken out of (2).

Define the attractive flow $\mathbf{F}_{gi}$ for each agent i in (2) as in (3). Then the group of agents:

1) aggregates in the region of goal locations if the repulsive flows for each agent i in (2) are defined as in (5),
2) navigates along collision-free trajectories and each agent converges to its assigned goal location if the repulsive flows for each agent i in (2) are defined as:

$$\mathrm{F}^i_{xoj} = \frac{x_i - x_j}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}}, \tag{10a}$$

$$\mathrm{F}^i_{yoj} = \frac{y_i - y_j}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}}. \tag{10b}$$

Proof: It is straightforward to verify that under the angular velocity control law (9) the orientation θ_i of agent i is globally exponentially stable to the orientation ϕ_i of the vector field (2). Consider agent i, and the set of its neighbor agents $\mathcal{N}_i$ defined as the set of agents $j \neq i$ lying in its communication region $\mathcal{C}_i$. Each agent i considers the subset of its neighbor agents $j \in \mathcal{N}_i$ towards whom it is moving, i.e., those for which $I_j < 0$, that is, not necessarily all its neighbor agents, but rather only those with whom it is susceptible to collide.

To see why we resort to this choice, consider the time derivative of the collision avoidance constraint $c_{ij} = (x_i - x_j)^2 + (y_i - y_j)^2 - {d_m}^2 \geq 0$ evaluated at the reference angles ϕ_i, ϕ_j of the vector field (2) for agent i and j, respectively, which reads:

$$\frac{d}{dt}\, c_{ij} = \underbrace{2u_i\, \boldsymbol{r}_{ji}{}^T \begin{bmatrix}\cos\phi_i \\ \sin\phi_i\end{bmatrix}}_{I} \underbrace{-2u_j\, \boldsymbol{r}_{ji}{}^T \begin{bmatrix}\cos\phi_j \\ \sin\phi_j\end{bmatrix}}_{J}, \tag{11}$$

where $\boldsymbol{r}_{ji} \triangleq \boldsymbol{r}_i - \boldsymbol{r}_j$ and the velocities u_i, u_j are positive by construction (this is justified later on). Denote $\boldsymbol{\eta}_i \triangleq \begin{bmatrix}\cos\phi_i & \sin\phi_i\end{bmatrix}^T$, $\boldsymbol{\eta}_j \triangleq \begin{bmatrix}\cos\phi_j & \sin\phi_j\end{bmatrix}^T$. To proceed with the analysis, let us first provide the following definitions:

Definition 1: Assume that ${\boldsymbol{r}_{ji}}^T \boldsymbol{\eta}_i \geq 0$ and ${\boldsymbol{r}_{ji}}^T \boldsymbol{\eta}_j \leq 0$: Then $I \geq 0$ and $J \geq 0$, which implies that both agents i, j contribute in satisfying the collision avoidance condition. We say that collision avoidance is *"fully cooperative."*

Definition 2: Assume that ${\boldsymbol{r}_{ji}}^T \boldsymbol{\eta}_i \geq 0$ and ${\boldsymbol{r}_{ji}}^T \boldsymbol{\eta}_i > 0$: Then $I \geq 0$ and $J < 0$, which implies that agent i contributes towards avoiding collision, whereas agent j does not. If $I + J \geq 0$, we say that collision avoidance is *"semi-cooperative by agent i"*; otherwise, *"collision"* occurs.

Definition 3: Assume that ${\boldsymbol{r}_{ji}}^T \boldsymbol{\eta}_i < 0$ and ${\boldsymbol{r}_{ji}}^T \boldsymbol{\eta}_j \leq 0$: Then $I < 0$ and $J \geq 0$, which implies that agent j contributes towards avoiding collision, whereas agent i does not. If $I + J \geq 0$, we say that collision avoidance is *"semi-cooperative by agent j"*; otherwise, *"collision"* occurs.

Definition 4: Assume that ${\boldsymbol{r}_{ji}}^T \boldsymbol{\eta}_i < 0$ and ${\boldsymbol{r}_{ji}}^T \boldsymbol{\eta}_j > 0$: Then $I < 0$ and $J < 0$, which implies that *"collision"* occurs.

Let us now consider the following cases:

- $I \geq 0$: This physically means that agent i moves away from or maintains fixed distance w.r.t., agent j. Collision avoidance may then be either "fully cooperative" or "semi-cooperative by agent i", depending on the effect of agent j in (11) via the term J. Collision occurs if and only if J is negative enough to render the condition (11) negative. We let agent i ignore agent j and move with positive linear velocity given out of (8) towards its destination $\boldsymbol{r}_{gi}$. The case of agent j moving so that $I + J < 0$ is excluded through the coordination imposed below.
- $I < 0$: This physically means that agent i moves towards agent j. Collision is avoided if and only if the term J renders the condition (11) positive, i.e., avoiding collision is, at best, "semi-cooperative by agent j." Nevertheless, agent i ignores the intentions of agent j. Hence, a way to ensure $I + J \geq 0$ is to have agent i suitably adjust its linear velocity u_i. We assume that agent i communicates with agent j, acquires its linear velocity u_j and orientation ϕ_j, and moves according to:

$$u_{i|j} = u_{ic} \frac{d_{ij} - d_m}{R_c - d_m} + u_{is|j} \frac{R_c - d_{ij}}{R_c - d_m}, \tag{12}$$

where:

$$u_{is|j} \leq u_j \frac{{\boldsymbol{r}_{ji}}^T \boldsymbol{\eta}_j}{{\boldsymbol{r}_{ji}}^T \boldsymbol{\eta}_i} \tag{13}$$

is the safe (i.e., collision avoiding) velocity for agent i w.r.t. agent j dictated by the condition (11), and d_{ij} is the distance between i and j. A straightforward option is to set $u_{is|j}$ satisfying the equality in (13). The velocity profile $u_{i|j}$ in (12) is depicted in Fig. 1.

Under this choice, it is easy to verify that:

1) If ${\boldsymbol{r}_{ji}}^T \boldsymbol{\eta}_j > 0$, i.e., if agent j is moving towards agent i, then $u_{is|j} < 0$, which implies that the linear velocity u_i in (12) decreases. Depending also on the magnitude of $u_{ic} = k_i \tanh(\|\boldsymbol{r}_i - \boldsymbol{r}_{gi}\|)$, the linear velocity u_i may become negative. The use of the maximum function between zero and the minimum over $u_{i|j}$ is to guarantee that agent i does not move with negative linear velocity, i.e., backwards, for reasons that become evident later on.
2) If ${\boldsymbol{r}_{ji}}^T \boldsymbol{\eta}_j < 0$, i.e., if agent j is moving away from agent i, then $u_{is|j} > 0$. This implies that the linear velocity u_i in (12) may increase. Nevertheless, when $d_{ij} = d_m$, the velocity u_i is equal to the safe velocity $u_{is|j}$, implying that collision with agent j is avoided.

Denote now $\mathcal{N}_i$ the set of neighbor agents for agent i. A sufficient condition for agent i to avoid collisions is to adjust its linear velocity as:

$$u_i = \min_{j \in \mathcal{N}_i | I_j < 0} u_{i|j}, \tag{14}$$

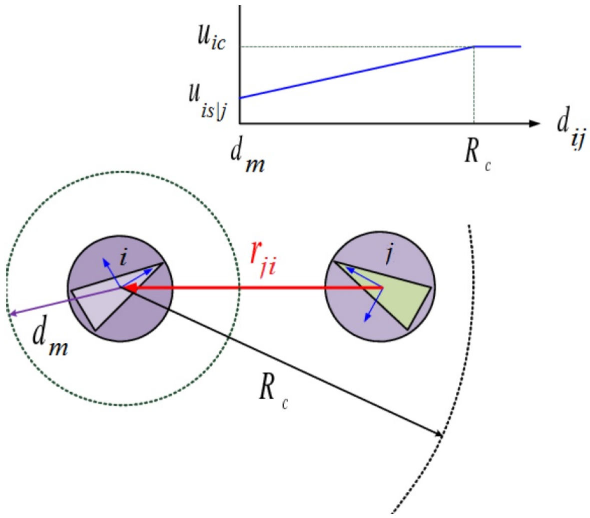


Fig. 1. If $\boldsymbol{r}_{ji}{}^T\boldsymbol{\eta}_i < 0$, i.e., if agent i moves towards agent j, then agent i adjusts its linear velocity according to the velocity profile shown here, given analytically by (12).

where $j \in \mathcal{N}_i | I_j < 0$ denotes the neighbor agents j of agent i for which the term I_j in (11) is negative; geometrically, this describes the subset of neighbor agents $j \in \mathcal{N}_i$ which lie in front of agent i, i.e., the subset of neighbor agents j towards whom agent i is moving. The safe velocity $u_{i|j}$ for agent i per neighbor agent j implies that the distance d_{ij} between agents i and j is lower bounded by the distance d_m. *Therefore, taking the minimum over all safe velocities $u_{i|j}$, $j \in \mathcal{N}_i$ implies that agent i adjusts its linear velocity so that it remains collision-free w.r.t. all its neighbors j.* The maximum function between zero and the minimum over $u_{i|j}$ is to guarantee that agent i does not move with negative linear velocity, and hence that there will not be any collision among two agents which may happen to move backwards while participating in a collision avoiding maneuver.

We continue with the analysis on convergence of the system trajectories by considering the two distinct scenarios of interest separately:

1) Aggregation: So far we have that each agent i adjusts its linear velocity u_i so that no collisions occur with any of its neighbor agents j. The relative motion of agent i w.r.t. agent j and vice versa depends on the form of the integral curves of (2), i.e., on the repulsive flows around each agent $j \neq i$. The following cases may occur:
   (i) The orientation ϕ_j is such that agent j moves away from agent i; then the velocity u_i of agent i is adjusted according to (6), collision is avoided, and both agents keep moving towards their goal locations.
   (ii) The orientation ϕ_j is such that agent j moves towards agent i; then both agents exchange information on their current states $\boldsymbol{q}_i$, $\boldsymbol{q}_j$ and linear velocities u_i, u_j, and both adjust their linear velocities according to (6), respectively, to avoid collision; this physically reads that both agents slow down until the inter-agent distance d_{ij} converges to d_m. Since the vector field (5) are tangential around the agents i, j, see also [23], it follows that the terms $\boldsymbol{r}_{ji}{}^T\boldsymbol{\eta}_j$ and $\boldsymbol{r}_{ji}{}^T\boldsymbol{\eta}_i$ are zero on the subset of the state space defined as $\Omega_{ij} = \{\boldsymbol{r}_i, \boldsymbol{r}_j \mid \|\boldsymbol{r}_i - \boldsymbol{r}_j\| = d_m\}$. The linear velocities (6) of the agents are also zero on that set, implying that Ω_{ij} is the largest invariant set for the system. Hence the system trajectories converge to Ω_{ij} and remain there ever after, which physically reads that agents converge to arbitrary locations forming the aggregation shape such that their inter-agent distance is d_m.

2) Avoidance: In a similar spirit to aggregation, the following cases may occur:
   (i) The orientation ϕ_j is such that agent j moves away from agent i; the same reasoning holds as in case 1(i) and agents keep moving towards their destinations.
   (ii) The orientation ϕ_j is such that agent j moves towards agent i; then both agents exchange information on their current states $\boldsymbol{q}_i$, $\boldsymbol{q}_j$ and linear velocities u_i, u_j, and adjust their linear velocities according to (6), respectively, so that collision is avoided; this physically reads that both agents slow down to avoid collision. In this case the integral curves of (5) are radial around the agents and hence the terms $\boldsymbol{r}_{ji}{}^T\boldsymbol{\eta}_j$ and $\boldsymbol{r}_{ji}{}^T\boldsymbol{\eta}_i$ are non-zero. Thus the subset Ω_{ij} of the state space defined as $\Omega_{ij} = \{\boldsymbol{r}_i, \boldsymbol{r}_j \mid \|\boldsymbol{r}_i - \boldsymbol{r}_j\| = d_m\}$ is *not* an invariant set for the system, since by construction the linear velocities (6) are non-zero there, implying that the system trajectories will escape the set. This physically reads that system trajectories perform sliding-like behavior across the surface Ω_{ij}. Unless system trajectories get stuck on a possible chattering Zeno point on the sliding surface (geometrically this reads that the goal locations and the current locations of the locally connected interacting agents happen to lie on the same line), the agents converge to their goal locations while avoiding collisions.

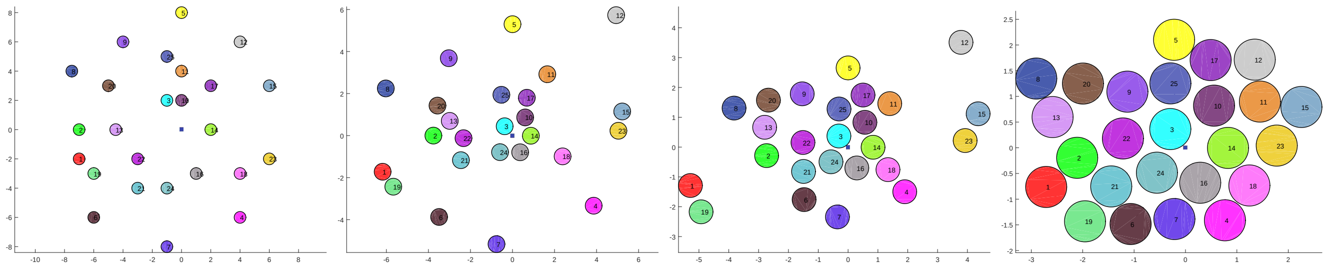
■

IV. Simulation Results

The efficacy of the proposed coordination control algorithms both in aggregation and in navigation scenarios is demonstrated via simulation results. In both cases the radii of all agents are $\rho = 0.4$ m, the parameters in the definition of the function (4) are $d_m = 0.820$ m, $d_r = 0.902$ m and $d_r = R_c = 1.025$ m, the linear velocity control gains in (8) are chosen so that $2.25 \leq k_i \leq 3.75$ and the angular control gain in (9) is set the same for all agents, $\lambda_i = 2$.

Aggregation: We initially consider a scenario involving N = 25 agents which are assigned with the task to aggregate w.r.t. the point of interest $\boldsymbol{r}_0 = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$. The agents are randomly assigned their initial positions shown in Fig. 2(a).

The evolution of the motion of the agents along with the final aggregation shape are depicted throughout Fig. 2, where the point of interest $\boldsymbol{r}_0$ is shown with the square marker.



(a) t=0 sec (b) t=1 sec (c) t=2 sec (d) t=7 sec

Fig. 2. Aggregation of 25 agents under the proposed control strategy.

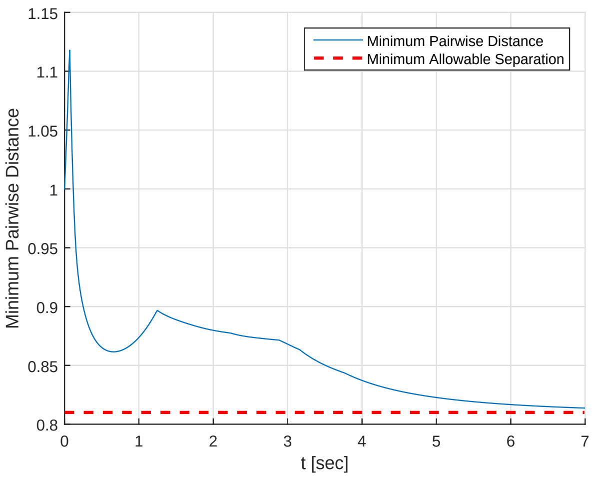


Fig. 3. Evolution of the minimum pairwise distance over time in the case of aggregation. Since the agents are forced under the proposed control strategy to slow down in order to maintain minimum separation, it follows that the minimum distance eventually converges to the minimum separation.

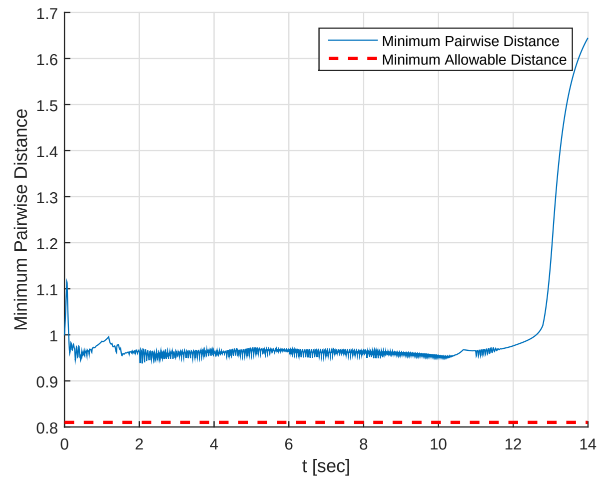


Fig. 4. Evolution of the minimum pairwise distance over time in the case of navigation to goal locations.

The agents coordinate their velocities according to proposed coordination protocol, i.e., each agent i slows down w.r.t. the neighbor agent j with whom the rate of decrease of the inter-agent distance d_{ij} is maximum. The evolution of the pairwise minimum distance over time is shown in Fig. 3; as expected, the minimum pairwise inter-agent distance converges to the minimum allowable separation, as the aggregation is formed.

Let us stress that the proposed algorithm does not prescribe any specific pattern on the agents' final locations; all we know is that agents will move towards their goal location, and will converge to this region while respecting the minimum separation w.r.t. their neighbors. Achieving specific patterns might be beneficial in having multiple agents moving through confined environments, and is thus ongoing research.

Avoidance: We then consider $N = 20$ agents which are assigned to navigate towards goal locations (depicted with square markers) starting from goal positions (depicted with "x" markers) while avoiding collisions (Fig. 5(f)). The goal locations are defined sufficiently far apart so that the communication regions do not overlap when agents lie on their goal locations. The evolution of their motion is depicted throughout Fig 5(a)-5(e). The evolution of the pairwise minimum distance over time is shown in Fig. 4, demonstrating that it never violates minimum separation.

V. Conclusions

This paper presented a unified view of coordination control strategies which achieve either aggregation or navigation of multiple unicycle robots using only local information and directed interactions. The planning method is built upon vector fields serving as reference motion plans for each agent. It was shown that the same linear velocity control protocol accompanied with slightly different angular velocity control protocols for aggregation and navigation, respectively, achieves the desired behaviors (aggregation and navigation, respectively) for the unicycle multi-robot system along collision-free trajectories. The coordination protocol builds upon the notion of semi-cooperative coordination, which can be also seen as implicit prioritization among locally connected agents so that each one avoids only the neighbor agents towards whom it is moving.

Current work focuses on the definition of coordination protocols accounting for complex dynamics and input constraints, such as curvature bounds, which may be more appropriate for aircraft and car-like vehicles.

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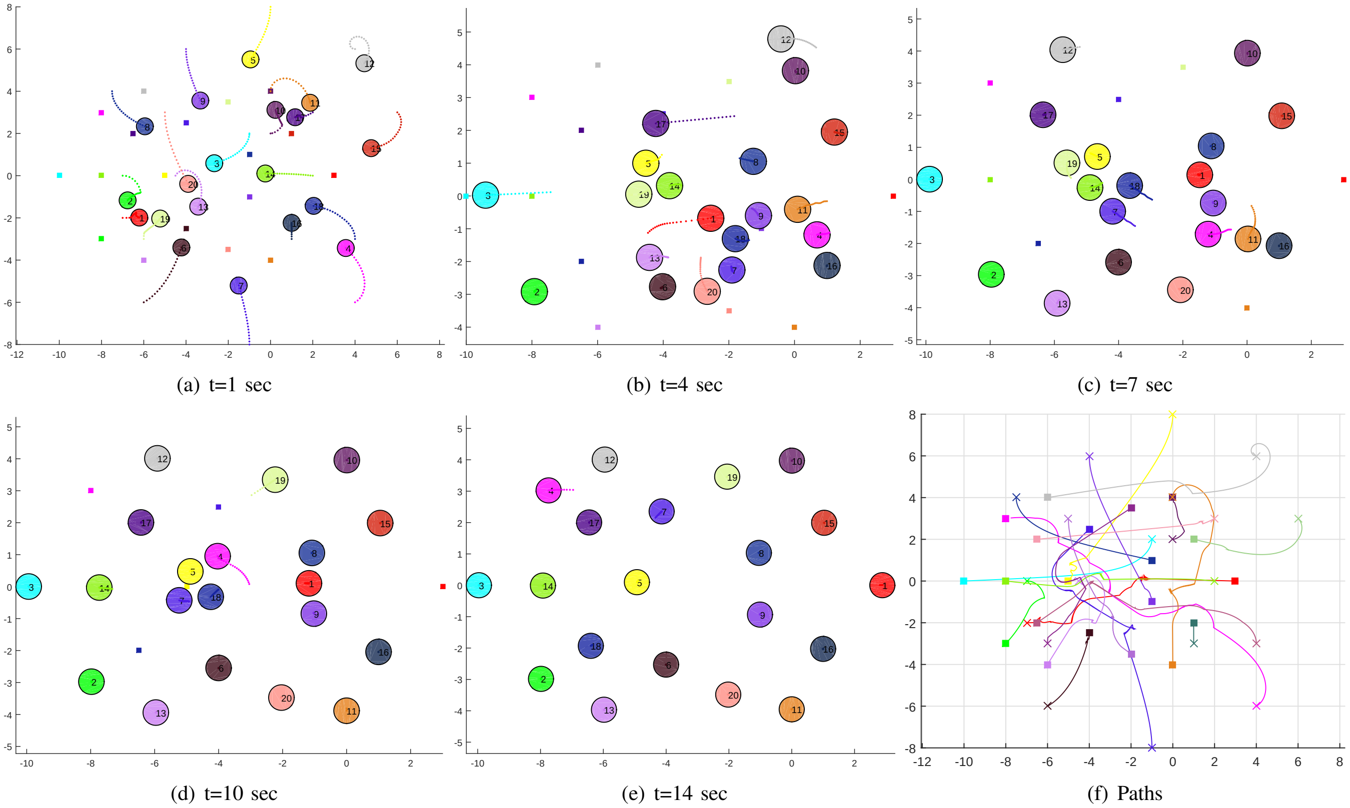

(a) t=1 sec (b) t=4 sec (c) t=7 sec

(d) t=10 sec (e) t=14 sec (f) Paths

Fig. 5. Navigation of 25 agents under the proposed control strategy.

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