Control Strategies for Multiplayer Target-Attacker-Defender Differential Games with Double Integrator Dynamics

Mitchell Coon and Dimitra Panagou

***Abstract*— This paper presents a method for deriving optimal controls and assigning attacker-defender pairs in a target-attacker-defender differential game between an arbitrary numbers of attackers and defenders, all of which are modeled using double integrator dynamics. It is assumed that each player has perfect information about the states and controls of the players within a certain range of themselves, but they are unaware of any players outside of this range. Isochrones are created based on the time-optimal trajectories needed for the players to reach any point in the shortest possible time. The intersections of the players' isochrones are used to determine whether a defender can intercept an attacker before the attacker reaches the target. Sufficient conditions on the detection range of the defenders and the guaranteed capture despite perturbations of the attackers off the nominal trajectories are derived. Then, in simulations with multiple players, attacker-defender pairs are assigned so that the maximum number of attackers are intercepted in the shortest possible time.**

I. INTRODUCTION

Differential pursuit-evasion games is a classical research topic in optimal control and game theory [1]–[3]. The target-attacker-defender differential game involves one or more attacking players who are seeking to reach a target, and one or more defending players whose goal is to prevent the attackers from reaching said target. In [4], the optimal controls for each player in a single-attacker-single-defender game are derived using constrained optimization, and the critical speed above which the target can always escape is determined analytically. The approach in [4] is extended to a scenario with two defenders in [5]. In both papers, the players are modeled as Dubins vehicles with constant linear velocities and bounded turning rates.

Focusing on aircraft defense and missile guidance, line-of-sight guidance for the defender is studied in [6], with an additional cooperative guidance law for the target to maximize the attacker-to-defender lateral acceleration ratio. The three-body pursuit-evasion problem is also addressed in [7] and [8], where the authors provide sufficient conditions for the missile to hit the target while evading the defender. Similarly, the authors in [9] and [10] study various guidance laws based on the rate of change of the line-of-sight between the players in aircraft defense games. Cooperative pursuit-evasion guidance strategies are presented in [11] for the target-attacker and the attacker-defender pairs, again for aircraft missile defense, as well as in [12]–[16]. In [17], the effects of limiting the defender's acceleration command in a target-attacker-defender game are studied, while [18] compares the performance of different guidance strategies, including proportional navigation, command-to-line-of-sight, and pure pursuit, for aerial defended targets.

The construction of barriers for differential games is particularly relevant to target-attacker-defender games, see in [19]–[21]. In these papers, barriers for two-player games that provide sufficient conditions for each player's victory, are derived, along with the optimal controls that lead to victory for each player in the game. The authors of [22] and [23] derive barriers for visibility-based pursuit-evasion games.

Another significant research topic is that of control strategies for differential games involving multiple players. In [24], a method for determining controls for a game with an arbitrary number of players is presented. In this method, a solution is determined for each possible attacker-defender pair, then a maximum matching algorithm is used to assign pairings that prevent the maximum number of attackers from reaching their target. The authors of [25] consider a game where one evader is trying to escape through the gap between two pursuers. In [26], the authors analyze the value function level sets for a game involving one evader and two pursuers, and present a method for constructing optimal feedback controls. In [27], the authors use a decentralized control scheme based on a Voronoi partition of a game involving one evader and multiple pursuers in a plane. Similarly, in [28], dynamic Voronoi diagrams are used to determine optimal controls for a multi-agent game. In [29], an integral cost function is used to capture the synergy between two evaders trying to evade a single pursuer. In [30], a game is simulated between two evaders and a single evader, where the evaders must tradeoff between evading the pursuer and herding together.

There are several other approaches to two-player pursuit-evasion differential games, such as the use of approximate dynamic programming in [31], and the use of a multistage influence diagram game to model one-on-one air combat [32]. In [33], the isochrones for Dubins vehicles are constructed, and the intersections of those isochrones are used to determine the dominance regions of each player, as well as the time-optimal controls which enable each player's victory.

In this paper, while we do not derive explicit barriers for the games discussed, we provide a method for determining the winner of a game based on the initial positions of the players, as well as the controls that enable victory. Our method builds upon the isochrones approach in [33],

Mitchell Coon is with the Department of Aerospace Engineering, University of Michigan, Ann Arbor; macoon@umich.edu.

Dimitra Panagou is with the Department of Aerospace Engineering, University of Michigan, Ann Arbor; dpanagou@umich.edu.

The authors would like to acknowledge the support of the Automotive Research Center (ARC) in accordance with Cooperative Agreement W56HZV-14-2-0001 U.S. Army TARDEC in Warren, MI.

and provides sufficient conditions on the minimum sensing (detection) range for the defenders, and capture guarantees despite a class of possible perturbations for the attacker. In addition, and in contrast to all above mentioned papers, here we consider double integrator dynamics for the agents. The double integrator model is a better representation for multi-copter aircraft compared to the Dubins model, since it captures the aircraft's ability to change direction quickly without making wide turns.[1] Furthermore, unlike to many of the previously mentioned papers which only deal with a small number of players, the method presented here is applicable to differential games with arbitrary numbers of players. The multi-player case is addressed by assigning pairs of attackers and defenders using a minimum cost bipartite matching algorithm similar to that of [24], where the associated cost of each attacker-defender pair is the time required for the defender to intercept the attacker.

The rest of the paper is structured as follows: Section II describes the mathematical formulation of the problem while Section III presents the adopted approach to the solution, and outlines the algorithm used in simulations. Section IV presents the results of the simulations. Finally, Section V summarizes the paper and discusses future work.

II. PROBLEM FORMULATION

In this paper we consider a game between a team of M defenders, a team of N attackers, and a single stationary target. It is assumed that the players have perfect information about the other players' states and controls when they are within a certain range R of each other, but they are otherwise unaware of players outside of this range. Both the attackers and defenders are modeled as point agents. The equations of motion for players are as follows:

$$\dot{x}(t) = v_x(t), \quad (1a) \qquad \dot{v}_x(t) = u_x(t), \quad (2a)$$

$$\dot{y}(t) = v_y(t), \quad (1b) \qquad \dot{v}_y(t) = u_y(t), \quad (2b)$$

$$\sqrt{u_x(t)^2 + u_y(t)^2} \leq a_{max}, \tag{3}$$

where x and y are the position coordinates of an agent, v_x and v_y are the velocity components, u_x and u_y are the acceleration commands of the agent, all w.r.t. a Cartesian inertial coordinate frame, t is the current time, and a_{max} is the maximum acceleration command possible for the agents.

Let r_c denote a finite radius of capture. Then, the scenario that any of the attackers reaches the target at some time t is described as:

$$\min_i \sqrt{(x_{A,i}(t) - x_T(t))^2 + (y_{A,i}(t) - y_T(t))^2} \leq r_c, \tag{4}$$

where $x_{A,i}$ and $y_{A,i}$ are the position coordinates of attacker i, and x_T and y_T are the target's coordinates.

[1]Note that in [34], optimal controls which drive a vehicle under double integrator dynamics to rest at the origin are derived; however, in this paper, we are concerned with the time-optimal controls that allow a double integrator vehicle to reach a point as quickly as possible, instead of coming to rest at that point.

Similarly, the scenario under which an attacker i is intercepted by a defender j at some time t is described as:

$$\min_j \sqrt{(x_{A,i}(t) - x_{D,j}(t))^2 + (y_{A,i}(t) - y_{D,j}(t))^2} \leq r_c, \tag{5}$$

where $x_{D,j}$ and $y_{D,j}$ are the position coordinates of defender j. Our goal for the single-attacker-single-defender game is to determine sufficient conditions under which a defender is guaranteed to be able to intercept an attacker before the attacker reaches the target, as well as sufficient conditions under which an attacker is guaranteed to reach the target without being intercepted. Furthermore, we seek the time-optimal control strategies for the players in each of these cases. We then apply these strategies to games with multiple attackers and defenders by considering defender-attacker assignments based on bipartite graph matching, similar to the approach of [24].

Let the payoff of the game be defined as the minimum distance of all attackers to the target after either all targets have been intercepted, or the target has been reached, i.e.,

$$d_{min} = \min_i \sqrt{(x_{A,i}(T) - x_T)^2 + (y_{A,i}(T) - y_T)^2}, \tag{6}$$

where T is the time at which the game ends, either in favor of the attackers or the defenders. The goal of the defenders is to maximize this payoff, while the attackers are seeking to minimize it.

III. APPROACH

A. Isochrone Equations

In order to determine the outcome of a game between a single attacker-defender pair, we first find sets of isochrones which determine how far the players can travel in a given amount of time. Once these are known, we can find intersections of the attackers' and defenders' isochrones to determine all possible interception points. These points represent a terminal surface, and if the attacker cannot reach the target without passing through this surface, the defender will be able to intercept the attacker before it reaches the target. These equations will not be explicitly derived here. The time-optimal controls for the agents are given by:

$$u_x = a_{max} \cos(\theta), \tag{7a}$$

$$u_y = a_{max} \sin(\theta), \tag{7b}$$

where θ is the counterclockwise angle from the x-axis to the acceleration vector. Since θ is constant in this case, given the initial position and velocity of an agent, the final position is:

$$x(T) = x_0 + v_{x,0}T + \frac{1}{2}a_{max} \cos(\theta)T^2, \tag{8a}$$

$$y(T) = y_0 + v_{y,0}T + \frac{1}{2}a_{max} \sin(\theta)T^2, \tag{8b}$$

where x_0 and y_0 are the initial Cartesian coordinates of the player, and $v_{x,0}$ and $v_{y,0}$ are the initial velocity components.

The equations derived above can be used to create isochrones for a single player. Fig. 1 shows a set of isochrones for an agent starting at the origin with an initial velocity of 1 m/s in the y-direction, and a maximum acceleration of 1 m/s^2 in any direction. The isochrones shown in this figure represent the set of all possible locations where the player could be at the indicated time, using the controls described above. Prior to $t = 2$ seconds in this example, some isochrones will overlap, since the agent may be moving in the positive or negative y-direction at a single point, but at different times. After $t = 2$ seconds, each point on the isochrone corresponds to a unique control input. If the player's desired location is on one of the overlapping isochrones, the player should choose the control that will allow it to reach that point at the same time as an opposing player. However, if the player is an attacker and it can reach the target without being intercepted, it should simply choose the control corresponding to the isochrone with the smallest time value, in order to reach the target as quickly as possible.

B. Single Attacker - Single Defender Games

The intersection of the isochrones of two or more opposing players can be used to determine the terminal surface where the defenders may intercept the attackers. An attacker's goal is to minimize their distance to the target at the time of interception, so its optimal control is the one that will bring it either to the target, or to the closest point on the isochrones' intersection surface to the target.

However, it should be noted that an attacker using a certain control input may be intercepted at multiple different locations at different times, depending on the defender's control input. Since the defender's goal is to intercept the attacker as far from the target as possible, it will choose whichever control input allows them to intercept the attacker at a location further from the target, unless the time required to intercept the attacker at such a location is greater than the time required for the attacker to reach the target.

The isochrones are only calculated for times up to the amount of time required for an attacker to reach the target. If the attacker cannot be intercepted before this time, there is nothing the defenders can do, and there is no reason to calculate the isochrones for later times. Furthermore, if an attacker can be intercepted within this time period, a defender will never attempt to intercept it at a later time, since doing so would allow the attacker to reach the target.

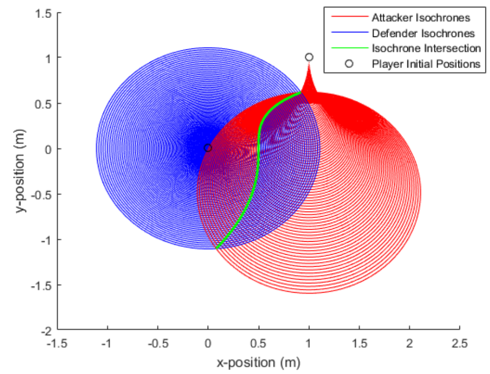

Fig. 2: Intersection of two players' isochrones.

1) Example: Fig. 2 shows the intersection of two players' isochrones. In this example, the defender starts from rest at the origin, while the attacker starts at $(1, 1)$ with an initial velocity of 1 m/s in the negative y-direction. Both players have a maximum acceleration magnitude of 1 m/s^2. Note that the intersection surface is not closed, and in fact extends to infinity in this case. In the simulations, the intersections are only calculated over a finite amount of time equal to the time that it would take for an attacker to reach the target.

Fig. 3 shows the trajectories taken by the players in the scenario described in Fig. 2, if the target is located at $(0.5, 0.5)$. The isochrones are left out of this figure for clarity. In this case, the defender is able to intercept the attacker before it can reach the target, thus the attacker chooses the closest point on the intersection surface to the target as its goal. Note that the trajectory of the defender is a straight line since it initiates from rest, while the attacker's trajectory is curved due to its non-zero initial velocity.

C. Performance Guarantees

1) Capture despite attacker deviation: We now consider perturbations (i.e., deviations) of the attackers from their

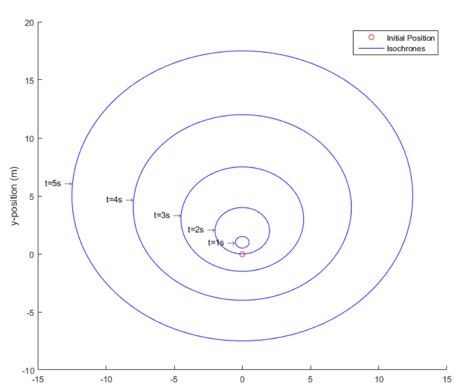

Fig. 1: Isochrones for an agent starting at the origin moving in the positive y-direction; the outermost isochrone corresponds to $t = 5s$, and the time difference between successive isochrones is 1 sec.

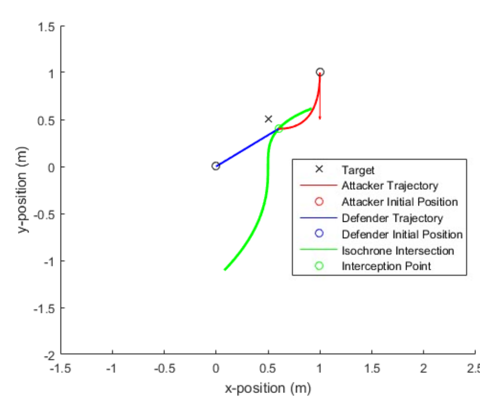

Fig. 3: Trajectories in a two-player game; the red arrow represents the attacker's initial velocity.

nominal trajectories that would result from their attempt to avoid the defenders. In order to guarantee that capture still occurs regardless of deviations from the nominal path, it is sufficient to show that the attackers' deviations are within a certain bound.

Proposition 3.1: Let $\theta_{A,i}$ be the direction of attacker i's acceleration vector that would result in interception according to the method above. Suppose in reality it accelerates with its maximum possible acceleration in a direction $\theta_{A,i} + \Delta\theta$. In order for the defender to still be within the desired capture radius at the expected interception time, it is sufficient to show that the attacker's deviation from the ideal acceleration angle satisfies the following condition:

$$\Delta\theta \leq \cos^{-1}\left(1 - \frac{2r_c^2}{T^4 a_{max}^2}\right). \tag{9}$$

Proof: If an attacker accelerates in the direction $\theta_{A,i} + \Delta\theta$, then assuming constant acceleration, the distance between the positions of the two different trajectories at any given time is:

$$\Delta d = \frac{a_{max}t^2}{2}\sqrt{\Delta x^2 + \Delta y^2}, \tag{10a}$$

$$\Delta x = \cos(\theta_{A,i} + \Delta\theta) - \cos(\theta_{A,i}), \tag{10b}$$

$$\Delta y = \sin(\theta_{A,i} + \Delta\theta) - \sin(\theta_{A,i}), \tag{10c}$$

Without loss of generality, assume $\theta_{A,i} = 0$. Then through some simple algebra:

$$\Delta d = \frac{\sqrt{2}}{2} a_{max} t^2 \sqrt{1 - \cos(\Delta\theta)}. \tag{11}$$

Substituting the capture radius r_c for Δd and the interception time T for t, then solving for $\Delta\theta$ yields:

$$\Delta\theta = \cos^{-1}\left(1 - \frac{2r_c^2}{T^4 a_{max}^2}\right). \tag{12}$$

This is the maximum deviation for the attacker that will still result in the attacker being captured, assuming constant acceleration for the defender in the same (i.e., its nominal) direction. Therefore, as long as $\Delta\theta$ is less than or equal to this value for all $t \leq T$, capture will still occur. ■

Remark 1: Note that as the time required to intercept an attacker decreases, the allowable deviation increases, so it is beneficial for the defenders to intercept the attackers as quickly as possible.

Remark 2: If an attacker's deviation exceeds this limit, the defenders would need to recalculate their trajectories in order to intercept that attacker. The frequency of recalculation required to guarantee capture is a potential topic for future research, but is not covered in this paper.

2) Limited sensing (detection) radius for guaranteed capture: We are now interested in obtaining bounds on the limited sensing (detection) radii of the defenders, so that interception of the detected attackers is guaranteed before the attackers reach the target. Given the initial position of a defender, if the initial velocity of an attacker is known, it is possible to use the isochrone intersections to determine the region in which the attacker is guaranteed to be able to reach the target without being intercepted. In order to do this, we first calculate the backwards isochrones from the target, assumed to be at the origin. The x- and y-coordinates of the backwards isochrones for attacker i are as follows:

$$x_{B,A,i}(t) = (V_i(T-t) - \frac{1}{2}(T-t)^2)\cos(\theta_{A,i}), \tag{13a}$$

$$y_{B,A,i}(t) = (V_i(T-t) - \frac{1}{2}(T-t)^2)\sin(\theta_{A,i}), \tag{13b}$$

$$V_i = v_{0,A,i} + a_{max}T, \tag{13c}$$

where $v_{0,A,i}$ is the magnitude of attacker i's initial velocity. The initial distance from an attacker to the target for a given value of T is:

$$r_{0,A,i} = \sqrt{x_{B,A,i}(0,T)^2 + y_{B,A,i}(0,T)^2}, \tag{14}$$

Note that larger values of T correspond to starting positions further away from the target. We then find the intersection of the defender's isochrones and the attacker's backwards isochrones using various starting positions $r_{0,A,i}$.

Proposition 3.2: Given the initial position of a defender and the initial velocity of an attacker, the outer edge of the region within which an attacker is guaranteed to be able to reach the target without being intercepted is defined by the points where intersection occurs between the defender's isochrones and the attacker's backwards isochrones for the smallest possible value of $r_{0,A,i}$.

Proof: The backwards isochrones represent the trajectories of attackers coming from various directions that can reach the target without being intercepted. The attackers are assumed to be moving toward the target initially, and they accelerate directly towards the target, because this results in the shortest time to reach the target. Therefore, if an attacker following such a trajectory can be intercepted before reaching the target, any attacker starting further away from the target with the same velocity, or starting at the same location with a velocity not in the direction of the target will also be able to be intercepted, since it will take a longer time to reach the target. Therefore, for a given value of $\theta_{A,i}$, the point where the defender's isochrones and the attacker's backwards isochrones intersect using the smallest value of $r_{0,A,i}$ represents the furthest point from the target in the $\theta_{A,i}$ direction that the attacker can start and still reach the target without being intercepted. ■

Examples of these regions are shown in Fig. 4: The blue region corresponds to a stationary defender, while the red region corresponds to a defender moving away from the target.

The above analysis on the regions determining victory assumes infinite sensing radius for the defenders, i.e., that the attackers can be detected at any time and anywhere on the plane. A sufficient bound on the sensing radii R of the defenders so that attackers can be detected and intercepted prior to reaching the target is then obtained as follows:

Proposition 3.3: If the defenders' sensing radii R are such that $R \geq R_{min}$, where:

$$R_{min} = r_{DT} + r_{AT}, \tag{15a}$$

$$r_{AT} = v_{0,A,i} t_{DT} + \frac{1}{2} a_{max} t_{DT}^2, \tag{15b}$$

where r_{DT} is the distance from defender j to the target, t_{DT} is the minimum time it would take the defender to reach the target, and r_{AT} is the distance that the attacker can travel over time t_{DT}, with t_{DT} given as the positive solution to the following set of equations:

$$x_{0,D,j} + v_{x,0,D,j} t_{DT} + \frac{1}{2} a_{max} t_{DT}^2 \cos\theta_{D,j} = 0 \tag{16a}$$

$$y_{0,D,j} + v_{y,0,D,j} t_{DT} + \frac{1}{2} a_{max} t_{DT}^2 \sin\theta_{D,j} = 0 \tag{16b}$$

where: $x_{0,D,j}$ and $y_{0,D,j}$ are defender j's initial position coordinates, $v_{x,0,D,j}$ and $v_{y,0,D,j}$ the defender j's initial velocity components, and $\theta_{D,j}$ is defender j's acceleration angle, then: the defenders are guaranteed to be able to intercept any attackers first detected outside of the regions described above.

Proof: A defender will never be able to intercept an attacker that can reach the target before $t = t_{DT}$. Therefore, the boundary of the region described above will be at most r_{AT} from the target. Furthermore, in order for the defender to detect an attacker at such a point on the boundary, it is necessary for the defender's sensing radius to satisfy $R \geq R_{min}$, where $R_{min} = r_{DT} + r_{AT}$. ∎

Remark 3: Note that the region for which an attacker is guaranteed to reach the target is larger if the defender is moving away from the target. Therefore, it is beneficial for a defender to come to a stop after intercepting a target in case any additional attackers come within its sensing radius.

D. Assignment of Attacker-Defender Pairs

When multiple attackers and defenders are present, the defenders must consider not only where to intercept an attacker, but also which attacker they should pursue. In this paper we assume all defenders communicate with each other to collectively decide which attacker each defender will pursue. The process for assigning pairs of attackers and defenders is as follows [24]:

1) Construct a bipartite graph with two sets of nodes, $\{D_j\}_j^M$ and $\{A_i\}_i^N$, where each node represents one of the defenders or attackers, respectively
2) For each D_j, determine for each A_i within the sensing range of D_j whether or not D_j can intercept A_i before A_i reaches the target using the isochrone method described in the previous sections.
3) Draw an edge in the bipartite graph from D_j to A_i if D_j can intercept A_i before A_i reaches the target. If interception is possible, let the time required for D_j to intercept A_i be the cost of this edge.
4) Run any minimum-cost matching algorithm to find a maximum matching in the graph that minimizes the associated cost. This can be done using the Hungarian matching algorithm [35], for example.

Remark 4: Note that while this matching guarantees that the defenders will intercept as many attackers within their range as possible, it does not guarantee that any attackers outside of this range will be intercepted. Because of this, the defenders will slow to a stop, i.e. they will accelerate in the direction opposite of their current velocity, after intercepting an attacker if there are no other nearby attackers that need to be intercepted.

Remark 5: If the agents only shared information about a small subgroup of nearby agents, this would reduce computation time, but possibly result in suboptimal pairings.

E. Implementation in Simulation

In the simulations presented in this paper, the defenders are initially at rest near the target, while the attackers start further away with some initial cruise speed v in the direction of the target. The simulation runs until either one of the attackers reaches the target, or until all attackers have been intercepted. If any attackers are within a distance R of any defenders at the beginning of the game, the optimal trajectories will be calculated as described above for those players. The trajectories will be recalculated every time a new player comes within range of an opposing player that was previously out of range.

After an attacker is intercepted, it will stop moving and no longer be considered in calculations of the trajectories. However, the defenders can still move after intercepting an attacker, and potentially intercept additional attackers. If the defenders have not been assigned to any attackers, they will simply slow to a stop and wait in case another attacker comes in range of them. Since the defenders do not know the locations of the attackers outside of their sensing range, it is best for them to remain at rest, because as mentioned above, stationary defenders are guaranteed to be able to intercept attackers in a larger area than defenders moving away from the target.

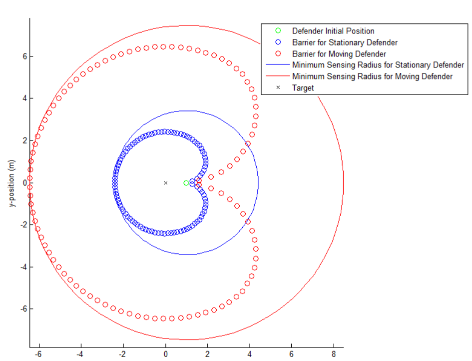

Fig. 4: Comparison of barriers for a stationary and moving defender. The solid lines show the minimum sensing radius required for the defender to intercept all defenders outside of the barriers.

IV. SIMULATION RESULTS

1) Equal Number of Attackers and Defenders: In the case where $M = N$, the end outcome of the game is largely

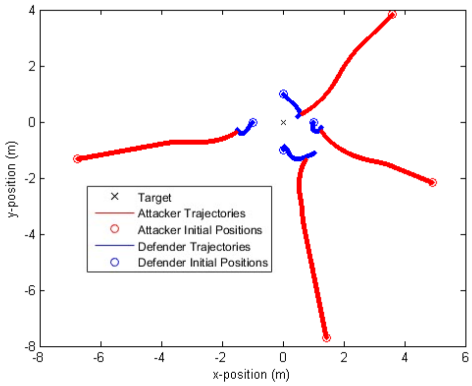

Fig. 5: Trajectories in a 4 defender vs. 4 attacker game; defenders win.

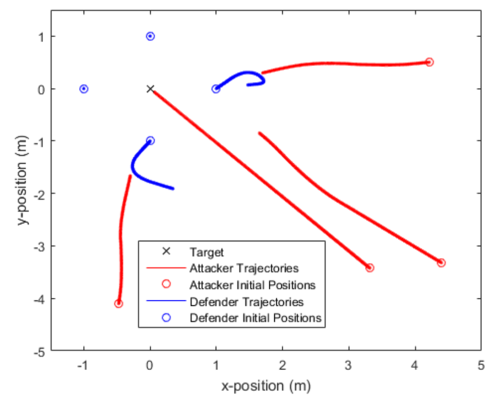

Fig. 6: Trajectories in a 4 defender vs. 4 attacker game; attackers win.

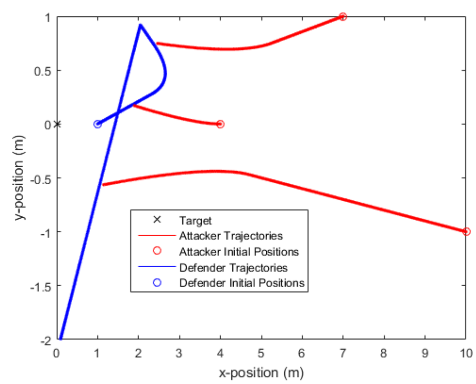

Fig. 7: Trajectories in a 1 defender vs. 3 attacker game; defender wins.

determined by the initial placement of the players. In the simulations presented here, the defenders are positioned in a diamond formation around the target, while the attackers are placed randomly in the space further away from the target.

Fig. 5 shows the trajectories of the players in a game between four attackers and four defenders. The attackers are not initially in range of the defenders, so they simply move towards the target at a constant cruise speed. When they get close enough to the defenders, they plan their trajectories using the method described in the previous sections. It is interesting to note that whenever the attackers recalculate their trajectories, even though they are still moving towards the same defender, they tend to start accelerating in a different direction, causing them to swerve back and forth slightly. In this case, the defenders are still able to intercept them.

Fig. 6 shows a similar game where the attackers win instead. Because the attackers are initially much closer to each other, they are able to overwhelm the defenders. While the first two attackers are intercepted, the defenders simply aren't close enough to perceive and intercept both of the remaining attackers in time. This situation could easily be prevented by having more defenders on one side of the target, but since the defenders don't know what direction the attackers will come from in this scenario, they are placed evenly around the target, making them unable to respond effectively to a concentrated assault such as this. Because of the lack of communication and cooperation between defenders, this example effectively reduces to the case where there are more attackers than defenders.

2) More Attackers than Defenders: In the case where $M < N$, it is much more difficult for the defenders to intercept all attackers. Unlike the case where there are equal numbers of defenders and attackers, and the target is guarded evenly on all sides, it is more advantageous for the attackers to start far from each other and come from opposite directions in this case. Since there are not enough defenders to intercept the attackers simultaneously, at least one of the attackers has a good chance of reaching the target.

Fig. 7 shows a game between a single defender and three attackers. In this game, the three attackers all come from a similar direction, and they don't all come at the same time, so the defender is able to intercept each of them as they come. This is one of the few cases where a large group of attackers would not be able to succeed against a smaller group of defenders. This is because the attackers are first detected outside of the regions specified in Section III.C, and the defender's sensing radius is large enough for it to respond in time.

3) Fewer Attackers than Defenders: In the case where $M > N$, the defenders have a clear advantage and are usually able to intercept all attackers with ease. Even if an attacker managed to get past a handful of defenders, there are still more ready to intercept it should the need arise. As with the case where $M = N$, the best chance that the attackers have is to group up and approach the target simultaneously in an attempt to overwhelm the defenders.

Fig. 8 shows a game between 8 defenders and 4 attackers. In this game, most of the defenders do not even need to move from their initial positions before all of the attackers have been intercepted. In cases where the integrity of the target is critical, it would be beneficial to have more defenders than necessary, but otherwise the additional defenders are redundant.

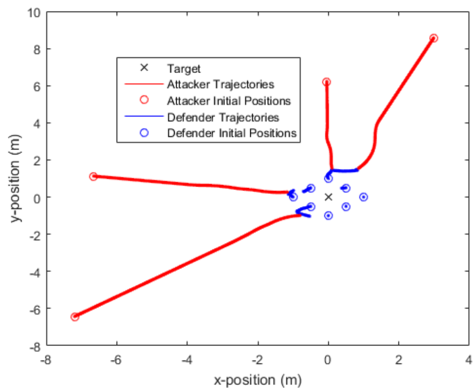

Fig. 8: Trajectories in a 8 defender vs. 4 attacker game; defenders win.

V. CONCLUSIONS

This paper presents a method for calculating optimal controls and assigning attacker-defender pairs for target-attacker differential games between agents with double integrator dynamics. The isochrone intersections of opposing players are used to determine the optimal trajectories for different attacker-defender pairs, then pairs are assigned using a minimum cost bipartite graph matching algorithm so that the maximum number of attackers are intercepted in the shortest possible time. Future work will focus on extending the methods used in this paper to games involving both players using Dubins vehicle dynamics and players using double integrator dynamics. We would also like to investigate scenarios where the defenders cooperate with each other to intercept the attackers more quickly.

REFERENCES

[1] P. T. Kabamba and A. R. Girard, *Fundamentals of Aerospace Navigation and Guidance*. New York: Cambridge University Press, 2014.
[2] R. Isaacs, *Differential Games: a Mathematical Theory with Applications to Warfare and Pursuit, Control and Optimization*. Courier Corporation, 1999.
[3] D. E. Kirk, *Optimal Control Theory: an Introduction*. Courier Corporation, 2012.
[4] M. Pachter, E. Garcia, and D. W. Casbeer, "Active target defense differential game," in *Communication, Control, and Computing (Allerton), 2014 52nd Annual Allerton Conference on*. IEEE, 2014, pp. 46–53.
[5] D. W. Casbeer, E. Garcia, and M. Pachter, "The target differential game with two defenders," in *Unmanned Aircraft Systems (ICUAS), 2016 International Conference on*. IEEE, 2016, pp. 202–210.
[6] A. Ratnoo and T. Shima, "Line-of-sight interceptor guidance for defending an aircraft," *AIAA Journal of Guidance, Control, and Dynamics*, vol. 34, no. 2, pp. 522–532, 2011.
[7] S. Rubinsky and S. Gutman, "Three body guaranteed pursuit and evasion," in *AIAA Guidance, Navigation, and Control Conference*, 2012, pp. 1–24.
[8] ——, "Three-player pursuit and evasion conflict," *AIAA Journal of Guidance, Control, and Dynamics*, vol. 37, no. 1, pp. 98–110, 2013.
[9] T. Yamasaki and S. Balakrishnan, "Triangle intercept guidance for aerial defense," in *AIAA Guidance, Navigation, and Control Conference*, 2010, pp. 1–22.
[10] T. Yamasaki, S. Balakrishnan, and H. Takano, "Modified command to line-of-sight intercept guidance for aircraft defense," *AIAA Journal of Guidance, Control, and Dynamics*, vol. 36, no. 3, pp. 898–902, 2013.
[11] A. Perelman, T. Shima, and I. Rusnak, "Cooperative differential games strategies for active aircraft protection from a homing missile," *AIAA Journal of Guidance, Control, and Dynamics*, vol. 34, no. 3, pp. 761–773, 2011.
[12] T. Shima, "Optimal cooperative pursuit and evasion strategies against a homing missile," *AIAA Journal of Guidance, Control, and Dynamics*, vol. 34, no. 2, pp. 414–425, 2011.
[13] V. Shaferman and T. Shima, "Cooperative multiple-model adaptive guidance for an aircraft defending missile," *AIAA Journal of Guidance, Control, and Dynamics*, vol. 33, no. 6, pp. 1801–1813, 2010.
[14] O. Prokopov and T. Shima, "Linear quadratic optimal cooperative strategies for active aircraft protection," *AIAA Journal of Guidance, Control, and Dynamics*, vol. 36, no. 3, pp. 753–764, 2013.
[15] E. Garcia, D. W. Casbeer, and M. Pachter, "Cooperative strategies for optimal aircraft defense from an attacking missile," *AIAA Journal of Guidance, Control, and Dynamics*, vol. 38, no. 8, pp. 1510–1520, 2015.
[16] ——, "Active target defense differential game with a fast defender," in *2015 American Control Conference*, 2015, pp. 3752–3757.
[17] I. Rusnak, H. Weiss, and G. Hexner, "Guidance laws in target-missile-defender scenario with an aggressive defender," *IFAC Proceedings Volumes*, vol. 44, no. 1, pp. 9349–9354, 2011.
[18] A. Ratnoo and T. Shima, "Guidance strategies against defended aerial targets," *AIAA Journal of Guidance, Control, and Dynamics*, vol. 35, no. 4, pp. 1059–1068, 2012.
[19] W. Getz and G. Leitmann, "Qualitative differential games with two targets," *Journal of Mathematical Analysis and Applications*, vol. 68, no. 2, pp. 421–430, 1979.
[20] W. M. Getz and M. Pachter, "Capturability in a two-target "game of two cars"," *AIAA Journal of Guidance, Control, and Dynamics*, vol. 4, no. 1, pp. 15–21, 1981.
[21] W. Grimm and K. Well, "Modelling air combat as differential game recent approaches and future requirements," in *Differential Games-Developments in Modelling and Computation*. Springer, 1991, pp. 1–13.
[22] S. Bhattacharya, T. Başar, and N. Hovakimyan, "On the construction of barrier in a visibility based pursuit evasion game," in *2014 European Control Conference*, 2014, pp. 1894–1901.
[23] S. Bhattacharya, "On the construction of barrier in a connectivity maintenance game," in *2013 European Control Conference*, 2013, pp. 3338–3345.
[24] M. Chen, Z. Zhou, and C. J. Tomlin, "Multiplayer reach-avoid games via low dimensional solutions and maximum matching," in *2014 American Control Conference*, 2014, pp. 1444–1449.
[25] W. Zha, J. Chen, Z. Peng, and D. Gu, "Construction of barrier in a fishing game with point capture," *IEEE Transactions on Cybernetics*, Apr. 2016.
[26] S. A. Ganebny, S. S. Kumkov, S. Le Ménec, and V. S. Patsko, "Model problem in a line with two pursuers and one evader," *Dynamic Games and Applications*, vol. 2, no. 2, pp. 228–257, 2012.
[27] H. Huang, W. Zhang, J. Ding, D. M. Stipanović, and C. J. Tomlin, "Guaranteed decentralized pursuit-evasion in the plane with multiple pursuers," in *50th IEEE Conference on Decision and Control and European Control Conference*, 2011, pp. 4835–4840.
[28] E. Bakolas and P. Tsiotras, "Optimal pursuit of moving targets using dynamic voronoi diagrams," in *49th IEEE Conference on Decision and Control*, 2010, pp. 7431–7436.
[29] Z. E. Fuchs, P. P. Khargonekar, and J. Evers, "Cooperative defense within a single-pursuer, two-evader pursuit evasion differential game," in *49th IEEE Conference on Decision and Control*, 2010, pp. 3091–3097.
[30] W. Scott and N. E. Leonard, "Pursuit, herding and evasion: A three-agent model of caribou predation," in *2013 American Control Conference*, 2013, pp. 2978–2983.
[31] J. S. McGrew, J. P. How, B. Williams, and N. Roy, "Air-combat strategy using approximate dynamic programming," *AIAA Journal of Guidance, Control, and Dynamics*, vol. 33, no. 5, pp. 1641–1654, 2010.
[32] K. Virtanen, J. Karelahti, and T. Raivio, "Modeling air combat by a moving horizon influence diagram game," *AIAA Journal of Guidance, Control, and Dynamics*, vol. 29, no. 5, pp. 1080–1091, 2006.
[33] D. W. Oyler and A. R. Girard, "Dominance regions in the homicidal chauffeur problem," in *2016 American Control Conference*, 2016, pp. 2494–2499.
[34] D. Feng and B. Krogh, "Acceleration-constrained time-optimal control in n dimensions," *IEEE Transactions on Automatic Control*, vol. 31, no. 10, pp. 955–958, 1986.
[35] H. W. Kuhn, "The hungarian method for the assignment problem," *Naval research logistics quarterly*, vol. 2, no. 1-2, pp. 83–97, 1955.