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Robust Multi-task Formation Control via Parametric Lyapunov-like Barrier Functions

Dongkun Han and Dimitra Panagou

***Abstract*— An essential problem in the coordination of multiple agents is formation control. Significant challenges to the theoretical design may arise when the multi-agent system is subject to uncertainty. This paper considers the robust multi-task formation control problem for multiple agents whose communication and measurements are disturbed by uncertain parameters. The control objectives include 1. achieving the desired configuration; 2. avoiding collisions; 3. preserving the connectivity of uncertain topology. To achieve these objectives, we first provide conditions in terms of Linear Matrix Inequalities (LMIs) for checking the connectivity of uncertain topologies. Then, we propose a new type of Lyapunov-like barrier functions, called parametric Lyapunov-like barrier functions, that is applicable to multi-agent systems with uncertainties in communication and measurements. It is shown that this new type of Lyapunov-like barrier functions guarantees the robust multi-task formation and displays advantages over parameter-independent Lyapunov-like barrier functions. The efficacy of the proposed method is demonstrated via simulation results.**

I. Introduction

Multi-agent systems consist of a group of intelligent agents or subsystems that cooperate to achieve collective goals through mutual communication. Due to its broad applications in various areas, a number of cooperative problems have been introduced, such as consensus, flocking, rendezvous and coverage [1]–[5]. Among these problems, the *formation control* aims to achieve a desired configuration specified by relative inter-agent distances. This problem attracts much attention in recent years because of its wide range of practical implementations, e.g., aircraft coordination and multi-robot path planning; for more applications, the reader is referred to [6].

In addition, much attention has been recently paid to practical implementations of multi-agent systems that are subject to *uncertainty*, such as packet dropouts, capacity constraints, sensing disturbances, channel fading, transmission noises, just to name a few [7]. Due to the great amount of related literature in formation control, here our review focuses on whether the multi-agent network is considered to be under uncertainty or not, rather than on providing an exhaustive review of formation control from all aspects; excellent surveys on formation control can be found in [8]–[10]. The case of uncertainty-free network is typically treated with methods using algebraic graph theory, Lyapunov stability theory, LaSalle's invariance principle, behavior-based approaches, see for instance [11], [12]. Nevertheless, in principle, methods developed under the assumption of no uncertainty are not applicable or not robust in practice [13].

In order to improve robustness against uncertainty, different classes of uncertain models and methods have been proposed recently [13]–[18]. Based on the mathematical description of uncertainty, these models can be roughly divided into two categories: models with *norm-bounded* uncertainties, and models with *stochastic* uncertainties. In the former case, it is often assumed that the uncertainty terms are of bounded l_2-norm or l_∞-norm, and affect the control input of each agent; then the Lyapunov stability theorem is employed for control design and analysis [14]–[16], [19]. In the latter case, Guassian noises are assumed in the available measurements or in the control input, reflecting unreliable information exchange among the agents [13], [17], [20]; then, mean square or almost sure consensus can be achieved via a stochastic-approximation type of gain [17].

However, the existing literature mainly focuses on merely additive, or merely multiplicative, uncertainties, which do not offer a wide description of disturbances that are encountered in practical implementation. For example, in several cases, communication among agents might be disturbed by parameters such as temperature, radiation, magnetic field intensity, so that the values of information exchanged among the agents can be modeled as a polynomial function of these parameters. In addition, the aforementioned approaches assume that the network is connected, or a spanning tree is always existing. In practice, this condition is not automatically satisfied if no extra control input is provided for *connectivity maintenance*. Moreover, *collision avoidance* is a key requirement for the safety of agents, however it is not often considered in the design of uncertain multi-agent systems.

Elegant approaches have been developed for connectivity maintenance control, both in the presence and in the absence of uncertainty. These approaches can be generally categorized into *centralized connectivity control methods* and *distributed connectivity control methods*. The former one uses global information of networks and ensures the second-smallest eigenvalue of the Laplacian matrix to be positive [21]. For distributed connectivity control methods, one strategy is to constrain all the agents in a selected workspace by using Lyapunov-like barrier functions [22]. Another strategy is to use a potential function to preserve edges once they appear in the network [12], [23]–[29]. The latter strategy does not require constraining the agents in a common workspace, hence it is less conservative than the first one. In [27], a decentralized navigation function is introduced for connectivity maintenance and collision avoidance, but without the consideration of un-

The authors are with the Department of Aerospace Engineering, University of Michigan, 1320 Beal Ave, Ann Arbor, MI 48109, USA. E-mail: {dongkunh,dpanagou}@umich.edu.

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certain terms in the system model. In [24], a robust rendezvous problem is studied under unknown dynamics and bounded disturbances, but without considering collision avoidance. For more related work in connectivity maintenance, the reader is referred to the survey paper [30] and references therein.

In addition, while uncertain holonomic models, such as single and double integrators, have been extensively investigated, this is not the case for *uncertain* non-holonomic models; to the best of our knowledge, there is not much work on the robust formation control problem for uncertain non-holonomic systems with anisotropic sensing. Considering in addition the challenges of connectivity maintenance and collision avoidance, that are particularly significant for multi-agent systems under uncertainty, the following three questions arise naturally: (1) how do we check the connectivity of uncertain networks with parametric uncertainties? (2) how do we ensure the connectivity and collision avoidance for a non-holonomic model with anisotropic sensing? and (3) how do we design a distributed controller to achieve the robust multi-task formation with uncertainties both in communication and measurement? To the best of our knowledge, these issues have not been addressed yet and still remain challenging. It is, therefore, the main purpose of this paper to explore and contribute on solving these issues.

Motivated by the aforementioned results and based on our earlier work [22], this paper considers a robust multi-task formation control problem for uncertain networks. The class of uncertainty considered in this paper can be interpreted in two ways: First, it models disturbances in communication among agents, where the coupling weights are assumed to be generic polynomial functions of uncertain parameters. In addition, it can also be interpreted as measurement uncertainty, where the on-board sensor of each agent is subject to an external perturbation which is a polynomial function of uncertain parameters.

Compared to the conference version of our work, which addresses the formation control for agents of holonomic motion model with cyclical sensing and parameter-independent Lyapunov-like barrier functions, [31], the novelties of this paper lie in the following aspects:

- We provide necessary and sufficient conditions for checking the connectivity of networks under a class of communication and measurement uncertainty. More specifically, based on the real Positivestellensatz, relaxed conditions are given via sum-of-squares programming. Then, by using the square matrix representation, solvable conditions of LMIs are proposed for checking the connectivity of the uncertain network.
- Motivated by [32], [33], which extend parameter-independent Lyapunov functions into a more general case of parameter-dependent Lyapunov functions, we introduce a new type of Lyapunov-like barrier functions, called parametric Lyapunov-like barrier functions, which guarantee the robust multi-task formation with a bounded control input for multi-agent systems under uncertainty in communication and measurement. This is different to the approach of using parameter-independent Lyapunov-like barrier functions, which are widely adopted in the related area [11], [12], [25], [28].
- We propose a distributed control strategy based on a parameter-dependent Lyapunov-like barrier function, which achieves robust multi-task formation for non-holonomic agents with anisotropic sensing. More specifically, the proposed controller ensures that the desired configuration can be obtained with guaranteed collision avoidance and connectivity maintenance.

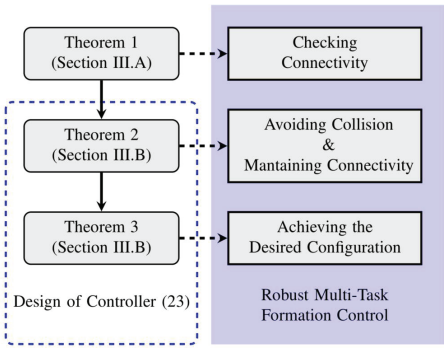

Fig. 1. Roadmap of robust multi-task formation controller design.

The rest of this paper is organized as follows: In Section II, we give the model description and formulate the problem of robust multi-task formation. Section III introduces the parametric Lyapunov-like barrier functions based on which a robust distributed controller is provided for multi-agent systems with parametric uncertainties. Section IV illustrates the effectiveness of the proposed method with two numerical examples. Section V summarizes the paper with some concluding remarks. A roadmap is given in Fig. 1 for a clear understanding of the structure of this paper.

II. Preliminaries

Notations: $\mathbb{N}, \mathbb{R}$ denote natural and real number sets, respectively; $\mathbb{R}^+$ denotes the positive real number set; A^T denotes the transpose of A; $A > 0$ ($A \geq 0$) denotes the symmetric positive definite (semidefinite) matrix A; $A \otimes B$ denotes the Kronecker product of matrices A and B; $\mathrm{diag}(a)$ denotes a square diagonal matrix with the elements of vector a on the main diagonal; $\|v\|$ denotes the Euclidean norm or l_2 norm of vector v; 1_n denotes a $n \times 1$ vector with all the entries equal to 1; I denotes an identity matrix (of size defined by the context); atan_2 denotes the arctangent function; $\deg(f)$ denotes the degree of polynomial function f; $\mathrm{Deg}(M)$ denotes the degree of matrix polynomial function M, i.e., $\max(\deg(M_{ij}))$; We denote $(*)^T AB$ in a form of Square Matrix Representation as $B^T AB$. Let $\mathcal{P}$ be the set of polynomials and $\mathcal{P}^{n\times m}$ be the set of matrix polynomials with dimension $n \times m$. A polynomial $p(x) \in \mathcal{P}$ is nonnegative if $p(x) \geq 0$ for all $x \in \mathbb{R}^n$. A useful way of establishing $p(x) \geq 0$ consists of checking whether $p(x)$ can be described as a sum of squares of polynomials (SOS), whose definition is given as follows.

Definition 1 (Sum of Squares of Polynomials): A polynomial $p(x)$ is a sum of squares of polynomials if $p(x) = \sum_{i=1}^{k} p_i(x)^2$ for some $p_1, \ \ldots, \ p_k \in \mathcal{P}$. □

The set of SOS polynomials is denoted by $\mathcal{P}_{\text{SOS}}$. A symmetric matrix polynomial $M(x) \in \mathbb{R}^{l\times l}$ is SOS if there exist matrix polynomials $M_1(x), M_2(x), \ldots$ such that $M(x) = \sum_i M_i(x)^T M_i(x)$. The set of SOS matrix polynomials is denoted by $\mathcal{P}_{\text{SOS}}^{l\times l}$.

A. Model Formulation

We consider that the motion of each agent is governed by a second-order non-holonomic model, given as follows:

$$\begin{aligned} \dot{x}_i(t) &= v_i(t)\cos(\theta_i(t)), \\ \dot{y}_i(t) &= v_i(t)\sin(\theta_i(t)), \\ \dot{\theta}_i(t) &= \omega_i(t), \\ \dot{v}_i(t) &= u_i(t),\ \forall i \in \mathcal{N}, \end{aligned} \tag{1}$$

in which i is the index of agent A_i, $\mathcal{N} = \{1, \ \ldots, \ N\}$, N is the number of agents, $p_i(t) = [x_i(t) \ \ y_i(t)]^T \in \mathbb{R}^2$ denotes the position vector, $v_i(t) \in \mathbb{R}$ is the translational speed, $\theta_i(t)$ is the angle of orientation with regard to a global Cartesian coordinate frame, and $u_i(t)$, $\omega_i(t) \in \mathbb{R}$ are the linear acceleration and the angular velocity, respectively, as *control inputs* on agent i. Denote $q_i(t) = [p_i(t)^T \ \ \theta_i(t)]^T$ the state vector of A_i, $p(t) = [p_1(t)^T \ \ \ldots \ \ p_N(t)^T]^T$ and $q(t) = [q_1(t)^T \ \ \ldots \ \ q_N(t)^T]^T$. Note that system (1) is a practical model widely adopted in autonomous driving and wheeled swarm robotics [4], and the model with holonomic kinematics will also be discussed in Section IV. Hereinafter, we will omit the arguments t, p and q of functions whenever possible for the brevity of notations.

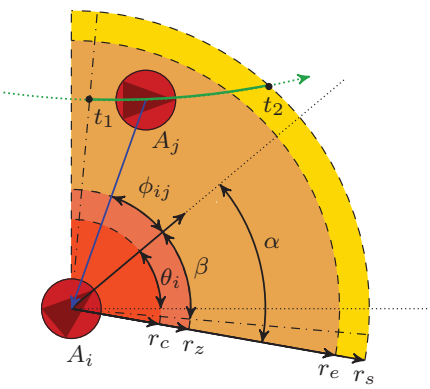

Fig. 2. The model of a single agent A_i when agent A_j enters in the sensing area of A_i: r_c is the radius of collision avoidance region; r_z is the radius of region that the control with collision avoidance objective is active; r_s is the radius of sensing range; θ_i is the orientation of A_i; $\phi_{ij} = \text{atan}_2(y_j - y_i, x_j - x_i) - \theta_i$ denotes the relative angle between A_j and A_i; 2α is the sensing angle of A_i; $r_e = r_s - \varepsilon_c$ and $\beta = \alpha - \varepsilon_a$ are used for the hysteresis in adding new edges; ε_c and ε_a are given constants. A_i detects A_j and receives the information from A_j, thus creating the edge (A_j, A_i) shown as a blue solid line. The green solid line for $t \in (t_1, t_2)$ depicts the part of trajectory when there is an edge (A_j, A_i) between the two agents.

In contrast to existing work with circular sensing and undirected networks [11], [26], [28], this paper studies a more general multi-agent model with anisotropic sensing in a directed network, which can be described by a *weighted directed dynamic graph* $\mathcal{G}(t) = (\mathcal{A}, \mathcal{E}(t), G)$ with the set of nodes $\mathcal{A} = \{A_1, \ \ldots, \ A_N\}$, the *weighted adjacency matrix* $G = (G_{ij})_{N\times N}$, and the set of directed edges $\mathcal{E}(t) = \{(A_i, A_j) |\ A_i, A_j \in \mathcal{A},\ i \neq j\}$ starting from A_i and ending at A_j, where A_i and A_j are called parent node and child node, respectively. For distinct agents A_{ik}, $k = 1, \ \ldots, \ l$, we call a sequence of edges (A_i, A_{i1}), (A_{i1}, A_{i2}), $\ldots$, (A_{il}, A_j) a *directed path* from A_i to A_j. If for some agent A_i, there is a directed path from A_i to any other agent, the agent A_i is denoted as a *root* of $\mathcal{G}$. A *directed tree* is a direct graph $\mathcal{G}$ in which exactly one root exists, and except the root, every node in $\mathcal{G}$ has exactly one parent. A *directed spanning tree* is a directed tree with $N-1$ edges which connect all the N agents.

We also assume that agent A_i has an on-board forward-looking sensor, which provides a limited angle and range of view. We realize this sensing area $\mathcal{S}_i$ by a circular sector of radius r_s and angle 2α (Fig. 2). In this paper, we assume all the agents have the same sensing ability. A *hysteresis* is introduced for creating new links in undirected networks with circular sensing [12], [24], [25], [28]. Based on a similar idea, we introduce constants ε_a and ε_c for the hysteresis of anisotropic sensing, which is shown in Fig. 2 and illustrated by the following rules:

1) Initial edges are provided by $\mathcal{E}(0) = \{(A_j, A_i) |\ \|x_i(0) - x_j(0)\| < r_e, \|\phi_{ij}\| < \beta,\ A_i, A_j \in \mathcal{A},\ i, j \in \mathcal{N}\}$.
2) If $((A_j, A_i) \notin \mathcal{E}(t^-)) \cap ((\|x_i(t) - x_j(t)\| \geq r_e) \cup (|\phi_{ij}(t)| \geq \beta))$, then, $(A_i, A_j) \notin \mathcal{E}(t)$.
3) If $((A_j, A_i) \notin \mathcal{E}(t^-)) \cap ((\|x_i(t) - x_j(t)\| < r_e) \cap (|\phi_{ij}(t)| < \beta))$, then, $(A_i, A_j) \in \mathcal{E}(t)$.
4) If $((A_j, A_i) \in \mathcal{E}(t^-)) \cap ((\|x_i(t) - x_j(t)\| \geq r_s) \cup (|\phi_{ij}(t)| \geq \alpha))$, then, $(A_i, A_j) \notin \mathcal{E}(t)$.
5) If $((A_j, A_i) \in \mathcal{E}(t^-)) \cap ((\|x_i(t) - x_j(t)\| < r_s) \cap (|\phi_{ij}(t)| < \beta))$, then, $(A_i, A_j) \in \mathcal{E}(t)$.

Note that rules 1)-5) characterize the switching policy of the edge from A_j to A_i. Specifically, a new edge (A_j, A_i) is added if rule 3) holds, while an existing edge (A_j, A_i) is removed if rule 4) holds.

A digraph $\mathcal{G}(t)$ is *quasi-strongly connected* at time t if for any pair of distinct nodes A_i and A_j, there exists an agent A_r such that there is a directed path from A_r to A_i and a directed path from A_r to A_j [34]. The *Laplacian matrix* is given as:

$$L(t) = \Delta(t) - G(t), \tag{2}$$

where $\Delta(t) = \text{diag}(\sum_{j=1}^{N} G_{ij}(t))$. A preliminary result reveals the relationship between the topology of networks and the quasi-strong connectivity of $\mathcal{G}(t)$ as follows:

Lemma 1 ([35]): A graph $\mathcal{G}(t)$ is quasi-strongly connected if and only if it contains a directed spanning tree at t.

Whether the network contains a spanning tree can be established by checking the eigenvalues of the Laplacian matrix, as shown by the following result.

Lemma 2 ([3]): The Laplacian matrix $L(t)$ has a simple eigenvalue zero, and all the other eigenvalues have positive real parts if and only if the directed network $\mathcal{G}(t)$ has a directed spanning tree at t.

Note that the above result is only valid for the cases where entries of the weighted adjacency matrix satisfy $G_{ij} \geq 0$. For uncertain networks with parametric uncertainties, this condition does not always hold. The next result provides a way to check the quasi-strong connectivity for a more general case.

Lemma 3: Define matrix $\tilde{G}(t) = (|G_{ij}(t)|)_{N\times N}$, i.e., each entry of $\tilde{G}(t)$ is the absolute value of the corresponding entry of $G(t)$. Define matrix

$$\tilde{L}(t) = \tilde{\Delta}(t) - \tilde{G}(t),$$

where $\tilde{\Delta}(t) = \mathrm{diag}(\sum_{j=1}^{N} \tilde{G}_{ij}(t))$. $\tilde{L}(t)$ has a simple eigenvalue zero, and all the other eigenvalues have positive real parts if and only if the directed network $\mathcal{G}(t)$ has a directed spanning tree at t.

Proof: For any $i, j \in \mathcal{N}$, if $G_{ij} \neq 0$, we have that $(A_j, A_i) \in \mathcal{E}$, i.e., there is a edge starting from G_j to G_i, and $\tilde{G}_{ij} > 0$. Now considering the Laplacian matrix $\tilde{L}$, from Lemma 2, we have that the matrix $\tilde{L}$ has a simple eigenvalue zero, and all the other eigenvalues have positive real parts if and only if the directed network $\mathcal{G}(t)$ has a directed spanning tree at t, which completes this proof. □

B. Problem Formulation

We consider a class of parametric uncertainties that affect communications and measurements in the multi-agent network. To model communication uncertainty, we assume that the weighted adjacency matrix is for the form $G(t, \delta)$, where $\delta \in \mathbb{R}^r$ is an uncertain vector constrained as:

$$\delta \in \Omega,\ \Omega = \{\delta \in \mathbb{R}^r :\ s_i(\delta) \geq 0,\ \forall i = 1,\ \ldots,\ h\}, \quad (3)$$

for some functions $s_1,\ \ldots,\ s_h : \mathbb{R}^r \to \mathbb{R}$. Hereinafter, we will assume that the entries of $G(t, \delta)$ and $s_1(\delta),\ \ldots,\ s_h(\delta)$ are polynomials.

Remark 1: New challenges brought by parametric uncertainties in multi-agent systems are beginning to be studied recently, and some pioneering explorations have already been done in designing robust controllers for various parametric uncertainties in multi-agent systems [15], [16], [36]–[39]. The main motivation to use parametric uncertainties is that a more practical and accurate model can be built with considering the modeling uncertainties and the disturbance in control gains.□

Remark 2: The type of uncertain model introduced above is more general compared to the additive norm-bounded uncertainties in [14]–[16]. Besides the motivating example given in Section I, more practical implementations in intelligent transportation and underwater vehicles control can be found in [8]. □

Distributed controller protocols are in principle designed based on the relative information among agents, i.e., $p_i - p_j$, $\theta_i - \theta_j$, and $v_i - v_j$. However, in practice, the exact relative information is not easy to obtain, while uncertainty in measurement is present in numerous implementations [13]. We assume that the on-board sensor of each agent A_i is affected by a multiplicative function $\zeta_i(\sigma)$, which is dependent on the uncertain parameter σ with

$$\begin{aligned}\sigma \in \Upsilon,\ \Upsilon = \Big\{\sigma \in \mathbb{R}^m :\ \zeta_i(\sigma) \geq 0,\ \rho_j(\sigma) \geq 0,\\ \forall j = 1,\ \ldots,\ h_\sigma\Big\},\end{aligned} \quad (4)$$

for some functions $\rho_1,\ \ldots,\ \rho_{h_\sigma} : \mathbb{R}^m \to \mathbb{R}$. To this end, we consider $\zeta_1,\ \ldots,\ \zeta_N$ and $\rho_1,\ \ldots,\ \rho_{h_\sigma}$ as polynomial functions.

Remark 3: For the measurement uncertainty introduced above, it is worth noting that multiplicative uncertainty in measurement is considered, which means that agent A_i can only receive relative information $\zeta_i(\sigma)(p_i - p_j)$, $\zeta_i(\sigma)(v_i - v_j)$ and $\zeta_i(\sigma)(\theta_i - \theta_j)$ from agent A_j in the sensing area of A_i. For a more practical setup, we assume that ζ_i is bounded as $\zeta_i \leq \zeta_i^{\max}$, as the limited amplification or the scaling down of relative information in measurements. Additive uncertainty in measurement is not considered in that it may lead to a *bounded formation control problem* which is out of the scope of this paper. □

The distributed controller of agent i relies on the uncertain local information of agent i, i.e., is of the general form:

$$\begin{aligned} u_i &= \sum_{j\in\mathcal{N}_i^{\mathrm{s}}(t)} f_u\Big(\zeta_i(\sigma)p_{ij},\ \zeta_i(\sigma)v_{ij},\ \zeta_i(\sigma)\theta_{ij},\ G_{ij}(t,\delta)\Big),\\ \omega_i &= \sum_{j\in\mathcal{N}_i^{\mathrm{s}}(t)} f_\omega\Big(\zeta_i(\sigma)p_{ij},\ \zeta_i(\sigma)v_{ij},\ \zeta_i(\sigma)\theta_{ij},\ G_{ij}(t,\delta)\Big).\end{aligned} \quad (5)$$

where $p_{ij} = p_i - p_j$, $v_{ij} = v_i - v_j$, $\theta_{ij} = \theta_i - \theta_j$, and the neighborhood set of agent i is denoted by $\mathcal{N}_i^{\mathrm{s}}(t) = \{j|\ (A_j, A_i) \in \mathcal{E}(t)\}$, which is obtained based on the rules we introduced in the last subsection. We define the following sets: $\mathcal{N}_i^{\mathrm{f}}$ is the neighborhood set to agent i in the desired formation configuration, i.e., $\mathcal{N}_i^{\mathrm{f}} = \{j|\ (A_j, A_i) \in \mathcal{E}^{\mathrm{f}},\ p_i - \tau_i - (p_j - \tau_j) = 0, v_i = v_j, \theta_i = \theta_j\}$, where τ_i is the ideal displacement of agent i in the desired formation configuration, whose edge set is $\mathcal{E}^{\mathrm{f}}$. We also define sets $\mathcal{N}_i^{\mathrm{sf}}(t) = \{j|\ j \in \mathcal{N}_i^{\mathrm{s}}(t),\ j \in \mathcal{N}_i^{\mathrm{f}}\}$ and $\mathcal{N}_i^{\mathrm{sz}}(t) = \{j|\ j \in \mathcal{N}_i^{\mathrm{s}}(t),\ \|p_i - p_j\| < r_z\}$, which will be used in the following sections.

Now, let us propose the *robust multi-task formation control problem* in the presence of both communication and measurement uncertainties:

Problem 1 (Robust Multi-Task Formation Control Problem): Under the dynamics (1) and the effects of δ and σ given in (3) and (4), respectively, find controllers u_i and ω_i in the form of (5) such that

1) $\lim_{t\to\infty}\|(p_i(t)-\tau_i)-(p_j(t)-\tau_j)\| = 0$, $\lim_{t\to\infty}|\theta_i(t)-\theta_j(t)| = 0$, and $\lim_{t\to\infty}|v_i(t) - v_j(t)| = 0$, for all $i = 1,\ \ldots,\ N$, $j \in \mathcal{N}_i^{\mathrm{f}}$.
2) $\mathcal{G}(t, \delta, \sigma)$ is quasi-strongly connected, for all $t > t_0$, $\delta \in \Omega$, and $\sigma \in \Upsilon$.
3) $\|p_i(t) - p_j(t)\| > d_s$, for all $t > t_0$, $\delta \in \Omega$, and $\sigma \in \Upsilon$ where $d_s \geq 2r_c$ denotes a user-defined safety distance defined in Section II. □

It is worth noting that under parametric uncertainties, item 1) ensures the convergence to the desired configuration; item 2) ensures the connectivity maintenance; item 3) ensures the collision avoidance. For this problem, we assume that:

- Assumption 1: The desired configuration given by τ_i is achievable, i.e., $r_z \leq \|\tau_i - \tau_j\| \leq r_s - \varepsilon_c$, and the desired relative angle satisfies $|\phi_{ij}^{\mathrm{d}}| < \alpha - \varepsilon_a$ for all $i \in \mathcal{N},\ j \in \mathcal{N}_i^{\mathrm{f}}$.
- Assumption 2: The neighbor set of agent i at time t_0 satisfies $\mathcal{N}_i^{\mathrm{f}} \subseteq \mathcal{N}_i^{\mathrm{s}}(t_0)$, which means that the desired edge set $\mathcal{E}_{\mathrm{f}}$ is contained in the initial edge set $\mathcal{E}(t_0)$.
- Assumption 3: To achieve both objectives of collision avoidance and connectivity maintenance, we require $r_s - \|\tau_{ij}\| > d_s + \|\tau_{ij}\|$, for all $i, j \in \mathcal{N}$.
- Assumption 4: The desired displacement τ_i and the desired neighborhood $\mathcal{N}_i^{\mathrm{f}}$ are assumed to be known for agent i.

III. Main Result

In this section, a condition for checking the quasi-strong connectivity of uncertain networks is provided, and a distributed controller is designed to solve Problem 1. The main idea is to preserve the quasi-strong connectivity of the initial uncertain graph, and then use a gradient-based controller based on properly constructing parametric Lyapunov-like barrier functions.

A. Checking the Connectivity of Uncertain Networks

The quasi-strong connectivity of uncertainty-free network can be established by checking the eigenvalues of $L(t)$ from Lemma 2. However, it is not easy to calculate $\lambda_i(L(t^*, \delta, \sigma))$ at a chosen time t^* under the effect of the disturbance δ and σ given in (3) and (4). In this subsection, we propose a method of Linear Matrix Inequalities (LMIs) to check the connectivity of an uncertain graph $\mathcal{G}(t^*, \delta, \sigma)$ at a fixed time t^*. We omit the argument t for the brevity of expressions.

First, we propose a definition of connectivity for networks under communication uncertainty:

Definition 2 (Robust Quasi-Strongly Connected): An uncertain digraph $\mathcal{G}(t, \delta)$ is called *robust quasi-strongly connected* at time t if for any pair of distinct nodes A_i and A_j, there exists an agent A_r such that there is a directed path from A_r to A_i and a directed path from A_r to A_j, for all δ given in (3). □

Remark 4: Note that quasi-strong connectivity does not imply robust quasi-strong connectivity, since the weighted adjacency matrix G is dependent on δ and the existing edges and paths can be removed under the effect of δ. □

For the sake of clarity, a simple example is provided to illustrate this definition:

Example 1: Consider a 3-agent system, A_2 is in the sensing area of A_1, and A_3 is in the sensing area of A_2, thus the existing edges are (A_2, A_1) and (A_3, A_2). Assume this network is affected by communication uncertainty $\delta = [\delta_1\ \delta_2]$ with $\Omega = \{\delta \in \mathbb{R}^2 :\ \delta_1^2 + \delta_2^2 \leq 4\}$. Let the entries of weighted adjacency matrix $G(\delta)$ be $G_{12} = 1 - \delta_1$ and $G_{23} = \delta_1^2 \delta_2$. It is obvious that A_1 cannot receive information from A_2 if $\delta_1 = 1$. Therefore, the network is not robust strongly connected because of the existence of δ. □

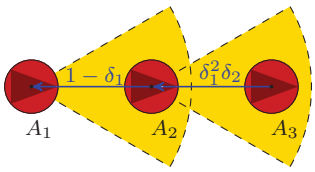

Fig. 3. Example 1: An illustration for robust quasi-strong connectivity.

Then, we propose the definition of connectivity for networks under communication and measurement uncertainty:

Definition 3 (Robust-2 Quasi-Strongly Connected): An uncertain digraph $\mathcal{G}(t, \delta, \sigma)$ is called *robust-2 quasi-strongly connected* at time t if for any pair of distinct nodes A_i and A_j, there exists an agent A_r such that there is a directed path from A_r to A_i and a directed path from A_r to A_j for all δ and σ given in (3) and (4), respectively. □

Remark 5: It is worth noting that robust quasi-strong connectivity does not imply robust-2 quasi-strong connectivity given above, due to the fact that edges of $\mathcal{G}$ are not merely dependent on δ but also on σ. □

To investigate robust quasi-strong connectivity, we introduce two definitions based on the uncertain sensing of agents:

Definition 4: Matrix $\tilde{G}(t, \delta, \sigma) = (\tilde{G}_{ij}(t, \delta, \sigma))_{N \times N}$ is called *uncertain measurement weighted adjacency matrix* with $\tilde{G}_{ij}(t, \delta, \sigma) = \zeta_i(\sigma) G_{ij}(t, \delta)$. Based on $\tilde{G}(t, \delta, \sigma)$, the *uncertain measurement Laplacian matrix* $\tilde{L}(t, \delta, \sigma)$ can be obtained by using (2). □

For the sake of clarity, a simple example is given for illustration:

Example 2: Let us extend Example 1 into the case with measurement uncertainty $\sigma = \sigma_1$ constrained in $\Upsilon = \{\sigma \in \mathbb{R} :\ \zeta_1(\sigma) \geq 0,\ \zeta_2(\sigma) \geq 0,\ \rho_1(\sigma) \geq 0\}$, where $\zeta_1 = 1 - \sigma^2$, $\zeta_2 = \sigma - 0.5$, and $\rho_1 = 2 + \sigma - \sigma^2$. Given the entries of weighted adjacency matrix $G(\delta)$ as $G_{12} = 1 - \delta_1$ and $G_{23} = \delta_1^2 \delta_2$, it is not difficult to see that agent A_1 cannot receive information from A_2 if $\sigma = 1$ and $\zeta_1 = 0$. Therefore, the network is not robust-2 quasi-strongly connected even though it is robust quasi-strongly connected. Note that measurement uncertainties not merely affect the connectivity of networks, but also the design of the barrier functions. □

In the following result, a necessary and sufficient condition is given for checking the robust-2 quasi-strong connectivity of uncertain networks:

Lemma 4: Consider a matrix $M \in \mathbb{R}^{N \times N-1}$ satisfying $\mathrm{img}(M) = \ker(1_N^T)$, then, a new matrix is constructed based on the uncertain Laplacian matrix as:

$$\check{L}(\delta, \sigma) = M^T \tilde{L}(\delta, \sigma) M. \tag{6}$$

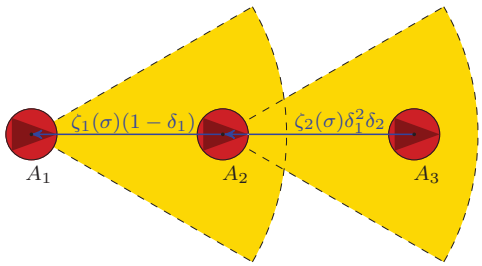

Fig. 4. Example 2: An illustration for robust-2 quasi-strong connectivity.

The graph $\mathcal{G}(\delta,\sigma)$ is robust-2 quasi-strongly connected under the effect of δ and σ if and only if there exists a symmetric definite matrix function $P(\delta,\sigma): \mathbb{R}^r \to \mathbb{R}^{N-1\times N-1}$ such that

$$P(\delta,\sigma)\check{L}(\delta,\sigma)+\check{L}(\delta,\sigma)^T P(\delta,\sigma)>0,\ \forall\delta\in\Omega,\sigma\in\Upsilon. \quad (7)$$

Proof: From Lemma 1, Lemma 2, and (2), we have that 1_N is a right eigenvector of $\tilde{L}(\delta,\sigma)$ corresponding to the eigenvalue zero. In addition, observe that $M^T\tilde{L}(\delta,\sigma)M$ has the same eigenvalues of $\tilde{L}(\delta,\sigma)$ except that the algebraic multiplicity of the eigenvalue zero has been decreased of one. i.e. $\mathrm{spc}(\check{L}(\delta,\sigma))\cup\{0\}=\mathrm{spc}(\tilde{L}(\delta,\sigma))$, where $\mathrm{spc}(A)$ is the spectrum of matrix A. Consider the following dynamical system

$$\dot{\hat{x}}(t)=-\check{L}(\delta,\sigma)\hat{x}(t), \quad (8)$$

we have that the linear hull of vector 1_{N-1}, i.e., $\mathrm{span}(1_{N-1})$ is the equilibrium point of (8). It yields that the condition that all the eigenvalues of $\check{L}(\delta,\sigma)$ have positive real parts is equivalent to the statement that (8) is asymptotically stable. Let us consider a parameter-dependent Lyapunov function $V(\hat{x},\delta,\sigma)=\hat{x}^T P(\delta,\sigma)\hat{x}$ with $P(\delta,\sigma)>0$. From Lyapunov stability theorem, we have that (8) is asymptotically stable for all $\delta\in\Omega$ if and only if $V(\hat{x},\delta)>0$ and $\dot{V}(\hat{x},\delta)<0$. Moreover, observe $\dot{V}(\hat{x},\delta,\sigma)=-\hat{x}^T(P(\delta,\sigma)\check{L}(\delta,\sigma)+\check{L}(\delta,\sigma)^T P(\delta,\sigma))\hat{x}<0$ is equivalent to the condition of (7), it yields that (7) holds if and only if $L(\delta,\sigma)$ has exactly one simple eigenvalue 0 and all the other eigenvalues have positive real parts, i.e., $\mathcal{G}(\delta,\sigma)$ is robust quasi-strongly connected for all $\delta\in\Omega$ and for all $\sigma\in\Upsilon$, which completes this proof. □

Note that the matrix polynomial inequalities (7) is not easy to be established under the constraints (3) and (4). To solve this problem, a tractable condition is provided resort to Real Positivestellensatz [40] as follows:

Lemma 5: Condition (7) holds if there exist matrix polynomials $P(\delta,\sigma)$, $R_i(\delta,\sigma)$, for $i=1,\ \ldots,\ h$, $Q_j(\delta,\sigma)$ for $j=1,\ \ldots,\ h_\sigma$, $F_k(\delta,\sigma)$ for $k=1,\ \ldots,\ N$ and positive scalars $c_1>0$ and $c_2>0$ such that

$$\begin{cases} R_i(\delta,\sigma)\in\mathcal{P}_{\mathrm{SOS}}^{N-1\times N-1} \\ Q_j(\delta,\sigma)\in\mathcal{P}_{\mathrm{SOS}}^{N-1\times N-1} \\ F_k(\delta,\sigma)\in\mathcal{P}_{\mathrm{SOS}}^{N-1\times N-1} \\ P(\delta,\sigma)-c_1 I_{N-1}\ \in\mathcal{P}_{\mathrm{SOS}}^{N-1\times N-1} \\ H(\delta,\sigma)-c_2 I_{N-1}\ \in\mathcal{P}_{\mathrm{SOS}}^{N-1\times N-1} \end{cases} \quad (9)$$

where $\mathcal{P}_{\mathrm{SOS}}^{N-1\times N-1}$ is the set of SOS matrix polynomials introduced in Section II,

$$\begin{aligned} H(\delta,\sigma)=&P(\delta,\sigma)\check{L}(\delta,\sigma)+\check{L}(\delta,\sigma)^T P(\delta,\sigma)\\ &-\sum_{j=1}^{h_\sigma}Q_j(\delta,\sigma)\rho_j(\delta)-\sum_{k=1}^{N}F_k(\delta,\sigma)\zeta_k(\delta)\\ &-\sum_{i=1}^{h}R_i(\delta,\sigma)s_i(\delta), \end{aligned} \quad (10)$$

in which δ, s_i are defined in (3), and σ, ζ_k, ρ_j are given in (4).

Proof: Given that (9) holds, we have $P(\delta,\sigma)>0$ and $H(\delta,\sigma)>0$. In addition, $R_i(\delta,\sigma)$, $Q_j(\delta,\sigma)$, $F_k(\delta,\sigma)$ are SOS matrix polynomials for $i=1,\ \ldots,\ h$, $j=1,\ \ldots,\ h_\sigma$, $k=1,\ \ldots,\ N$, and $s_i(\delta,\sigma)>0$ from (3), $\rho_j(\delta,\sigma)>0$, $\zeta_k(\delta,\sigma)$ from (4). Based on the Real Positivestellensatz, it yields that $P(\delta,\sigma)\check{L}(\delta,\sigma)+\check{L}(\delta,\sigma)^T P(\delta,\sigma)>0$, which completes this proof. □

Note that (9) requires to find appropriate c_1, c_2, $P(\delta)$ and $H(\delta)$ at the same time which is difficult and computationally demanding. Next, we propose an efficient method by using the Square Matrix Representation (SMR). Given a polynomial $f_0(\delta)$ of degree $\deg(f_0)$, $\delta\in\mathbb{R}^r$ and $f_0(\delta)\in\mathcal{P}_{\mathrm{SOS}}$, its SMR is as follows:

$$f_0(\delta)=(*)^T(\bar{F}_0+C(\xi))\phi(r,d_{f_0}), \quad (11)$$

where $(*)^T AB$ is short for $B^T AB$ given in Section II, $\bar{F}_0$ is denoted by the SMR matrix of $f_0(\delta)$, r is the dimension of δ, d_{f_0} is the smallest integer not less than $\frac{\deg(f_0)}{2}$, i.e., $d_{f_0}=\lceil\frac{\deg(f_0)}{2}\rceil$, $\phi(r,d_{f_0})\in\mathbb{R}^{l(r,d_{f_0})}$ denotes a power vector which contains all monomials of degree less or equal to d_{p_0}, $C(\xi)$ is a parameterization of the space $\mathscr{C}=\{C(\xi)\in\mathbb{R}^{l(r,d_{f_0})\times l(r,d_{f_0})}: C(\xi)=C^T(\xi),\ (*)^T C(\xi)\phi(r,d_{f_0})=0\}$, in which $\xi\in\mathbb{R}^{\vartheta(r,d_{f_0})}$ is a vector of free parameters. The values of $l(r,d_{f_0})$ and $\vartheta(r,d_{f_0})$ can be obtained similarly as in [40]. For the ease of understanding, an illustration is given:

Example 3: Given the polynomial $f_1(\delta)=7\delta^4+2\delta^3+4\delta^2+6\delta+9$, we have $d_{f_1}=2$, $r=1$ and $\phi(r,d_{f_1})=(\delta^2,\delta,1)^T$. Then, $f_1(x)$ can be written as follows:

$$\bar{F}_1=\begin{pmatrix}7&1&0\\1&4&3\\0&3&9\end{pmatrix},\quad C_1(\xi)=\begin{pmatrix}0&0&-\xi\\0&2\xi&0\\-\xi&0&0\end{pmatrix}.\ \square$$

This technique can be extended to matrix polynomials. Specifically, let $M(\delta)\in\mathcal{P}_{\mathrm{SOS}}^{s\times s}$ be a symmetric matrix polynomial of size $s\times s$ of degree $\mathrm{Deg}(M)$ in $\delta\in\mathbb{R}^r$ (this means that the highest degree of all the entries of $M(\delta)$ is $\mathrm{Deg}(M)$ in δ), i.e., $\mathrm{Deg}(M)=\max(\deg(M_{ij}))$, $d_M=\lceil\frac{\mathrm{Deg}(M)}{2}\rceil$. Then, $M(\delta)$ can be written as

$$M(\delta)=\Phi(\bar{M}+D(\xi),d_M,s), \quad (12)$$

where Φ is the compact form of square matrix representation, i.e., $\Phi(\bar{M}+D(\xi),m,s)=(*)^T(\bar{M}+D(\xi))(\phi(r,d_M)\otimes I_s)$, $\bar{M}$ is a symmetric matrix, and $D(\xi)$ is a linear parametrization of the linear subspace $\mathscr{D}=\{D(\xi)\in\mathbb{R}^{l(r,d_M)s\times l(r,d_M)s}: D(\xi)=D^T(\xi),\ (*)^T D(\xi)(\phi(r,d_M)\otimes I_s)=0\}$. Now, we

can propose the condition of LMIs for checking the robust-2 quasi-strong connectivity of networks under communication uncertainty:

Theorem 1: Condition (7) holds if $c^* > 0$, where c^* is the solution of the convex optimization problem

$$c^* = \sup_{c,\bar{R}_i,\bar{Q}_j,\bar{F}_k,\bar{P},\xi} c$$
$$\text{s.t.} \begin{cases} \bar{R}_i \geq 0,\ \bar{Q}_j \geq 0,\ \bar{F}_k \geq 0 \\ \bar{P} \geq 0,\ \text{trace}(\bar{P}) = 1 \\ \bar{H} + D(\xi) - cI_{(N-1)\cdot l(r+m,d_H)} - \sum_{i=1}^{h} \bar{\Theta}_i(\bar{R}_i) \\ -\sum_{j=1}^{h_\sigma} \bar{Z}_j(\bar{Q}_j) - \sum_{k=1}^{N} \bar{\Gamma}_k(\bar{F}_k) \geq 0 \end{cases} \tag{13}$$

for $i = 1, \ \ldots, \ h$, $j = 1, \ \ldots, \ h_\sigma$, and $k = 1, \ \ldots, \ N$. The matrices involved in this problem are defined by

$$\begin{aligned} R_i(\delta,\sigma) &= \Phi(\bar{R}_i, d_{Ri}, N-1), \\ Q_j(\delta,\sigma) &= \Phi(\bar{Q}_j, d_{Qj}, N-1), \\ F_k(\delta,\sigma) &= \Phi(\bar{F}_k, d_{Fk}, N-1), \\ R_i(\delta,\sigma)s_i(\delta) &= \Phi(\bar{\Theta}_i(\bar{R}_i), d_{Rs}, N-1), \\ Q_j(\delta,\sigma)\rho_j(\sigma) &= \Phi(\bar{Z}_j(\bar{Q}_j), d_{Q\rho}, N-1), \\ F_k(\delta,\sigma)\zeta_k(\sigma) &= \Phi(\bar{\Gamma}_i(\bar{F}_k), d_{F\zeta}, N-1), \\ P(\delta,\sigma) &= \Phi(\bar{P}, d_P, N-1), \\ H(\delta,\sigma) &= \Phi(\bar{H} + D(\xi), d_H, N-1), \end{aligned}$$

in which $2d_{Ri}$, $2d_{Qj}$, $2d_{Fk}$, $2d_P$ and $2d_H$ are the degrees of $R_i(\delta,\sigma)$, $Q_j(\delta,\sigma)$, $F_k(\delta,\sigma)$, $P(\delta,\sigma)$, and $H(\delta,\sigma) - cI$, respectively.

Proof: Given that (13) holds, by pre- and post-multiplying the first inequality in (13) with $(\phi(r,d_{Ri}) \otimes I_{N-1})^T$ and $(\phi(r,d_{Ri}) \otimes I_{N-1})$, respectively, we have that

$$R_i(\delta) \geq 0, \forall \delta \in \Omega.$$

Thus, the first constraint in (9) holds by choosing a power vector $\phi(r,d_{Ri}) \otimes I_{N-1}$. Similarly, by pre- and post-multiplying the last inequality in (13) by $(\phi(r,d_H) \otimes I_{N-1})^T$ and $(\phi(r,d_H) \otimes I_{N-1})$, respectively, we have that

$$\begin{aligned} 0 &\leq H(\delta) - c(\phi(r,d_H) \otimes I_{N-1})^T(\phi(r,d_H) \otimes I_{N-1}), \\ &\leq H(\delta) - c((\phi(r,d_H)^T \cdot \phi(r,d_H)) \otimes I_{N-1}), \\ &\leq H(\delta) - (c \cdot \phi(r,d_H)^T \cdot \phi(r,d_H))I_{N-1}, \\ &\leq H(\delta) - \bar{c}_2 I_{N-1}, \end{aligned}$$

where $\bar{c}_2 = (c \cdot \phi(r,d_H)^T \cdot \phi(r,d_H))$. It directly yields that the last constraint in (9) is satisfied. Moreover, let us assume the second and the third constraints in (13) hold. One has that for all the eigenvalues of $\bar{P}$, $\lambda_k(\bar{P}) \geq 0$ and $\sum_{k=1}^{(N-1)\cdot l(r,d_P)} \lambda_k(\bar{P}) = 1$, which results in $\exists k \in \{1, \ 2, \ \ldots, \ (N-1)\cdot l(r,d_P)\}$ such that $\lambda_k(\bar{P}) > 0$. Then, pre- and post-multiplying the second inequality in (13) by $(\phi(r,d_{Ri}) \otimes I_{N-1})^T$ and $(\phi(r,d_{Ri}) \otimes I_{N-1})$, respectively, we have that $P(\delta) > 0$, which means there exists a positive constant $\hat{c} > 0$ and $\bar{c}_1 = (\hat{c} \cdot \phi(r,d_P)^T \cdot \phi(r,d_P))$ such that $P(\delta) - \bar{c}_1 I_{N-1} \geq 0$. Therefore, the condition (9) holds. From Lemma 5, the condition (7) also holds, which completes this proof. □

Remark 6: For this result, it is worth noting that

- Theorem 1 is essential in that an LMI condition is provided for checking the robust-2 quasi-strong connectivity of networks with both communication and measurement uncertainties, rather than calculating the eigenvalues of Laplacian matrix. It thus paves a way to design a distributed controller for maintaining the robust-2 quasi-strong connectivity in the next subsection.
- The quasi-strong connectivity does not imply the robust-2 quasi-strong connectivity. Unlike the quasi-strong connectivity which can be checked by some global information in topological condition, Theorem 1 provides a solvable method to check the robust-2 quasi-strong connectivity by also using the global information.
- Though Theorem 1 provides a solvable method, it is observed that only a sufficient condition is given. The conservatism originates from the fact that Real Positivestellensatz requires that the auxiliary polynomial matrix $R(\delta)$ in Lemma 5 has limited degree on δ. The necessity can be obtained if $R(\delta)$ has unbounded degree and there is a polynomial p such that $p^{-1}[0,+\infty)$ is compact in $\mathbb{R}^r$. For details, please see [41] and references therein. □

B. Controller Design based on Parametric Lyapunov-like Barrier Functions

Lyapunov-like barrier functions provide an efficient way to treat collision avoidance and connectivity maintenance [6], [22], [42]. In this subsection, we propose a new Lyapunov-like barrier function that is applicable to multi-agent systems under anisotropic sensing for each agent. For the brevity of expressions, let $v = (v_1, \ \ldots, \ v_N)^T$, $\theta = (\theta_1, \ \ldots, \ \theta_N)^T$, $\tilde{p}_i = p_i - \tau_i$, $\tilde{p} = (\tilde{p}_1^T, \ \ldots, \ \tilde{p}_N^T)^T$, $\tilde{p}_{ij} = \tilde{p}_i - \tilde{p}_j$, $p_{ij} = p_i - p_j$, $\tau_{ij} = \tau_i - \tau_j$, $\tilde{\phi}_{ij} = \phi_{ij} - \phi_{ij}^{\mathrm{d}}$ and $v_{ij} = v_i - v_j$.

The nominal designs based on Lyapunov-like barrier functions are not directly applicable to multi-agent systems under measurement uncertainty, since then collision avoidance and connectivity maintenance cannot be ensured based on uncertain relative information. To cope with this problem, motivated by Gahinet who introduced parameter-dependent Lyapunov functions for parameter-dependent systems [32], in this paper we develop a new class of Lyapunov-like barrier functions to incorporate measurement uncertainty in the control design.

First, let us introduce the definition of parametric Lyapunov-like barrier functions:

Definition 5: Consider the autonomous system $\dot{x} = f(x,\sigma)$, f is a locally Lipschitz map $\mathbb{R}^n \to \mathbb{R}^n$, $x \in \mathbb{R}^n$ denotes the states and $\sigma \in \mathcal{S} \in \mathbb{R}^m$ is a parameter vector constrained in set $\mathcal{S}$, and the origin 0_n is an equilibrium point of the system. $\mathcal{U} \in \mathbb{R}^n$ is an unsafe or undesired set. Let $V(x,\sigma) : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ be a continuously differentiable function on x such that: 1) $V(0,\sigma) = 0$ and $V(x,\sigma) > 0$ in $\mathbb{R}^n/\{0_n\}$ for all $\sigma \in \mathcal{S}$; 2) $\dot{V}(x,\sigma) \leq 0$ in $\mathbb{R}^n$ for all $\sigma \in \mathcal{S}$; 3) $V(x,\sigma) = \infty$, for all $x \in \mathcal{U}$ and $\sigma \in \mathcal{S}$. Then, $\bar{x} = 0_n$ is stable, and $V(x,\sigma)$ is called a *parametric Lyapunov barrier function*. In addition, if 3) holds and V satisfies the condition of LaSalle's theorem

for all $\sigma \in \mathcal{S}$, then $V(x,\sigma)$ is called a *parametric Lyapunov-like barrier function*.

As we explained in Section III, the input of a gradient-based controller may be unbounded if condition 3) holds in Definition 5. For a more practical manner, we modify condition 3) in Definition 5 as $V(x,\sigma) > \mu_{\rm v}$, for all $x \in \mathcal{U}$ and $\sigma \in \mathcal{S}$, where $\mu_{\rm v} < \infty$ is a pre-selected positive scalar.

For connectivity maintenance, the main idea is to preserve the desired topology $\mathcal{E}^{\rm f} \subseteq \mathcal{E}(t)$ such that the network is always robust-2 quasi-strongly connected for $t \geq t_0$, $\delta \in \Omega$ and $\sigma \in \Upsilon$. To do this, we would like to make the following conditions satisfied: $\|p_{ij}\| < r_s$, and $|\phi_{ij}| < \alpha$ for all $i \in \mathcal{N}$ and $j \in \mathcal{N}^{\rm sf}(t)$. Since A_i can only obtain the disturbed information $\zeta_i(\sigma)p_{ij}$, $\zeta_i(\sigma)v_{ij}$ and $\zeta_i(\sigma)\theta_{ij}$, different with the barrier function introduced in Section III, a parametric barrier function is proposed by solving a Sum-of-Squares feasibility test. Specifically, the following barrier function is considered:

$$\Psi^{\rm e}_{ij} = \psi^{\rm e}_{ij}(\|\hat{p}_{ij}\|) + \frac{|\tilde{\phi}_{ij}|^2}{\tilde{\alpha} - |\tilde{\phi}_{ij}| + \frac{2\tilde{\alpha}^2}{\mu_1}}, \tag{14}$$

for all $i \in \mathcal{N}$, $j \in \mathcal{N}^{\rm sf}_i(t)$, where $\hat{p}_{ij} = \zeta_i(\sigma)(p_i - p_j) - \zeta_i(\sigma)(\tau_i - \tau_j)$, $\tilde{\alpha} = \alpha - |\phi^{\rm d}_{ij}|$, recall $\mathcal{N}^{\rm sf}_i(t) = \{j|\ j \in \mathcal{N}^{\rm s}_i(t),\ j \in \mathcal{N}^{\rm f}_i\}$ defined in Section II, μ_1 is a positive scalar. Recall that $\phi_{ij} = {\rm atan}_2(y_j - y_i, x_j - x_i) - \theta_i = {\rm atan}_2(\zeta_i(\sigma)(y_j - y_i), \zeta_i(\sigma)(x_j - x_i)) - \theta_i$, thus the second item in (14) directly exploits a fixed and parameter-independent barrier function as the one given [31]. For the first terms,

$$\psi^{\rm e}_{ij}(\|\hat{p}_{ij}\|) = \frac{g^{\rm e}(\|\hat{p}_{ij}\|)}{h^{\rm e}(\|\hat{p}_{ij}\|)}, \tag{15}$$

both $g^{\rm e}(\|\hat{p}_{ij}\|)$ and $h^{\rm e}(\|\hat{p}_{ij}\|)$ are polynomial functions satisfying:

$$\begin{cases} g^{\rm e}_{ij}(\|\hat{p}_{ij}\|) \geq 0,\ g^{\rm e}_{ij}(0) = 0,\ h^{\rm e}_{ij}(\|\hat{p}_{ij}\|) > 0, \\ \underline{\mu}_1 \leq \dfrac{g^{\rm e}(\|\zeta_i(\sigma)\cdot\tilde{r}_s\|)}{h^{\rm e}(\|\zeta_i(\sigma)\cdot\tilde{r}_s\|)} \leq \dfrac{\mu_1}{2},\ \forall \sigma \in \Upsilon, \\ \dfrac{\partial \psi^{\rm e}_{ij}(\|\hat{p}_{ij}\|)}{\partial(\|\hat{p}_{ij}\|)} > 0,\ \forall 0 \leq \|\hat{p}_{ij}\| \leq \hat{r}_s, \\ \dfrac{\partial \psi^{\rm e}_{ij}(\|\hat{p}_{ij}\|)}{\partial(\|\hat{p}_{ij}\|)} \cdot \dfrac{1}{\|\hat{p}_{ij}\|} > 0,\ \forall j \in \mathcal{N}^{\rm sf}_i(t), \end{cases} \tag{16}$$

where $\tilde{r}_s = r_s - \|\tau_{ij}\|$, the values of $\underline{\mu}_1$ and μ_1 are pre-selected which will be discussed later. The following result provides a systematic way to construct a parameter-dependent barrier function by convex optimization:

Lemma 6: The condition (16) holds if there exist SOS polynomials $s_i(\sigma)$ and $p_i(\sigma)$, $i = 0, 1, \ldots, h_\sigma$, and positive scalars c_1, c_2, and c_3, such that the following SOS feasibility test problem is solved:

$$\begin{cases} g^{\rm e}_{ij}(\|\hat{p}_{ij}\|) \in \mathcal{P}_{\rm SOS},\ g^{\rm e}_{ij}(0) = 0,\ h^{\rm e}_{ij}(\|\hat{p}_{ij}\|) - c_1 \in \mathcal{P}_{\rm SOS}, \\ K_1(\tilde{r}_s, \zeta_i(\sigma)) - s_0\zeta_i - \displaystyle\sum_{j=1}^{h_\sigma} s_j\rho_j \in \mathcal{P}_{\rm SOS}, \\ K_2(\tilde{r}_s, \zeta_i(\sigma)) - p_0\zeta_i - \displaystyle\sum_{j=1}^{h_\sigma} p_j\rho_j \in \mathcal{P}_{\rm SOS}, \\ \dfrac{\partial \psi^{\rm e}_{ij}(\|\hat{p}_{ij}\|)}{\partial(\|\hat{p}_{ij}\|)} - c_2 \in \mathcal{P}_{\rm SOS},\ \forall 0 \leq \|\hat{p}_{ij}\| \leq \hat{r}_s, \\ \dfrac{\partial \psi^{\rm e}_{ij}(\|\hat{p}_{ij}\|)}{\partial(\|\hat{p}_{ij}\|)} \cdot \dfrac{1}{\|\hat{p}_{ij}\|} - c_3 \in \mathcal{P}_{\rm SOS}, \end{cases} \tag{17}$$

where ζ_i and ρ_j are introduced in (4), $K_1(\tilde{r}_s, \zeta_i(\sigma)) = \frac{\underline{\mu}_1}{2}h^{\rm e}(\|\zeta_i(\sigma)\cdot\tilde{r}_s\|) - g^{\rm e}(\|\zeta_i(\sigma)\cdot\tilde{r}_s\|)$, $K_2(\tilde{r}_s, \zeta_i(\sigma)) = g^{\rm e}(\|\zeta_i(\sigma)\cdot\tilde{r}_s\|) - \underline{\mu}_1 h^{\rm e}(\|\zeta_i(\sigma)\cdot\tilde{r}_s\|)$.

Proof: Given that there exist SOS polynomials $s_i(\sigma)$ and $p_i(\sigma)$, $i = 0, 1, \ldots, h_\sigma$, positive scalars c_1, c_2, c_3 and (17) holds, we have $h^{\rm e}_{ij}(\|\hat{p}_{ij}\|) > 0$ and $\frac{\partial \psi^{\rm e}_{ij}(\|\hat{p}_{ij}\|)}{\partial(\|\hat{p}_{ij}\|)} > 0$, and $\frac{\partial \psi^{\rm e}_{ij}(\|\hat{p}_{ij}\|)}{\partial(\|\hat{p}_{ij}\|)} \cdot \frac{1}{\|\hat{p}_{ij}\|} > 0$. In addition, $s_i(\sigma)$ and $p_i(\sigma)$ are SOS matrix polynomials for $i = 0, 1, \ldots, h_\sigma$, and $\zeta_i(\sigma) > 0$, $\rho_j(\sigma) > 0$ from (4). Based on the Real Positivestellensatz, we have that $K_1(\tilde{r}_s, \zeta_i(\sigma)) \geq 0$ and $K_2(\tilde{r}_s, \zeta_i(\sigma)) \geq 0$, which is equivalent to $\underline{\mu}_1 \leq \frac{g^{\rm e}(\|\zeta_i(\sigma)\cdot\tilde{r}_s\|)}{h^{\rm e}(\|\zeta_i(\sigma)\cdot\tilde{r}_s\|)} \leq \frac{\mu_1}{2}$.

Similarly, a parametric Lyapunov-like barrier function can be constructed for collision avoidance. A rational polynomial function is used to construct the barrier function:

$$\psi^{\rm c}_{ij}(\|\hat{p}_{ij}\|) = \frac{g^{\rm c}(\|\hat{p}_{ij}\|)}{h^{\rm c}(\|\hat{p}_{ij}\|)}, \tag{18}$$

where $g^{\rm c}(\|\hat{p}_{ij}\|)$ and $h^{\rm c}(\|\hat{p}_{ij}\|)$ are polynomial functions satisfying:

$$\begin{cases} g^{\rm c}_{ij}(\|\hat{p}_{ij}\|) \geq 0,\ g^{\rm c}_{ij}(0) = 0,\ h^{\rm c}_{ij}(\|\hat{p}_{ij}\|) > 0, \\ \underline{\mu}_2 \leq \dfrac{g^{\rm c}(\|\zeta_i(\sigma)\cdot\tilde{d}_s\|)}{h^{\rm e}(\|\zeta_i(\sigma)\cdot\tilde{d}_s\|)} \leq \mu_2,\ \forall \sigma \in \Upsilon, \\ \dfrac{\partial \psi^{\rm c}_{ij}(\|\hat{p}_{ij}\|)}{\partial(\|\hat{p}_{ij}\|)} < 0,\ \forall \|\hat{p}_{ij}\| \geq \tilde{d}_s, \end{cases} \tag{19}$$

where $\tilde{d}_s = d_s - \|\tau_{ij}\|$, the values of $\underline{\mu}_2$ and μ_2 are pre-selected which will be discussed later. Like Lemma 6, the following result provides a tractable way to construct $\psi^{\rm c}_{ij}$:

Lemma 7: The condition (19) holds if there exist SOS polynomials $r_i(\sigma)$ and $q_i(\sigma)$, $i = 0, 1, \ldots, h_\sigma$, and positive scalars c_4 and c_5, such that the following SOS feasibility test problem

is solved:

$$\begin{cases} g_{ij}^{\mathrm{c}}(\|\hat{p}_{ij}\|) \in \mathcal{P}_{\mathrm{SOS}},\ g_{ij}^{\mathrm{c}}(0)=0,\ h_{ij}^{\mathrm{c}}(\|\hat{p}_{ij}\|) - c_4 \in \mathcal{P}_{\mathrm{SOS}}, \\ J_1(\tilde{r}_s, \zeta_i(\sigma)) - r_0\zeta_i - \sum_{j=1}^{h_\sigma} r_j\rho_j \in \mathcal{P}_{\mathrm{SOS}}, \\ J_2(\tilde{r}_s, \zeta_i(\sigma)) - q_0\zeta_i - \sum_{j=1}^{h_\sigma} q_j\rho_j \in \mathcal{P}_{\mathrm{SOS}}, \\ -\dfrac{\partial \psi_{ij}^{\mathrm{c}}(\|\hat{p}_{ij}\|)}{\partial(\|\hat{p}_{ij}\|)} - c_5 \in \mathcal{P}_{\mathrm{SOS}}, \end{cases} \tag{20}$$

where ζ_i and ρ_j are introduced in (4), $J_1(\tilde{r}_s, \zeta_i(\sigma)) = \mu_2 h^{\mathrm{c}}(\|\zeta_i(\sigma)\cdot\tilde{r}_s\|) - g^{\mathrm{c}}(\|\zeta_i(\sigma)\cdot\tilde{r}_s\|)$, $J_2(\tilde{r}_s, \zeta_i(\sigma)) = g^{\mathrm{c}}(\|\zeta_i(\sigma)\cdot\tilde{r}_s\|) - \underline{\mu}_2 h^{\mathrm{c}}(\|\zeta_i(\sigma)\cdot\tilde{r}_s\|)$.
Proof: Omitted due to limited space. □

Therefore, the barrier function of agent A_i for coping with uncertainties both in measurement and communication is as follows:

$$\hat{\Psi}_i = \sum_{j\in\mathcal{N}_i^{\mathrm{sf}}}\left(\psi_{ij}^{\mathrm{e}}(\|\hat{p}_{ij}\|) + \frac{|\tilde{\phi}_{ij}|^2}{\tilde{\alpha} - |\tilde{\phi}_{ij}| + \frac{2\tilde{\alpha}}{\mu_1}}\right) + \sum_{j\in\mathcal{N}_i^{\mathrm{sz}}} \psi_{ij}^{\mathrm{c}}(\|\hat{p}_{ij}\|), \tag{21}$$

where $\hat{p}_{ij} = \zeta_i(\sigma)p_{ij}$, and $\tilde{\alpha} = \alpha - |\phi_{ij}^{\mathrm{d}}|$.

Remark 7: The methods based on parameter-independent barrier functions are not applicable under measurement uncertainty, e.g. the barriers proposed in [23], [24] and the one used in Section III. Barrier function in (21) provides a well-suited way to cope with measurement uncertainty of on-board sensing in multi-agent systems. The main merit is that it ensures a barrier on an unsafe set by directly using disturbed information ($\|\hat{p}_{ij}\| = \|\zeta_i(\sigma)p_{ij}\|$) in construction. In addition, a systematic way to find a suitable parameter-dependent barrier function is proposed, and the parametric barrier function make the parameter-independent form widely used in [12], [23]–[25], [28] as a special case. □

Remark 8: We assume $\underline{\mu}_1$ and $\underline{\mu}_2$ satisfying $\underline{\mu}_1 > 2\mu_{\max}$ and $\underline{\mu}_2 > \mu_{\max}$ with $\mu_{\max} := \frac{1}{2}\sum_{i=1}^{N}(\sum_{j\in\mathcal{N}_i^{\mathrm{f}}} \Psi_{ij}^{\mathrm{e}}(\|\tilde{r}_s - \tilde{\varepsilon}\|, |\tilde{\alpha} - \tilde{\varepsilon}|) + v_i(t_0)^T v_i(t_0) + \theta_i(t_0)^T\theta_i(t_0)) + (N-1)N\Psi_{ij}^{\mathrm{c}}(\|d_s + \tilde{\varepsilon}\|)$, where $0 < \tilde{\varepsilon} < \min\{\frac{1}{2}d_s - r_c, \varepsilon_c, \varepsilon_a\}$. The value of $\mu_{\max}$ is to ensure the collision avoidance and connectivity maintenance, and we will discuss it later. □

A simple example is provided for illustration:

Example 4: Consider two agents A_1 and A_2, the position of A_1 is $(0,\ 0)^T$, $2 \in \mathcal{N}_1^{\mathrm{sf}}$, and A_2 is in the sensing area of A_1 with the parameters $\underline{\mu}_1 = 150$, $\underline{\mu}_2 = 80$, $\mu_1 = 200$, $\mu_2 = 100$, $\alpha = \frac{\pi}{3}$, $d_s = 3$, $r_z = 3.5$, $r_s = 10$, $\tau_1 = (0,\ 0)^T$ and $\tau_2 = (5,\ 0)^T$. A feasible barrier function in (21) is shown in Fig. 5, from which one can find that the barrier function is bounded by 200. □

Let us introduce $\Psi_i^{\mathrm{e}} = \sum_{j\in\mathcal{N}_i^{\mathrm{sf}}} \Psi_{ij}^{\mathrm{e}}$, and $\Psi_i^{\mathrm{c}} = \sum_{j\in\mathcal{N}_i^{\mathrm{sz}}} \psi_{ij}^{\mathrm{c}}$. Thus, $\Psi_i = \Psi_i^{\mathrm{e}} + \Psi_i^{\mathrm{c}}$. Define the vector and the orientation

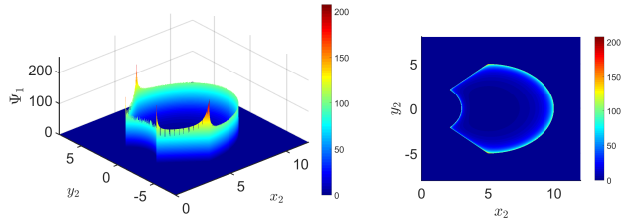

Fig. 5. Example 4: An illustration for the barrier function based on (21).

dictated by the Lyapunov-like barrier function on agent A_i:

$$\eta_i(\sigma) = \left(\frac{\partial\hat{\Psi}_i}{\partial x_i}, \frac{\partial\hat{\Psi}_i}{\partial y_i}\right),\ \theta_i^{\mathrm{L}} = \mathrm{atan}_2\left(\frac{\partial\hat{\Psi}_i}{\partial y_i}, \frac{\partial\hat{\Psi}_i}{\partial x_i}\right), \tag{22}$$

and the orientation error $e_{\theta_i} = \theta_i - \theta_i^{\mathrm{L}}$. A distributed parameter-dependent controller is provided as follows: A distributed controller is provided as follows:

$$u_i = \underbrace{-\|\eta_i\|\cos(e_{\theta_i})}_{u^{\mathrm{c}}} \underbrace{- \sum_{j\in\mathcal{N}_i^{\mathrm{s}}(t)} G_{ij}(t,\delta,\sigma)(v_i - v_j)}_{u^{\mathrm{v}}}, \tag{23a}$$

$$w_i = \underbrace{\theta_i - \sum_{j\in\mathcal{N}_i^{\mathrm{sf}}(t)} \frac{\mu_1|\tilde{\phi}_{ij}|^2 - 2\mu_1^2|\tilde{\phi}_{ij}|\tilde{\alpha} - 4\tilde{\alpha}\mu_1|\tilde{\phi}_{ij}|}{(\mu_1\tilde{\alpha} - \mu_1|\tilde{\phi}_{ij}| + 2\tilde{\alpha})^2}}_{\omega^{\mathrm{c}}} \underbrace{- \sum_{j\in\mathcal{N}_i^{\mathrm{s}}(t)} G_{ij}(t,\delta,\sigma)(\theta_i - \theta_j)}_{\omega^{\theta}}, \tag{23b}$$

where δ is introduced in (3), σ is introduced in (4), G_{ij} is the ij-th entry of weighted adjacency matrix. The terms u^{c} and ω^{c} are designed to achieve connectivity maintenance, collision avoidance and the convergence to the desired position; the parameter-dependent terms u^{v} is designed to achieve the convergence of velocities; the term ω^{θ} is designed to achieve the convergence of θ_{ij}. The information of communication parametric uncertainties are involved in $G_{ij}(t,\delta,\sigma)$ which are used in the design of u^{v} and ω^{θ}. The following result shows that under the proposed controller (23), the connectivity maintenance and the collision avoidance of uncertain networks can be ensured.

Theorem 2: Under Assumption 1-3, and provided that (13) is satisfied for $\mathcal{G}(t_0,\delta,\sigma)$, the uncertain graph $\mathcal{G}(t,\delta,\sigma)$ is robust-2 quasi-strongly connected, and collision avoidance is guaranteed for all $t \geq t_0$, $\delta \in \Omega$ and $\sigma \in \Upsilon$ by using (23).
Proof: This proof aims to show the concerned set is in a positively invariant set, which implies the quasi-strong connectivity and collision avoidance in the uncertain network. Specifically, we assume that the edge set $\mathcal{E}(t)$ changes at t_l, $l = 0,\ 1,\ 2,\ \ldots$. For each $[t_l, t_{l+1})$, $\mathcal{G}$ is fixed. Let us introduce a Lyapunov-like function

$$W = \frac{1}{2}\sum_{i=1}^{N}\left(\Psi_i(\|\tilde{p}_{ij}\|, |\tilde{\phi}_{ij}|) + \theta_i^2 + v_i^2\right). \tag{24}$$

Consider the time interval $[t_0, t_1)$, we have $\Psi_{ij}^{\mathrm{e}} \geq 0$ from (16), $\Psi_{ij}^{\mathrm{c}} \geq 0$ from (19), and $\theta_i^2 + v_i^2 \geq 0$. Thus, we have that $W_0 = W(t_0, \delta) > 0$ for all $\delta \in \Omega$, $\|\tilde{p}_{ij}\| \neq 0$, $|\tilde{\phi}_{ij}| \neq 0$, $v_i \neq 0$ and $\theta_i \neq 0$. Moreover, for $t \in [t_0, t_1)$, $G_{ij}(t, \delta)$ is fixed, from the definition of (24), we have

$$\begin{aligned}\dot{W} &= \frac{1}{2}\sum_{i=1}^{N}\left(\dot{\Psi}_i(\|\tilde{p}_{ij}\|, |\tilde{\phi}_{ij}|)\right) + \sum_{i=1}^{N} v_i \cdot \dot{v}_i + \sum_{i=1}^{N}\theta_i \cdot \dot{\theta}_i \\ &= \sum_{i=1}^{N}\left(\frac{\partial \Psi_i}{\partial x_i} v_i \cos(\theta_i) + \frac{\partial \Psi_i}{\partial y_i} v_i \sin(\theta_i) + \frac{\partial \Psi_i}{\partial \theta_i}\omega_i\right) \\ &\quad + \sum_{i=1}^{N} v_i \cdot u_i + \sum_{i=1}^{N}\theta_i \cdot \omega_i.\end{aligned} \tag{25}$$

By using the proposed controller (23), $\dot{W}$ from the above can be simplified as:

$$\begin{aligned}\dot{W} &= \sum_{i=1}^{N}\left(v_i\|\eta_i\|\cos(\theta_i - \theta_i^{\mathrm{L}}) + \frac{\partial \Psi_i}{\partial \theta_i}\omega_i\right) \\ &\quad + \sum_{i=1}^{N} v_i\left(-\|\eta_i\|\cos(e_{\theta_i}) - \sum_{j\in\mathcal{N}_i^{\mathrm{s}}(t)} G_{ij}(v_i - v_j)\right) \\ &\quad + \sum_{i=1}^{N}\theta_i(w^{\mathrm{c}} + w^{\theta}) \\ &= \sum_{i=1}^{N}\left(v_i\|\eta_i\|\cos(e_{\theta_i}) + \frac{\partial \Psi_i}{\partial \theta_i}\omega_i\right) \\ &\quad + \sum_{i=1}^{N} v_i\left(-\|\eta_i\|\cos(e_{\theta_i}) - \sum_{j\in\mathcal{N}_i^{\mathrm{s}}(t)} G_{ij}(v_i - v_j)\right) \\ &\quad + \sum_{i=1}^{N}\theta_i(w^{\mathrm{c}} + w^{\theta}) \\ &= -v^T L v - \theta^T L \theta \\ &\quad - \left(\sum_{j\in\mathcal{N}_i^{\mathrm{sf}}(t)} \frac{\mu_1|\tilde{\phi}_{ij}|^2 - 2\mu_1^2|\tilde{\phi}_{ij}|\tilde{\alpha} - 4\tilde{\alpha}\mu_1|\tilde{\phi}_{ij}|}{(\mu_1\tilde{\alpha} - \mu_1|\tilde{\phi}_{ij}| + 2\tilde{\alpha})^2} - \theta_i\right)^2.\end{aligned} \tag{26}$$

Taking into account that (13) is satisfied for $\mathcal{G}(t_0, \delta)$, we have $L(t_0, \delta) \geq 0$ for all $\delta \in \Omega$. Since $\mathcal{E}^{\mathrm{f}} \subseteq \mathcal{E}(t)$, it is not difficult to see $L(t, \delta) \geq 0$, which implies that $\dot{W} \leq 0$. Thus, $W(t, \delta) \leq W(t_0, \delta) \leq \mu_{\max}$, for $t \in [t_0, t_1)$. From (16), (19) and Remark 8, we have that $\Psi_{ij}^{\mathrm{e}}(\hat{r}_s, |\tilde{\phi}_{ij}|) \geq 0.5\underline{\mu}_1 > \mu_{\max}$, $\Psi_{ij}^{\mathrm{e}}(\|\tilde{p}_{ij}\|, |\tilde{\alpha}|) \geq 0.5\underline{\mu}_1 > \mu_{\max}$, and $\Psi_{ij}^{\mathrm{c}}|_{\|\tilde{p}_{ij}+\tau_{ij}\|=d_s} \geq \underline{\mu}_2 > \mu_{\max}$, which implies that no collision occurs during $[t_0, t_1)$, and no agent j has left the set $\mathcal{N}_i^{\mathrm{sf}}$ for agent i. Hence, the network $\mathcal{G}(t, \delta)$ is still robust quasi-strongly connected. Let us consider $t = t_1$, we assume that the number of new agents added in the set $\mathcal{N}_i^{\mathrm{sz}}$ is k_i for agent i. One has that $\sum_{i=1}^{N}(k_i + \mathrm{num}_i(\mathcal{N}_i^{\mathrm{sz}})) \leq N(N-1)$, and $\mathrm{num}_i(\mathcal{N}_i^{\mathrm{sz}})$ is the number of agents in $\mathcal{N}_i^{\mathrm{sz}}$. It yields

$$\begin{aligned}W(t_1, \delta) &\leq W(t_1^-, \delta) + \sum_{i=1}^{N} k_i\widetilde{\Psi} \\ &\leq W(t_0, \delta) + \sum_{i=1}^{N} k_i\widetilde{\Psi} \\ &\leq \frac{1}{2}\sum_{i=1}^{N}\Big(\sum_{j\in\mathcal{N}_i^{\mathrm{f}}} \Psi_{ij}^{\mathrm{e}}(\|\hat{r}_s - \tilde{\varepsilon}\|, |\tilde{\alpha} - \tilde{\varepsilon}|) + \theta_i(t_0)^2 \\ &\qquad + v_i(t_0)^2 + \sum_{j\in\mathcal{N}_i^{\mathrm{sz}}(t)} \Psi_{ij}^{\mathrm{c}}(\|\tilde{p}_{ij}\|)\Big) + \sum_{i=1}^{N} k_i\widetilde{\Psi} \\ &< \mu_{\max},\end{aligned} \tag{27}$$

where $\widetilde{\Psi} = \frac{1}{2}\sum_{j\in\mathcal{N}_i^{\mathrm{sz}}} \Psi_{ij}^{\mathrm{c}}(\|\hat{d}_s - \tilde{\varepsilon}\|)$. The above inequality implies that the total energy of the system, expressed by the Lyapunov-like function (24), is less than $\mu_{\max}$. Since $\underline{\mu}_1 > \mu_{\max}$ and $\underline{\mu}_2 > \mu_{\max}$, according to the construction of barrier functions, i.e., equations (16) and (19), we have that the connectivity and the collision avoidance can be ensured at time t_1.

The above argument can be applied to time intervals $[t_l, t_{l+1})$. The condition still holds that $\dot{W}(t, \delta) \leq 0$, and we have

$$W(t, \delta) \leq W(t_l) \leq \mu_{\max}, \tag{28}$$

which implies that no collision occurs during $[t_l, t_{l+1})$, and no agent j has left the set $\mathcal{N}_i^{\mathrm{sf}}$ for agent i. Hence, the graph $\mathcal{G}(t, \delta)$ is connected for $t \in [t_l, t_{l+1})$. □

Remark 9: Note that the controller (23) is derived from a parametric Lyapunov-like barrier function, while directly using controller based on a parameter-independent barrier function in [31] is not valid for the case under measurement uncertainty.□

The above result achieves the objectives of last two items in Problem 1. By the following result, the objective of the first item in Problem 1 can also be achieved.

Theorem 3: If Assumption 1-3 hold and (13) is satisfied for $\mathcal{G}(t_0, \delta, \sigma)$, then, under the controller (23), the following conditions hold for all $\delta \in \Omega$, $\sigma \in \Upsilon$ and $i \in \mathcal{N}$:

1) $\lim_{t\to\infty}|v_i - v_j| = 0$, for $j \in \mathcal{N}$;
2) $\lim_{t\to\infty}|\theta_i - \theta_j| = 0$, for $j \in \mathcal{N}$;
3) $\lim_{t\to\infty}\|p_i(t) - \tau_i - (p_j(t) - \tau_j)\| = 0$, for $j \in \mathcal{N}_i^{\mathrm{f}}$.

Proof: 1) For the first two statements, based on the construction of W in (24), we have that

$$\frac{1}{2}\sum_{i=1}^{N} v_i^2 \leq \mu_{\max}, \ \frac{1}{2}\sum_{i=1}^{N}\theta_i^2 \leq \mu_{\max}.$$

which results in $\|v\| \leq \sqrt{2\mu_{\max}}$ and $\|\theta\| \leq \sqrt{2\mu_{\max}}$. Since the uncertain network $\mathcal{G}$ is robust quasi-strongly connected for all $t \geq t_0$, we have $\|p_{ij}\| < (N-1)r_s$ for all $i, j \in \mathcal{N}$. Then, it yields

$$\|\tilde{p}_{ij}\| < (N-1)r_s - (N-1)\tilde{\tau},$$

where $\tilde{\tau} = \min_{\forall i,j\in\mathcal{N},\ i\neq j}\{\|\tau_{ij}\|\}$.

Let us consider the set

$$\begin{aligned}\Xi \;=\; & \Big\{\tilde{p}\in\mathbb{R}^{2N},\ v\in\mathbb{R}^N,\ \theta\in\mathbb{R}^N|\ W(\tilde{p},v,\theta)\le\mu_{\max},\\ & \|\tilde{p}_{ij}\|\le(N-1)r_s-(N-1)\tilde{\tau},\\ & \|v\|\le\sqrt{2\mu_{\max}},\ \|\theta\|\le\sqrt{2\mu_{\max}}\Big\},\end{aligned}$$

which is closed and bounded, and thus a compact set. Now, let us investigate the largest invariant set in $\mathcal{I}=\{\tilde{p}\in\mathbb{R}^{2N},\ v\in\mathbb{R}^N,\ \theta\in\mathbb{R}^N|\ \dot{W}=0\}$.

Based on (26), we have

$$\begin{aligned}\dot{W} &= -v^TLv-\theta^TL\theta\\ &\quad-\Bigg(\sum_{j\in\mathcal{N}_i^{\rm sf}(t)}\frac{\mu_1|\tilde{\phi}_{ij}|^2-2\mu_1^2|\tilde{\phi}_{ij}|\tilde{\alpha}-4\tilde{\alpha}\mu_1|\tilde{\phi}_{ij}|}{(\mu_1\tilde{\alpha}-\mu_1|\tilde{\phi}_{ij}|+2\tilde{\alpha})^2}-\theta_i\Bigg)^2\\ &= -\frac{1}{2}\sum_{i\in\mathcal{N},\ j\in\mathcal{N}_i^{\rm s}}G_{ij}\|v_i-v_j\|^2\\ &\quad-\frac{1}{2}\sum_{i\in\mathcal{N},\ j\in\mathcal{N}_i^{\rm s}}G_{ij}\|\theta_i-\theta_j\|^2\\ &\quad-\Bigg(\sum_{j\in\mathcal{N}_i^{\rm sf}(t)}\frac{\mu_1|\tilde{\phi}_{ij}|^2-2\mu_1^2|\tilde{\phi}_{ij}|\tilde{\alpha}-4\tilde{\alpha}\mu_1|\tilde{\phi}_{ij}|}{(\mu_1\tilde{\alpha}-\mu_1|\tilde{\phi}_{ij}|+2\tilde{\alpha})^2}-\theta_i\Bigg)^2,\end{aligned}$$

which implies that $\dot{W}=0$ if and only if $v_1=\cdots=v_N$ and $\theta_1=\cdots=\theta_N=\sum_{j\in\mathcal{N}_i^{\rm sf}(t)}\frac{\mu_1|\tilde{\phi}_{ij}|^2-2\mu_1^2|\tilde{\phi}_{ij}|\tilde{\alpha}-4\tilde{\alpha}\mu_1|\tilde{\phi}_{ij}|}{(\mu_1\tilde{\alpha}-\mu_1|\tilde{\phi}_{ij}|+2\tilde{\alpha})^2}$. By using LaSalle's invariance principle, we have that all the trajectories started in the set Ξ will converge to set $\mathcal{I}$, i.e., $v_1=\cdots=v_N$ and $\theta_1=\cdots=\theta_N$.

2) For the third statement, we assume that the edge set $\mathcal{E}(t)$ changes at t_l, $l=0,\ 1,\ 2,\ \ldots$, and there exists a time $\hat{t}_l$ such that the topology of $\mathcal{G}$ is fixed. From the above proof, consider $t\ge\hat{t}_l$, we have $v_i-v_j=0$ for all $i,j\in\mathcal{N}$. Then, (23) can be rewritten as

$$\begin{aligned}u_i &= -\|\eta_i\|\cos(e_{\theta_i})\\ &= -\langle(\frac{\partial\Psi_i}{\partial x_i},\frac{\partial\Psi_i}{\partial y_i}),(\cos(\theta_i),\sin(\theta_i))\rangle\\ &= -\langle(\frac{\partial\Psi_i(\|\tilde{p}_{ij}\|)}{\partial\|\tilde{p}_{ij}\|}\cdot\frac{1}{\|\tilde{p}_{ij}\|}\tilde{p}_{ij}),(\cos(\theta_i),\sin(\theta_i))\rangle.\end{aligned}\tag{29}$$

Observe that when $t>t_l$, $\Psi_i=\sum_{j\in\mathcal{N}_i^{\rm sf}}\Psi_{ij}^{\rm e}$, and $\frac{\partial\Psi_{ij}^{\rm e}(\|\tilde{p}_{ij}\|)}{\partial\|\tilde{p}_{ij}\|}\cdot\frac{1}{\|\tilde{p}_{ij}\|}$ is positive and bounded as $\|\tilde{p}_{ij}\|\to 0$, let us assume if there exist some $\tilde{i}$ and $\tilde{j}$ such that $\tilde{p}_{\tilde{i}\tilde{j}}\ne 0$. Then, from (23), it yields that there exist some $\tilde{i}$ and $\tilde{j}$ such that $\theta_{\tilde{i}}\ne\theta_{\tilde{j}}$ which contradicts with the statement 2). Therefore, we have that $\lim_{t\to\infty}\tilde{p}=\mathrm{span}(1_{Nn})$, i.e., $p_i-\tau_i-(p_j-\tau_j)=0$, for all $i,j\in\mathcal{N}$, which completes this proof. □

IV. Examples

Two examples are provided for illustration. We execute the computation using MATLAB R2017a on a desktop with a 16GB DDR3 RAM and an Intel Xeon E3-1245 processor (3.4 GHz). MATLAB toolbox SeDuMi is used for solving semi-definite problems.

A. Example 5: A 6-Agent Case

An uncertain 6-agent system is considered, with the initial topology shown in Fig. 6 (a). The model of agents is set up with the parameters: $r_a=0.75$, $\tilde{r}_s=9$, $r_z=3.5$, $r_c=1.25r_a$, $d_s=2r_c$, $\alpha=\frac{\pi}{3}$, $\varepsilon_a=\frac{\pi}{60}$ and $\varepsilon_c=0.1$.

It is assumed that the network is affected by two uncertain parameters in communication, i.e. δ_1 and δ_2. Specifically, the uncertain matrix $G(t_0,\delta)$ is given by

$$\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 2-\delta_1\\ 3+2\delta_1\delta_2 & 0 & 0 & 0 & 0 & 0\\ 7-5\delta_1 & 0 & 0 & 0 & 0 & 0\\ 0 & 2\delta_1+\delta_2+5 & 0 & 0 & 0 & 0\\ 3 & 2\delta_1+\delta_2 & & 0 & 4-\delta_1 & 0\\ 0 & 0 & 3 & 0 & 0 & 0\end{bmatrix},$$

where $\delta\in\mathbb{R}^2$ is constrained in the set Ω chosen as $\Omega=\{\delta:\|\delta\|\le 1\}$. Hence, we have $N=6$ and $r=2$. For the newly added edge (i,j) for $t>t_0$, we assume $G(j,i)=1-\delta_1\delta_2$. Moreover, Ω can be described as in (3) with

$$s_1(\delta)=1-\delta_1^2-\delta_2^2.$$

We also assume the on-board sensor of each agent A_i is affected by a multiplicative function $\zeta(\sigma)=3-\sigma^2$, where σ is constrained in the set

$$\Upsilon=\{\sigma\in\mathbb{R}:\ 3-\sigma^2\ge 0,\ 3\sigma-2\ge 0\}.$$

Then, we use (13) to check the robust-2 quasi-strong connectivity of the initial network under both communication uncertainty and measurement uncertainty. By solving the LMI (13), we obtain $c^*=1.423$ which means that $\mathcal{G}(t_0,\delta,\sigma)$ is robust-2 quasi-strongly connected. The computational time is shown in Tab. I.

Next, we apply the proposed controller (23), and the results are shown in Fig. 7. Let us observe that $\min(\|p_{ij}\|)$ is always larger than d_s in Fig. 8. Thus, the collision avoidance is ensured. For the connectivity maintenance, by only preserving connections of $j\in\mathcal{N}_i^{\rm sf}$ (Green directed edges shown in Fig. 6.), the controller allows breaks of edges when system evolves (blue edges are not guaranteed by the distributed controller (23)). As demonstrated by Fig. 6-7, the robust formation is achieved with collision avoidance and connectivity maintenance.

TABLE I
The computational time t_c [sec] for Example 5 and Example 6, and different degrees of $P(x)$.

	Example 5			Example 6		
	$d_{\rm P}$=2	$d_{\rm P}$=4	$d_{\rm P}$=6	$d_{\rm P}$=2	$d_{\rm P}$=4	$d_{\rm P}$=6
Solving (13)	11.83	16.91	25.73	62.71	82.74	112.33

B. Example 6: A 30-Agent Case

In this scenario, a 30-agent system is considered with the parameters: $r_a=0.75$, $\tilde{r}_s=9$, $r_z=3.5$, $r_c=1.25r_a$,

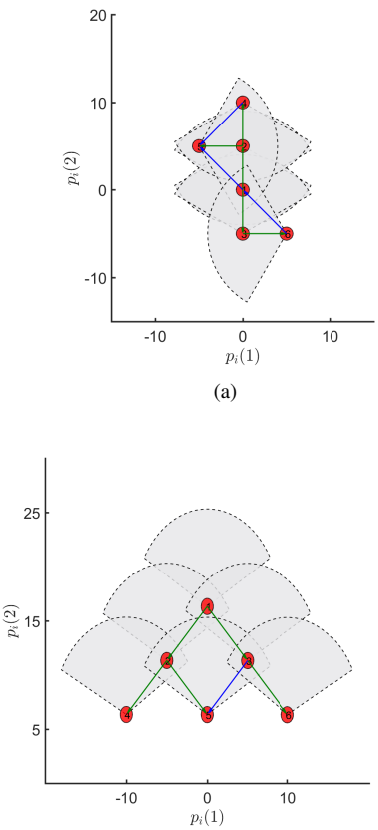

Fig. 6. Example 5: The initial and final topologies of the 6-agent system.

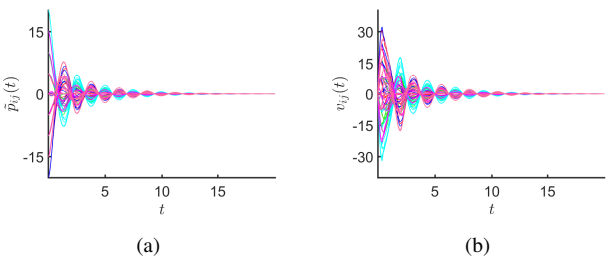

Fig. 7. Example 5: The differences of transformed positions and velocities.

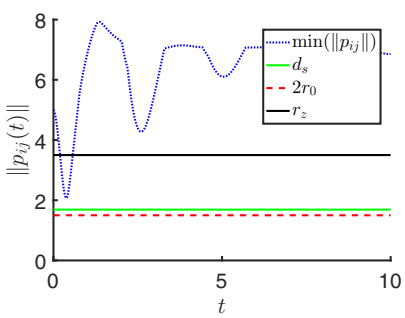

Fig. 8. Example 5: The minimal distance between agents under uncertainties.

$d_s = 2r_c$, $\alpha = \frac{\pi}{3}$, $\varepsilon_a = \frac{\pi}{60}$ and $\varepsilon_c = 0.1$. The initial positions are selected on x-axis as shown in Fig. 9. The desired configuration is a two-segment on which all agents are symmetrically distributed. The communication is affected by three parameters. Due to limited space, we omit the expression of Ω, Υ and $G(\delta, \sigma)$. By solving (13) we find $c^* = 0.924$, which ensures that $\mathcal{G}(t_0, \delta, \sigma)$ is robust-2 quasi-strongly connected. The proposed controller (23) guarantees the collision avoidance, and the robust formation is achieved as shown in Fig. 10-11. Finally, it is worth noting that the proposed method is scalable for systems with even larger number of agents.

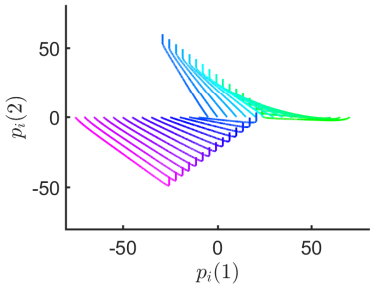

Fig. 9. Example 6: The initial topology and states.

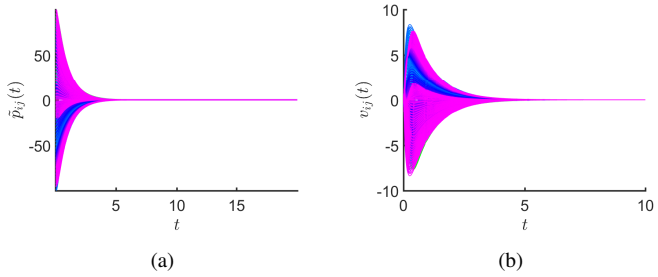

Fig. 10. Example 6: The trajectories of agents and the differences of velocities.

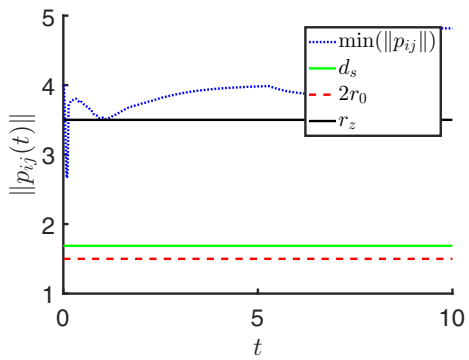

Fig. 11. Example 6: The minimal distance between agents.

V. Conclusion and Discussion

This paper studies the robust formation control problem of multiple agents under both parametric communication un-

certainty and measurement uncertainty. For a non-holonomic multi-agent model with anisotropic sensing, a distributed controller is proposed for robust multi-task formation in the presence of both communication and measurement uncertainties. A necessary and sufficient condition is proposed for checking the connectivity of uncertain networks under communication uncertainties. Based on this condition, a solvable condition consisting of LMIs is given by employing the square matrix representation. Furthermore, a Lyapunov-like barrier function is constructed for anisotropic sensing, and a gradient-based controller with bounded input is designed such that the collision avoidance and connectivity maintenance can be ensured. In addition, in order to solve the robust formation control problem regarding to the measurement uncertainty, a parametric Lyapunov-like barrier function is introduced, which provides a well-suited way to solve this problem where the classical construction of Lyapunov-like barrier functions is not working. The main benefit of this class of Lyapunov-like barrier functions is that the artificial barrier and the corresponding robust controller are changing with regard to the effect of uncertainty, making the artificial barrier robust against measurement disturbances. These benefits are illustrated by some numerical examples.

Along with Remark 3, the main conservativeness of this approach stems from the fact that some selected links (edges of spanning tree) are maintained by barrier functions. Though the robust quasi-strong connectivity can be formally proven, the result can be relaxed by only considering global connectivity maintenance requirement, where these selected links can be broken and the overall graph remains connected [30]. Promising methods to reduce the conservativeness include exploiting k-hop communication [43], path planning strategy [44], local connectivity estimation [45], but it is still an open question that how to encode these elegant approaches and objectives by barrier functions such that a gradient-based distributed controller can be designed, to which our future efforts will be devoted. Another source of conservatism arises from the fact that the parametric Lyapunov-like barrier function in this paper is fixed, without considering the performance of distributed controllers. To get less conservative results, a systematic way to generate barrier functions via sum-of-squares programming is a promising method [42]. Followed by the discussion in Remark 5, checking the robust global connectivity by only using local information is a promising way to cope with fully distributed large-scale networks. In addition, more efforts will be devoted to the comparisons to analytic vector field methods [46], [47], and contraction theory methods [48].

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Dongkun Han is currently a post-doctoral research fellow with the Department of Aerospace Engineering at the University of Michigan. He earned his Ph.D. degree in Electrical and Electronic Engineering in 2014 from the University of Hong Kong. From 2014 to 2016, he was a research associate at the Department of Informatics, Technical University of Munich, Germany. From 2013 to 2014, he was an exchange research student at Information Systems Laboratory, Stanford University, USA. In 2015, he was a visiting academic faculty at the Department of Electrical Engineering, German Institute of Science and Technology-TUM Asia, Singapore. He joined the Department of Aerospace Engineering at the University of Michigan in 2016. He was a recipient of the Distinct Ph.D. Graduation Award of Electrical and Electronic Engineering Department, the University of Hong Kong, in 2014, the Excellent Paper Award with the First Prize, Journal of Power Distribution and Utilization, in 2009, and the Best Paper Award in the Annual Meeting of Chinese Electrical Engineering Society, in 2008.

Dimitra Panagou received the Diploma and PhD degrees in Mechanical Engineering from the National Technical University of Athens, Greece, in 2006 and 2012, respectively. Since September 2014 she has been an Assistant Professor with the Department of Aerospace Engineering, University of Michigan. Prior to joining the University of Michigan, Dr. Panagou was a postdoctoral research associate with the Coordinated Science Laboratory, University of Illinois, Urbana-Champaign (2012-2014), a visiting research scholar with the GRASP Lab, University of Pennsylvania (June 2013, fall 2010) and a visiting research scholar with the University of Delaware, Mechanical Engineering Department (spring 2009). Her research interests include the fields of planning, coordination and distributed control of complex systems, with applications in unmanned aerial systems, robotic networks and autonomous multi-vehicle systems (ground, marine, aerial, space). She was a recipient of the NASA 2016 Early Career Faculty Award and of the Air Force Office of Science Research 2017 Young Investigator Award.