

Worksheet 6 for Math 115

Dondi Jay Ellis, Jr.

March 12, 2013

I found a discarded textbook on calculus in a wastebasket and read it from cover to cover. - John Pople

Work on the following functions. As usual, it is my sincerest apologies if these problems unintentionally agitate you.

1. If $3x^2 + 2xy + y^2 = 2$, then what is the value of $\frac{dy}{dx}$ at $x = 1$?
2. Find the inflection points of the graph of $y = 5x^4 - x^5$.
3. For what values of k will $x + \frac{k}{x}$ have a relative maximum at $x = -2$?
4. The following questions relate to the implicit function $y^2 + 4x = 4xy^2$.
 - a. Compute $\frac{dy}{dx}$.
 - b. Find the equation for the tangent line to this curve at the point $(\frac{1}{3}, 2)$.
 - c. Find the x - and y -coordinates of all points at which the tangent line to this curve is vertical.
5. Consider the family of functions $f(x) = ax - e^{bx}$, where a and b are positive constants.
 - a. Any function $f(x)$ in this family has only one critical point. In terms of a and b , what are the x - and y -coordinates of that critical point?
 - b. Is the critical point a local maximum or a local minimum? Justify your answer with either the first-derivative test or the second-derivative test.
 - c. For which values of a and b will $f(x)$ have a critical point at $(1, 0)$?

6. Find the inflection points of the graph of $y = 5x^4 - x^5$.
7. A point moves on the x -axis in such a way that its velocity at time t ($t > 0$) is given by $v = \frac{\ln t}{t}$. At what values of t does v attain its maximum?
8. A right triangle with hypotenuse of length a is rotated about one of its legs to generate a right circular cone. Find the greatest possible volume of such a cone.
9. Show that the sum of a positive number and its reciprocal is at least 2.
10. Approximate the value of $y = \sqrt{4 + \sin x}$ at $x = 0.12$ using tangent line approximation at $x = 0$. Do so without using a calculator.
11. If $x^3 + 3xy + 2y^3 = 17$, then find $\frac{dy}{dx}$ in terms of x and y .
12. Find a value of x for which $y = \frac{1}{x^2} - \frac{1}{x^3}$ has a point of inflection.
13. How many critical points does the function $f(x) = (x + 2)^5(x - 3)^4$ have?

Mathematics is a game played according to certain simple rules with meaningless marks on paper.

-David Hilbert