

Worksheet 6 for Math 115

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“Which is more musical: a truck passing by a factory or a truck passing by a music school?”

- John Cage

Work on the following functions. My sincerest apologies if these problems unintentionally agitate you.

1. Differentiate $f(x) = e^{\arcsin(\frac{1}{\ln x})}$.
2. Differentiate $f(x) = e^{\ln(10^{\cos x})}$.
3. For the following questions, say True if the statement is always true, and otherwise say False. No justification is necessary.
 - a. If $f(x)$ is a function with a local maximum at $x = c$, then $f'(c) = 0$.
 - b. If $g'(55) = g'(65) = 0$, then $g(x)$ is constant on the interval $55 \leq x \leq 65$.
 - c. The point $(\pi, 1)$ is on the curve defined by the implicit function $5 \sin(xy) = \ln(y)$.
 - d. The function $A(x) = \frac{1}{R^2} \cos(Rx) + \frac{1}{2}x^2$ has an inflection point at $x = 0$, where R is a nonzero constant.
 - e. If $h'(x) < 0$ for all x in the interval $[2, 8]$, then the global maximum of $h(x)$ on that interval occurs at $x = 2$.
4. The following questions relate to the implicit function $y^2 + 4x = 4xy^2$.
 - a. Compute $\frac{dy}{dx}$.
 - b. Find the equation for the tangent line to this curve at the point $(\frac{1}{3}, 2)$.

- c. Find the x - and y -coordinates of all points at which the tangent line to this curve is vertical.
5. Consider the family of functions $f(x) = ax - e^{bx}$, where a and b are positive constants.
- a. Any function $f(x)$ in this family has only one critical point. In terms of a and b , what are the x - and y -coordinates of that critical point?
- b. Is the critical point a local maximum or a local minimum? Justify your answer with either the first-derivative test or the second-derivative test.
- c. For which values of a and b will $f(x)$ have a critical point at $(1, 0)$?
- 6.
- a. Find the local linearization $L(x)$ of the function $f(x) = (1 + x)^k$ near $x = 0$, where k is a positive constant.
- b. For which values of k does this local linearization give underestimates of the actual value of $f(x)$? (Show your work.)
- c. Suppose you want to use $L(x)$ to find an approximation of the number $\sqrt{1.1}$. What number should k be, and what number should x be?
- d. Approximate $\sqrt{1.1}$ using $L(x)$.
- e. What is the error in the approximation from part (d)?
7. The following questions relate to the implicit curve $2x^2 + 4x - x^2y^2 + 3y^4 = -1$.
- a. Calculate $\frac{dy}{dx}$.
- b. Q is the only point on the curve that has a y -coordinate of 1. Find the x -coordinate of Q .
- c. Find the equation of the tangent line to the curve at Q .

8. Consider the family of functions $y = ax^b \ln x$ where a and b are nonzero constants.

a. Calculate $\frac{dy}{dx}$ in terms of the constants a and b .

b. Find specific values of a and b so that the resulting function has a local maximum at the point $(e, 1)$. You must show that $(e, 1)$ is a local maximum to receive full credit.

9. Two smokestacks d miles apart deposit soot on the ground between them. The concentration of the combined soot deposits on the line joining them, at a distance x from one stack, is given by

$$S = \frac{c}{x^2} + \frac{k}{(d-x)^2}$$

where c and k are positive constants which depend on the quantity of smoke each stack is emitting. If $k = 27c$, find the x -value of the point on the line joining the stacks where the concentration of the deposit is a minimum. Justify that the point you found is actually a global minimum.

“Boulez, who is everything I don’t want art to be... Boulez, who once said in an essay that he is not interested in how a piece sounds, only how it is made.”

- Morton Feldman, “predeterminate/indeterminate”