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Math 216 — a note about last class

So I promised the second section class that I would write up the integration by parts method known as tabular method. This method will make your life a little easier, and if you are not able to "just see" the answer, or guess and and check, or if in general you aren't naturally really good at integrating, then I recommend looking over this paper. Okay, so remember that when you do integration by parts you have to choose a u and a dv. Also, notice that sometimes you have to repeat the method over and over again. Okay, so this will make it easier to keep track of stuff, and under the hood you are really doing integration by parts. Suppose for example you are given the following:

$$\int (x^2 + 2x + 3) \sin x dx.$$

So you would choose your u to be $(x^2 + 2x + 3)$ and you would choose your dv to be $\sin x dx$. Now make a 2 column table. Label the first column of your table **Derivative** and the second column label **AntiDerivative**. In the first entry of column one write down your u. In the first entry of column two write down your dv term (don't bother writing the dx part). Okay, so you should get a table like this:

Derivative	AntiDerivative
$(x^2 + 2x + 3)$	$\sin x$

Next you want to take repeated derivatives in the first column and you want to do repeated integrations in the second column. In our case we will get the following picture:

Derivative	AntiDerivative
$(x^2 + 2x + 3)$	$\sin x$
$(2x + 2)$	$-\cos x$
2	$-\sin x$
0	$\cos x$
0	$\sin x$

You may stop whenever you get a zero in the first column. However, you may never get a zero in the first column. In general you may not ever get a zero in the righthand column. This is okay. It is possible to stop at any point. However, you want to stop whenever you get to a point where the adjacent column entries multiply to something which can easily be integrated or which is the same thing (up to a constant multiple) which is inside your original integral. We will do each of these cases in what follows. However, our current situation is one in which the righthand side achieves zero. Write in arrows as follows:

$$\begin{tikzcd}
\mathit{Derivative} & \mathit{AntiDerivative} \\
(x^2+2x+3) \arrow[dr, "+"] & \sin x \\
(2x+2) \arrow[dr, "-"] & -\cos x \\
2 \arrow[dr, "+"] & -\sin x \\
0 & \cos x \arrow[l, "-"]
\end{tikzcd}$$

This means that:

$$\int (x^2+2x+3)\sin x dx = (x^2+2x+3)(-\cos)-(2x+2)(-\sin x)+(2)(\cos x)-\int (0)(-\cos x)dx.$$

Now suppose you were asked to integrate $\int x^2 \ln x dx$. In this case we would choose our u to be $\ln x$ since we don't know the antiderivative of this guy, whereas we do know the antiderivative of x^2. We choose our dv to be x^2dx. Now we write down a table as before and we get:

$$\begin{tikzcd}
\mathit{Derivative} & \mathit{AntiDerivative} \\
\ln x \arrow[dr, "+"] & x^2 \\
\frac{1}{x} & \frac{1}{3}x^3 \arrow[l, "-"]
\end{tikzcd}$$

We stop at the second line since $\frac{1}{x}$ and $\frac{1}{3}x^3$ multiply to $\frac{1}{3}x^2$ which is easy to integrate. In general, you want to draw diagonal arrows from right to left alternating signs and starting with a plus sign. The line at which you stop, draw a horizontal arrow from right to left of sign opposite of the previous diagonal arrow. You may stop at any point you like, at which point you draw the horizontal arrow.

Now to finish the problem, we read off the diagram and we get $(\ln x)(\frac{1}{3}x^3) - \int(\frac{1}{x})(\frac{1}{3}x^3)dx = (\ln x)(\frac{1}{3}x^3) - \frac{1}{9}x^3$.

For our final example, consider $\int e^x \cos x dx$. Again, we write down our lovely little table. This time it doesn't matter which of the two functions $\cos x$ and e^x we choose to be our u. I will choose u to be $\cos x$ and my dv to be $e^x dx$. In this case I get:

$$Derivative \qquad AntiDerivative$$

$$\begin{tikzcd}
\cos x \arrow[dr, "+"] & e^x \\
-\sin x \arrow[dr, "-"] & e^x \\
-\cos x & e^x \arrow[l, "+"]
\end{tikzcd}$$

I stopped where I did since $\cos x$ and e^x multiply together to what we originally where integrating. This will give me a "Solve for Y" type setup. We read off the table and find:

$$\int e^x \cos x dx = (\cos x)(e^x) - (-\sin x)(e^x) + \int(-\cos x)(e^x)dx.$$

The result is clear from here.