Important Trigonometric Identities

(1) The trigonometric functions satisfy several identities. Most are a consequence of the very important:

Fundamental Identity $(cos(t))^2 + (sin(t))^2 = 1.$

This holds because $(cos(t), sin(t))$ is defined to be a point on the unit circle $x^2 + y^2 = 1$.

(2) We can also define some other trigonometric functions using sine and cosine.

Definition of: $tan(t) := \dfrac{sin(t)}{cos(t)} \qquad cot(t) := \dfrac{cos(t)}{sin(t)} \qquad sec(t) := \dfrac{1}{cos(t)} \qquad csc(t) := \dfrac{1}{sin(t)}.$

(3) The following identities can be established by looking at the unit circle or at the graphs of sine and cosine.

Periodicity: $cos(t + 2\pi) = cos(t)$ **and** $sin(t + 2\pi) = sin(t)$

Even/Odd Properties: $cos(-t) = cos(t) \qquad sin(-t) = -sin(t) \qquad tan(-t) = -tan(t)$

Corelations: $sin\left(\frac{\pi}{2} - t\right) = cos(t), \quad cos\left(\frac{\pi}{2} - t\right) = sin(t), \quad tan\left(\frac{\pi}{2} - t\right) = cot(t), \quad cot\left(\frac{\pi}{2} - t\right) = tan(t).$

(4) These final identities are also very useful. They come from previous identities.

Difference of sines: $sin(a) - sin(b) = 2sin\left(\frac{a-b}{2}\right) cos\left(\frac{a+b}{2}\right)$

Difference of cosines: $cos(a) - cos(b) = -2sin\left(\frac{a-b}{2}\right) sin\left(\frac{a+b}{2}\right)$

Sine of a sum: $sin(s + t) = sin(s)cos(t) + sin(t)cos(s)$

Sine of a difference: $sin(s - t) = sin(s)cos(t) - sin(t)cos(s)$

Cosine of a sum: $cos(s + t) = cos(s)cos(t) - sin(s)sin(t)$

Cosine of a difference: $cos(s - t) = cos(s)cos(t) + sin(s)sin(t)$

Tangent of a sum: $tan(s + t) = \dfrac{tan(s) + tan(t)}{1 - tan(s)tan(t)}$

Tangent of a difference: $tan(s - t) = \dfrac{tan(s) - tan(t)}{1 - tan(s)tan(t)}$

Double angles: $sin(2t) = 2sin(t)cos(t) \qquad cos(2t) = cos^2(t) - sin^2(t) \qquad tan(2t) = \dfrac{2tan(t)}{1 - (tan(t))^2}.$

Half angles: $sin(t/2) = \pm\sqrt{\dfrac{1 - cos(t)}{2}} \qquad cos(t/2) = \pm\sqrt{\dfrac{1 + cos(t)}{2}} \qquad tan(t/2) = \pm\sqrt{\dfrac{1 - cos(t)}{1 + cos(t)}}.$

For the half angle identities, you must think about whether the result should be positive or negative.

Fundamental inequalities: $0 < cos(t) < \dfrac{sin(t)}{t} < sect$ **whenever** $0 < t < \dfrac{\pi}{2}.$