

Bargaining power and supply base diversification

Zhixi Wan • Damian R. Beil

Ross School of Business, University of Michigan, Ann Arbor, Michigan 48109
wanzhixi@umich.edu • dbeil@umich.edu

Abstract

We examine a supply base diversification problem faced by a buyer who periodically holds auctions to award short term supply contracts among a cohort of suppliers (i.e., the supply base). To mitigate significant cost shocks to procurement, the buyer can diversify her supply base by selecting suppliers from different regions. We find that the optimal degree of supply base diversification depends on the buyer's bargaining power, i.e., the buyer's ability to choose the auction mechanism. At one extreme, when the buyer has full bargaining power and thus can dictatorially implement the optimal mechanism, she prefers to fully diversify. At the other extreme, when the buyer uses a reverse English auction with no reserve price due to her lack of bargaining power, she may consider protecting herself against potential price escalation from cost-advantaged suppliers by using a less diversified supply base. We also examine cases where the buyer has intermediate bargaining power and can employ a reserve price and/or a first-price sealed-bid auction, and we find that in general the more bargaining power the buyer has to control price escalation from cost-advantaged suppliers the more she prefers a diversified supply base.

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1. Introduction

It is common for buyers (procurement managers) responsible for procuring an item to identify a *supply base*, a group of qualified suppliers that are capable of producing the item. A supply base is a well-known tool for managing risks. For specialized items where availability is the main objective, buyers can place orders with multiple suppliers to manage non-delivery risks (e.g., Anupindi and Akella 1993). But, as is our focus in this paper, a supply base can also be a crucial strategic tool for purchasing commodity-type items where cost, not availability, is the central issue.

Buyers typically do not know the true costs of suppliers, who possess private information about their cost drivers (inventory level, capacity utilization, financial status, etc.). To find a low price, buyers increasingly employ procurement auctions aimed at price discovery (Jap 2003). As the practitioner survey Beall et al. (2003) page 49 points out, “If a qualified supply base is identified, and the market for a particular commodity/purchase family group changes rapidly, [procurement auctions] are an excellent tool to award business for short duration and re-auction regularly. For example, one company interviewed purchases highly engineered printed circuit boards quarterly through [procurement auctions].” In a procurement auction, competition between suppliers can come down to cents or fractions of a cent, yet these small differences can translate into millions of dollars of savings to the buyer given large volumes — a high tech firm we interacted with runs quarterly auctions in which commodity (cables, connectors, etc.) suppliers compete on unit prices in increments of one tenth of a cent.

When margins are razor-thin, factors such as transportation costs, taxes and tariffs become non-negligible (Pederson 2004). Buyers are increasingly aware of the need make sourcing decisions based on *total cost*, which from the buyer’s perspective measures the total cost of procuring from the supplier. In addition to the supplier’s price, total cost includes non-price costs such as duties and tariffs, transportation costs, shipping insurance and commissions (Ariba 2005). In this paper we introduce the idea of strategic supply base design to mitigate total procurement cost shocks, and examine how the buyer’s optimal supply base design is affected by the buyer’s bargaining clout. We now motivate and introduce both these concepts.

Supply base design to mitigate cost shocks. The “non-price costs” associated with a supplier can be closely related to the supplier’s geographic region, and thus subject to *cost shocks* affecting that region. For example, shipping costs associated with procuring from a supplier are largely affected by local logistics markets and regulations within the supplier’s region, and can be

dramatically increased by labor strikes or regulation changes. In February 2007, the CN Railway strike disabled almost three quarters of Canada's rail capacity, forcing companies such as Ford to use much more expensive alternatives like truck freight for shipments from its Canadian suppliers. Seeking heightened security for the Olympics in the summer of 2008, the Chinese government forbade a wide range of hazardous materials at six major ports; affected buyers incurred significant rerouting costs. Other examples of regional cost shocks include ocean shipping insurance rates (which are based on geopolitical and geosecurity elements along shipping routes¹) and — of course — tariff increases or punitive duties.

Ideally, a buyer could respond to regional cost shocks by instantly augmenting her supply base with new suppliers from unaffected regions. However, for some buyers this can be impractical (for all but the most catastrophic scenarios), because finding and qualifying a new supplier is usually time-consuming and costly. The process of vetting suppliers, called supplier qualification screening (Wan and Beil 2008), typically involves reference checks, financial audits, site visits to supplier facilities abroad, approval and buy-in from the buyers' internal customers, etc. At a Fortune 100 manufacturer we interacted with it takes an average of 8 to 26 weeks to find and qualify a new supplier — even for commodity parts.

Instead of frequently finding and qualifying totally new suppliers, buyers, including the large manufacturer we interacted with, build their supply base as a long-term strategic decision, and then frequently auction off short-term supply contracts among them to find the current lowest-total-cost supplier. For such buyers, therefore, an important strategic decision arises when forming their supply base: Facing potential regional cost shocks, should the buyer's supply base include *similar* suppliers (selected from the same region) or *diversified* suppliers (selected from different regions)?

Intuitively, geographically diversifying the supply base, i.e., selecting suppliers from different regions, can mitigate regional cost shocks. For example, once a prolonged labor strike at the ports in region A drives up the cost of transporting goods from the supplier in region A, a buyer who sources a large and expensive-to-transport component can avoid a high transportation cost if she has a supplier in an unaffected region B. However, a buyer seeking to minimize total procurement cost needs to take into account the impact of diversifying on her contract payment: Will the supplier in region B strategically mark up his price to make a windfall profit based on his cost advantage over the supplier in region A? If so, how should the buyer design her supply base in the first place to manage both regional-costs risks and supplier-windfall-profit-taking risks?

¹A recent example is the thousand-percent increase in shipping insurance premiums for Asia to Europe ocean transport, as freighters funneling through the Suez canal face a gauntlet of pirates and kidnappers based in an increasingly destabilized Somalia (Costello 2008).

Bargaining power and supply base design decision. In our study the buyer’s contract payment is determined through a competitive bidding process (i.e., auction). Thus, it is crucial to understand how the buyer’s ability to design auctions (i.e., choose auction format and rules) should be taken into account when she designs her supply base. We term such ability the buyer’s *bargaining power*. For forward auctions, Bulow and Klemperer (1996) point out that an auctioneer with no bargaining power can only run an English auction with no reserve price while an auctioneer with full bargaining power can utilize an optimal auction mechanism.

Similarly, in this paper, at one extreme we model a buyer with no bargaining power — such a buyer cannot make credible take-it-or-leave-it offers and must solely rely on supplier competition for price concessions, utilizing a simple reverse English auction with no reserve price. In such an auction, the lowest-total-cost supplier charges the buyer a price that is set according to second-lowest supplier’s total-cost, creating the risk of severe windfall-profit taking. Returning to our example two paragraphs above, the supplier in region B could take windfall profits and consequently the buyer’s total cost could be the total cost of the supplier in region A, which includes A’s regional cost shock! Thus, the imperative to diversify the supply base (i.e., choose suppliers from different regions) is mitigated by the need for cost parity among suppliers. We find that the optimal amount of diversification depends on the total number of suppliers and the likelihood of regional cost shocks.

At the other extreme, we model a buyer having full bargaining power, who thus can design an optimal procurement mechanism within which suppliers compete for the buyer’s business (e.g., could promise to bias against the supplier in B who has regional cost advantage). Between the two extremes are various intermediate cases, where for example the buyer is unable to use an optimal mechanism but can commit to using a reserve price and/or a first-price sealed-bid auction format. We find that supplier cost parity is less crucial for buyers with more bargaining power — such buyers are better served by a diversified supply base — and the optimal supply-base-design strategy can depend on the supplier cost distribution, and the likelihood and anticipated severity of regional cost shocks.

The next section reviews related literature, and §3 introduces the model and assumptions. Section 4 analyzes the buyer’s optimal supply-base-design problem and the effect of bargaining power. Sections 5 and 6 extend the analysis to codependent and generally distributed regional costs, respectively, and §7 concludes. All proofs are provided in the electronic companion.

2. Literature Review

Our paper analytically studies how buyers should select suppliers to mitigate regional cost risks, and is thus related to the supply risk management literature. However, our paper differs from the majority of the literature in two main aspects. First, we focus on supply risks that can be modelled as “cost shocks,” while the existing literature mainly focuses on catastrophic “supply shocks” that cause supply shortages. Such “supply shocks,” more commonly referred to as supply disruptions, include natural disasters (fire, hurricane, earthquake, etc.), supplier bankruptcy, etc. Researchers have studied various mitigation and contingency strategies to manage supply disruption risks; readers are referred to Tomlin (2006), which categorizes these strategies as stockpiling, multi-sourcing, using backup options, managing demand, and others. Among these categories, multi-sourcing and using backup options are related to supply base design. Studies on multi-sourcing to mitigate supply disruptions typically focus on buyers’ inventory management decisions (e.g., determining the optimal ordering quantity and split of quantities among suppliers) and model the impact of disruptions by various random yield models. Recent examples include Dada et al. (2007), Federgruen and Yang (2006, 2007), etc.; readers are referred to Tomlin (2006), which provides a detailed survey of early work of this stream. For works including backup options in the supply base, see, for example, Yang et al. (2008) and references therein.

Second, this paper studies price escalation risks (e.g., windfall-profit taking by suppliers), while the majority of supply risk management literature presumes exogenous contract prices (or unit procurement costs) and ignores suppliers’ strategic pricing behavior. One exception is Babich et al. (2007), which endogenizes suppliers’ pricing decisions in a multi-sourcing problem where a buyer allocates ordering quantities among suppliers with correlated default risks. They assume that suppliers have full information of competitors’ costs, and show how suppliers’ pricing decisions can be affected by their default risk correlations. In particular, they find that the buyer prefers suppliers with positively correlated default risks despite the loss of diversification benefits, because default risk correlation increases supplier competition. In our paper, which studies the supply base design problem in the presence of suppliers’ regional cost risks, we model supplier competition via procurement auctions in which suppliers possess private cost information, and we show how supplier competition can be affected by correlations across suppliers’ cost shocks. We find that the buyer’s bargaining power dictates her preference for the supply base design, namely, a buyer with stronger bargaining power prefers a more diversified supply base, which effects less correlation across suppliers’ cost shocks.

The term “bargaining power” is probably one of the most widely used but vaguely defined concepts in the literature of bargaining models. The asymmetric Nash bargaining model (Roth 1979), which extends the classic symmetric Nash bargaining model (Nash 1950) by including weighting scalars in the calculation of utility products, is widely used by modelers to “capture some imprecisely defined ‘bargaining power’ ” (Binmore et al. 1986). Although the weighting scalars of the asymmetric Nash bargaining models do help map the degree of a player’s bargaining power to bargaining outcomes — a larger scalar represents higher bargaining power and leads to a more favorable outcome — such mappings cannot easily be extended to more complicated settings where bargaining players have incomplete information. Instead, the literature on bargaining games with incomplete information focuses on analyzing bargaining outcomes given different bargaining mechanisms without explicitly defining players’ “bargaining power.” Readers are referred to Ausubel et al. (2002) for a detailed survey of literature on bargaining with incomplete information. In the present paper, we interpret the term “bargaining power” as the buyer’s ability to impose an auction mechanism that she favors, an interpretation that can be traced to the prominent work of Bulow and Klempeper (1996). In other words, we use the term “bargaining power” as a way to rank the auction mechanisms that we study in this paper.

Extensive work has examined procurement cost reduction via supply base competition; Elmaghraby (2000) provides a comprehensive survey of early work on competitive sourcing strategies including auctions. Our paper considers a buyer who finds the lowest-price provider by auctioning off short-term supply contracts. Methodologically, our paper is related to the auction and mechanism design literature; readers are referred to the books by Krishna (2002) and Milgrom (2004), which provide excellent treatments and detailed references on auction theory.

3. Model and Preliminaries

We study a stylized model in which a risk-neutral buyer selects a cohort of N qualified suppliers to form a supply base, to which short-term supply contracts will be auctioned off in periods $t = 1, 2, \dots, L$, where L represents the length of the buyer’s planning horizon. At period $t = 0$, the buyer designs the supply base as a one-time decision; in other words, we assume that no suppliers are removed from or added to the supply base in the following L periods. To focus on the supply base diversification decision, we assume that N is exogenously given. Suppliers can be selected from different geographic *regions*. The buyer’s decision variables are the number of regions to select suppliers from, R , and the number of suppliers to select from each region, denoted by n_1, n_2, \dots, n_R

for region 1, region 2. . . , and region R , respectively, where $\sum_{r=1}^R n_r = N$. We assume that there are at least N symmetric regions available and within each region up to N suppliers can be found (the analysis changes in a straightforward way if these assumptions are relaxed; see our discussion in §7). Thus, the number of regions R can be any integer from 1 to N ; in particular, $R = 1$ means selecting all suppliers from only one region, which we call the *pooling strategy*, and $R = N$ means selecting each supplier from a different region, which we call the *fully diversifying strategy*.

After establishing her supply base (finding and pre-qualifying the suppliers), in each period the buyer runs an auction to award an indivisible short-term contract to one of the suppliers in the supply base. This setup is most appropriate when the buyer procures commodity parts from suppliers, who do not fully rely on the buyer's contract to keep afloat. To keep the analysis focused and tractable, we assume that the buyer does not store inventory and does not have in-house production, hence she must contract with one supplier in every period. This setup could model, for example, a buyer who produces high tech, short life-cycle products, relies on suppliers for key components, and holds quarterly supply auctions. When analyzing auction outcomes we assume that the suppliers are risk-neutral and fully rational players following a Bayesian Nash bidding equilibrium, as is standard in the auction literature.

Two types of costs are associated with each supplier $i = 1, \dots, N$ in each period t . The first type of cost is an idiosyncratic production cost, $x_i^t \in [0, 1]$, which as typical in auction models is assumed to be independently and identically distributed across suppliers according to a commonly known distribution F . Cost x_i represents supplier i 's firm-specific and privately known cost of fulfilling the contract offered in period t , per supplier i 's inventory level, capacity utilization, working capital position, debt status, etc. For simplicity we assume F has a positive and continuous density f and is stationary over time (this assumption can be relaxed; see our discussion in §7). As is standard in the auction theoretic literature, we also assume that $x + \frac{F(x)}{f(x)}$ and $x - \frac{1-F(x)}{f(x)}$ are increasing in x .² These technical assumptions are satisfied, for example, if f is logconcave, which includes uniform, normal, logistic, and exponential distributions; see Bagnoli and Bergstrom (2005).

The second type of cost is a region-specific cost, y_r^t , which represents (from the buyer's perspective) common costs affecting all suppliers in region r in period t . In our analysis, we assume such regional costs are not related to suppliers' production costs but are the additional procurement expenses the buyer incurs when doing business with a supplier in the region, for instance, covering logistics costs, local taxes, and trade tariffs. (Our results easily extend to cases where regional

²Technically, the assumption on $x + \frac{F(x)}{f(x)}$ is only needed for the optimal mechanism analyses, while the assumption on $x - \frac{1-F(x)}{f(x)}$ is only needed for the asymmetric sealed-bid first-price analysis in Proposition 2.

factors also influence suppliers' production costs; see our discussion in §7.) Because we are interested in significant cost shocks (such as a labor strike at a port, or a large import tariff increase), we assume that y_r^t 's are commonly observable at the outset of period t and are independently distributed according to a two-point distribution with a probability mass $p \in (0, 1)$ at $s > 0$ and a probability mass $1 - p$ at 0. Namely, $y_r^t = s$ models a scenario where some significant cost shock occurs in region r and hence drives up the regional cost to a *high* level; $y_r^t = 0$ models a scenario where no significant cost shock occurs in region r and the regional cost is at a *low* level. Hence, p models the likelihood of regional cost shocks, and s models the *relative scale* of regional cost shocks compared to the dispersion of suppliers' production cost distribution (given that the range of x_i 's is normalized to one).³ Sections 5 and 6 discuss how our results extend to cases where regional costs are possibly codependent or follow a general distribution. We let a_i^t denote the regional cost of supplier i in period t , that is, $a_i^t = y_r^t$ if supplier i is located in region r .

The buyer seeks to minimize her expected long-term total procurement cost. Thus, the supply base design problem can be formulated as follows:

$$\begin{aligned} \min_{n_1, \dots, n_R} E \left[\sum_{t=1}^L \beta^t \pi(\mathbf{x}^t; \mathbf{a}^t) \right] &= \frac{\beta(1 - \beta^L)}{1 - \beta} E [\pi(\mathbf{x}^1; \mathbf{a}^1)] & (P) \\ \text{s.t. } R \in \{1, \dots, N\}, \quad n_i \in \mathbb{N} \quad \forall i \in \{1, \dots, R\}, \quad \text{and } n_1 + \dots + n_R &= N, \end{aligned}$$

where $\mathbf{x}^t \stackrel{\text{def}}{=} (x_1^t, x_2^t, \dots, x_N^t)$ is the vector of realized supplier production costs in period t , $\mathbf{a}^t \stackrel{\text{def}}{=} (a_1^t, a_2^t, \dots, a_N^t)$ is the vector of realized regional costs in period t , β is the discount factor, and $\pi(\cdot)$ is the buyer's per period total procurement cost. Since x_i^t 's and a_i^t 's are assumed to be identically distributed from period to period, the buyer's objective is simplified to minimizing the expected one-period total procurement cost. Therefore, we omit the superscript t for notational convenience in the rest of this paper. Throughout the paper, $\bar{X}_{k:N}$ denotes the expected k^{th} -lowest order statistic out of N independent and identical random draws from distribution F , and $\mathbb{I}_{\{A\}}$ denotes the indicator function of event A .

Because the buyer must contract with a supplier, if the buyer can use a reserve price, it is always optimal to set it at the worst possible supplier cost type, i.e., at 1. This reserve price can be instrumental in mitigating windfall profits by a winning supplier, and we explore its effect — along with other auction format decisions — on the buyer's supply base design problem. More

³For example, suppose suppliers' actual production costs for the contract fall between \$180,000 and \$220,000, and a regional shock — perhaps the need for air shipments if the region's ocean port is on strike — would add \$30,000 to the total cost of doing business with a supplier. Because F 's domain is normalized to the unit interval and suppliers' production cost dispersion is $(\$220,000 - \$180,000) = \$40,000$, s would be correspondingly normalized to $(\$30,000 \div \$40,000) = 0.75$.

specifically, to explore the effect of the buyer’s bargaining power, we examine the buyer’s optimal supply-base-design strategy under the following auction mechanisms.

Reverse English auction without/with reserve (RE/RER). In the extreme case where the buyer does not have any bargaining power, she can only demand price concessions on the basis of competing offers from suppliers and cannot credibly impose a reserve price. Thus, the contract award and payment decisions can be modelled as outcomes of a reverse English total-cost auction without a reserve price (RE). In such an auction, a supplier’s regional cost is added to his price bid to yield a total-cost bid. For instance, a supplier with a regional cost of \$8,000 who bids \$100,000 for the contract has a total-cost bid of \$108,000. During the auction, suppliers can see the current lowest total-cost bid, and can respond by lowering their own price bid if they are not the current lowest total-cost bidder. The auction ends when bidding stops, and the lowest total-cost bidder at the end of the auction wins the contract and is paid his price bid. (In this paper we assume that ties in an auction are broken randomly.) In such an auction, it is a weakly dominant strategy for a supplier to bid down to his true total cost $x_i + a_i$ (although he may not have to); see, for example, Maskin and Riley (2000). We also study cases where the buyer has some bargaining power and can impose a credible reserve price whereby price bids above 1 are rejected (RER); in such a case, it remains optimal for bidders to bid down to their true costs before dropping out.

First-price sealed-bid auction without/with reserve (FS/FSR). When the buyer has the ability to make a binding commitment to reject re-negotiation offers, she can hold a first-price sealed-bid auction with no reserve price (FS). In such an auction, each supplier i makes a single, “best and final” sealed price bid, denoted by b_i . The contract winner is supplier j with the lowest total-cost sealed bid, that is, $j = \arg \min_{i=1 \dots N} \{b_i + a_i\}$, and the winner j is paid his price bid b_j . We will let the Bayesian Nash equilibrium strategy followed by supplier i be denoted by $b_i = \beta_i^{FS}(x_i)$. In our analysis, we also study cases where the buyer can impose a credible reserve price when using a first-price sealed-bid auction whereby price bids above 1 are rejected (FSR). In such a case, the equilibrium bidding strategy is denoted by $b_i = \beta_i^{FSR}(x_i)$. In general, strategies β^{FS} and β^{FSR} are quite difficult to derive in closed-form due the ex ante (regional cost) asymmetry between bidders. Fortunately, we are interested in characterizing the buyer’s optimal diversification decision, for which a closed-form expression for suppliers’ bidding strategies is not always needed.

Optimal mechanism (OPT). When the buyer has full bargaining power, she can offer suppliers a join-or-leave-it mechanism such that all suppliers will participate and meanwhile the buyer's expected total procurement cost is minimized. We refer to such a mechanism as the *optimal mechanism* (OPT). Let $\psi(x_i) \stackrel{def}{=} x_i + \frac{F(x_i)}{f(x_i)}$, which is commonly referred to as supplier i 's *virtual cost* in the mechanism design literature, and let $\psi(x_i) + a_i$ denote supplier i 's *adjusted virtual cost*, that is, supplier i 's virtual cost adjusted by the additive regional cost a_i . Since $\psi(x_i)$ is increasing the optimal mechanism has a pure strategy implementation as follows. The buyer awards the contract to supplier j having the lowest adjusted virtual cost, i.e., $j = \arg \min_{i=1 \dots N} \{\psi(x_i) + a_i\}$, breaking ties evenly, and pays the contract winner $\min\{\psi^{-1}[\psi(x_{j_1}) + a_{j_1} - a_j], 1\}$, where $j_1 = \arg \min_{i=1 \dots N, i \neq j} \{\psi(x_i) + a_i\}$ is the losing supplier with the lowest adjusted virtual cost. The payment is truncated from above at 1 by using a reserve price of 1. The optimality of this mechanism is straightforward to prove using the biased procurement auctions analysis of Rezende (2008).

4. Analysis

4.1 General framework: Expected total cost function $T_N(N_0)$

Since suppliers' production costs are ex ante symmetric, we have $E[\pi(\mathbf{x}, \hat{\mathbf{a}})] = E[\pi(\mathbf{x}, \tilde{\mathbf{a}})]$ for any regional costs realizations $\hat{\mathbf{a}}$ and $\tilde{\mathbf{a}}$ such that $\sum_{i=1}^N \mathbb{I}_{\{\hat{a}_i=0\}} = \sum_{i=1}^N \mathbb{I}_{\{\tilde{a}_i=0\}}$.

Definition 1 We define $T_N(N_0) \stackrel{def}{=} E \left[\pi(\mathbf{x}; \mathbf{a}) \middle| N_0 = \sum_{i=1}^N \mathbb{I}_{\{a_i=0\}} \right]$ as the *expected total cost function*.

In words, $T_N(N_0)$ is the buyer's conditional expected total procurement cost given N_0 suppliers with low regional cost and $N - N_0$ suppliers with high regional cost. Given any N_0 , the value of $T_N(N_0)$ depends on the auction mechanism used, the cost shock scale s , and the suppliers' production cost distribution F , but is independent from the buyer's supply-base-design strategy (n_1, \dots, n_R) and the cost shock likelihood p . However, the supply base design strategy determines the correlation between the a_i 's, and hence intimately affects N_0 ; in particular, $N_0 = \sum_{r=1}^R n_r \cdot \mathbb{I}_{\{y_r=0\}}$. Thus, the buyer's supply base design strategy, together with the cost shock likelihood p , determines the distribution of N_0 . For example, the pooling strategy generates a distribution with probability mass p at $N_0 = 0$ and with probability mass $1 - p$ at $N_0 = N$, while the fully diversifying strategy generates a distribution with probability mass $\binom{N}{N_0} p^{N-N_0} (1-p)^{N_0}$ at N_0 , for $N_0 = 0, 1, \dots, N$.

Therefore, the buyer's supply base design problem (P) in §3 can be reformulated as follows:

$$\begin{aligned}
& \min_{n_1, \dots, n_R} E[T_N(N_0)] & (P') \\
& s.t. \quad N_0 = \sum_{r=1}^R n_r \cdot \mathbb{I}_{\{y_r=0\}}, \\
& R \in \{1, \dots, N\}, \quad n_i \in \mathbb{N} \quad \forall i \in \{1, \dots, R\}, \text{ and } n_1 + \dots + n_R = N.
\end{aligned}$$

4.2 Expected total cost function $T_N(N_0)$ for the five mechanisms

Using the setup of §4.1, we can explore the effect of the buyer's bargaining power on the optimal supply-base-design strategy by examining the buyer's conditional expected total procurement cost function $T_N(N_0)$ under a reverse English auction with no reserve price, a reverse English auction with reserve price, a first-price sealed-bid auction with no reserve price, a first-price sealed-bid auction with reserve price, and the optimal mechanism, denoted by $T_N^{RE}(N_0)$, $T_N^{RER}(N_0)$, $T_N^{FS}(N_0)$, $T_N^{FSR}(N_0)$, and $T_N^{OPT}(N_0)$, respectively.

Given an auction format, we will find the buyer's expected payment by computing the expected payment of an associated, outcome-equivalent direct mechanism in which suppliers truthfully reveal their production cost. The existence of such an outcome-equivalent, truthful direct mechanism is guaranteed by the revelation principle (e.g., Gibbard 1973, Green and Laffont 1977, Dasgupta et al. 1979). In a direct mechanism each supplier $i = 1, \dots, N$ chooses a "message" $z_i \in [0, 1]$ to send to the auctioneer, who then determines the contract winner and payment(s). A direct mechanism can be described by specifying, for each supplier i , given the message vector $\mathbf{z} = (z_1, \dots, z_N)$ and regional costs \mathbf{a} , a mapping $Q_i(\mathbf{z}; \mathbf{a}) \rightarrow [0, 1]$ describing the probability that supplier i wins the contract, and a mapping $M_i(\mathbf{z}; \mathbf{a}) \rightarrow \mathbb{R}$ describing the expected payment from the buyer to supplier i . For example, for a first-price sealed-bid auction without a reserve price, its outcome-equivalent truthful direct mechanism (in which $\mathbf{z} = \mathbf{x}$) sets $Q_j^{FS}(\mathbf{z}; \mathbf{a}) = 1$ and $M_j^{FS}(\mathbf{z}; \mathbf{a}) = \beta_j^{FS}(z_j)$ if $j = \arg \min_{i=1, \dots, N} \{\beta_i^{FS}(z_i) + a_i\}$, and sets Q_j and M_j to zero otherwise.

For any supplier i , let \mathbf{x}_{-i} be the vector \mathbf{x} excluding the element x_i , and let \mathbf{F}_{-i} be the joint distribution of \mathbf{x}_{-i} . Given a regional costs realization \mathbf{a} , let $m_i(z_i; \mathbf{a}) \stackrel{def}{=} \int_{\mathbf{x}_{-i}} M_i(z_i, \mathbf{x}_{-i}; \mathbf{a}) d\mathbf{F}_{-i}(\mathbf{x}_{-i})$ be the expected payment to supplier i when i reports his cost as z_i and all other suppliers report their true costs, and likewise define $q_i(z_i; \mathbf{a}) \stackrel{def}{=} \int_{\mathbf{x}_{-i}} Q_i(z_i, \mathbf{x}_{-i}; \mathbf{a}) d\mathbf{F}_{-i}(\mathbf{x}_{-i})$ to be the probability that supplier i is awarded the contract when i reports his cost as z_i and all other suppliers report their true costs. We will use this machinery to derive expressions for the buyer's expected total procurement cost, that is, the expected contract payment plus the expected regional costs.

Expected regional costs. Given that a_i is the regional cost associated with supplier i , and $Q_i(\mathbf{x}; \mathbf{a})$ is the probability that supplier i is awarded the contract, it is easy to see that for a truthful direct mechanism the buyer's expected regional cost equals

$$\sum_{i=1}^N \int_{\mathbf{x}} a_i Q_i(\mathbf{x}; \mathbf{a}) d\mathbf{F}(\mathbf{x}). \quad (1)$$

Expected contract payment. Given regional cost shock realizations \mathbf{a} , supplier i 's expected profit in a truthful direct mechanism can be written as

$$U_i(x_i; \mathbf{a}) \stackrel{def}{=} m_i(x_i; \mathbf{a}) - x_i q_i(x_i; \mathbf{a}) = \max_{z_i} \{m_i(z_i; \mathbf{a}) - x_i q_i(z_i; \mathbf{a})\}.$$

Applying the envelope theorem yields $\frac{\partial U_i(x_i; \mathbf{a})}{\partial x_i} = -q_i(x_i; \mathbf{a})$ for $x_i \in [0, 1]$. Treating this as a differential equation and using the expected profit at $x_i = 1$ as an integration constant yields an equation for supplier i 's equilibrium expected profit:

$$m_i(x_i; \mathbf{a}) - x_i q_i(x_i; \mathbf{a}) = \int_{z_i=x_i}^1 q_i(z_i; \mathbf{a}) dz_i + U_i(1; \mathbf{a}). \quad (2)$$

The term $U_i(1; \mathbf{a})$ represents the equilibrium expected profit for supplier i 's worst possible production cost type, namely 1. Clearly, for any of the five auction formats, this expected profit will be zero for all i whenever $N_0 \neq 1$ — in such cases, a supplier with worst production cost type is certain to face a competitor having no worse regional cost and no worse production cost, and hence will always lose the auction (or earn zero profit). Also, for any auction with a reserve price of 1 (RER, FSR, and OPT) we conclude that $U_i(1; \mathbf{a})$ will be zero — price bids above 1 are rejected, so any supplier whose cost equals 1 will earn zero profit. However, the story is different for auctions RE and FS when $N_0 = 1$. Consider a case in which $N_0 = 1$ and $a_i = 0$, so even if supplier i is of worst production cost type, $x_i = 1$, he still has a regional cost advantage because he is the only supplier whose region did not experience a cost shock. In auction RE, for instance, with $x_i = 1$ supplier i wins the auction if all $N - 1$ other suppliers have production costs exceeding $1 - s$, and thus in expectation supplier i earns a positive expected profit $E[\max\{0, X_{1:N-1} + s - 1\}]$. Furthermore, this positive expected profit “inflates” the expected profit of all cost types of such a supplier i , per equation (2). Similar reasoning applies to format FS.

Thus, we define $\Gamma \stackrel{def}{=} U_i(1; \mathbf{a})$ as the *status rent* for the supplier i having $a_i = 0$ when $N_0 = 1$. As shorthand we will simply refer to such a supplier as “advantaged.” In words, status rent represents the profit accruing to an advantaged supplier (all cost types) based upon his unilateral status as the only supplier not experiencing a cost shock.

Solving equation (2) for m_i , the expected contract payment to supplier i , and then integrating over \mathbf{x} and summing over all suppliers yields an expression for the buyer's expected contract payment:

$$\sum_{i=1}^N \int_{\mathbf{x}} Q_i(\mathbf{x}; \mathbf{a}) \psi(x_i) d\mathbf{F}(\mathbf{x}) + \sum_{i=1}^N U_i(1; \mathbf{a}). \quad (3)$$

Combining equations (1) and (3), and applying the definition of Γ yields the following lemma. Without loss of generality, we label the suppliers such that $a_i = 0$ for $i = 1, \dots, N_0$ and $a_i = s$ for $i = N_0 + 1, \dots, N$.

Lemma 1 *Given a realized regional cost vector \mathbf{a} , the buyer's expected total procurement cost is*

$$T_N(N_0) = \sum_{i=1}^{N_0} \int_{\mathbf{x}} Q_i(\mathbf{x}; \mathbf{a}) \psi(x_i) d\mathbf{F}(\mathbf{x}) + \sum_{i=N_0+1}^N \int_{\mathbf{x}} Q_i(\mathbf{x}; \mathbf{a}) [\psi(x_i) + s] d\mathbf{F}(\mathbf{x}) + \Gamma \cdot \mathbb{I}_{\{N_0=1\}},$$

where allocation probability Q_i and status rent Γ for mechanisms RE, RER, FS, FSR, and OPT are as follows:

Mechanism	$Q_i(\mathbf{x}; \mathbf{a}) = 1$, if	Status rent, Γ
RE	$x_i + a_i < x_j + a_j, \forall j \neq i$	$E[\max\{0, X_{1:N-1} + s - 1\}]$
RER	$x_i + a_i < x_j + a_j, \forall j \neq i$	0
FS	$\beta_i^{FS}(x_i) + a_i < \beta_j^{FS}(x_j) + a_j, \forall j \neq i$	$(\bar{b} - 1)[1 - F(\bar{b} - s)]^{N-1} \dagger$
FSR	$\beta_i^{FSR}(x_i) + a_i < \beta_j^{FSR}(x_j) + a_j, \forall j \neq i$	0
OPT	$\psi(x_i) + a_i < \psi(x_j) + a_j, \forall j \neq i$	0

$$\dagger \bar{b} = \max\left\{s, 1 + \frac{1 - F(\bar{b} - s)}{(N-1)f(\bar{b} - s)}\right\}.$$

Hence, given an auction mechanism, the buyer's expected total cost is determined by the allocation rule and the status rent. When an advantaged supplier emerges (i.e., when $N_0 = 1$), the buyer expects to forfeit a *windfall profit* to the supplier. Windfall profit consists of two parts. First, the advantaged supplier wins more often (for more values of x_{-i}) with his regional cost advantage (than without such), per any of the five auction formats' allocation rules; thus, he has more chance to earn profits. Second, the advantaged supplier extracts a status rent Γ . However, the auction format affects the windfall profit-taking. For example, we compare formats OPT and RE. First, note that the advantaged supplier wins less often (for fewer values of x_{-i}) under OPT compared with RE. This is because OPT optimally biases *against* the advantaged supplier.⁴ Second, the advantaged supplier earns a positive status rent in RE, but earns zero status rent in format OPT.

⁴Under format OPT the buyer compares suppliers via adjusted virtual cost $\psi(x_i) + a_i$. This biases against the advantaged supplier — for example, when $F \sim U[0, 1]$, $a_1 = 0$, and $a_2 = s$, the buyer compares $2x_1$ with $2x_2 + s$, which means that only half of supplier 1's regional cost advantage is admitted by the buyer.

In the remainder of this section we will see how the buyer's optimal supply base design decision is driven by her auction format's ability to curb windfall profit-taking. We begin with a description of the buyer's diversification tradeoff in the next subsection. We then apply this to characterize the optimal supply base design decision for two-suppliers (§4.4) and N -suppliers (§4.5).

4.3 Diversification Tradeoff

Suppose the buyer considers changing her R -region diversification strategy (n_1, n_2, \dots, n_R) to the $(R+1)$ -region strategy $(n_1, n_2, \dots, n_{R-1}, \tilde{n}_R, \tilde{n}_{R+1})$ such that $n_R = \tilde{n}_R + \tilde{n}_{R+1}$. With probability $p^2 + (1-p)^2$, regions R and $(R+1)$ experience equal regional costs — in such a case the $(R+1)$ -region strategy would yield the same cost as the R -region strategy. However, with probability $p(1-p)$ region R experiences a cost shock but region $(R+1)$ does not — in such a case the $(R+1)$ -region strategy would be expected to save the buyer money, resulting in a *diversifying upside*. Conversely, with probability $(1-p)p$ region R experiences no cost shock but region $(R+1)$ does — in such a case, the $(R+1)$ -region strategy would result in an expected disbenefit for the buyer, the *diversifying downside*. Let $\hat{N}_0 \stackrel{def}{=} \sum_{r=1}^{R-1} n_r \cdot \mathbb{I}_{\{y_r=0\}}$, a random variable representing the number of suppliers with low regional cost in regions 1 through $(R-1)$. The diversifying upside and downside depend on \hat{N}_0 . For a given \hat{N}_0 we can write

$$\begin{aligned} \text{diversifying upside} &= T_N(\hat{N}_0) - T_N(\hat{N}_0 + \tilde{n}_{R+1}), \text{ and} \\ \text{diversifying downside} &= T_N(\hat{N}_0 + \tilde{n}_R) - T_N(\hat{N}_0 + n_R). \end{aligned}$$

Definition 2 Let $\Delta(\hat{N}_0) \stackrel{def}{=} T_N(\hat{N}_0) - T_N(\hat{N}_0 + \tilde{n}_{R+1}) - T_N(\hat{N}_0 + \tilde{n}_R) + T_N(\hat{N}_0 + n_R)$ denote the *diversification tradeoff*, that is, the *diversifying upside* minus the *diversifying downside*.

Since both the upside and the downside occur with probability $p(1-p)$, the buyer's expected change in profit as a result of switching to the $(R+1)$ -region strategy is $p(1-p)E_{\hat{N}_0}[\Delta(\hat{N}_0)]$ and the buyer prefers the $(R+1)$ -region strategy if $E_{\hat{N}_0}[\Delta(\hat{N}_0)] \geq 0$. Thus, Δ encapsulates the diversification tradeoff for the buyer, as we will see in the following sections.

4.4 Optimal supply base design with two suppliers

4.4.1 Optimal supply base design driven by shape of $T_2(N_0)$

When a buyer designs a supply base with two suppliers, she chooses either to select both suppliers from the same region — the pooling strategy, or to select two suppliers from two different regions — the diversifying strategy. Per the diversification tradeoff discussed in §4.3, we have $\hat{N}_0 \equiv 0$ and

$E_{\hat{N}_0}[\Delta(\hat{N}_0)] = \Delta(0) = T_2(0) - 2T_2(1) + T_2(2)$. Consequently, the buyer’s diversification preference is driven solely by the size of $T_2(1)$ versus $T_2(0)$ and $T_2(2)$.

Lemma 2 *With two suppliers, for any cost shock likelihood p , it is optimal to pool ($R = 1$) if $T_2(1) > \frac{T_2(0)+T_2(2)}{2}$, while it is optimal to diversify ($R = 2$), otherwise.*

4.4.2 Characterizations of $T_2(N_0)$ for the five mechanisms

Lemma 1 reveals (see e-companion EC.4) that

$$T_2^{Mech}(0) = \bar{X}_{2:2} + s \quad \text{and} \quad T_2^{Mech}(2) = \bar{X}_{2:2}, \quad \text{for all } Mech \in \{RE, RER, FS, FSR, OPT\}. \quad (4)$$

In words, the five auction mechanisms are cost-equivalent when both suppliers have high or both have low regional costs, that is, when $N_0 = 0$ or 2. When suppliers’ regional costs “are a wash” they instead must compete against each other solely on the basis of production costs. In such a case, each of the five auction formats will allocate the contract to the lowest-production-cost supplier and no windfall profit will be made. It is straightforward to see this as an occurrence of the revenue equivalence theorem. In contrast, the auction format drives the buyer’s expected total cost if one supplier has a high regional cost and the other (the advantaged supplier) has a low regional cost, that is, when $N_0 = 1$. In such a case, formats RE and FS yield a status rent and hence a large windfall profit. Formats RER, FSR and OPT use a reserve price to avoid paying a status rent, and moreover format OPT applies the optimal allocation rule to bias against the advantaged supplier, and hence outperforms the suboptimal mechanism RER and FSR.⁵ Combining Lemma 2 with equation (4) gives

Corollary 1 *For any of the five auction mechanisms, it is optimal to pool if $T_2(1) > \bar{X}_{2:2} + \frac{s}{2}$; otherwise, it is optimal to diversify.*

The buyer’s supply base diversification decision depends on the total procurement cost it expects to pay when facing an advantaged supplier, versus the cost it would expect to pay when both, or neither, suppliers experience a cost shock. *Note that Corollary 1 holds for any supplier cost distribution F and any regional cost shock probability p .* In the following, we first characterize the optimal supply-base-design strategy for the two extreme cases of buyer bargaining power, that is, RE and OPT. We then examine the cases where the buyer has intermediate bargaining power and can impose a reserve price and/or run a first-price sealed-bid auction (RER, FS, and FSR). In

⁵Note that, for example, $T_2^{OPT}(1) \leq T_2^{RER}(1) < T_2^{RE}(1)$ does not contradict the revenue equivalence theorem, whose ex ante symmetric bidder assumption does not apply when $N_0 = 1$.

so doing, we obtain a view of the optimal supply-base-design strategy across the spectrum of the buyer’s bargaining power.

4.4.3 Optimal supply base design: Mechanisms RE and OPT

The following proposition completely characterizes the buyer’s optimal supply-base design decision under zero and full bargaining power, corresponding to mechanisms RE and OPT, respectively.

Proposition 1 *With two suppliers, for any p , s , or F , it is optimal to pool if mechanism RE is used, and it is optimal to diversify if mechanism OPT is used.*

Surprisingly, with two suppliers, Proposition 1 shows that the buyer *always* prefers to diversify if she has full bargaining power and will *always* prefer the opposite if she has no bargaining power. Perhaps more surprising, this complete dichotomy between strategies persists for any supplier cost distribution, any cost shock probability, and any cost shock size. Why is it never optimal to diversify when the buyer uses a reverse English auction without a reserve, even if large supply shocks are very likely? The intuition is that, without a reserve price, the English auction mechanism fully exposes the buyer to windfall profit-taking by an advantaged supplier, who largely absorbs what would have been the buyer’s upside benefit of diversifying. The diversifying strategy saddles the buyer with downside risk but very little upside benefit, prompting her to prefer the pooling strategy. In contrast, a buyer with strong bargaining power can curtail windfall profit-taking by the advantaged supplier, and as a result enjoys a robust upside benefit from diversifying — so much so that a buyer with full bargaining power always finds it preferable to diversify.

We now study the performance gap between pooling and diversifying under these two mechanisms. Figure 1 assumes $F \sim U[0, 1]$ and plots the *optimality gap* versus the cost shock likelihood p for two different cost shock sizes s , where the optimality gap is defined as:

$$\text{Optimality gap} = \frac{\text{Expected total cost under the suboptimal strategy}}{\text{Expected total cost under the optimal strategy}} - 1. \quad (5)$$

The rainbow-shape of Figure 1’s plots reveals that under both mechanisms the optimality gap between the pooling and diversifying strategies is maximized when the cost shock likelihood p is moderate, which is intuitive since when p is either very small or very large the two strategies effectively differ little from each other (in terms of the generated distribution of N_0). Figure 1 also reveals that the optimality gap increases in the cost shock size s . This is because the diversification tradeoff (diversifying upside minus downside), $2\bar{X}_{2:2} + s - 2T_2(1)$, is monotonic in the cost shock size s . For format OPT, the tradeoff is positive and increases in s because the biasing against the

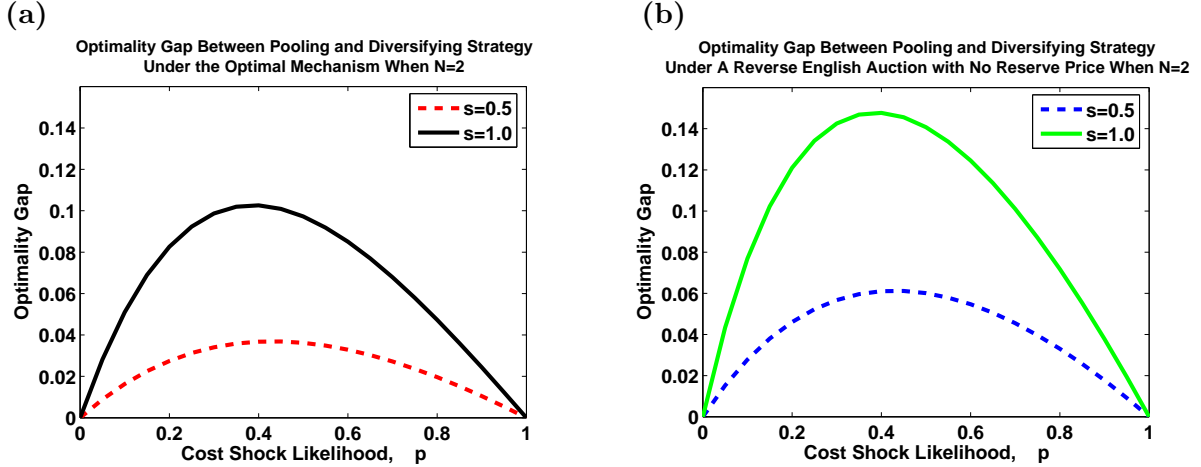


Figure 1: Optimality gap between pooling and diversifying when $N = 2$, per equation (5), assuming $F \sim U[0, 1]$. Panel (a) shows the optimality gap under the optimal mechanism, and panel (b) shows the optimal gap under a reverse English auction with no reserve price.

advantaged supplier (see Footnote 4) and the reserve price mute the advantaged supplier’s profits (hence, mutes the impact of s on $T_2^{OPT}(1)$); in contrast, under format RE the advantaged supplier’s profits balloon with the size of his regional cost advantage s . Because s normalizes regional cost relative to the dispersion of suppliers’ production costs, the implication is that the supply base design decision is particularly important when regional cost shocks can overwhelm the differences in production costs — when $s = 1$, the pooling strategy saves the buyer as much as 14% when she uses the RE format.

4.4.4 Optimal supply base design: Mechanisms FS, RER and FSR

§4.4.3 showed that the buyer prefers to pool under format RE because she is unable to prevent severe windfall profit-taking by the advantaged supplier, i.e., $T_2^{RE}(1)$ is large. Although the buyer may be unable to implement the pairwise bias schedules necessary to employ the optimal mechanism (see Footnote 4), it is reasonable to suspect that other mechanisms which employ some degree of “bargaining power” in the form of credible threats may help the buyer curtail windfall supplier profits. One such mechanism is the first-price sealed-bid format, in which the buyer requires “best and final” bids from suppliers and refuses to renegotiate after opening the bids and declaring a winner. This puts pressure on the advantaged supplier, who must balance a desire for a larger profit margin (placing a high bid) against the risk of losing the contract altogether (being underbid).

Effect of using a sealed-bid auction (FS). The following proposition shows that the buyer

can make herself better off by running a first-price sealed-bid auction rather than a reverse English auction but she may still need to pool her supply base to prevent severe windfall profit-taking by an advantaged supplier.

Proposition 2 *With two suppliers, the buyer prefers using mechanism FS to mechanism RE when $s \geq 1 + \frac{1}{f(0)}$. Whenever FS is used, it is optimal to pool when $s \geq 2\bar{X}_{2:2}$.*

When facing an advantaged supplier ($N_0 = 1$), the buyer would prefer using FS instead of RE if the former yields a lower expected total procurement cost, that is, $T_2^{FS}(1) < T_2^{RE}(1)$. (Recall that by (4) all formats yield the same expected total procurement cost for the buyer when $N_0 \neq 1$.) Proposition 2 proves that the buyer indeed prefers FS when the cost shock scale s is sufficiently large; when $s \geq 1 + \frac{1}{f(0)}$, the advantaged supplier in the FS auction finds it optimal to use a “lock in” bidding strategy (ensure he wins the contract) by submitting a bid exactly equal to s . In so doing, the advantaged supplier knows he leaves money on the table (his competitor’s total-cost is at least s), but he does so anyways to avoid losing his chance at an already sizeable profit margin (recall that the advantaged supplier’s own cost is between zero and one, so bidding $s > 1$ guarantees him profit). For small s the same reasoning applies and the advantaged supplier strategically curtails his own profit, but analytical comparisons of the FS and RE’s expected total costs are precluded by the intractability of suppliers’ bidding strategies in FS auctions, which involve non-trivial differential equations (see e-companion EC.1). However, our numerical studies indicate that the buyer still prefers FS to RE no matter what the size of s ; for example, see Figure 2, which demonstrates this for various supplier cost distributions.

Despite the fact that Proposition 2 shows that a first-price sealed-bid auction can make a buyer better off in the presence of an advantaged supplier, it nonetheless shows a persistence in the buyer’s preference for pooling. This is because windfall-profit taking is still severe if a first-price sealed-bid auction is used without reserve price — an advantaged supplier will always bid no less than s , and thus when s is sufficiently large, e.g., $s \geq 2\bar{X}_{2:2}$, we have $T_2^{FS}(1) > s > \bar{X}_{2:2} + \frac{s}{2}$, which by Corollary 1 implies that pooling is optimal. Our numerical studies show that pooling remains optimal under the FS format even when the cost shock scale s is small; see, for example, Figure 2 which shows that the pooling condition $T_2^{FS}(1) > \bar{X}_{2:2} + \frac{s}{2}$ persists for a wide range of s .

Effect of using a reserve price (RER, FSR). Setting a reserve price is another way that a buyer might curtail windfall supplier profits. However, to be effective the buyer must promise to not award the contract to any supplier if the reserve is not met. This means that the buyer commits

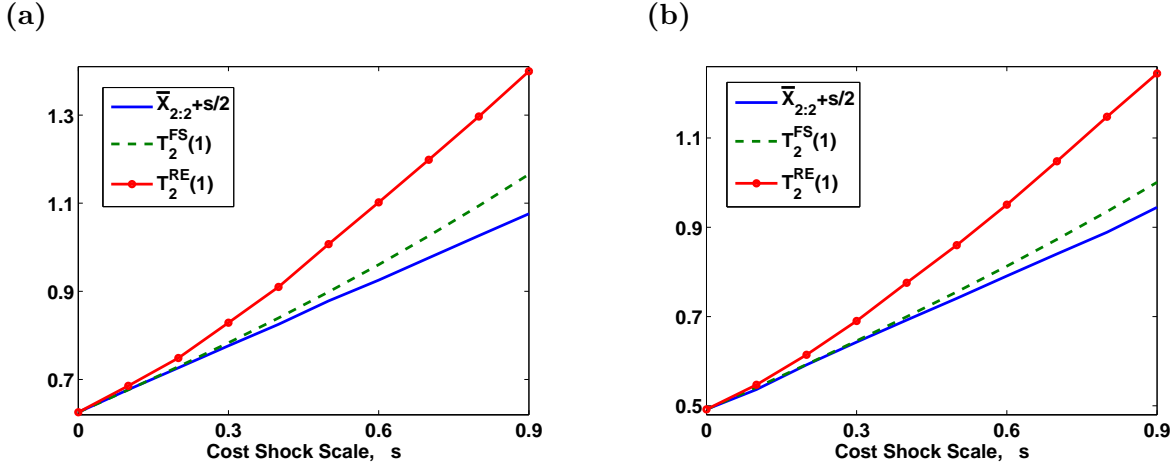


Figure 2: Examples showing that, while the sealed-bid format curtails windfall profit-taking compared to the reverse English format (i.e., $T_2^{FS}(1) \leq T_2^{RE}(1)$), the buyer still finds it optimal to pool her supply base (i.e., $\bar{X}_{2:2} + \frac{s}{2} < T_2^{FS}(1)$). Panel (a) assumes that the supplier production cost distribution F is a truncated normal with mean 0.5 and standard deviation 0.25, and panel (b) assumes that F is a truncated exponential distribution with mean 0.5.

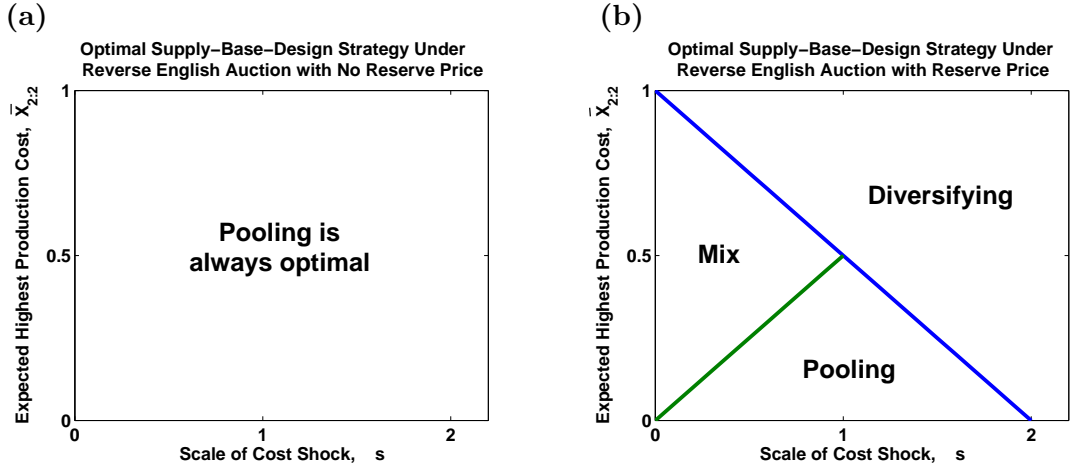


Figure 3: Effect of using a reserve price. Given the cost shock scale s and the expected production cost of the more expensive supplier, $\bar{X}_{2:2}$, panel (a) shows the optimal supply-base-design strategy under a reverse English auction with no reserve price, and panel (b) shows the optimal supply-base-design strategy under a reverse English auction with reserve price.

to reject bids that might be ex post attractive to her. With this severe “walk away” threat, we find that the buyer can quite effectively curtail suppliers’ profits, and the ability to employ a reserve price in a reverse English or first-price sealed-bid format can push the buyer to prefer diversifying.

Proposition 3 *With two suppliers, when mechanism RER or FSR is used, it is optimal to diversify if $s > 2(1 - \bar{X}_{2:2})$, but it is optimal to pool if $2\bar{X}_{2:2} < s < 2(1 - \bar{X}_{2:2})$. When $s < \min\{2\bar{X}_{2:2}, 2(1 - \bar{X}_{2:2})\}$, either strategy can be optimal, depending on the cost distribution F .*

For the reverse English auction format, Figure 3 illustrates how the buyer’s supply-base-design decision is affected by her ability to employ a reserve price. Although pooling is always optimal if a reverse English auction is used with no reserve price (Proposition 1), the employment of a reserve price greatly shrinks the region where pooling is optimal. In particular, when the cost shock scale is large (i.e., $s > 2(1 - \bar{X}_{2:2})$), a reserve price allows the buyer to capture significant cost savings when sourcing from an advantaged supplier and consequently the buyer prefers diversifying. However, when the cost shock size shrinks (i.e., $2\bar{X}_{2:2} < s < 2(1 - \bar{X}_{2:2})$), diversifying faces the buyer with considerable downside risk, but at the same time her upside benefit is largely lost to windfall profits because the auction price is likely to be set purely by pricing competition rather than the reserve. Thus, even with a reserve price we find that a pooled supply base can still be optimal for the buyer. The effectiveness of the reserve price weakens as the cost shock scale shrinks even smaller (i.e., $s < \min\{2\bar{X}_{2:2}, 2(1 - \bar{X}_{2:2})\}$), but the diversifying downside is also smaller. While F ’s expected order statistic $\bar{X}_{2:2}$ does not always capture all information needed to predict the buyer’s preference in such cases, numerical studies suggest that it is optimal to pool when F is such that low-cost types are very likely (e.g., F is a truncated normal with mean 0.2 and standard deviation 0.25 when $s = 0.3$), but it is optimal to diversify when F is such that low-cost types are not likely (e.g., if in the previous example the mean is increased from 0.2 to 0.8). This is consistent with our intuition that pooling is preferred when price concessions are less apt to be gleaned through a reserve price.

Summary of results for supply base with two suppliers. The results of §4.4 suggest that the buyer should carefully evaluate her bargaining clout relative to the suppliers before deciding to diversify her supply base. If the buyer has extremely little channel power and cannot prevent advantaged suppliers from making significant windfall profits, there is little benefit to diversifying and the buyer should instead simply pool her risk by choosing both suppliers in a single region. While such a buyer is inevitably more vulnerable to cost shocks, by pooling she ensures greater cost parity between suppliers, which she needs to drive down suppliers’ price bids. On the other hand,

if the buyer has bargaining power and can make credible “walk-away” threats to the suppliers, she can choose auction formats that curb the windfall-profits of advantaged suppliers. Consequently, greater bargaining power pushes the buyer away from a pooled supply base strategy and towards a diversifying strategy. By diversifying, the buyer mitigates her exposure to regional cost shocks, while her bargaining power allows her to capture significant benefits from having a supplier in a low-cost region.

4.5 Optimal supply base design with N suppliers

The supply-base-design problem with three or more suppliers is analytically more challenging than that with two suppliers in two aspects. First, the buyer has more than two options: Other than pooling ($R = 1$) and fully diversifying ($R = N$), there are various *partially diversifying* strategies ($2 \leq R \leq N - 1$). Second, with two suppliers the buyer’s problem boils down to evaluating $T_2(1)$ (Corollary 1), but with $N \geq 3$ suppliers we potentially need to evaluate $T_N(N_0)$ for $N_0 = 1, \dots, N - 1$, which involves order statistics of non-identically distributed random variables and can be analytically intractable. We again use a two-step approach to study the optimal supply-base-design problem, mirroring §4.4: In the following subsection we characterize the optimal strategy when the buyer’s expected total procurement cost function $T_N(\cdot)$ exhibits two types of shapes. Then, in §4.5.2 we apply these general characterizations to study the buyer’s optimal strategy under the five mechanisms, RE, RER, FS, FSR, and OPT.

4.5.1 Characterizations of the optimal supply-base-design strategy

We say that $T_N(\cdot)$ is convex over a discrete support S if $\frac{T_N(N_{02}) - T_N(N_{01})}{N_{02} - N_{01}} \leq \frac{T_N(N_{03}) - T_N(N_{02})}{N_{03} - N_{02}}$ for all $N_{01}, N_{02}, N_{03} \in S$ such that $N_{01} < N_{02} < N_{03}$.

Condition 1 *The cost shock scale s , cost distribution F , and the chosen mechanism are such that $T_N(\cdot)$ is convex and decreasing over the support $\{0, 2, 3, \dots, N\}$ and the support $\{1, \dots, N\}$.*

Under Condition 1, $T_N(\cdot)$ exhibits either of two types of shapes, depending on the value of $T_N(1)$. When $T_N(1) < \frac{T_N(0) + T_N(2)}{2}$, the function $T_N(\cdot)$ is convex over the support $\{0, \dots, N\}$ (Figure 4(a)). However, when $T_N(1) > \frac{T_N(0) + T_N(2)}{2}$, $T_N(\cdot)$ is concave over the support $\{0, 1, 2\}$ and is convex over $\{1, \dots, N\}$ (Figure 4(b)). For these two types of shapes of $T_N(\cdot)$, we can characterize the buyer’s optimal supply base design strategy. The insights mirror those of Lemma 2, which indicated that with two suppliers the buyer diversifies when function T_2 is convex.

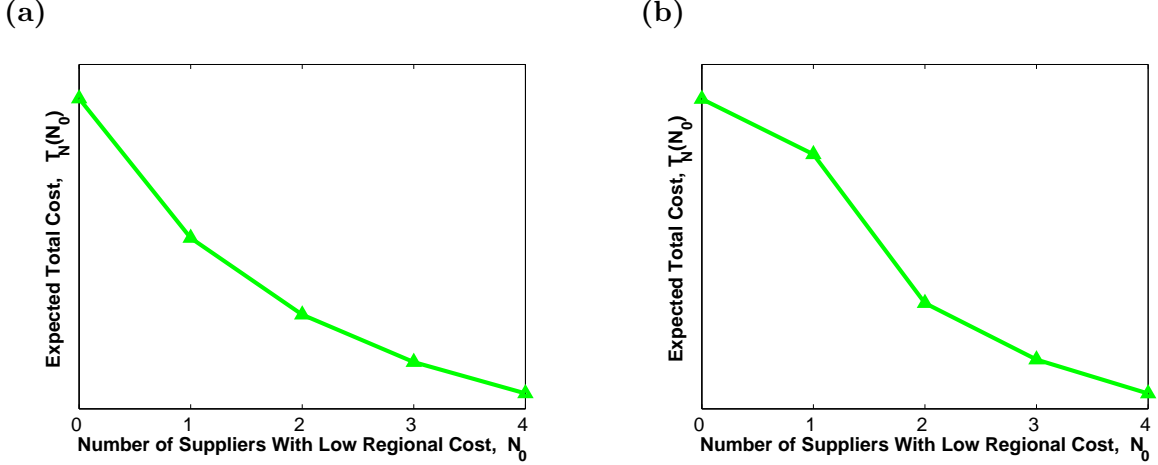


Figure 4: The two possible shapes of $T_N(\cdot)$ under Condition 1 when $N = 4$. Panel (a) shows that $T_N(\cdot)$ is convex over the support $N_0 = 0, \dots, N$ when $T_N(1) < \frac{T_N(0)+T_N(2)}{2}$, and panel (b) shows that $T_N(\cdot)$ is concave over the support $N_0 = 0, 1, 2$ and convex over the support $N_0 = 1, \dots, N$ when $T_N(1) > \frac{T_N(0)+T_N(2)}{2}$.

To understand how convexity of the expected total cost function $T_N(N_0)$ encourages diversification, recall the diversification tradeoff discussion in §4.3. Without loss of generality, suppose that $\tilde{n}_R \geq \tilde{n}_{R+1}$. For $\tilde{n}_{R+1} \geq 2$, Condition 1 implies that $\Delta(\hat{N}_0) \geq 0$ for all \hat{N}_0 . This in turn implies that, for instance when N is even, the buyer prefers to have regions with *at most* two suppliers each.

However, when deciding whether to “split” a two-supplier region (and thus $\tilde{n}_R = \tilde{n}_{R+1} = 1$), we must pay careful attention to the convexity of T_N over the set $\{0, 1, 2\}$. In particular, if T_N is convex over $\{0, 1, 2\}$, then by Condition 1 it is convex everywhere and hence $\Delta(\hat{N}_0) \geq 0$ even if $\tilde{n}_R = \tilde{n}_{R+1} = 1$; as a result, the buyer prefers to have regions with *at most* a single supplier each — that is, fully diversifying. On the other hand, if T_N is concave over $\{0, 1, 2\}$, then $\Delta(0) < 0$ if $\tilde{n}_R = \tilde{n}_{R+1} = 1$. Consequently, fully diversifying is discouraged when it is very likely that $\hat{N}_0 = 0$. Because $Prob(\hat{N}_0 = 0) = p^{R-1}$ increases in p , the buyer prefers not to fully diversify precisely when p is large. Thus, surprisingly, a riskier environment (more chance of cost shocks) actually pushes the buyer away from fully diversifying and towards *partially* diversifying her supply base by having several regions with two suppliers each. For instance, with four suppliers this strategy diversifies her cost shock risk (by having two separate regions) and simultaneously curbs the profit-taking ability of suppliers in an advantaged region (even if only one region has low cost, there will be two suppliers within that region). The following result summarizes these insights.

Lemma 3 *With $N \geq 3$ suppliers, under Condition 1,*

- (i) *If $T_N(1) < \frac{T_N(0)+T_N(2)}{2}$, it is optimal to fully diversify ($R = N$).*
- (ii) *If $T_N(1) > \frac{T_N(0)+T_N(2)}{2}$, it is optimal to fully diversify ($R = N$) when the cost shock likelihood p is sufficiently small, but it is optimal to partially diversify when the cost shock likelihood p is sufficient large — in particular, for N even, it is optimal to have $R = \frac{N}{2}$ regions with two suppliers each; for N odd, it is optimal to have $\frac{N-3}{2}$ regions with two suppliers each plus a region with three suppliers, or to have $\frac{N-1}{2}$ regions with two suppliers each plus a region with one supplier [see conditions in the e-companion, equation (EC.4)].*

4.5.2 Optimal supply-base-design strategies under the five mechanisms

We now apply Lemma 3 to analyze the optimal supply-base-design strategy under the five mechanisms, RE, FS, RER, FSR, and OPT.

Large cost shocks. When cost shock size s is sufficiently large, we can explicitly derive the values of $T_N(N_0)$, $N_0 \geq 2$. Per Lemma 1, for any of the five mechanisms $Mech \in \{RE, FS, RER, FSR, OPT\}$, there exists a threshold s_1^{Mech} such that when $s \geq s_1^{Mech}$ only suppliers with low regional cost can possibly win the auction. Consequently, the auction can be analyzed as having N_0 symmetric suppliers competing for the contract. It is then straightforward that $T_N^{RE}(N_0) = \bar{X}_{2:N_0}$ for $N_0 = 2, \dots, N$. Because $T_N^{RE}(0) = s + \bar{X}_{2:N}$ and revenue equivalence holds under symmetric suppliers, we can conclude that $T_N^{Mech}(0) = s + \bar{X}_{2:N}$ and $T_N^{Mech}(N_0) = \bar{X}_{2:N_0}$ for $N_0 = 2, \dots, N$ when $s \geq s_1^{Mech}$ for all five mechanisms.

Given these expressions of $T_N^{Mech}(N_0)$, for any of the five mechanisms we can show that there exists a threshold s_2^{Mech} such that Condition 1 holds if $s \geq s_2^{Mech}$ and the cost distribution F is such that the sequence $\{1, \bar{X}_{2:2}, \dots, \bar{X}_{2:N}\}$ is convex and decreasing⁶ (e.g., power-function distributions $F(x) = x^v$ with $v > 0$, including the uniform distribution; see e-companion⁷). Moreover, because $T_N^{Mech}(1)$ increases in s unless the buyer can impose a reserve price, there exists a threshold s_3^{Mech} such that when $s > s_3^{Mech}$ we have $T_N^{Mech}(1) > \frac{T_N^{Mech}(0)+T_N^{Mech}(2)}{2}$ for $Mech \in \{RE, FS\}$ and $T_N^{Mech}(1) < \frac{T_N^{Mech}(0)+T_N^{Mech}(2)}{2}$ for $Mech \in \{RER, FSR, OPT\}$.

⁶In other words, the second-lowest order statistic decreases in the number of draws from F , with the rate of decrease slowing as draws begin “piling up” near the left endpoint of the domain, zero.

⁷While closed-form expressions for order statistics do not (to our knowledge) exist for other distributions over $[0, 1]$, numerical studies under a wide range of parameters indicate that this condition also holds, for example, under truncated normal and exponential distributions. However, it can be shown to fail if the distribution F has a “heavy enough right tail,” e.g., if F is bernoulli distributed with a high ($> .8$) probability mass at one, in which case the condition fails for small N .

For any of the five mechanisms, Lemma 1 allows us to explicitly derive s_1^{Mech} , s_2^{Mech} , and s_3^{Mech} ; see Table 1 in the Appendix for these expressions. For any such mechanism, when $s > \max\{s_1^{Mech}, s_2^{Mech}, s_3^{Mech}\}$ we can invoke Lemma 3, yielding the following result.

Proposition 4 *With $N \geq 3$ suppliers, if the cost shock scale s is sufficiently large and the cost distribution F is such that the sequence $\{1, \bar{X}_{2:2}, \dots, \bar{X}_{2:N}\}$ is convex and decreasing, then:*

- (i) *Under mechanisms RER, FSR, and OPT, it is optimal to fully diversify ($R = N$) regardless of p .*
- (ii) *Under mechanisms RE and FS, it is optimal to partially diversify (as described in Lemma 3) when the cost shock likelihood p is sufficiently large, and it is optimal to fully diversify when the cost shock likelihood p is sufficiently small.*

Proposition 4 shows that, as in the two-supplier case, the auction mechanism (i.e., the buyer's bargaining power) drives $T_N(1)$, $N \geq 3$, which in turn drives the optimal supply-base-design strategy. In particular, in settings with large cost shocks, a buyer who can employ a reserve price (RER, FSR, or OPT) to contain windfall profit-taking when $N_0 = 1$ prefers to fully diversify her supply base. Intuitively, in such cases having more regions allows the buyer to better spread out her risks. However, a buyer who cannot employ a reserve price (RE or FS) would expect severe windfall profit-taking when $N_0 = 1$ had the fully diversifying strategy been used. Thus, she backs away from fully diversifying when the cost shock likelihood is large, as prescribed by Lemma 3.

Using the pooling strategy as a benchmark, the performance of the fully and partially diversifying strategies can be measured via their *percent improvement*, which is defined as:

$$\text{Percent improvement of a strategy} = 1 - \frac{\text{Expected total cost under the (fully or partially) diversifying strategy}}{\text{Expected total cost under the pooling strategy}}. \quad (6)$$

For format RE with four suppliers having uniformly distributed production costs, Figure 5 plots the percent improvement of the partially ($R = 2$) and fully ($R = 4$) diversifying strategies, versus the cost shock likelihood p . For a large cost shock size ($s = 1$, which is sufficiently large to invoke Proposition 4 for the considered case) Figure 5(a) shows that Proposition 4 holds: Fully diversifying is preferred when p is sufficiently small ($p < 0.42$) and partially diversifying is preferred when p is sufficiently large ($p > 0.42$). Similar to Figure 1, Figure 5(a) implies that the supply base design decision is particularly important when the regional cost shock likelihood p is medium: When $0.1 \leq p \leq 0.7$, the preferred strategy outperforms the pooling strategy by 10-16%.

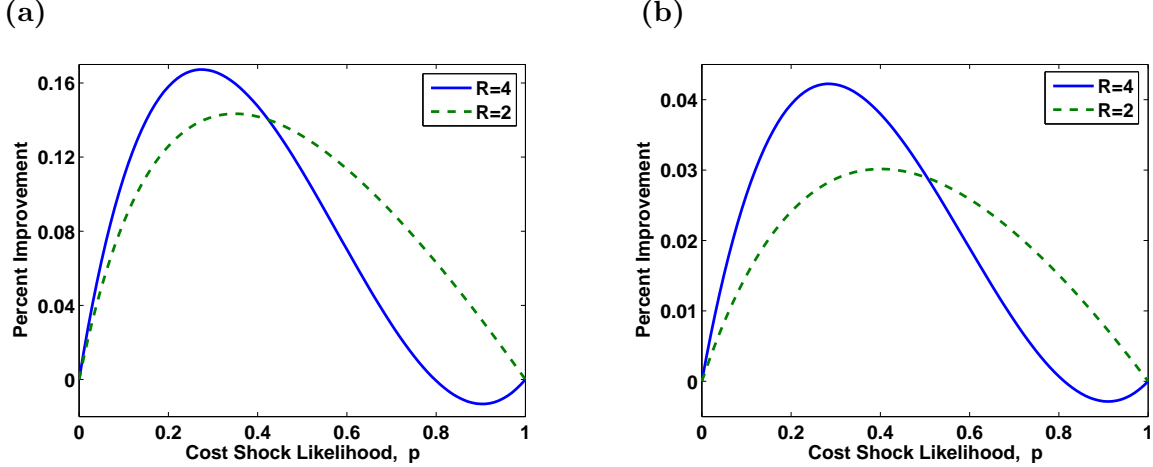


Figure 5: Percent improvement, per equation (6), of the fully diversifying strategy and the partially diversifying strategy. Plots assume $N = 4$, $F \sim U[0, 1]$, and that RE is used. Panel (a) assumes $s = 1$, and panel (b) assumes $s = 0.5$.

Small cost shocks. When cost shock size s is not sufficiently large, both the suppliers with low regional cost and those with high regional cost can potentially win the auction. As a result, closed-form expressions of $T_N(N_0)$ involve order statistics of non-identically distributed random variables and hence are in general difficult to obtain. However, our numerical studies indicate that, for the five mechanisms, Condition 1 can hold even with small s under commonly used distributions, including the uniform distribution, and the truncated normal and exponential distributions with a wide range of parameters. Thus, the insights of Lemma 3 appear robust: The buyer's preference between fully and partially diversifying depends on the cost shock likelihood and the buyer's ability to contain windfall profit-taking. However, to perform an exact analysis the buyer would need to compute $T_N(N_0)$ for $N_0 = 0, \dots, N$ according to Lemma 1 so as to verify Condition 1 and check the convexity/concavity of $T_N(N_0)$ over $N_0 = 0, 1, 2$. For example, suppose the buyer uses format RE, has four suppliers with uniformly distributed production costs, and the cost shock size is $s = 0.5$; we find numerically that $T_4^{RE}(0) = 0.90$, $T_4^{RE}(1) = 0.79$, $T_4^{RE}(2) = 0.61$, $T_4^{RE}(3) = 0.49$, and $T_4^{RE}(4) = 0.40$. It is easy to see that Condition 1 holds and $T_4^{RE}(N_0)$ is concave over $N_0 = 0, 1, 2$. Thus, part (ii) of Lemma 3 applies: The buyer will want to partially diversify ($R = 2$) if p is large and fully diversify if p is small; this is illustrated in Figure 5(b). Practically speaking, a buyer can take the insight of Lemma 3 and conclude that if p is not too large then she will prefer to fully diversify; however, if p is moderate to large then it is worth computing the values of $T_N(N_0)$ to see if she would be better off partially diversifying.

4.6 General takeaways

Sections 4.4–4.5 reveal that the buyer prefers adding more regions to her supply base portfolio (moving from an R -region strategy to a more diversified $(R + 1)$ -region strategy) only if an advantaged supplier is unlikely to emerge and largely absorb the resulting benefits of the diversification.

Buyers with strong bargaining power can use reserve prices and/or biasing rules to prevent suppliers from absorbing the benefits of diversification. Consequently, such buyers always find it optimal to have just one supplier per region (fully diversifying).

In contrast, buyers with weak bargaining power cannot prevent absorption of benefits by an advantaged supplier. Therefore, such buyers will only add more regions to their portfolio if doing so is unlikely to forfeit the benefits of diversification to an advantaged supplier. Because an advantaged supplier absorbs the benefits that accrue to his unilateral status as the only supplier without a cost shock, the buyer can avoid the emergence of an advantaged supplier by keeping two suppliers in each region (partially diversifying). A buyer with weak bargaining power will diversify a two-supplier region into two single-supplier regions only if neither region is likely to contain an advantaged supplier that can absorb the benefits of the diversification. Because the benefit of this diversification accrues precisely when just one of the two regions experiences a cost shock, this pushes the buyer to prefer diversifying only if, in such a case, she has supplier(s) in a third region which is unlikely to experience a cost shock. Consequently, buyers with weak bargaining power prefer to fully diversify only if there are at least three suppliers in the supply base and the cost shock likelihood is small.

5. Codependent regional costs

Our analyses thus far assumed independent regional costs. However, one can easily imagine situations where regions exhibit vulnerability to common shock factors. This subsection considers regional codependence, where regions are ex ante symmetric. For any two regions i and j we can write down the pairwise joint regional cost distribution, $Prob(y_i, y_j)$, $y_i, y_j \in \{0, s\}$. Letting p continue to denote the marginal probability of a regional cost shock, we get $Prob(0, 0) = 1 - 2p + Prob(s, s)$. Hence, given p , the value $Prob(s, s)$ uniquely characterizes the pairwise joint regional cost distribution and captures the amount of pairwise codependence across regions.

Diversification tradeoff. To accommodate codependence, the diversification tradeoff analysis in §4.3 requires a few small changes. First, when switching to the $(R + 1)$ -region strategy, the diversifying upside and downside each occur with probability $[p - Prob(s, s)]$ instead of $p(1 - p)$.

Second, when we evaluate the buyer's expected change in profit as a result of switching to the $(R + 1)$ -region strategy, we need to take this expectation over the distribution of \hat{N}_0 conditional on having unequal cost shocks in regions R and $R + 1$, $y_R \neq y_{R+1}$ (the condition under which diversification affects the buyer's profits).

Diversification decision. Thus, with two suppliers, the buyer's expected change in profit as a result of switching from pooling to diversifying is $[p - Prob(s, s)][T_2(0) - 2T_2(1) + T_2(2)]$. Consequently, the buyer prefers diversifying if and only if $[T_2(0) - 2T_2(1) + T_2(2)] \geq 0$ — precisely the same condition used with independent cost shocks in §4.4. Hence, codependence does not affect the diversification decision with two suppliers. Similarly, with $N \geq 3$ suppliers the buyer's expected change in profit as a result of switching from the R -region strategy to the $(R + 1)$ -region strategy is $[p - Prob(s, s)]E_{\hat{N}_0|y_R \neq y_{R+1}}[\Delta(\hat{N}_0)]$. Consequently, the buyer prefers diversifying if and only if $E_{\hat{N}_0|y_R \neq y_{R+1}}[\Delta(\hat{N}_0)] \geq 0$ — the same condition used with independent cost shocks in §4.5, except that the expectation over \hat{N}_0 uses a conditional distribution. Because $\Delta(\cdot)$ is positive for all \hat{N}_0 when $T_N(\cdot)$ is convex (formats RER, FSR and OPT), we have the following result.

Proposition 5 *Under codependent regional costs, Lemma 1 holds as before, and*

- (i) *With two suppliers, all results (Lemma 2, Propositions 1–3) continue to hold as before.*
- (ii) *With $N \geq 3$ suppliers, Lemma 3 part (i) and Proposition 4 part (i) hold as before.*

Thus, codependence affects the magnitude of benefits accruing to diversification but generally leaves the buyer's diversification decision unchanged. Proposition 5 shows that all our earlier results are robust to codependence, except for Lemma 3 part (ii) and Proposition 4 part (ii). These latter results deal with cases where there are at least three suppliers and the buyer is susceptible to severe windfall profit-taking by advantaged suppliers (formats RE and FS). In these cases it is easy to show that the buyer still always prefers at least partially diversifying; however, codependence can affect the distribution of \hat{N}_0 conditional on $y_R \neq y_{R+1}$. Intuitively, greater codependence will encourage fully diversifying if it reduces the risk of an advantaged supplier emerging, but will discourage fully diversifying if codependence instead escalates the risk of an advantaged supplier emerging; the e-companion illustrates both possibilities with a stylized example, §EC.12.

6. Generally distributed regional costs

The preceding sections show that buyers with strong bargaining power should diversify more, while buyers with weak bargaining power should diversify less due to concerns about advantaged suppliers. In this section, we present evidence that these insights — obtained from the two-point regional cost model — persist when the regional cost shocks $y_r, r = 1 \dots R$, are independent draws from a general distribution G with density (if it exists) g .

Two-supplier case. We begin by providing results for the two-supplier case. Consider a buyer choosing between pooling and diversifying. She knows that one of the regions $r = 1, 2$ will have the lower cost $\min\{y_1, y_2\}$ and the other will have the higher cost $\max\{y_1, y_2\}$. Diversifying hedges the risk of high regional cost by guaranteeing access to the low-cost region (the diversifying upside), but relinquishes the natural hedge of cost parity between suppliers (the diversifying downside). In particular, the expected diversifying upside and downside can be written as $E_s[T_2(0|s) - T_2(1|s)]$ and $E_s[T_2(1|s) - T_2(2|s)]$, respectively, where $s = \max\{y_1, y_2\} - \min\{y_1, y_2\}$, and $T_N(N_0|s)$ is the expected total procurement cost given a two-point regional cost distribution with cost shock size s and N_0 suppliers without cost shock. In other words, for the two-supplier case with general regional costs, the intuition developed in §4 applies with the “advantaged supplier” simply being the supplier with the *lower* regional cost and the “cost shock” simply being the *difference* between the two regions’ costs. As a result, the expected profit change by switching from pooling to diversifying is $E_s[T_2(0|s) - 2T_2(1|s) + T_2(2|s)]$.

Proposition 6 *Suppose that regional costs follow a general distribution. With two suppliers, it is optimal to pool if RE is used, while it is optimal to diversify if OPT is used.*

Proposition 6 is able to fully characterize the diversification decision for RE and OPT because $T_2(0|s) - 2T_2(1|s) + T_2(2|s)$ is negative and hence pooling is preferred (positive and hence diversifying is preferred) regardless of s when the buyer uses RE (OPT) — the same insight used for proving Proposition 1. For format FS, Proposition 2 proved that pooling is preferred for large values of s , with numerical evidence (Figure 2) suggesting that the pooling preference also holds for small s . This insensitivity of the preference to the size of s suggests that pooling would be preferred under FS even with generally distributed regional costs (although proving this analytically is precluded by the same difficulties that limited Proposition 2 to large s). In contrast, for RER and FSR, Proposition 3 showed that the pooling versus diversification preference depends on the value of s . Hence, for these

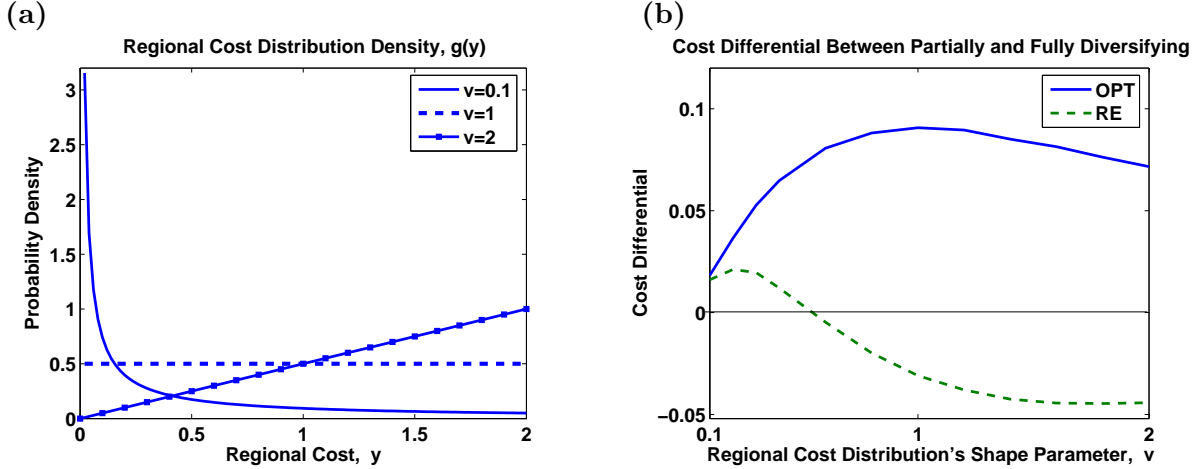


Figure 6: Effect of regional cost distribution’s shape on partially versus fully diversifying preference. Panel (a) illustrates the power-function distribution’s density, $g(y) = va^{-v}y^{v-1}$, with scale parameter $a = 2$ and shape parameters $v = 0.1, 1, 2$. Panel (b) plots the difference between the buyer’s expected total procurement cost under partially ($R = 2$) and fully ($R = 4$) diversifying, assuming $G(y) = a^{-v}y^v$, $a = 2$, $N = 4$, and suppliers’ production costs follow $F \sim U[0, 1]$.

formats the buyer’s preference is sure to depend on the distribution of $s = \max\{y_1, y_2\} - \min\{y_1, y_2\}$; in particular, given Figure 3(b), we expect that under RER (or FSR) diversifying is optimal only if $\max\{y_1, y_2\} - \min\{y_1, y_2\}$ is likely to be relatively large compared to $\bar{X}_{2;2}$.

N -supplier case. Asymmetries between regions’ cost shocks preclude using N_0 as a sufficient statistic for computing the buyer’s expected total cost. Consequently, analytical comparisons of the diversifying upside and downside (per §4.5) become intractable in the N -supplier case. Nonetheless, the basic intuition remains: Diversification hedges regional costs at the peril of losing price parity across suppliers, and buyers with strong bargaining power are better able to withstand the risks of the latter. Thus, with continuous cost shocks one might still expect to see buyers favoring fully diversifying unless price parity is a serious concern — that is, unless the buyer has little bargaining power and it is likely that an “advantaged” supplier emerges.

To provide a baseline, using an argument analogous to Proposition 6 above, we can prove (see §EC.14) that when Condition 1 holds the buyer prefers to have at least two regions when she can have at least two suppliers in each such region. Having at least two suppliers in each region eliminates unilateral regional cost advantages and thus removes the possibility of an advantaged supplier emerging and absorbing the benefits of the diversification. (The case $N=3$ is similar but requires an extra condition, see §EC.14.)

We now discuss how the buyer’s preference for further diversification depends on her bargaining power. Figure 6 examines this question when regional costs are distributed according to a power-function distribution $G(y) = a^{-v}y^v$ ($a > 0$ is the scale parameter), whose density $g(y)$ takes various shapes according to the shape parameter $v > 0$ (Figure 6(a)). To investigate how the shape of $g(y)$ affects the buyer’s diversification decision, Figure 6(b) plots, for various shape parameters v , the difference between the buyer’s expected total cost under the partially ($R = 2$) and the fully ($R = 4$) diversifying strategies. Figure 6(b) reveals that under format OPT the buyer always prefers fully diversifying, but under format RE the buyer prefers fully diversifying only if v is small.

This insight mirrors that of Proposition 4, only now the shape parameter v plays the role of the cost shock probability p . Note from Figure 6(a) that as shape parameter v becomes smaller, the regional cost distribution becomes very concentrated near the left endpoint, which is akin to a two-point regional cost-shock distribution with small p . Under such a distribution, the intuition for the two-point analysis carries over: The buyer prefers fully diversifying because, when diversification gives her access to a low-regional-cost supplier, this supplier’s profit-taking is mitigated by cost-parity with other regions’ suppliers — the regional cost distribution is simply “piled up” near the low-cost end. In contrast, as v increases, the density of the regional cost distribution flattens out. In such a case, when diversification gives the buyer access to a low-regional-cost supplier, this supplier is likely to enjoy a significant regional-cost advantage; if the buyer has little bargaining power this advantaged supplier can largely absorb this upside of diversification, leaving the buyer with only the diversification downside.

Just as Proposition 4 was enabled by taking the cost shock size s to be large, the continuous cost analysis is enabled by taking the variability of regional costs y_r ’s to be large relative to that of the supplier production costs x_i ’s. In particular, for the extreme case where x_i has zero variability (is a constant for all i) and y_r is variable, the following proposition formalizes our insights.

Proposition 7 *Consider the N -supplier case with no variability across supplier production costs, that is, x_i is a constant for all i . The buyer’s preference between fully and partially diversifying depends on her bargaining power:*

- (i) *If the buyer has full bargaining power and can run format OPT, she always prefers to fully diversify, regardless of the regional cost distribution G .*
- (ii) *Suppose the buyer has no bargaining power and can only run format RE, and G is a power-function distribution with shape parameter v . There exists a $\underline{v} > 0$ such that the buyer prefers fully diversifying if $v < \underline{v}$, and partially diversifying if $v \geq \underline{v}$.*

While our above presentation assumed independently-distributed regional costs, the proofs of Propositions 6 and 7 part (i) also accommodate codependence. Additionally, results analogous to Proposition 7 part (ii) can be derived for other distributions having closed-form order statistics (namely, exponential and Pareto — note that the uniform distribution is subsumed in the power-function family) but for brevity we omit the details. In summary, our main insight persists under continuously distributed regional costs: Buyers with strong bargaining power favor fully diversifying, and buyers with weak bargaining power may instead prefer partially diversifying.

7. Conclusions

A buyers' total procurement cost includes not only the contract payment to a supplier but also other costs (logistics costs, duties and tariffs, etc.), which are often subject to various regional cost shocks such as labor strikes, regulation changes, and geopolitical events. To mitigate regional cost risks, a buyer seeking to minimize her total procurement cost can strategically reduce the cost correlation across suppliers by diversifying her supply base (i.e., choosing suppliers from different regions). However, in settings where the buyer's payment to her supplier is determined by a competitive bidding process (i.e., an auction), the buyer's upside benefit of diversification — having a significantly cost-advantaged supplier — can be undermined by this supplier's windfall profit-taking. This paper models the interaction between the buyer's supply-base-design strategy and the risks of windfall profit-taking by suppliers, and characterizes the optimal supply-base-design strategy under various auction mechanisms. To our knowledge, this paper is the first study of supply base design to mitigate regional cost shocks.

We find that the buyer needs to make a tradeoff between the benefit from diversifying her risk of exposure to cost shocks, and the risk that suppliers will absorb such benefits for themselves by taking windfall profits. The ability of suppliers to take windfall profits depends upon the buyer's bargaining power, that is, the buyer's ability to choose an auction mechanism to suppress supplier profits. In particular, at one extreme, when the buyer has full bargaining power and thus can impose the optimal mechanism (i.e., the optimal reserve price plus the optimal contract allocation rule that biases against cost-advantaged suppliers), windfall profit-taking is curbed and consequently the buyer finds it optimal to fully diversify her supply base (i.e., select each supplier from a different region). However, at the other extreme, when the buyer has no bargaining power and solely relies on supplier competition for price concessions (i.e., uses a reverse English auction with no reserve price), supplier windfall profit-taking can be severe and consequently the buyer diversifies less.

With two suppliers she always finds it optimal to pool both suppliers in a single region. With more suppliers she prefers a blended strategy: She diversifies by using multiple regions, but keeps two suppliers per region to hedge her bets and eliminate the risk that any supplier possesses a unilateral regional cost advantage.

We also study cases where the buyer has intermediate bargaining power and thus can use a first-price sealed-bid auction format and/or impose a reserve price. The first-price sealed-bid format can make the buyer better off because suppliers enjoying a regional cost advantage tend to bid lower when they must make a “best and final offer” and want to reduce their risk of losing the contract. Nonetheless, such suppliers still take significant windfall profits, and this pushes the buyer away from full diversification when using the first-price sealed-bid format. On the other hand, imposing a reserve price allows the buyer to truncate large supplier profits. When the cost shock size is large the buyer prefers full diversification when she can use a reserve price.

We find that introducing codependence across regional cost shocks affects the magnitude of benefits accruing to diversification but generally leaves the buyer’s diversification decision unchanged, and only affects the buyer’s decision in cases where there are at least three suppliers and the buyer is susceptible to severe windfall profit-taking by advantaged suppliers (formats RE and FS). In these cases, greater codependence can encourage fully diversifying if it reduces the risk of an advantaged supplier emerging, that is, reduces the risk of windfall profit-taking.

We also show how our results extend to broad settings where regional costs follow a general distribution. Once again, we find that buyers with strong bargaining power prefer to diversify more, while buyers with less bargaining power prefer to diversify less due to concerns about windfall profit-taking. For example, with two suppliers we prove that a buyer able to implement the optimal mechanism always finds it optimal to fully diversify her supply base, while a buyer using a reverse English format without a reserve always finds it optimal to pool her supply base. This result holds for any supplier production cost distribution and any distribution over regional costs.

Our study was motivated by focusing on shocks to the buyers’ “non-price” costs (logistics costs, tariffs, etc.), but our results can easily be extended to cases where suppliers share regional cost drivers, such as costs associated with a small local labor force, regional energy market, or a common second-tier supply base. More precisely, all analyses in this paper follow if regional cost y_r is re-interpreted as a commonly known cost factor shared by all suppliers within region r .

Although we assume that the distributions capturing production costs and regional costs remain static over time, Propositions 1 and 6 directly extend to cases where these distributions vary over time, given that these results hold regardless of production cost distribution F or cost shock

likelihood p and size s (or cost shock distribution G in the case of Proposition 6). Proposition 4 holds for time-varying F 's satisfying its mild condition (essentially that F 's order statistics are convex and decreasing). For Proposition 3, which says the optimal strategy depends on $\overline{X}_{2:2}$ (i.e., the cost distribution F), we suspect that the optimal supply-base design decision depends on some “average” level of $\overline{X}_{2:2}$ if F is time-variant.

We examined five auction mechanisms meant to reflect a broad array of mechanisms one might use in practice. Of course, buyers may use other auction mechanisms or unstructured bargaining processes — for example, the buyer may negotiate with the advantaged supplier. For such cases, we suspect that the key insight of our paper will continue to apply: The more bargaining clout the buyer has to control windfall-profit taking by cost-advantaged suppliers, the more she will prefer building a diversified supply base. Our results can also extend to the cases where the buyer has only a limited number of regions to choose suppliers from — suppose there are only $\overline{R} < N$ regions available — the buyer tends to use all \overline{R} regions if fully diversifying is optimal in the unconstrained case, or tends to use $\min\{\overline{R}, \lfloor \frac{N}{2} \rfloor\}$ regions if partially diversifying is optimal in the unconstrained case. Some other extensions are possible, for example, imposing a fixed cost of using additional regions, or allowing asymmetry among regions. Our preliminary numerical studies suggest that these issues affect the buyer’s supply base design decisions in quite straightforward ways, but we leave further examinations for future work.

Finally, to keep our analysis focused and tractable we ignore the buyer’s inventory decisions. To the extent that the buyer can anticipate regional cost shocks, she may choose to speculatively purchase inventory to avoid future cost spikes, e.g., impending tariffs increases in a certain country. Interestingly, our analysis suggests that the usefulness of such a strategy depends on the buyer’s bargaining clout. Speculative inventory might behoove a buyer with little bargaining clout, who might use it to help avoid paying windfall profits to cost advantaged suppliers. On the other hand, speculative inventory would likely be of much less benefit to a buyer with strong bargaining clout, who could contract with cost-advantaged suppliers without paying an undue price premium, thereby reducing the speculative benefits of holding inventory. We leave a detailed analysis of the interplay between inventory decisions, bargaining power and supply base design to our future work.

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Appendix: Table 1

Mechanism	s_1	s_2	s_3
RE, FS	1	$\max\{1, 3\bar{X}_{2:2} - 2\bar{X}_{2:3} - \bar{X}_{2:N}\}$	$\max\{1, \bar{X}_{2:2} + \bar{X}_{2:N}\}$
RER, FSR	1	$\max\{1, 2 - \bar{X}_{2:2} - \bar{X}_{2:N}\}$	(same as s_2)
OPT	$\psi(1)$	$\max\{\psi(1), 2 - \bar{X}_{2:2} - \bar{X}_{2:N}\}$	(same as s_2)

Table 1: s_1^{Mech} , s_2^{Mech} , and s_3^{Mech} for $Mech \in \{RE, FS, RER, FSR, OPT\}$.

Electronic Companion for Bargaining Power and Supply Base Diversification

Zhixi Wan • Damian R. Beil

Ross School of Business, University of Michigan, Ann Arbor, Michigan 48109

wanzhixi@umich.edu • dbeil@umich.edu

EC.1. Bidding in First-price Sealed-bid Auctions

Our analytical results regarding first-price sealed-bid auctions (with or without reserve price), i.e., Proposition 2, 3, and 4, do not rely on solving the equilibrium bidding functions of first-price sealed-bid auctions. However, our numerical studies — Figure 2 and the discussion on page 24 — involve evaluations of $T_N^{FS}(\cdot)$ and $T_N^{FSR}(\cdot)$ through numerically solving the differential equations that characterize these bidding functions. Thus, we provide the derivations of such differential equations. Along the way we also derive format FS's status rent, which is used in Lemma 1, and boundary conditions which are used in proving Propositions 2, 3 and 4.

When all suppliers are symmetric, i.e., $N_0 = 0$ or N , the bidding equilibrium is well-known; readers are referred to Krishna (2002). Thus, we focus on cases where suppliers are asymmetric, i.e., $1 \leq N_0 \leq N - 1$. We assume that in equilibrium the N_0 suppliers with low regional cost (referred to as low-regional-cost suppliers) share the same bidding function, denoted by β_1 , and the $N - N_0$ suppliers with high regional cost (referred to as high-regional-cost suppliers) share the same bidding function, denoted by β_2 . We assume β_1 and β_2 are increasing functions on $[0, 1]$. Let $\phi_1 \triangleq \beta_1^{-1}$ and $\phi_2 \triangleq \beta_2^{-1}$.

Boundary conditions. Let $\bar{b} = \beta_1(1)$, the bid placed by a low-regional-cost supplier who is of worst production cost type. Clearly, $\bar{b} \geq 1$ because low-regional-cost suppliers with production cost 1 need to make a non-negative profit. When FSR is used, $\bar{b} = 1$ because of the reserve price. When FS is used and $N_0 \geq 2$, $\bar{b} = 1$ because otherwise a low-regional-cost supplier with production cost 1 could improve his payoff by unilaterally deviating to a bid slightly lower than \bar{b} . With $\bar{b} = 1$, we can get a boundary condition for high-regional-cost suppliers as well: When $s > 1$, high-regional-cost suppliers are irrelevant (they can never win); when $s \leq 1$, in equilibrium, $1 = \beta_2(1 - s) + s$, and high-regional-cost suppliers with production cost greater than $1 - s$ are irrelevant.

When FS is used and $N_0 = 1$, $\bar{b} \in [\max\{1, s\}, 1 + s]$. First, if $s > 1$ the low-regional-cost supplier with cost type 1 is guaranteed to win if $\bar{b} \leq s$ and among such bids $\bar{b} = s$ gives the highest profit. Second, $\bar{b} > 1 + s$ guarantees zero profit for a low-regional-cost bidder with production cost 1: A

high-regional-cost supplier with production cost 1 could earn positive expected profit by bidding in interval $(1, \bar{b}-s)$, and since β_2 is increasing, the low-regional-cost bidder with production cost 1 could never win the auction. Thus, $\bar{b} \leq 1 + s$. We conclude that, in equilibrium, the boundary condition for high-regional-cost suppliers is provided by $\bar{b} = \beta_2(\bar{b} - s) + s$. Consequently, when $N_0 = 1$ and FS is used, the low-regional-cost supplier with production cost 1 chooses $\bar{b} = \arg \max_{b \in [\max\{1, s\}, 1+s]} (b - 1)[1 - F(b - s)]^{N-1}$, where $(b - 1)$ is his profit upon winning and $[1 - F(b - s)]^{N-1}$ is his equilibrium winning probability. Clearly, $(b - 1)[1 - F(b - s)]^{N-1}$ is zero for $b = 1$ and $b = 1 + s$. Furthermore, our assumption $x - \frac{1-F(x)}{f(x)}$ increases in x implies that $(b - 1)[1 - F(b - s)]^{N-1}$ is unimodal in b . Together, these observations imply that there is a unique solution $\bar{b} = \max \left\{ s, 1 + \frac{1-F(\bar{b}-s)}{(N-1)f(\bar{b}-s)} \right\}$.

Let $\underline{b}_1 \triangleq \beta_1(0)$ and $\underline{b}_2 \triangleq \beta_2(0) + s$. In equilibrium, it must be that $\underline{b}_2 \geq s$ because high-regional-cost suppliers with production cost 0 need to make non-negative payoff. When $N_0 = 1$ and FS is used, it must be that $\underline{b}_1 \geq s$, because if $\underline{b}_1 < s$ the low-regional-cost supplier with production cost 0 can strictly improve by deviating to bid s . Similarly, we can conclude that $\underline{b}_1 \geq s$ when $N_0 = 1$, $s < 1$, and FSR is used. Clearly, $\underline{b}_1 = 1$ when $N_0 = 1$, $s \geq 1$, and FSR is used.

Differential equations. We solve for equilibria in which $\underline{b}_1 \leq \underline{b}_2$, that is, low-regional-cost suppliers with production cost 0 bid lower than high-regional-cost suppliers with production cost 0 (this is in the spirit of the equilibrium bidding function increasing in the bidder's cost). The expected profit of a low-regional-cost bidder with production cost x who bids b is given by $\pi_1 = (b - x)[1 - F(\phi_1(b))]^{N_0-1}[1 - F(\phi_2(b - s))]^{N-N_0}$ if $b \geq \underline{b}_2$, or $\pi_1 = (b - x)[1 - F(\phi_1(b))]^{N_0-1}$ if $b < \underline{b}_2$; and the expected profit of a high-regional-cost bidder with production cost x who bids b is given by $\pi_2 = (b - x)[1 - F(\phi_1(b + s))]^{N_0}[1 - F(\phi_2(b))]^{N-N_0-1}$. Suppliers choose bids to maximize their expected profits. By first order conditions, we have $\frac{d\phi_1}{db}(b) = \frac{K4-K2}{K1K4-K2K3}$ if $b \geq \underline{b}_2$, or $\frac{d\phi_1}{db}(b) = \frac{1}{K1}$ if $b < \underline{b}_2$; and $\frac{d\phi_2}{db}(b) = \frac{K1-K3}{K1K4-K2K3}$ if $b \geq \underline{b}_2$. Here $K1 \triangleq \frac{(N_0-1)f[\phi_1(b)][b-\phi_1(b)]}{1-F[\phi_1(b)]}$, $K2 \triangleq \frac{(N-N_0)f[\phi_2(b-s)][b-\phi_1(b)]}{1-F[\phi_2(b-s)]}$, $K3 \triangleq \frac{N_0f[\phi_1(b+s)][b-\phi_2(b)]}{1-F[\phi_1(b+s)]}$, and $K4 \triangleq \frac{(N-N_0-1)f[\phi_2(b)][b-\phi_2(b)]}{1-F[\phi_2(b)]}$.

In our numerical studies, we evaluate the buyer's expected payment (i.e., $T_N^{FS}(N_0)$ or $T_N^{FSR}(N_0)$) by numerically solving the above differential equations. Readers are referred to Press et al. (1992) for details of numerical methods to solve differential equations.

EC.2. Proof of Lemma 1

The derivation of $T_N(N_0)$ follows immediately from equations (1) and (3) and the definition of Γ . Values for Q_i follow from the definitions of the auction formats in §3. Γ^{RE} , Γ^{RER} , Γ^{FSR} , and Γ^{OPT}

were derived in the text. Γ^{FS} , and the value of \bar{b} , follow from §EC.1's derivation of \bar{b} when $N_0 = 1$ and format FS is used.

EC.3. Proof of Lemma 2

With two suppliers, $\hat{N}_0 \equiv 0$ and $E_{\hat{N}_0}[\Delta(\hat{N}_0)] = \Delta(0) = T_2(0) - 2T_2(1) + T_2(2)$; hence, $E_{\hat{N}_0}[\Delta(\hat{N}_0)] < 0$ (it is optimal to pool) if and only if $T_2(1) > [T_2(0) + T_2(2)]/2$.

EC.4. Proof of Equation (4)

When N_0 equals 0 or 2, all five mechanisms are efficient (allocate to the lowest total-cost supplier) and have zero status rent; thus by Lemma 1, revenue equivalence holds. Finally, the buyer's expected total procurement cost under $N_0 = 0, 2$ can be easily derived under format RE — in such an auction suppliers find it a dominant strategy to bid down to their true costs (although they may not have to).

EC.5. Proof of Proposition 1

By Lemma 1, we have $T_2^{OPT}(1) = \int_{\mathbf{x}} \min\{\psi(x_1), \psi(x_2) + s\} d\mathbf{F}(\mathbf{x})$ and $T_2^{RE}(1) = \int_{\mathbf{x}} \max\{x_1, x_2 + s\} d\mathbf{F}(\mathbf{x})$. One can check the following: $\frac{\partial[T_2^{OPT}(1)]}{\partial s} = \int_{\mathbf{x}} \mathbb{I}_{\{\psi(x_1) > \psi(x_2) + s\}} d\mathbf{F}(\mathbf{x}) < \frac{1}{2}$, $\frac{\partial[T_2^{RE}(1)]}{\partial s} = \int_{\mathbf{x}} \mathbb{I}_{\{x_1 < x_2 + s\}} d\mathbf{F}(\mathbf{x}) > \frac{1}{2}$, $\frac{\partial[T_2^{OPT}(0)]}{\partial s} = \frac{\partial[T_2^{RE}(0)]}{\partial s} = 1$, and $\frac{\partial[T_2^{OPT}(2)]}{\partial s} = \frac{\partial[T_2^{RE}(2)]}{\partial s} = 0$. Therefore, $\frac{\partial[T_2^{OPT}(0) - 2T_2^{OPT}(1) + T_2^{OPT}(2)]}{\partial s} > 0$ and $\frac{\partial[T_2^{RE}(0) - 2T_2^{RE}(1) + T_2^{RE}(2)]}{\partial s} < 0$. Given that $T_2^{OPT}(0) - 2T_2^{OPT}(1) + T_2^{OPT}(2) = T_2^{RE}(0) - 2T_2^{RE}(1) + T_2^{RE}(2) = 0$ when $s = 0$, we get $T_2^{OPT}(0) - 2T_2^{OPT}(1) + T_2^{OPT}(2) > 0$ and $T_2^{RE}(0) - 2T_2^{RE}(1) + T_2^{RE}(2) < 0$ when $s > 0$. Thus, the proposition follows from Lemma 2.

EC.6. Proof of Proposition 2

By equation (4), the buyer prefers FS to RE if and only if $T_2^{FS}(1) < T_2^{RE}(1)$. Per §EC.1, $\bar{b} = \max\{s, 1 + \frac{1-F(\bar{b}-s)}{f(\bar{b}-s)}\}$ when $N = 2$, $N_0 = 1$, and FS is used. One can check that $\bar{b} = s$ if $s \geq 1 + \frac{1}{f(0)}$. Since $\underline{b}_1 \geq s$ (from §EC.1 when $N_0 = 1$ and FS is used) and $\bar{b} = s$, the monotonicity of β_1 implies that $\beta_1(x) = s$ for all production costs x . Thus, when $s \geq 1 + \frac{1}{f(0)}$, the low-regional-cost supplier (the advantaged supplier) wins the auction for sure by bidding s , which implies that $T_2^{FS}(1) = s < T_2^{RE}(1) = s + E[x_2 | x_2 > 1 - s]$, that is, FS is preferred to RE.

Next we show that the buyer prefers to pool when format FS is used and $s > 2\bar{X}_{2,2}$. Note that $\beta_1(x) \geq s$ for all x (since $\underline{b}_1 \geq s$ per §EC.1) and $\beta_2(x) + s \geq x + s$ (since $\beta_2(x) \geq x$). Thus, we have

$T_2^{FS}(1) \geq s$. Furthermore, when $s > 2\bar{X}_{2:2}$, we also have $s > \bar{X}_{2:2} + \frac{s}{2}$. Thus, when $s > 2\bar{X}_{2:2}$ we get $T_2^{FS}(1) > s > \bar{X}_{2:2} + \frac{s}{2}$, which by Corollary 1 implies pooling is optimal when FS is used.

EC.7. Proof of Proposition 3

When $s > 2(1 - \bar{X}_{2:2})$, we have $\bar{X}_{2:2} + \frac{s}{2} > 1 \geq T_2^{Mech}(1)$ for $Mech \in \{RER, FSR\}$, where the second inequality follows from the reserve price of 1. Hence, by Corollary 1 diversifying is preferred under formats RER and FSR when $s > 2(1 - \bar{X}_{2:2})$.

When $2\bar{X}_{2:2} < 2(1 - \bar{X}_{2:2})$, we have $\bar{X}_{2:2} < \frac{1}{2}$. Further, $2\bar{X}_{2:2} < s$ implies $\bar{X}_{2:2} + \frac{s}{2} < s$. Thus, when $2\bar{X}_{2:2} < s < 2(1 - \bar{X}_{2:2})$, we have $\bar{X}_{2:2} + \frac{s}{2} < s < 1$. By Corollary 1, to prove pooling is preferred when $2\bar{X}_{2:2} < s < 2(1 - \bar{X}_{2:2})$ (and complete the proposition's proof) it suffices to show for $Mech \in \{RER, FSR\}$ that $T_2^{Mech}(1) \geq s$ when $s < 1$. To see this for $Mech = RER$, note that $T_2^{RER}(1) = E \max\{x_1, \min\{x_2 + s, 1\}\}$. When $s < 1$, we have $\min\{x_2 + s, 1\} \geq s$ and hence $T_2^{RER}(1) \geq s$. For $Mech = FSR$, per §EC.1, we have that $\underline{b}_1 \geq s$ when $N_0 = 1$, $s < 1$, and FSR is used. Since $\underline{b}_2 \geq s$, we get that $T_2^{FSR}(1) \geq s$ when $s < 1$.

EC.8. Proof of Lemma 3

Part (i). Per the discussion in §4.3, the R -region strategy $(n_1, \dots, n_{R-1}, n_R)$ is outperformed by the $(R+1)$ -region strategy $(n_1, \dots, n_{R-1}, \tilde{n}_R, \tilde{n}_{R+1})$ with $\tilde{n}_R + \tilde{n}_{R+1} = n_R$ whenever $E[\Delta(\hat{N}_0)] \geq 0$. Under Condition 1, when $T_N(1) < \frac{T_N(0) + T_N(2)}{2}$, the function $T_N(\cdot)$ is convex over the support $\{0, \dots, N\}$. Since the diversification tradeoff Δ is nonnegative when $T_N(\cdot)$ is convex, the buyer always prefers the $(R+1)$ -region strategy; repeatedly applying this argument proves part (i).

Part (ii). We utilize three steps.

Step 1. We show that, in the optimal diversification strategy, either all regions have *at most* two suppliers, or one region has three suppliers and all other regions have exactly two suppliers. (For example, this implies that for N even the optimal diversification strategy is such that all regions have at most two suppliers.) This is shown by the following three sub-steps.

Step 1.1. We show that there are *at most* three suppliers in each region. This is true because, per the discussion in page 21, Condition 1 implies that the R -region strategy $(n_1, \dots, n_{R-1}, n_R)$ is outperformed by the $(R+1)$ -region strategy $(n_1, \dots, n_{R-1}, \tilde{n}_R, \tilde{n}_{R+1})$ with $\tilde{n}_R + \tilde{n}_{R+1} = n_R$ whenever $\tilde{n}_R, \tilde{n}_{R+1} \geq 2$.

Step 1.2. We now show that there is *at most* one region with three suppliers in the optimal diversification strategy. This is true because we can show any R -region strategy $(n_1, \dots, n_{R-1}, n_R)$

with $n_{R-1} = n_R = 3$, is dominated by the $(R+1)$ -region strategy $(n_1, \dots, n_{R-2}, \check{n}_{R-1}, \check{n}_R, \check{n}_{R+1})$ with $\check{n}_{R-1} = \check{n}_R = \check{n}_{R+1} = 2$. Let $\check{N}_0 \stackrel{def}{=} \sum_{r=1}^{R-2} n_r \cdot \mathbb{I}_{\{y_r=0\}}$, a random variable representing the number of suppliers with low regional cost in regions 1 through $(R-2)$. Thus, the expected total procurement cost under the R -region strategy is $E_{\check{N}_0}[p^2 T_N(\check{N}_0) + 2p(1-p)T_N(\check{N}_0 + 3) + (1-p)^2 T_N(\check{N}_0 + 6)]$ and that under the $(R+1)$ -region strategy is $E_{\check{N}_0}[p^3 T_N(\check{N}_0) + 3p^2(1-p)T_N(\check{N}_0 + 2) + 3p(1-p)^2 T_N(\check{N}_0 + 4) + (1-p)^3 T_N(\check{N}_0 + 6)]$. The latter is always smaller because we can show for any \check{N}_0 it is true that

$$p^3 T_N(\check{N}_0) + 3p^2(1-p)T_N(\check{N}_0 + 2) + 3p(1-p)^2 T_N(\check{N}_0 + 4) + (1-p)^3 T_N(\check{N}_0 + 6) \quad (\text{EC.1})$$

is smaller than

$$p^2 T_N(\check{N}_0) + 2p(1-p)T_N(\check{N}_0 + 3) + (1-p)^2 T_N(\check{N}_0 + 6). \quad (\text{EC.2})$$

To see this, note that (EC.2) minus (EC.1) equals $p(1-p)\{p[(T_N(\check{N}_0) - T_N(\check{N}_0 + 2)) - 2(T_N(\check{N}_0 + 2) - T_N(\check{N}_0 + 3))]\} + (1-p)[(2(T_N(\check{N}_0 + 3) - T_N(\check{N}_0 + 4)) - T_N(\check{N}_0 + 4) - T_N(\check{N}_0 + 6))]$, which is positive given Condition 1.

Step 1.3. We show that the optimal diversification strategy can not simultaneously have one region with one supplier and one region with three suppliers. This is true because any strategy (n_1, \dots, n_R) with $n_{R-1} = 3, n_R = 1$ is outperformed by a modified strategy $(n_1, \dots, n_{R-2}, \acute{n}_{R-1}, \acute{n}_R)$ with $\acute{n}_{R-1} = \acute{n}_R = 2$. Let $\acute{N}_0 \stackrel{def}{=} \sum_{r=1}^{R-2} n_r \cdot \mathbb{I}_{\{y_r=0\}}$, a random variable representing the number of suppliers with low regional cost in regions 1 through $(R-2)$. Thus, the expected total procurement cost under the strategy (n_1, \dots, n_R) is $E_{\acute{N}_0}[p^2 T_N(\acute{N}_0) + p(1-p)T_N(\acute{N}_0 + 1) + p(1-p)T_N(\acute{N}_0 + 3) + (1-p)^2 T_N(\acute{N}_0 + 4)]$ and that under the modified strategy is $E_{\acute{N}_0}[p^2 T_N(\acute{N}_0) + 2p(1-p)T_N(\acute{N}_0 + 2) + (1-p)^2 T_N(\acute{N}_0 + 4)]$. The latter is smaller because for any \acute{N}_0 we have $p^2 T_N(\acute{N}_0) + p(1-p)T_N(\acute{N}_0 + 1) + p(1-p)T_N(\acute{N}_0 + 3) + (1-p)^2 T_N(\acute{N}_0 + 4)$ is greater than $p^2 T_N(\acute{N}_0) + 2p(1-p)T_N(\acute{N}_0 + 2) + (1-p)^2 T_N(\acute{N}_0 + 4)$, given that $T_N(\acute{N}_0 + 1) + T_N(\acute{N}_0 + 3) > 2T_N(\acute{N}_0 + 2)$ (by Condition 1).

Step 2. We now show that fully diversifying is optimal when p is close to zero. Per the discussion in §4.3, the R -region strategy $(n_1, \dots, n_{R-1}, n_R)$ is outperformed by the $(R+1)$ -region strategy $(n_1, \dots, n_{R-1}, \tilde{n}_R, \tilde{n}_{R+1})$ with $\tilde{n}_R + \tilde{n}_{R+1} = n_R$ whenever $E[\Delta(\hat{N}_0)] \geq 0$. Let $\underline{\Delta} \stackrel{def}{=} \min\{\Delta(1), \dots, \Delta(N - n_R)\}$. By Condition 1, we have $\underline{\Delta} > 0$. Note that $E[\Delta(\hat{N}_0)]$ is greater than $p^{R-1}\Delta(0) + (1-p^{R-1})\underline{\Delta}$, which is positive when p is close enough to zero regardless of the sign of $\Delta(0)$. Namely, when p is close enough to zero the buyer always prefers the $(R+1)$ -region strategy to the R -region strategy. Repeated application of this argument implies that fully diversifying is optimal.

Step 3. We now show that the partially diversifying strategy (as prescribed in the lemma's statement) is optimal when p is close to one. This is shown in the following two sub-steps.

Step 3.1. We show that the optimal diversification strategy has at most one region with one supplier. Together with Step 1, this implies that it is optimal to have $\frac{N}{2}$ regions with two supplier each when N is even. We show this by showing any R -region strategy $(n_1, \dots, n_{R-1}, n_R)$ with $n_R = 2$ outperforms the $(R+1)$ -region strategy $(n_1, \dots, n_{R-1}, \tilde{n}_R, \tilde{n}_{R+1})$ with $\tilde{n}_R = \tilde{n}_{R+1} = 1$ when p is close to one. Per the discussion in §4.3, the buyer prefers the R -region strategy to the $(R+1)$ -region strategy whenever $E[\Delta(\hat{N}_0)] < 0$. Let $\bar{\Delta} \stackrel{def}{=} \max\{\Delta(1), \dots, \Delta(N - n_R)\}$. Note that $E[\Delta(\hat{N}_0)] < p^{R-1}\Delta(0) + (1 - p^{R-1})\bar{\Delta}$ and $\Delta(0) < 0$ when $\tilde{n}_R = \tilde{n}_{R+1} = 1$ if $T_N(1) > \frac{T_N(0) + T_N(2)}{2}$. These together imply that $E[\Delta(\hat{N}_0)] < 0$ when p is close to 1, regardless of the sign of $\bar{\Delta}$.

Step 3.2. We now derive the optimal diversification strategy when N is odd. By Steps 1 and 3.1, we conclude that when N is odd, the optimal diversification strategy is either an $(\frac{N+1}{2})$ -region strategy with $n_1 = \dots = n_{\frac{N-1}{2}} = 2$ and $n_{\frac{N+1}{2}} = 1$ or an $(\frac{N-1}{2})$ -region strategy with $n_1 = \dots = n_{\frac{N-3}{2}} = 2$ and $n_{\frac{N-1}{2}} = 3$. Let $\hat{N}_0 \stackrel{def}{=} \sum_{r=1}^{\frac{N-3}{2}} n_r \cdot \mathbb{I}_{\{y_r=0\}}$, a random variable representing the number of suppliers with low regional cost in regions 1 through $\frac{N-3}{2}$. The expected total procurement cost under the $(\frac{N-1}{2})$ -region strategy, equal to $E_{\hat{N}_0}[pT_N(\hat{N}_0) + (1-p)T_N(\hat{N}_0 + 3)]$, minus that under the $(\frac{N+1}{2})$ -region strategy, equal to $E_{\hat{N}_0}[p^2T_N(\hat{N}_0) + p(1-p)T_N(\hat{N}_0 + 1) + p(1-p)T_N(\hat{N}_0 + 2) + (1-p)^2T_N(\hat{N}_0 + 3)]$, is

$$p(1-p)E_{\hat{N}_0}[T_N(\hat{N}_0) - T_N(\hat{N}_0 + 1) - T_N(\hat{N}_0 + 2) + T_N(\hat{N}_0 + 3)]. \quad (\text{EC.3})$$

Note that when p is close to one (and thus \hat{N}_0 is most likely to be zero) the sign of (EC.3) is the same as the sign of $T_N(0) - T_N(1) - T_N(2) + T_N(3)$. Thus, when p is close to one and N is odd,

$$\begin{aligned} &\text{the } \left(\frac{N-1}{2}\right)\text{-region strategy is optimal if } T_N(0) - T_N(1) - T_N(2) + T_N(3) < 0; \\ &\text{the } \left(\frac{N+1}{2}\right)\text{-region strategy is optimal, otherwise.} \end{aligned} \quad (\text{EC.4})$$

EC.9. Proof that $\{1, \bar{X}_{2:2}, \dots, \bar{X}_{2:N}\}$ is convex and decreasing for power-function distributions

For power-function distributions $F(x) = x^v$ with $v > 0$, we have $\bar{X}_{2:N} = \frac{\Gamma(N+1)\Gamma(2+1/v)}{\Gamma(N+1+1/v)} = \prod_{n=2}^N \frac{n}{n+1/v}$ for $N \geq 2$; see Malik (1967).

To see $\{1, \bar{X}_{2:2}, \dots, \bar{X}_{2:N}\}$ is convex and decreasing, first, note that for any $n \geq 2$, we have $\bar{X}_{2:n} - \bar{X}_{2:n+1} = \bar{X}_{2:n} - \bar{X}_{2:n} \frac{n+1}{n+1+1/v} = \bar{X}_{2:n} \frac{1/v}{n+1+1/v}$, which decreases in n (because both $\bar{X}_{2:n}$

and $\frac{1/v}{n+1+1/v}$ decrease in n). This implies that $\{\bar{X}_{2:2}, \dots, \bar{X}_{2:N}\}$ is convex and decreasing. Then, note that $\{1, \bar{X}_{2:2}, \bar{X}_{2:3}\}$ is convex and decreasing because we can check that $1 + \bar{X}_{2:3} - 2\bar{X}_{2:2} = \frac{1/v+1/v^2}{(2+1/v)(3+1/v)} > 0$.

EC.10. Proof of Proposition 4

We now explicitly derive the thresholds s_1^{Mech} , s_2^{Mech} , and s_3^{Mech} . For any of the five mechanisms, when $s > \max\{s_1^{Mech}, s_2^{Mech}, s_3^{Mech}\}$, the proposition follows from Lemma 3.

Thresholds s_1^{Mech} . Per the mechanisms' allocation rules described in Lemma 1, if $N_0 \geq 2$, only the N_0 suppliers with low regional cost can possibly win the auction when $s \geq \psi(1)$ under format OPT, or when $s \geq 1$ under formats RE and RER. Under formats FS and FSR, when $N_0 \geq 2$, we have $\beta_1(x) \leq \bar{b} = 1$ per the discussion in §EC.1, which implies that the $N - N_0$ high-regional-cost suppliers can never win the auction when $s \geq 1$. Thus, we have $s_1^{OPT} = \psi(1)$, and $s_1^{Mech} = 1$ for $Mech \in \{RE, RER, FS, FSR\}$.

Thresholds s_2^{Mech} . Given that $T_N^{Mech}(N_0) = \bar{X}_{2:N_0}$ when $N_0 \geq 2$ and $s \geq s_1^{Mech}$, and that the sequence $\{1, \bar{X}_{2:2}, \dots, \bar{X}_{2:N}\}$ is assumed convex, we find the thresholds s_2^{Mech} for Condition 1 by examining $T_N^{Mech}(0)$ and $T_N^{Mech}(1)$. Per the mechanisms' allocation rules described in Lemma 1 and the effect of a reserve price, we have $T_N^{Mech}(1) = 1$ if $Mech = OPT$ and $s \geq \psi(1) = s_1^{Mech}$, or if $Mech \in \{RER, FSR\}$ and $s \geq 1 = s_1^{Mech}$. Thus, for $Mech \in \{OPT, RER, FSR\}$, $T_N^{Mech}(N)$ is convex over $\{1, \dots, N\}$ when $s \geq s_1^{Mech}$. Note that $T_N^{Mech}(0) = s + \bar{X}_{2:N}$. Thus, when $s \geq \max\{s_1^{Mech}, 2 - \bar{X}_{2:2} - \bar{X}_{2:N}\}$, $T_N^{Mech}(N)$ is convex over $\{0, 1, 2\}$ and hence over the whole support $\{0, 1, \dots, N\}$. Hence, $s_2^{Mech} = \max\{s_1^{Mech}, 2 - \bar{X}_{2:2} - \bar{X}_{2:N}\}$, for $Mech \in \{OPT, RER, FSR\}$.

Per the discussion in §EC.1, we have $\underline{b}_1, \underline{b}_2 \geq s$ when $N_0 = 1$ and FS is used; thus, we have $T_N^{FS}(1) \geq s$. Clearly, we have $T_N^{RE}(1) \geq s$. Because $T_N^{Mech}(1) \geq s$ for $Mech \in \{FS, RE\}$ and $\{1, \bar{X}_{2:2}, \dots, \bar{X}_{2:N}\}$ is assumed convex, when $s \geq 1 = s_1^{Mech}$ we have $T_N^{Mech}(N)$ is convex over $\{1, \dots, N\}$. Then, given that $T_N^{Mech}(0) = s + \bar{X}_{2:N}$, we conclude that $T_N^{Mech}(N)$ is convex over $\{0, 2, 3, \dots, N\}$ when $s \geq \max\{s_1^{Mech}, 3\bar{X}_{2:2} - 2\bar{X}_{2:3} - \bar{X}_{2:N}\}$ for $Mech \in \{RE, FS\}$. Namely, $s_2^{Mech} = \max\{s_1^{Mech}, 3\bar{X}_{2:2} - 2\bar{X}_{2:3} - \bar{X}_{2:N}\}$ for $Mech \in \{RE, FS\}$.

Thresholds s_3^{Mech} . For $Mech \in \{OPT, RER, FSR\}$, when deriving s_2^{Mech} we showed that $T_N^{Mech}(N)$ is convex over the entire support $\{0, 1, \dots, N\}$ when $s \geq s_2^{Mech}$; thus, $s_3^{Mech} = s_2^{Mech}$. For $Mech \in \{RE, FS\}$, given that $T_N^{Mech}(0) = s + \bar{X}_{2:N}$, $T_N^{Mech}(1) \geq s$, and $T_N^{Mech}(2) = \bar{X}_{2:2}$ when

$s \geq s_1^{Mech} = 1$, we have $T_N^{Mech}(1) > \frac{T_N^{Mech}(0) + T_N^{Mech}(2)}{2}$ when $s \geq \max\{1, \bar{X}_{2:2} + \bar{X}_{2:N}\}$. Hence, $s_3^{Mech} = \max\{1, \bar{X}_{2:2} + \bar{X}_{2:N}\}$ for $Mech \in \{RE, FS\}$.

EC.11. Proof of Proposition 5

The proposition follows immediately from the discussion on page 26.

EC.12. Codependence can dis/encourage fully diversifying under RE and FS.

We illustrate this possibility with a stylized example. Suppose each region has its own outbound port and a cost shock reflects a prolonged port labor strike which would necessitate costly re-routing (e.g., air shipments). To capture codependence across ports, suppose with probability β the ports can form a coalition whereby all ports agree to adhere to an umbrella organization's decision to strike or not. If a coalition is formed, let q be the probability that it would strike. In the absence of a coalition, let γ be the probability that a port independently decides to strike. Because a coalition removes differences between ports' regional costs, the diversification decision is made based on γ the probability of strikes absent a coalition.⁸ Because the marginal probability that a region strikes is fixed at $p = \beta q + (1 - \beta)\gamma$, an increase in the codependence level β monotonically affects the size of $\gamma = \frac{p - \beta q}{1 - \beta}$. If $p < q$, then γ decreases with β . In this case, larger β accentuates the fact that individual ports are less aggressive than the coalition ($p < q$) and hence unlikely to strike unilaterally, diminishing the risk of an advantaged supplier. Thus, more coordination across ports (larger β) favors fully diversifying under formats RE/FS when $p < q$. Conversely, more coordination discourages fully diversifying under these formats when $p > q$.

EC.13. Proof of Proposition 6

Let $T_N(N_0|s)$ denote the expected total procurement cost given a two-point regional cost distribution with cost shock scale s and N_0 suppliers without a cost shock. Given the regional costs of supplier 1 and supplier 2, denoted by a_1 and a_2 , respectively, we let $Z(a_1, a_2)$ denote the expected total procurement cost. Then, for $y_1 \leq y_2$ we can write $Z(y_1, y_2) = Z(y_2, y_1) = y_1 + T_2(1|y_2 - y_1)$; $Z(y_1, y_1) = y_1 + T_2(2|y_2 - y_1)$; $Z(y_2, y_2) = y_1 + T_2(0|y_2 - y_1)$.

The expected cost difference between pooling and diversifying equals $E_{(y_1, y_2)}[Z(y_1, y_1) - Z(y_1, y_2)]$, which equals the diversifying upside $E_{(y_1 \geq y_2)}[Z(y_1, y_1) - Z(y_1, y_2)]$, minus the diversifying downside

⁸Formally, the distribution of \hat{N}_0 conditional on $y_R \neq y_{R+1}$ is binomial($N - n_R, 1 - \gamma$).

$E_{(y_1 \leq y_2)}[Z(y_1, y_2) - Z(y_1, y_1)]$. Note that the diversifying upside equals $E_{(y_1 \geq y_2)}[y_2 + T_2(0|y_1 - y_2) - (y_2 + T_2(1|y_1 - y_2))]$, and the diversifying downside equals $E_{(y_1 \leq y_2)}[y_1 + T_2(1|y_2 - y_1) - (y_1 + T_2(2|y_2 - y_1))]$. Thus, the expected cost difference equals $E_s[T_2(0|s) - 2T_2(1|s) + T_2(2|s)]$ where $s = \max\{y_1, y_2\} - \min\{y_1, y_2\}$.

Consequently, the proposition follows from the fact that $T_2(0|s) - 2T_2(1|s) + T_2(2|s)$ is positive (negative) regardless of $s > 0$ when the buyer uses OPT (RE), per the proof in §EC.5. Finally, note that all of the above arguments are valid as long as y_1 and y_2 are symmetric; thus, the proposition holds even if y_1 and y_2 are correlated.

EC.14. Optimal to have $R \geq 2$ with $N \geq 3$ suppliers and general regional cost distribution

Using the same approach as in §EC.13, for $N \geq 3$ suppliers we compare pooling against a two-region strategy having $n_1 = \lfloor \frac{N}{2} \rfloor = N - n_2$. We let $Z_N(a_1, a_2)$ denote the expected total procurement cost when n_1 suppliers have regional cost a_1 and n_2 suppliers have regional cost a_2 . Then, for $y_1 \leq y_2$ we can write $Z(y_1, y_2) = y_1 + T_N(n_1|y_2 - y_1)$, $Z(y_2, y_1) = y_1 + T_N(n_2|y_2 - y_1)$, $Z(y_1, y_1) = y_1 + T_N(N|y_2 - y_1)$; $Z(y_2, y_2) = y_1 + T_N(0|y_2 - y_1)$. Note that the expected cost difference between pooling and the two-region strategy equals $E_{(y_1, y_2)}[Z(y_1, y_1) - Z(y_1, y_2)] = E_{(y_1 \leq y_2)}[Z(y_1, y_1) - Z(y_1, y_2)] + E_{(y_1 \geq y_2)}[Z(y_1, y_1) - Z(y_1, y_2)] = E_{(y_1 \leq y_2)}[y_1 + T_N(N|y_2 - y_1) - y_1 - T_N(n_1|y_2 - y_1)] + E_{(y_1 \geq y_2)}[y_2 + T_N(0|y_1 - y_2) - y_2 - T_N(n_2|y_1 - y_2)] = E_s[T_N(0|s) - T_N(n_1|s) - T_N(n_2|s) + T_N(N|s)]$, where $s = \max\{y_1, y_2\} - \min\{y_1, y_2\}$. Under Condition 1, the buyer prefers the two-region strategy when $N \geq 4$ because in such cases $T_N(0|s) - T_N(n_1|s) - T_N(n_2|s) + T_N(N|s) > 0$ for all s . For a three-supplier case, the two-region strategy is preferred when $E_s[T_3(0|s) - T_3(1|s) - T_3(2|s) + T_3(3|s)] > 0$; otherwise, pooling is preferred.

EC.15. Proof of Proposition 7

When all suppliers have equal constant production cost, say x_0 , the format OPT awards the contract to the supplier with the lowest regional cost, breaking ties randomly, and pays the contract winner his production cost. Thus, when the buyer uses OPT, her expected total procurement cost equals the constant production cost plus the expected lowest regional cost, i.e., $x_0 + \bar{Y}_{1:R}$. Since $\bar{Y}_{1:R}$ decreases in R even when $y_r, r = 1 \dots R$, follow a general distribution G and are possibly correlated, fully diversifying ($R = N$) is optimal. Namely, part (i) of the proposition holds for any regional cost distribution G and any codependence across regional costs.

We now turn to format RE, and examine the buyer's preference between fully diversifying ($R = N$) and partially diversifying (for N even, $R = \frac{N}{2}$ with two suppliers in each region; for N odd, $R = \frac{N-1}{2}$ with two suppliers in each region but one region with three suppliers). If the buyer fully diversifies, the supplier with the lowest regional cost wins and receives a payment equal to the difference between the second-lowest regional cost and his regional cost. Consequently, the buyer's expected total procurement cost equals $x_0 + \bar{Y}_{2:R}$. On the other hand, if the buyer partially diversifies, one supplier from the lowest-cost region wins (ties are broken randomly) and is paid his production cost x_0 . In such a case, the buyer's expected total procurement cost equals $x_0 + \bar{Y}_{1:R}$.

When the regional costs are independent draws from a power-function distribution $G(y) = a^{-v}y^v$ with scale parameter $a > 0$ and shape parameter $v > 0$, we have $\bar{Y}_{1:R} = \frac{a\Gamma(R+1)\Gamma(1+1/v)}{\Gamma(R+1+1/v)}$ and $\bar{Y}_{2:R} = \frac{a\Gamma(R+1)\Gamma(2+1/v)}{\Gamma(R+1+1/v)}$; see Malik (1967). Thus, for $N \geq 4$ even, we have

$$\frac{\bar{Y}_{1:\frac{N}{2}}}{\bar{Y}_{2:N}} = \frac{(N+1/v)(N-1+1/v)\cdots(N/2+1+1/v)}{(1+1/v)N(N-1)\cdots(N/2+1)}.$$

We now show that there exists a threshold $\underline{v} > 0$ such that the above fraction is greater than 1 when $v < \underline{v}$ and less than 1 when $v > \underline{v}$. To see this, note that the numerator minus the denominator can be written as $-b_1v^{-1} + b_2v^{-2} + \dots + b_{\frac{N}{2}}v^{-\frac{N}{2}}$ with $b_1, \dots, b_{\frac{N}{2}} > 0$. Thus, the threshold \underline{v} is the unique positive root of $b_1 = b_2v^{-1} + \dots + b_{\frac{N}{2}}v^{-\frac{N}{2}+1}$. Note \underline{v} is unique because $b_2v^{-1} + \dots + b_{\frac{N}{2}}v^{-\frac{N}{2}+1}$ is strictly decreasing, approaches positive infinity as v approaches zero, and approaches zero as v approaches positive infinity. For $N \geq 3$ odd, we can similarly prove that $\bar{Y}_{1:\frac{N-1}{2}}/\bar{Y}_{2:N}$ is greater (less) than 1 if v is greater (less) than a threshold $\underline{v} > 0$. Namely, part (ii) of the proposition holds.

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