Total-Cost Procurement Auctions with Sustainability Audits to Inform Bid Markups

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Sustainability is a differentiator-type supplier attribute which can affect the total cost of ownership and is of increasing concern to buyer firms. To make a more informed total-cost procurement decision, a buyer can choose to employ one of the proliferating sustainability auditing agencies to audit her potential suppliers, and select a supplier based on the price bids and the unsustainability cost markup terms informed by the audits. However, sustainability audits are costly and whether to use them is at the discretion of the buyer. Hence, the buyer can instead choose to forgo the sustainability audits and select a supplier based on price only. In this paper, we explore this tradeoff. We find that although the audits are used for resolving uncertainty about suppliers' sustainability levels, greater uncertainty about supplier sustainability levels and a less sustainable supplier base do not necessarily make the audits more valuable for the buyer. Similarly, greater ex ante variability and an increase in suppliers' production costs — which intuitively would reduce the benefit of fine-tuning comparisons through sustainability audits – can actually increase the value of audits. Furthermore, we find that the current industry practice of "all or none" audits can be improved upon by selectively auditing just a subset of suppliers, even if they are ex ante symmetric. Our results are generalizable to cases where the buyer has the option to invest in costly information acquisition on her potential suppliers' generic, differentiator-type attributes.

Key words: Procurement Auctions, Total Cost of Ownership, Information Acquisition, Supplier Audits, Sustainable Procurement

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1. Introduction

Total cost of ownership (TCO) is an important consideration when making procurement decisions (Ellram 1994). However, TCO analyses are costly for a buyer to perform. For example, a buyer may need to create detailed cost models to properly assess the inventory implications of a supplier's location, conduct tests to estimate how well a supplier's part performs when finally integrated into the buyer's product, or hire auditing agencies to evaluate a supplier's social and environmental sustainability level. Hence, the buyer needs to exert effort, spend time and divert valuable resources for a TCO analysis. Consequently, a TCO analysis, although informative, represents a form of transaction cost. In fact, transaction costs have long been recognized as an important challenge in deciding whether to or not to source from a supplier (Coase 1937). In this paper, we address this fundamental trade-off in the supplier selection context, and examine to what extent a buyer with an outside option (e.g., internal production) should invest in gaining better evaluations on non-price attributes associated with her potential suppliers.

Non-price attributes can be divided into two categories — requirements and differentiators. A buyer must ensure that her requirements will be met by the chosen supplier prior to signing a contract. Hence, she has no discretion over the requirement evaluations. In contrast, the buyer has discretion over how much to invest in evaluating the differentiator-type attributes that can lead to a higher or lower TCO.

There are a number of differentiators that a buyer could choose to evaluate. Many involve intrinsic features of a supplier (which interact with the intrinsic features of the buyer in numerous cases), which the suppliers typically cannot quantify nor can change in the time frame of the buyer's imminent contract. For example, a supplier's proximity to a seaport and the quality of infrastructure connecting the supplier to the seaport affect the degree of uncertainty regarding delivery times, the likelihood of damage during transport, and the buyer's ability to rely on ocean shipping — leading to different cost implications for the buyer under different inventory policies. Similarly, the likelihood of field-failures, when the supplier's part is integrated into the buyer firm's final product, can be estimated by extensive lab and field testing conducted by the buyer and depends on both the buyer's and the supplier's products' intrinsic features and interactions. The supplier's degree of sustainability on a multitude of different dimensions — e.g., ecological footprint, human and social impacts, etc. — may drive the buyer firm's anticipated costs associated with loss of goodwill, supply chain disruptions, and damage to brand image due to unsustainable procurement. Note that these anticipated costs depend on a number of intrinsic features of the buyer (e.g., industry segment, country of operation, level of public scrutiny on the buyer due to poor labor practices in the past, etc.), and hence the specific considerations used when generating a meaningful supplier sustainability assessment/rating depends on the buyer's own context (see Gibson et al. (2013) for a detailed discussion on the context-dependent nature of sustainability assessments).

The results from our paper are applicable to a wide variety of differentiator-type attributes. However, motivated by growing concerns on sustainable procurement (for example, see BSR Press Release (2013), Lacey et al. (2010), J&J Press Release (2011), and Primark Press Release (2013)), the extent of challenges involved with implementing the TCO approach in sustainable procurement (BSR Press Release 2013), our own interactions with a major sustainability auditing firm (EcoVadis), and the growing discussion around the sustainability auditing agencies' role in procurement, we will focus on sustainability as our prime example.

To make a TCO-based procurement decision, the buyer would like to rate her potential suppliers in terms of sustainability and assess the associated additive costs (unsustainability costs) the buyer would anticipate incurring if she does business with them. However, it can be difficult to do so. An informed rating and assessment requires gathering an understanding of supplier practices, which are then viewed through various lenses to evaluate how these impact different sustainability dimensions like water and energy usage, carbon foot print, social well-being, etc., while taking into account the buyer's specific context.

This evaluation process requires deep, domain-specific knowledge on various technical (e.g., aquifer resources and biodiversity) and socio-political (e.g., social norms and political landscape) issues. Given these specialized knowledge requirements, third-party sustainability auditing agencies provide supplier sustainability auditing and rating services for buyer firms (Bruel et al. 2013), albeit at a cost.

Auditing agencies possess deep domain expertise that neither the buyer nor the supplier firms possess. Furthermore, as explained above, the sustainability assessments also depend on the buyer's intrinsic features and/or subjective preferences. Hence without the audits neither the supplier nor the buyer can observe the sustainability ratings and the associated unsustainability costs. Having an auditing agency assess a supplier allows the buyer to form a TCO assessment for that supplier, which in turn helps the buyer make a more informed procurement decision. Both the sustainability audits and suppliers' proposal preparation (deciding how much to bid) take time. Hence, in practice (e.g., at EcoVadis) carrying out the audits in parallel to suppliers' proposal preparation, and prior to the actual bidding stage, ensures the timely completion of the procurement process. Similarly, audits for different suppliers are carried out simultaneously (rather than sequentially) in order to save time.

Buyer firms today face ever larger supplier pools with the use of online procurement platforms (for example, a search on alibaba.com for printed circuit boards in Guangdong, China produces a list of over 2,800 suppliers). Note that the costs of auditing the suppliers in the supply pool, especially when facing a large supplier pool, can be prohibitive. Due to this auditing cost and the fact that evaluating differentiator attributes is by definition discretionary, a buyer may choose to simply forgo a costly assessment of them. That is, a buyer may proceed with the procurement event without having obtained detailed supplier sustainability assessments. This presents a tradeoff for the buyer. On one hand, she would like to audit her potential suppliers in order to make a more informed supplier selection decision taking into account the total cost of ownership. On the other hand, sustainability audits are expensive, and they might not even lead to changing the buyer's sourcing decision (this may happen, for example, if the audits reveal that the suppliers are roughly equal in terms of their sustainability levels).

To capture this tradeoff, we utilize a stylized model wherein if the buyer chooses to conduct sustainability audits, the winning supplier is selected based on total cost. If the buyer chooses not to conduct sustainability audits, then the winning supplier is chosen based on the price bids (without information on the unsustainability cost assessments). If the buyer chooses not to award the business to any supplier, she defaults to her "outside option" (e.g., in-house production). We address the following research questions:

1. Under what conditions is the buyer better off using the costly sustainability audits?
2. How does the answer to the above question depend on the buyer's ex ante uncertainty about the suppliers' production costs and sustainability levels?
3. Is it ever preferable to audit some suppliers and not others, even if they are ex ante symmetric?

In answering these questions we derive the buyer's optimal sourcing mechanism given a set of sustainability levels gleaned from the audits (if any) that were conducted. We find that although the audits are used for resolving uncertainty about suppliers' relative sustainability levels, more uncertainty about supplier sustainability levels and a less sustainable supplier base do not necessarily make the audits more valuable for the buyer. Similarly, greater ex ante variability and an increase in suppliers' production costs — which intuitively would reduce the benefit of fine-tuning comparisons through sustainability audits – can actually increase the benefits of such audits. Furthermore, we find that the use of "all or none" audits (currently used in practice) can be improved upon by selectively auditing just a subset of suppliers, even if they are ex ante symmetric.

Another related and interesting challenge that buyer firms face is not knowing how to convert audit results (i.e., sustainability ratings) into unsustainability costs for a TCO assessment (see, for example, BSR Press Release (2013)). This conversion should capture the buyer's level of sensitivity to sustainability (which depends on, for example, the proportion of green consumers) — on which buyer firms generally lack information (and which the buyer firms can choose to learn through market research, consumer surveys, etc.). In fact, how to assess the cost implications of suppliers' non-price attributes is a major concern for buyer firms when implementing a TCO analysis in many other contexts (e.g., how to convert the level of compatibility between the supplier's and the buyer's parts into a cost term). Hence, in this paper we also address a fourth research question:

4. When should the buyer seek to resolve uncertainty (if any) on how to convert the sustainability ratings into unsustainability costs terms for a TCO assessment?

The non-price attributes we study are intrinsic to the buyer-supplier pair, moreover the suppliers do not have the in-house expertise to assess and quantify these attributes. Hence, in a procurement auction the suppliers cannot bid these attributes alongside price (hence these attributes are "non-biddable"). Several previous studies on total-cost auctions (see for example the scoring auctions in Che (1993), Branco (1997), Bichler (2000), Beil and Wein (2003), and Asker and Cantillon (2008)) focus on "biddable" non-price attributes that can be instantaneously controlled by the suppliers (with an underlying assumption that the suppliers can observe all the non-price attributes). In our paper however, similar to Engelbrecht-Wiggans et al. (2007), Kostamis et al. (2009), and Haruvy and Katok (2013), the non-price attribute of a supplier is an exogenous, non-biddable attribute that cannot be changed in the imminent contract term. Although both the practitioner and the academic literature (e.g., Ellram (1994), Degraeve and Roodhooft (1999), Ferrin and Plank (2002), Ellram (1993), Engelbrecht-Wiggans et al. (2007), and Tunca et al. (2013)) acknowledge the fact that the buyer would need assessments to evaluate the non-biddable attributes and cost markups associated with her potential suppliers, any treatment of such an assessment step (i.e., the TCO analysis step) is ignored in previous studies on total-cost auctions. Hence, in contrast to the existing literature on total-cost auctions with exogenous quality (e.g., Kostamis et al. (2009), Haruvy and Katok (2013), and

Engelbrecht-Wiggans et al. (2007)), we forgo the assumption that the buyer is endowed with information on the non-biddable attributes of the competing suppliers. Note that this reflects the realistic situation where the buyer needs to invest in TCO analysis (e.g., by using audits) to assess these attributes.

A large body of literature exists on the topic of quality inspection in operations management, and several recent studies in procurement auctions look at how buyers should most cost-effectively acquire information on competing suppliers' requirement-type attributes (supplier qualification screening, Wan and Beil (2009), Wan et al. (2012)). Despite this rich body of research on quality inspection (focusing on requirement-type attributes), and total-cost auctions (assuming that non-price attributes are biddable and/or readily observable) there is a lack of understanding about procurement process management in the presence of uncertain, non-biddable, differentiator-type attributes (for example, suppliers' sustainability levels and the related unsustainability costs), and the characteristics of optimal auditing policies to assess these attributes in a total-cost procurement setting. In this paper, we aim to fill this gap.

2. Related Literature

The main features of our model are as follows: (a) suppliers submit price-only bids; (b) however, the competition is over two dimensions: purchase price and a non-biddable differentiator-type attribute (e.g., unsustainability cost); (c) there is intrinsic uncertainty around the non-biddable attributes of suppliers (for both the buyer and the suppliers themselves); and (d) the buyer has the option to reduce this uncertainty at a cost.

Features (a) and (b) are self-explanatory. Feature (c) arises in settings where evaluating a non-price attribute requires expertise (such as sustainability assessments through audits) and/or depends on opaque criteria (e.g., when evaluating the buyer's personal rapport with counterparts at a supplier). Feature (d) simply says that learning more (e.g., through interviews, audits, etc.) can help the buyer perform this evaluation but is costly. Features (c)-(d) are in this sense related to the widely studied notion of transaction costs, dating back to the work of Coase (1937).

Since the buyer may collect information on the non-price attribute through audits, our work is related to the "informed principle" literature, tracing back to the seminal work Myerson (1983). Skreta (2011) provides a recent overview of informed principle problems in an auction framework, as well as a result we find useful for our setting: In an optimal mechanism, the auctioneer cannot increase her payoff by not sharing the information she has with the bidders on competing bidders' exogenous characteristics.

Kostamis et al. (2009) study a procurement auction setting, where the auction features are identical to features (a) and (b) above. However, in their setting the buyer is perfectly informed about the exogenous non-price attributes (called "cost adjustments") of all suppliers to start with, and the paper focuses on the buyer's choice between sealed bid or open descending auction formats. The auction formats studied are suboptimal, and the buyer may do better by hiding some information from suppliers about how they compare to their competitors, unlike the case for optimal auctions (Skreta (2011)). In a similar vein to Kostamis et al.

(2009), the papers Engelbrecht-Wiggans et al. (2007), Elmaghraby et al. (2012) and Haruvy and Katok (2013) compare the buyer's outcomes from different sub-optimal auction formats (given that the buyer is readily informed about suppliers' cost adjustments). In all these papers, the underlying assumption is that the buyer is informed up-front about the exogenous characteristics of suppliers. In our paper, we forgo this assumption. Consequently, in our setting, the buyer needs to exert effort (e.g., through audits) to acquire information on the exogenous characteristics of the suppliers. With or without audits, in our paper the buyer uses an optimal auction.

Motivated by the increasing importance of sustainable supply chain management, there are a number of recent papers on supplier inspection policies. Kim (2013) studies how to best ensure compliance with environmental laws, by examining the interaction between a regulator's inspection schedule (random or periodic) and the supplier's non-compliance disclosure policy. Plambeck and Taylor (2012) study a setting where the buyer decides on how much effort to exert on auditing a supplier, and in return the supplier decides on how much effort to exert on hiding information from the buyer during the auditing process. In both these papers, there is a single supplier and the focus is on that supplier's decision on hiding information and how this affects auditing decisions, an aspect akin to a law enforcement setting. Our paper focuses on a different and important issue, namely the implementation of an auditing policy in a procurement setting where there are multiple suppliers, and the buyer can conduct audits on her potential suppliers to determine which supplier (if any) she prefers from a total-cost perspective.

Research on information acquisition in auctions has generally focused on bidders' acquisition of information, where a key question is to what extent the auctioneer should make information available to the bidders. Examples include work on the linkage principle and auction format choice (e.g., Milgrom and Weber (1982)), and more recently Bergemann and Pesendorfer (2007) and Shi (2012). Information acquisition by auctioneers (buyers) in procurement auctions is extremely relevant in practice but has only recently started to gain research attention. In an attempt to explain the unconsummated procurement auctions documented in Snir and Hitt (2003), Carr (2003) studies an equilibrium analysis with suppliers' endogenous entry decisions (due to costly entry and risk of auction cancellation) and buyer's decision on whether to cancel the auction or not (due to the cost of bid evaluation). Yin et al. (2013) study the role of cost modeling in procurement auctions where the buyer can acquire information on the suppliers' production costs. The buyer then uses this information to optimally set a reserve price to put downward pressure on the suppliers' price bids. This helps the buyer reduce the information rent paid to suppliers. Chen et al. (2008) compare two procurement settings: scoring auctions (such as in Asker and Cantillon (2008) and Che (1993)) and a profit sharing setting where only the winning supplier is audited to verify his production cost. After verifying the production cost of the winning supplier, the buyer shares the supply chain profit with the winning supplier accordingly. So, similar to cost modeling in Yin et al. (2013), in Chen et al. (2008) the audit is used for acquiring information on the production cost (however, in a simplified setting where the information acquisition is only on the winning supplier). Our paper studies a different problem: the buyer's information acquisition policy on her potential suppliers' non-biddable non-price attributes in order to make a more informed total-cost decision in supplier selection. To the best of our knowledge, we are the first to identify and study this important problem.

In fact, the role of TCO analysis in procurement and the importance of evaluating the non-biddable non-price attributes associated with potential suppliers in supplier selection have long been recognized (e.g., Degraeve and Roodhooft (1999), Ferrin and Plank (2002), Ellram (1993), and Ellram (1994)). However, it is often difficult and costly to evaluate suppliers' non-price attributes. Therefore, in this paper, we aim to understand if, when, and to what extent a buyer should invest in assessing her potential suppliers' non-price attributes in a total-cost procurement setting.

3. Procurement Auctions with Unsustainability Cost Markups

In our model a risk-neutral buyer wishes to award an indivisible contract to one of N competing suppliers through a reverse auction. As explained above, the suppliers submit price-only bids in the auction. However, the buyer cares about the total cost of procurement, which equals the purchase price paid to the chosen supplier plus the unsustainability cost associated with that supplier. The unsustainability cost represents, from the buyer's perspective, the expected additional "non-price" costs the buyer incurs when doing business with a supplier. This could include, for example, the additional costs due to potential environmental or social sustainability issues with the supplier during the contract term. The unsustainability cost of a supplier is an attribute that, at least in the near term, is not changeable. As an example, consider worker safety at a supplier. While the buyer can require her potential suppliers to implement standard practices (such as hanging signs in the production facility describing emergency evacuation procedures), she cannot in the near term have a supplier move to another production facility with better structural properties that make it less likely to collapse. Unsustainability cost captures this latter type of risk and related costs inflicted on the buyer firm due to doing business with a given supplier.

Valuating the unsustainability cost attribute requires information about the supplier's sustainability practices, gleaned through an audit. Assuming that such a valuation approach exists, the mechanics of exactly how the buyer turns the audit data into an unsustainability cost attribute are unimportant for the purposes of our analysis. In practice, the auditing agency we interacted with reports that buyer firms typically use a linear weighting between the sustainability rating (output of the sustainability audit) and the price bid when comparing suppliers from a total-cost perspective. Rather than assuming a particular valuation approach, we model a setting where auditing a supplier simply gives rise to an unsustainability cost attribute for that supplier, with two exceptions where we are more explicit: In Section 5.1 we look at non-linear functional forms to highlight how one could model the buyer's risk attitudes towards unsustainability, and in Section 5.2 we model the buyer's uncertainty about how precisely to map the audit data to an unsustainability cost.

The sustainability levels, and hence the related unsustainability costs of suppliers can only be assessed using a costly auditing process (such as on-site audits, and 3rd party assessments). Let us denote by Δ_i the unsustainability cost of supplier i, where Δ_i is a non-negative random variable identically and independently distributed according to publicly known distribution F with finite mean $\bar{\Delta}$ and support $[\Delta_{(l)}, \Delta_{(u)})$ where $\Delta_{(l)} \geq 0$ and $\Delta_{(u)} \leq \infty$. The distribution F over unsustainability cost captures situations such as a buyer seeking to outsource production of a new product category or a buyer outsourcing an existing product to a new set of suppliers. In both situations, the buyer is not intimately familiar with each supplier in the supply base, and it is difficult ex ante to distinguish between their sustainability levels.

Note that in this setting neither the buyer, nor the supplier i can directly observe the realization δ_i of Δ_i without an audit. This is meant to capture the realistic case in which technical capability is required to assess the sustainability ratings of suppliers. This type of technical capability is what sustainability auditing agencies offer. The auditing agency collects detailed data from suppliers on a number of sustainability-related criteria, and using technical expertise maps this data into sustainability ratings which the buyer then converts into unsustainability cost terms. The buyer incurs a constant auditing cost k per supplier; this can be thought of as the per-supplier auditing fee charged by the auditing agency.

On one hand, if the buyer conducts an audit on supplier i, the buyer will learn the value of unsustainability cost δ_i. On the other hand, if the buyer does not conduct an audit on supplier i, the buyer will only know that Δ_i is distributed according to F. An interesting challenge faced by the buyer is the need to identify an auditing policy on whether to use the costly sustainability audits in her procurement auction to inform her supplier selection decision. This challenge and the related trade-off is the focus of our paper.

We now explain the information held by the suppliers. Of course, suppliers have information regarding their own practices. However, as explained above, without an audit there is opacity regarding the unsustainability costs for the suppliers as well. For example, a supplier may roughly know his annual water usage level, but may not appreciate or comprehend how his water usage affects downstream users hundreds of miles away in an area of water scarcity. We assume that before the auction, the buyer communicates to each of the audited suppliers what their unsustainability cost is, as is done in practice for transparency purposes (e.g., transparency policy regarding cost markups is also documented in previous studies (Ellram (1994), page 71)). Hence, if supplier i is audited, before the auction the buyer informs supplier i of his unsustainability cost δ_i. If a supplier is not audited, we assume he just knows that his unsustainability cost is distributed according to F. (Section 5.3 examines cases where suppliers have private information about their sustainability level.)

For each supplier i, c_i denotes the supplier's production cost. This cost represents the minimum price at which the supplier would accept the contract, capturing things like the supplier's production efficiency, current book of business, inventory levels, strategic objectives, etc. We take c_i to be statistically independent from Δ_i, $\forall i$. For example, a supplier doing standard but water-intensive work such as corn processing may be associated with a high unsustainability cost simply because it happens to be located upstream of an area with impending water scarcity (e.g., see Gassert et al. (2013)). Additionally, we assume that the buyer uses the auditing process as a way to evaluate the unsustainability costs, rather than an attempt to discover suppliers' production costs. (We relax these assumptions in §5.3.) Cost c_i is the supplier's private information, but the buyer knows its distribution G, which has finite support $[c_{(l)}, c_{(u)}]$. The purpose of the auction is to reveal the suppliers' production cost information. As is common in the auctions literature, we assume that the c_i's are independently and identically distributed and that $J(c) \triangleq c + \frac{G(c)}{g(c)}$ is strictly increasing in c (in the remainder of the paper, unless otherwise mentioned "increasing" and "decreasing" are used in the weak sense). This regularity condition is satisfied, for example, by log-normal distributions, including uniform, exponential, and normal; see Bagnoli and Bergstrom (2005). Suppliers are assumed to be risk-neutral and they seek to maximize their expected payoffs.

Timeline. Our model unfolds as follows. In the first step the buyer carries out supplier audits. As a part of standard industry practice, sustainability audits precede the bidding process. This is mainly because both the audits and preparing for the price bids take time, and having these two processes run in parallel ensures the timely completion of procurement. Following the auditing process, the buyer learns the unsustainability costs associated with doing business with the audited suppliers. The buyer then communicates their own unsustainability costs to each of the suppliers. If the buyer chooses not to carry out the supplier audits, all she knows about the unsustainability costs is that they are distributed according to F. In the final step, the suppliers participate in a total-cost auction. The buyer wishes to minimize her total overall cost, that is, the sum of her auditing cost and her procurement cost. The audit cost equals k times the number of audited suppliers; the procurement cost equals the payment to the winning supplier plus its unsustainability cost.

Optimal Auction. We will begin our analysis backwards, starting at the auction step. At this point, the buyer faces N suppliers, and associates an unsustainability cost δ_i with each audited supplier i. The unsustainability cost represents the additional costs that the buyer would incur due to doing business with an audited supplier i. Note that if the buyer chooses to do business with an unaudited supplier, she will observe the realization of the unsustainability cost associated with this supplier only after the auction stage, hence at the time of the auction we treat the unsustainability cost of an unaudited supplier i as random variable Δ_i drawn from cumulative distribution function F. The production cost of supplier i, c_i, is his private information. The buyer wishes to minimize her expected procurement cost, that is, the payment to the winning supplier plus the unsustainability cost associated with the winning supplier.

In searching for the optimal auction mechanism, the revelation principle (Myerson 1981) allows us to focus without loss of optimality on direct mechanisms where each supplier truthfully reveals their private information, namely their production cost. Let p denote an assignment rule, and t a transfer rule: $p_i(\mathbf{c})$ is the probability that the supplier i wins the auction given production cost vector $\mathbf{c} = (c_1, \ldots, c_N)$; $t_i(\mathbf{c})$ is the payment to supplier i given $\mathbf{c}$. We let $\mathbf{c}_{-i}$ denote the vector of production costs excluding c_i. We denote by c_0 the buyer's cost of non-transaction (outside option); for example, this could correspond to the cost of forgoing the contract, or the cost of in-house production. Using a mechanism design analysis (e.g., Myerson (1981)), the buyer's optimal mechanism (p^*, t^*) is characterized as follows:

PROPOSITION 1. *In the optimal mechanism, the buyer assigns each bidder an unsustainability cost markup as follows: $s_i = \delta_i$ if bidder i was audited, $s_i = \bar{\Delta}$ if bidder i was not audited. The buyer announces the following allocation and payment rules:*

The buyer allocates the contract to the supplier with the lowest adjusted virtual cost:

$$p_i^*(\mathbf{c}) = \begin{cases} 1 & \text{if } s_i + J(c_i) \leq s_j + J(c_j), \forall j \neq i \text{ and } s_i + J(c_i) < c_0, \\ 0 & \text{otherwise}. \end{cases}$$

The buyer pays the winning supplier $t_i^*(\mathbf{c}) = p_i^*(\mathbf{c}) c_i + \int_{c_i}^{c(u)} p_i^*(t, \mathbf{c}_{-i}) dt$. The buyer's expected total cost from this optimal mechanism is $E_{\mathbf{c}}[\min\{s_1 + J(c_1), \ldots, s_N + J(c_N), c_0\}]$.

The assignment rule requires choosing a supplier with the lowest adjusted virtual cost, which is the sum of the **unsustainability cost markup** s_i, and the "virtual production cost" $J(c_i)$. A modified reverse clock auction (see Ausubel and Cramton (2006) for discussions about clock auctions in practice) can be used to implement the optimal mechanism. The auction calling price opens at c_0 and continuously drops. At each point in time bidders choose whether to remain in the auction or drop out. Suppose the auction ends at calling price ν and supplier i is the last remaining bidder in the auction; supplier i wins the contract and is paid $J^{-1}(\nu - s_i)$; ties are broken evenly. All suppliers find it a dominant strategy to remain in the auction until the calling price drops below their adjusted virtual cost.

4. Auditing Policies

Section 4.1 studies the buyer's auditing policy when audits lead to perfect observations on the unsustainability costs of the suppliers. Section 4.2 relaxes this assumption and considers a setting where the auditing process may lead to imperfect observations on the unsustainability costs, and more precise observations can be obtained at a higher cost.

4.1. Perfect Learning

Through an audit the buyer obtains a perfect observation on the unsustainability cost of a supplier (without an observation or measurement error). If the buyer chooses to audit supplier i, she will observe the unsustainability cost δ_i; if instead the buyer chooses not to audit supplier i, as shown in Proposition 1, she uses $\bar{\Delta}$ (instead of Δ_i) to stand for the unsustainability cost markup of supplier i. Note that if the unsustainability cost can take large values (e.g., if supplier sustainability failures can have quite severe consequences), then $\bar{\Delta}$ would be high as a consequence.

Per Proposition 1, after the audits (given a vector of unsustainability cost markups $\mathbf{s}$) and prior to running the auction, the buyer's expected total cost from the optimal mechanism is $E_{\mathbf{c}}[\min\{s_1 + J(c_1), \dots, s_N + J(c_N), c_0\}]$. Without loss of generality, let us order the suppliers such that suppliers $i = \{1, \dots, M\}$ are the suppliers to be audited. Then, prior to the audits the buyer's expected total cost in the optimal mechanism is $E_{\mathbf{\Delta},\mathbf{c}}[\min\{\Delta_1 + J(c_1), \dots, \Delta_M + J(c_M), \bar{\Delta} + J(c_{M+1}), \dots, \bar{\Delta} + J(c_N), c_0\}]$.

If the buyer decides not to audit any of the suppliers, the total cost expectation prior to audits is $E_{\mathbf{c}}[\min\{\bar{\Delta} + J(c_1), \dots, \bar{\Delta} + J(c_N), c_0\}]$. It follows that the audits are valuable to the buyer to the degree that they lead to a decrease in this ex ante (pre-audit) total cost expectation. Hence, in §4.1.1 and §4.1.2 we analyze the effect of different auditing policies in changing the buyer's pre-audit total cost expectation.

4.1.1. All Or None Auditing Policies It is currently common practice (e.g., based on discussions with sustainability auditing agencies) that buyers simply audit either all of the participating suppliers or none of them. We refer to such policies as "All or None" auditing policies. It follows that if the buyer chooses to audit all suppliers, the winning supplier is selected based on both unsustainability cost markups and price bids. If the buyer chooses not to audit any of the suppliers, the suppliers are treated as if they all have the mean unsustainability cost $\bar{\Delta}$. Hence, the competition would take place only through the price bids.

We have seen above that the audits are valuable to the buyer to the degree that they help change the expected minimum total cost $E[\text{Cost}|\text{Audit None}] = E_{\mathbf{c}}[\min\{\bar{\Delta} + J(c_1), \dots, \bar{\Delta} + J(c_N), c_0\}]$ that the buyer would incur without the audits. In the rest of the paper, for notational convenience we use J_i to denote supplier i's virtual production cost $J(c_i)$. We denote by $\tilde{G}$ the virtual production cost distribution.

To quantify the value of information from auditing all suppliers with perfect observations, we compare the expected total cost when auditing all suppliers with the expected total cost when auditing none. Letting $1:N$ denote the lowest order statistic out of N draws, we define the expected audit value (*EAV*) as:

$$\textit{EAV} \triangleq E[\text{Cost}|\text{Audit None}] - E[\text{Cost}|\text{Audit All}] = E[\min\{\bar{\Delta} + J_{1:N}, c_0\}] - E[\min\{(\Delta + J)_{1:N}, c_0\}].$$

To evaluate whether audits are worthwhile, the buyer would take into account the cost of audits, and thus she would conduct the audits if $\textit{EAV} - N \cdot k$ is positive.

Having defined the expected audit value, we now seek to understand how this value changes with the underlying business environment, namely the unsustainability cost and the virtual production cost distributions, and the size of the outside option cost c_0. We will use H_1 and H_2 to denote the cumulative distribution functions of the random variables $\bar{\Delta} + J$ and $\Delta + J$, respectively.

PROPOSITION 2. *EAV is positive for all distributions F and $\tilde{G}$ defined on non-negative intervals, and it is bounded above by $\bar{\Delta} - \Delta_{(l)}$.*

Proposition 2 confirms that more information is always beneficial. When the buyer conducts audits, she learns the true unsustainability cost of each supplier, and this helps her pick whichever supplier has the lowest total cost. However, even when the buyer audits all the suppliers, the above proposition shows that her expected savings is bounded above by the expected unsustainability cost of any one supplier minus the lower bound on the unsustainability cost distribution. Next we explore the factors that affect the *EAV*.

Intuitively, when c_0 is small, regardless of the outcome of the audits, the buyer will probably still prefer her outside option, so she probably will not benefit much from auditing her potential suppliers. As c_0 grows, the likelihood that the buyer will wish to contract with one of the potential suppliers increases, hence one would expect that the value of audits should always increase. However, this is not the case. Figure 1 illustrates the regions where auditing is preferred to not auditing as a function of the outside option and the auditing cost (for $N = 2$). Shaded regions are where the expected audit value (*EAV*) exceeds the auditing cost. These audit/no audit regions are plotted for two cases A and B where suppliers have the same virtual production cost distribution ($\tilde{G} = \text{Normal}(2100, 700)$), same mean unsustainability cost and the same mean adjusted virtual cost. However, the unsustainability cost distributions for the two cases are different ($F_1 = 6000 \cdot \text{Beta}(0.5, 0.5)$ and $F_2 = \text{Gamma}(0.2, 15000)$ for Cases A and B, respectively). [1]

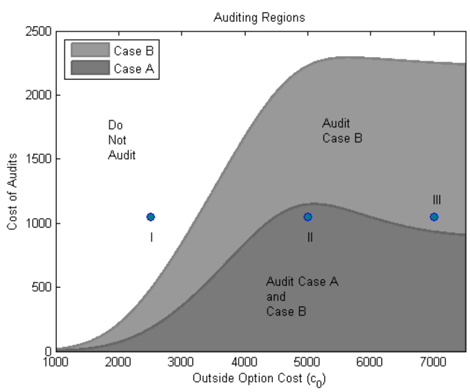

Figure 1 Audit regions (shaded)

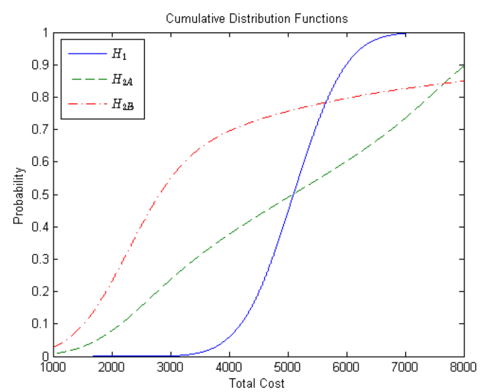

Figure 2 Adjusted virtual cost distributions

Consider the points labeled I, II, and III in Figure 1. We see that for both Case A and B, point I is in the no-audit region. This is intuitive, as for small c_0 we expect that audits will not be needed. Moving to the right, we see that for both cases point II is in the audit region; this is also intuitive because the buyer has a good chance of transacting with one of the suppliers and would find it beneficial to identify the supplier with the lowest total cost. But as c_0 increases further to point III, the above intuition fails and audits are no longer beneficial in Case A. Thus, even though one might expect that the value of audits should be increasing in c_0, this is not the case.

[1] The probability density function at x for Beta(a, b) is $f(x; a, b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1}$, and for Gamma$(a, b)$ is $f(x; a, b) = \frac{x^{a-1} e^{\frac{-x}{b}}}{b^a \Gamma(a)}$, where $\Gamma(\cdot)$ is the Gamma function.

In fact, for both Case A and Case B, the expected audit value (i.e., audit region frontiers) start declining in the outside option cost, c_0 beyond particular points. To understand why this happens, note that, as c_0 continues to increase, the outside option becomes a less viable alternative relative to the suppliers. In this region, audits are still valuable, however less so than before. This is because they no longer serve in deciding whether or not to use the outside option, but merely in determining which supplier to choose. Formally, we can prove the following result:

PROPOSITION 3. *There exists a $x_0 \in \mathbb{R}^+$ such that EAV increases in c_0 for $c_0 < x_0$, and EAV decreases in c_0 for $c_0 > x_0$.*

Now returning to Figure 1, we see that in contrast to Case A, point III is still in the audit region for Case B. Furthermore, the audit regions for the two cases peak at different points. Our next result explains what dictates the outside option cost where the audit regions peak.

PROPOSITION 4. *The value x_0 described in Proposition 3 is where the cumulative distribution functions of $\bar{\Delta} + J$ and $\Delta + J$ cross.*

To see an illustration of this result, examine Figure 2, which plots the cumulative distribution functions $H_1 = T_{\{3000\}} \circledast \tilde{G}$, $H_{2A} = F_1 \circledast \tilde{G}$, and $H_{2B} = F_2 \circledast \tilde{G}$. (Here $T_{\{3000\}}$ is the step function that takes unit value at 3000, and $\circledast$ is the convolution operator.) We see that the cumulative distribution functions H_{2A} and H_{2B} have a single crossing point with H_1. We note that the crossing point of H_{2A} and H_1 is to the left of the crossing point of H_{2B} and H_1. Indeed, the crossing points correspond exactly to where the respective audit regions peak in Figure 1.

Intuitively, audits spread out the probability distribution of suppliers' adjusted virtual costs — in particular, auditing suppliers make observing costs below x_0 more likely, and observing costs above x_0 more likely, than without audits. In other words, audits can reveal both good and bad news, and x_0 is simply the break point. The benefit of audits is largest when the buyer can leverage the good news to the greatest extent possible, namely when the outside option cost equals the break point x_0.

We also note that although the mean virtual production cost, mean unsustainability cost, and the mean adjusted virtual cost are the same for the two cases, the corresponding auditing regions are different. In order to understand how *EAV* changes with the underlying distributions, we now study the effect of scaling the unsustainability costs and the virtual production costs by positive constants γ, and κ, respectively.

In Figure 3, we plot *EAV* for two cases which have the same virtual production cost distribution but different unsustainability cost distributions. The unsustainability cost distribution for the second case is the scaled version of the unsustainability cost distribution of the first case, with $\gamma = 1.5$. Similarly, in Figure 4, we plot *EAV* for two cases which have the same unsustainability cost distribution but different virtual production cost distributions with a scale factor $\kappa = 1.5$.

Intuitively, one would expect that the value of sustainability audits would increase if the magnitude and dispersion of Δ increases, since the unsustainability markup would play more of an important role in the overall total cost. However, as shown in Figure 3, this is not necessarily the case. For small values of c_0, the *EAV* for the first case exceeds the *EAV* for the second case (where the Δ distribution is scaled by $\gamma = 1.5$). The reason behind this stems from Proposition 3: Scaling up the unsustainability cost markup can make the outside option more attractive relative to the suppliers, diminishing the value of supplier audits. Hence, greater uncertainty about supplier sustainability levels and a less sustainable supplier base do not necessarily make the audits more valuable for the buyer.

Similarly, as an increase in the magnitude and the dispersion of the virtual production cost diminishes the role of the unsustainability cost markup in the total cost, one would intuitively expect that the value of audits would decrease as the virtual production cost scales up. Yet, as shown in Figure 4, this is not necessarily true either. For moderate values of c_0, the *EAV* for the second case (where the virtual production cost distribution is scaled by $\kappa = 1.5$) can exceed the *EAV* for the first case. This is because increasing the magnitude of the virtual production cost shifts the *EAV* peak point further to the right – which, for moderate values of c_0, leads to an increase in *EAV*. Hence, greater ex ante variability and an increase in suppliers' production costs — which intuitively should reduce the benefit of audits – can actually increase the value of audits. We therefore conclude that when evaluating the benefits of audits, naïve intuition is not always correct.

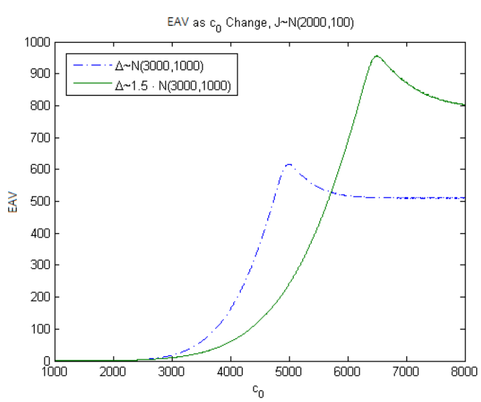

Figure 3 ***EAV* may decrease in γ**

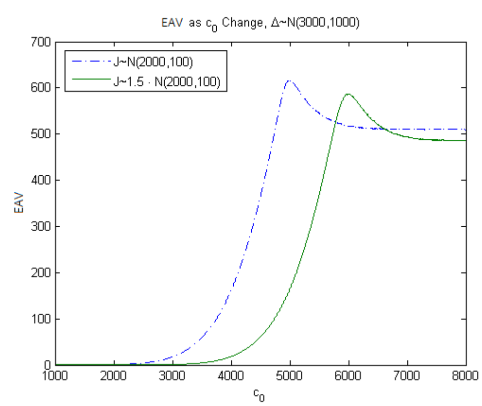

Figure 4 ***EAV* may increase in κ**

In settings where the buyer does not have an outside option (or has a very expensive outside option that can effectively be ignored), we can show that *EAV* always decreases in the virtual production cost multiplier κ, and increases in the unsustainability cost multiplier γ. This result is formalized below in Proposition 5.

PROPOSITION 5. *In the absence of an outside option, keeping everything else the same, for all i*

- *replace Δ_i by $\gamma \cdot \Delta_i$, where $\gamma > 0$; EAV is always increasing in γ.*

- *replace J_i by $\kappa \cdot J_i$, where $\kappa > 0$; EAV is always decreasing in κ.*
- *replace Δ_i by $\Delta_i + \tau$, for some $\tau \in \mathbb{R}$; EAV is constant in τ.*
- *replace J_i by $J_i + \tau$, for some $\tau \in \mathbb{R}$; EAV is constant in τ.*

Proposition 5 says that when the buyer does not have an outside option, as the dispersion of the unsustainability costs increases, the benefit from audits increases. This is because the buyer has a higher chance of finding a highly sustainable supplier with a low total cost as the unsustainability cost dispersion increases. Conversely, as the dispersion of the virtual production cost increases, the benefit from audits decreases. Furthermore, the *EAV* is not affected by distributional shifts, hence the auditing policy decisions should not change as F and $\tilde{G}$ are shifted to the right or left. This result suggests that, in the absence of an outside option, carrying out audits in a region which is notorious for unsustainable practices (e.g., Savar Bangladesh where the Rana Plaza incident took place) may not be any more preferable than carrying out audits in a region with highly sustainable practices (e.g., Western Europe) as long as the dispersions of the unsustainability cost and the dispersions of the virtual production cost distributions are similar in the two regions. However, in the presence of an outside option, shifting the unsustainability cost and the virtual production cost distributions does play a role in the auditing decisions. More specifically, in this case, any distributional shift would affect the position of the corresponding cumulative distribution functions' (H_1 and H_2) crossing points, and hence would lead to changes in *EAV*, exactly as explained after Propositions 3 and 4.

4.1.2. Differentiated Audits Now we consider the case where the buyer can choose how many of the ex ante symmetric suppliers to audit. As before, an audited supplier i's unsustainability cost markup term Δ_i plus the virtual production cost term J_i comprise the total cost associated with procuring from this supplier. The suppliers who are not audited can still participate in the auction; however, as their unsustainability cost markup is not measured, the buyer uses $\bar{\Delta}$ to substitute for the unsustainability cost markup of these suppliers. The buyer needs to decide on how many of the ex ante symmetric suppliers to audit. Define

$$\begin{aligned} \mathit{EAV}(M) &\triangleq E[\text{Cost}|\text{Audit None}] - E[\text{Cost}|\text{Audit } M] \\ &= E[\min\{\bar{\Delta} + J_{1:N}, c_0\}] - E[\min\{\Delta_1 + J_1, \ldots, \Delta_M + J_M, \bar{\Delta} + J_{M+1}, \ldots, \bar{\Delta} + J_N, c_0\}]. \end{aligned}$$

The following result shows that the value of auditing an additional supplier is decreasing in the number of audited suppliers. This result suggests that when the buyer is facing a high number of potential suppliers, and linear auditing costs, the optimal number of suppliers to audit M^* is an interior solution.

PROPOSITION 6. *EAV(M), the expected value of auditing M suppliers, is positive and concave increasing in M.*

Figure 5 plots the *EAV* as the number of audited suppliers (M, within a pool of $N = 10$ suppliers) increases for two cases (Case A: $\tilde{G} = \text{Normal}(2100, 700)$, $F_1 = \text{Gamma}(0.2, 15000)$, Case B: $\tilde{G} =$

Normal$(2100, 700)$, $F_2 = 1.5 \cdot \text{Gamma}(0.2, 15000)$; in both cases, $c_0 = 8000$). We see that as we increase the number of audited suppliers, *EAV* increases at a slower rate. We also note that *EAV* for Case B is higher than the *EAV* for Case A for all M. The intuition is the same as explained in Proposition 5 (note that Case B is a scaled-up version of Case A in the unsustainability cost distribution). In fact, we can prove that our earlier results continue to hold:

PROPOSITION 7. *Replace EAV with EAV(M); Propositions 2-5 hold as before.*

The takeaway from this subsection is twofold: First, the buyer may wish to audit just some, but not all, of her suppliers. That is, even if the suppliers are ex ante symmetric, the buyer may find it optimal to use an asymmetric auditing policy. This is because the expected benefit from an additional audit decreases in the number of audited suppliers. Second, the value of auditing a subset of suppliers changes with the underlying business environment in some surprising ways. For example, a less attractive outside option might actually make audits less valuable.

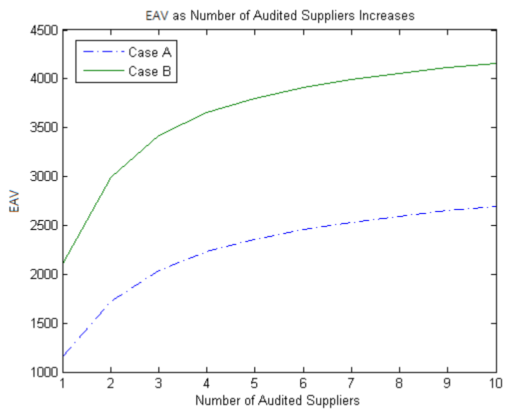

Figure 5 ***EAV* is concave increasing in the number of audited suppliers**

4.2. Imperfect Learning

Thus far we have supposed that an audit on supplier i provides a perfect observation on the unsustainability cost δ_i. However, there may be situations where the audits do not lead to perfect observations. In order to formally analyze such situations, in this section we introduce a parameter $\alpha \in \mathbb{R}^+$ to represent the level of auditing accuracy. Given an auditing accuracy level α, we now denote by F_α the distribution of the posterior mean unsustainability cost $E[\Delta_i|\alpha, \zeta_i]$ where ζ_i is the observation from the audit on supplier i, and by H_2^α the convolution of F_α and $\tilde{G}$. As before, H_1 denotes the distribution of $\bar{\Delta} + J$. Thus, by appropriately modifying the proof of Proposition 1 one can easily show that in the buyer's optimal mechanism the unsustainability cost markups can now be characterized as follows:

$$s_i = \begin{cases} E[\Delta_i|\alpha, \zeta_i] & \text{if the buyer audits supplier } i \text{ with accuracy } \alpha; \\ \bar{\Delta} & \text{if the buyer does not audit supplier } i. \end{cases} \tag{1}$$

To study a setting where higher auditing accuracy leads to more precise observations on the unsustainability cost, we adopt the following model: for $\alpha_1 > \alpha_2$, F_{α_1} is a mean-preserving spread of F_{α_2} (i.e., F_{α_2} second-order stochastically dominates F_{α_1} while the posterior mean unsustainability cost stays the same). Furthermore, the family of distributions $H_2^\alpha = F_\alpha \circledast \tilde{G}(x)$ is rotation ordered: $H_2^{\alpha'}$ cross H_2^α only once at the same crossing point $\forall \alpha' \neq \alpha$ (Johnson and Myatt (2006)). Hence, $\exists x_0 \in \mathbb{R}^+$ such that $\frac{\partial H_2^\alpha(x)}{\partial \alpha} \geq 0$ for $x < x_0$ and $\frac{\partial H_2^\alpha(x)}{\partial \alpha} \leq 0$ for $x > x_0, \forall \alpha$.

We now illustrate the effect of the auditing accuracy parameter α with an example. Suppose that by auditing supplier i, the buyer observes a signal $\zeta_i = \Delta_i + \varepsilon$, where $\varepsilon \sim \text{Normal}(0; 1/\alpha)$. Hence, higher levels of α correspond to more accurate and more informative audits. Furthermore, suppose that $\Delta_i \sim \text{Normal}(\bar{\Delta}, \frac{1}{\beta})$. Having observed ζ_i, the buyer updates her belief on supplier i's true unsustainability cost. The posterior distribution for supplier i's unsustainability cost given an observation ζ_i is $\tilde{\Delta}_i|\zeta_i \sim \text{Normal}(\frac{\beta\bar{\Delta}+\alpha\zeta_i}{\alpha+\beta}; \frac{1}{\alpha+\beta})$. In this example, we note that the posterior mean unsustainability cost $E[\Delta_i|\alpha, \zeta_i]$ (distributed with F_α), is $\frac{\beta\bar{\Delta}+\alpha\zeta_i}{\alpha+\beta}$. Hence, F_α is normal with mean $\bar{\Delta}$ and variance $\frac{\alpha}{\beta(\alpha+\beta)}$. Furthermore, F_{α_2} is a mean-preserving spread of F_{α_1} for $\alpha_2 > \alpha_1$, consistent with our general model stated above. We also note that for all α, $H_2^\alpha = F_\alpha \circledast \tilde{G}$ cross at the mean adjusted virtual cost. Hence, rotation ordering for all α is satisfied.

For a given auditing accuracy α, the expected value of auditing all suppliers can similarly be defined as in §4.1:

$$\begin{aligned} \textit{EAV}(\alpha) &\triangleq E[\text{Cost}|\text{Audit None}] - E[\text{Cost}|\text{Audit All with Accuracy } \alpha], \\ &= E[\min\{\bar{\Delta} + J_1, \ldots, \bar{\Delta} + J_N, c_0\}] - E[\min\{E[\Delta_1|\zeta_1] + J_1, \ldots, E[\Delta_N|\zeta_N] + J_N, c_0\}|\alpha]. \end{aligned}$$

Similarly, the expected value of auditing M suppliers can be defined as:

$$\begin{aligned} \textit{EAV}(M,\alpha) &\triangleq E[\text{Cost}|\text{Audit None}] - E[\text{Cost}|\text{Audit M with Accuracy } \alpha], \\ &= E[\min\{\bar{\Delta} + J_1, \ldots, \bar{\Delta} + J_N, c_0\}] \\ &\quad - E[\min\{E[\Delta_1|\zeta_1] + J_1, \ldots, E[\Delta_M|\zeta_M] + J_M, \bar{\Delta} + J_{M+1}, \bar{\Delta} + J_N, c_0\}|\alpha]. \end{aligned}$$

Intuitively, more accurate audits are more costly for the buyer to obtain. The following result establishes that more accurate audits are also more valuable for the buyer.

PROPOSITION 8. *EAV*(α) *and EAV*(M,α) *are increasing in* α.

The next result extends our previous results from §4.1.

PROPOSITION 9. *Replace EAV with EAV*(α) *and replace EAV(M) with EAV*(M,α)*, and* H_2 *with* H_2^α*; Propositions 2-7 hold as before.*

This result reveals how the accuracy of audits, together with the number of suppliers, magnitude and dispersion of the unsustainability and the virtual production costs affect the value of audits. Proposition 10 extends the intuition developed in §4.1 on the effect of changes in the outside option.

PROPOSITION 10. *The expected benefit from an increase in auditing accuracy can increase or decrease in the size of the outside option, c_0. Furthermore, there exists a point x_0 such that it is increasing in c_0 when $c_0 < x_0$, and decreasing in c_0 when $c_0 > x_0$, i.e.: $\frac{\partial^2 EAV(M,\alpha)}{\partial\alpha\partial c_0} \geq (\leq) 0$, for $c_0 < x_0$ ($c_0 > x_0$).*

Proposition 10 establishes that the expected benefit from an increase in auditing accuracy is non-monotonic in the outside option c_0. Interestingly, being more reliant on the prospective suppliers (larger c_0) can cause the buyer to derive less benefit from increases in the auditing accuracy, and increasing the auditing accuracy is more beneficial for the buyer when the outside option is already fairly good.

Next, we allow the accuracy of audits to be different for different suppliers, and analyze the buyer's willingness to pay for auditing accuracy on her potential suppliers. Intuitively, because all suppliers are ex ante symmetric, the buyer should audit all suppliers to the same level of accuracy. However, the next proposition establishes that there is a substitution effect between the accuracy of the audits across suppliers. It is worth noting that this substitution effect is in line with our previous result in Proposition 6, which shows that the benefit from auditing one more supplier is decreasing in the number of audited suppliers.

PROPOSITION 11. *The accuracy level of the audits on any two suppliers in a supplier pool with N suppliers are substitutes (i.e., the expected benefit from increasing the auditing accuracy on supplier i (α_i) decreases as the auditing accuracy on supplier j (α_j) increases, namely $\frac{\partial^2 E[Cost]}{\partial\alpha_i\partial\alpha_j} \geq 0$, $i \neq j$).*

Hence, the willingness to pay for a more accurate audit on supplier i is smaller if the audit on supplier j will be at a high accuracy, when compared to the case where the audit on supplier j will be at a low accuracy. This result suggests that when the buyer faces incremental set-up costs associated with the audits and variable costs associated with the accuracy of the audits (such as a staircase-shaped cost function), the current industry practice of blanket auditing all suppliers to the same depth may be improved on by auditing a subset of suppliers with a high accuracy, and auditing the rest with a lower level of accuracy. We note that the rotation ordering for the family of distributions H_2^α is a sufficient condition for Proposition 11, however it is not required for any of the results in Propositions 8-10.

5. Extensions

Above we ignored the specific way in which the buyer evaluates the unsustainability cost markup after an audit. Below in §5.1 we explicitly model a non-linear mapping to reflect risk-taking preferences of the buyer regarding supplier sustainability. We then study the case where the buyer's unsustainability cost function is linear, but the buyer does not know the true unsustainability cost multiplier. We investigate the value of information on this unsustainability cost multiplier from the buyer's perspective in §5.2. Finally §5.3 studies cases where there is correlation between the unsustainability costs the production costs, and where the suppliers are endowed with some information on their unsustainability costs even before audits.

5.1. Non-Linear Unsustainability Cost Function

Slightly abusing our earlier notation, here we recast Δ_i as the output of a sustainability audit, which the buyer then maps into an unsustainability cost term using a non-linear, strictly increasing function $V(\cdot)$. In this case, if the buyer does not audit supplier i, supplier i's unsustainability cost markup will be $s_i = E[V(\Delta)]$ (as can easily be seen by appropriately modifying Proposition 1 and its proof). If the buyer audits supplier i, then after having obtained an observation ζ_i with accuracy α_i, supplier i's unsustainability cost markup will be $s_i = E[V(\Delta_i)|\zeta_i, \alpha_i]$.

We note that $E[E[V(\Delta_i)|\zeta_i, \alpha_i]] = E[V(\Delta_i)|\alpha_i]$. We further note that the prior distribution of Δ is independent of α_i, hence $E[V(\Delta_i)|\alpha_i] = E[V(\Delta_i)]$. Therefore, $E[E[V(\Delta_i)|\zeta_i, \alpha_i]] = E[V(\Delta_i)]$. In fact, this leads to the following result:

LEMMA 1. *The distribution of $E[V(\Delta_i)|\zeta_i, \alpha_i] + J$ is a mean-preserving spread of the distribution of $E[V(\Delta)] + J$, for all $\alpha \in \mathbb{R}^+$.*

We note that establishing Lemma 1 is a sufficient condition for the results on imperfect learning from §4.2 to hold in this setting. Corollary 1 is immediate.

COROLLARY 1. *Propositions 8-10 continue to hold under the non-linear unsustainability cost function.*

Hence, we conclude that the risk taking preferences of the buyer (as reflected in a non-linear cost function $V(\Delta)$) does not change our main analysis above. We further note that the rotation ordering for the distributions of the posterior means under different accuracy levels is a sufficient condition for Proposition 11 to continue to hold even when the buyer's unsustainability cost function is non-linear.

5.2. Learning the Cost Multiplier

We now consider the case where the buyer faces a linear penalty $\Theta \cdot \Delta_i$ as a function of Δ_i. The buyer knows that the cost multiplier Θ is continuously distributed with commonly known cumulative distribution function P (and probability density function ρ) over the finite interval $[\Theta_{(l)}, \Theta_{(u)}]$ with mean $\bar{\Theta}$. However, she does not know the value of the cost multiplier. This multiplier reflects the buyer's underlying cost sensitivity to the chosen supplier's sustainability level, and can be thought of as a weighting rule which converts the sustainability ratings into unsustainability cost markups. The buyer has the option to learn about the cost multiplier (through consumer surveys, market research, etc.) prior to the audit stage after which she will observe the sustainability levels of all suppliers. Hence, if the buyer chooses to learn the cost multiplier Θ, she assigns $\Theta \cdot \Delta_i$ as the unsustainability cost markup to supplier i. If the buyer chooses not to learn the cost multiplier, it can be easily shown that (by appropriately modifying the proof of Proposition 1), the buyer should assign $\bar{\Theta} \cdot \Delta_i$ as the unsustainability cost markup to supplier i under the optimal mechanism.

We define the expected value of information on Θ as follows: *EVI* $\triangleq E_{\Delta,J}[\min\{\bar{\Theta}\Delta_1 + J_1, \ldots, \bar{\Theta}\Delta_N + J_N, c_0\}] - E_{\Delta,J,\Theta}[\min\{\Theta\Delta_1 + J_1, \ldots, \Theta\Delta_N + J_N, c_0\}]$. Let us now denote by H_1 the distribution of $\bar{\Theta}\Delta + J$, and by H_2 the distribution of $\Theta\Delta + J$.

Akin to Proposition 2, the next proposition shows that *EVI* is always positive. In Proposition 13, we establish the non-monotonicity of *EVI* as the outside option increases.

PROPOSITION 12. *EVI is positive for all distributions F, $\tilde{G}$, and P defined on non-negative intervals.*

PROPOSITION 13. *A larger outside option, c_0, can decrease or increase EVI. In particular, there exists a $x_0 \in \mathbb{R}^+$ such that EVI increases in c_0 for $c_0 < x_0$, and EVI decreases in c_0 for $c_0 > x_0$. x_0 is precisely where H_1 and H_2 cross.*

Thus far we have seen that the expected value of information on the cost multiplier (*EVI*) behaves very similarly to the expected value audits (*EAV*). However, below in Proposition 14, we observe that the behavior of *EVI* can significantly differ from *EAV* in a sensitivity analysis. In §4.1 we saw that, in the absence of the outside option, *EAV* behaves monotonically as we scale up the unsustainability costs. This means that scaling up the unsustainability costs can only make the buyer value the audits more. However, the same is not true when it comes to learning the cost multiplier. We show below in Proposition 14, among other results, that increasing the magnitude of the unsustainability costs can actually decrease the value of acquiring information about the cost multiplier.

To illustrate this, we consider the case where the buyer does not have an outside option, and for simplicity use normal distributions for Δ and J ($\Delta \sim \text{Normal}(\bar{\Delta}, \sigma_\Delta^2)$, and $J \sim \text{Normal}(\bar{J}, \sigma_J^2)$). We examine the behavior of the following: $EVI' \triangleq E[(\bar{\Theta} \cdot \Delta + J)_{1:N}] - E[(\Theta \cdot \Delta + J)_{1:N}]$.

Proposition 14 first shows that when $\sigma_\Delta = \sigma_J = \sigma$, EVI' is monotone increasing in σ. Hence, EVI' increases with σ_Δ and σ_J when they are exactly comparable in size. Now, consider multiplying Δ by a positive constant γ. Intuitively, one might expect that as γ increases, $\gamma\sigma_\Delta$ increases, and misestimating the buyer's sensitivity to sustainability cost (i.e., the size of Θ) becomes potentially very costly for the buyer. Thus, one would expect that as γ increases, the buyer's value of learning the true Θ also increases. However, we find that this is not necessarily the case, and in fact the expected benefit from learning Θ, EVI', is non-monotonic in γ. A similar result holds for the multiplicative constant κ for J. The following proposition formalizes these results.

PROPOSITION 14. *In the absence of an outside option, keeping everything else the same,*

- *Suppose $\sigma_\Delta = \sigma_J = \sigma$. $\frac{\partial EVI'}{\partial \sigma} > 0$, hence EVI' is increasing in σ*
- *Replace Δ_i, $\forall i$ by $\gamma \cdot \Delta_i$, where $\gamma > 0$. EVI' is non-monotonic in γ.*
- *Replace J_i by $\kappa \cdot J_i$, where $\kappa > 0$. EVI' is non-monotonic in κ.*

In particular, EVI' is increasing (decreasing) in γ when $\frac{\sqrt{2}\sigma_J}{\Theta_{(u)}} > \gamma\sigma_\Delta$ ($\frac{\sqrt{2}\sigma_J}{\Theta_{(l)}} < \gamma\sigma_\Delta$). EVI' is increasing (decreasing) in κ when $\sqrt{2}\Theta_{(l)}\sigma_\Delta > \kappa\sigma_J$ ($\sqrt{2}\Theta_{(u)}\sigma_\Delta < \kappa\sigma_J$).

Proposition 14 shows that EVI' from learning Θ is non-monotonic in $\gamma\sigma_\Delta$. The intuition behind this result is as follows: for small values of $\gamma\sigma_\Delta$ as γ increases, the variability in the unsustainability cost markups become more important in the buyer's supplier selection decision. Hence, learning Θ becomes increasingly valuable. However, when $\gamma\sigma_{\Delta}$ becomes large enough to govern the total cost, unsustainability cost markups become the main driver of the supplier selection decision. It follows that the buyer can now choose a supplier depending on the unsustainability cost markup realizations, the ranking of which does not change for different values of Θ. Hence, the value of learning Θ decreases.

Similarly, when $\kappa\sigma_J$ is low, learning Θ is not valuable to the buyer since the supplier ranking is largely dependent on the unsustainability cost markup realizations. However, as κ increases, virtual production cost realizations become increasingly important in the supplier selection decision. Hence, the supplier ranking now depends on the realizations of Θ. Conversely, when $\kappa\sigma_J$ becomes large enough to govern the total cost, the supplier selection decision depends mainly on the virtual production cost realizations, and the value of learning Θ decreases.

Recapping the above, we see that greater variability can diminish the value of information when one cost component – the unsustainability markup or the virtual production cost – tends to "dominate" the other. It is then interesting to ask what happens when neither dominates. In this case, when $\sigma_{\Delta} = \sigma_J = \sigma$, Proposition 14 shows that *EVI*$'$ monotonically increases in σ. The managerial takeaway is as follows: when deciding whether to exert effort to fine-tune her unsustainability cost multiplier, the buyer must take a holistic look at the cost drivers. More dispersion can increase or decrease the benefit of fine-tuning the unsustainability cost multiplier, depending on how the price and non-price cost drivers compare in size and dispersion.

5.3. Correlation Between the Production and the Unsustainability Cost Terms

There may be cases where specific production technologies are associated with a higher or lower unsustainability cost markup. For example, a supplier with waste water treatment facilities would be labelled as more sustainable than suppliers without such facilities, but would incur an increased production cost due to waste water treatment. Conversely, a supplier without such facilities enjoys a lower production cost, but the lack of waste water treatment facilities puts him in an unsustainable supplier category. In such settings, where the unsustainability cost markups are linked to certain production technologies (and hence the production costs), the suppliers can easily observe which supplier category they are in, and moreover there is correlation between a supplier's production cost and its sustainability level.

To model this we define supplier sustainability categories, $\tau = A$ and $\tau = B$ (for simplicity we treat two categories, but the results we present would generalize to multiple categories in a straightforward manner). Supplier i's production cost is $c(\tau_i) + \epsilon_i^c$. Random variable ϵ_i^c follows cumulative distribution function G, satisfying the standard regularity condition that $\epsilon_i^c + \frac{G(\epsilon_i^c)}{g(\epsilon_i^c)}$ is strictly increasing. The structural form of $c(\cdot)$ which maps the supplier category to a deterministic, additive production cost term is publicly known, but both ϵ_i^c and his category τ_i is supplier i's private information.

Similarly, the unsustainability cost associated with supplier i is comprised of two terms: $\Delta(\tau_i) + \epsilon_i^\Delta$. The structural form of $\Delta(\cdot)$ is publicly known, however as his category τ_i is supplier i's private information, it is only supplier i who can observe $\Delta(\tau_i)$ initially. Neither the buyer nor the supplier can readily observe ϵ_i^Δ without audits. This is because the suppliers do not have the in-house expertise to identify the precise unsustainability cost markup. ϵ_i^Δ is a random variable symmetrically distributed around its finite mean $\bar{\epsilon}^\Delta$ with cumulative distribution function F. To summarize, the supplier is endowed with knowledge of τ_i (e.g., whether the supplier has a waste water treatment facility or not), but audits are needed to observe ϵ_i^Δ (e.g., how effective the waste water treatment facility actually is at protecting sensitive downstream wetlands). Of course, upon auditing a supplier i, the buyer observes the supplier's category τ_i and the random term ϵ_i^Δ.

Audits allow the buyer to observe the supplier's category τ_i, and this has an important implication for a powerful principal, like the buyer we study in this paper. It gives the buyer the ability to ex post verify information that she could ask a supplier i to divulge, namely τ_i. This can be used to induce suppliers to truthfully report their categories. The logic behind this is simple: If the principal (the buyer) discovers the agent (the supplier) has lied, the principal will severely punish the agent (e.g., commit to never do business with the supplier again or badly damage the supplier's reputation by widely revealing their misrepresentation). A summary on an agent's incentives for truthful disclosure of ex post verifiable information under the possibility of random audits and sanctions in a principle-agent setting can be found in Laffont and Martimort (2009) (pg. 125), tracing back to the seminal paper Becker (1968). Since our powerful buyer can ensure truthful category revelation via random audits carried out with vanishingly small probability, for the rest of this subsection we will ignore these random audits, we assume that suppliers truthfully report their categories to the buyer, and from here onwards an "audit" refers to fine-tuning information about total cost of ownership given knowledge of the supplier categories. The timeline is:

1. For each supplier i, nature chooses τ_i, the supplier's sustainability category, and reveals it to the supplier. Nature also chooses ϵ_i^c and ϵ_i^Δ, and reveals ϵ_i^c to supplier i, while neither party can observe ϵ_i^Δ without supplier i undergoing an audit.
2. Each supplier i reports to the buyer his sustainability category τ_i (see explanation above).
3. The buyer conducts sustainability audits (if any) in order to further assess the supplier sustainability levels (to learn the ϵ_i^Δ's).
4. The buyer conducts an optimal total-cost procurement auction.

Note that after the suppliers disclose their categories, the buyer views the suppliers as ex ante asymmetric, as they are in either category A or B. For step 4, if the buyer does not audit supplier i, supplier i's unsustainability cost markup will be $s_i = \Delta(\tau_i) + \bar{\epsilon}^\Delta$ (as can easily be seen by appropriately modifying Proposition 1 and its proof). If the buyer audits supplier i, then after having observed ϵ_i^Δ, supplier i's unsustainability cost markup will be $s_i = \Delta(\tau_i) + \epsilon_i^\Delta$. In Propositions 15 and 16, we analyze auditing policies for step 3. After having observed the supplier categories and the number of suppliers in category-A and category-B (denoted by N_A and N_B, respectively) the buyer must decide which (if any) suppliers to audit in step 3. We define $EAV_{corr}(M_A, M_B)$ as the expected audit value when the buyer audits $M_A \leq N_A$ category-A and $M_B \leq N_B$ category-B suppliers, respectively. Let $\Lambda^{(A)} \triangleq c(A) + \Delta(A) \quad$ and $\quad \Lambda^{(B)} \triangleq c(B) + \Delta(B)$ denote the total base-cost of a category-A and category-B supplier, respectively.

Note that supplier i, $\forall i$ still earns information rents under the optimal mechanism, from his information on ϵ_i^c. Let J_i (distributed according to distribution $\tilde{G}$) now denote $\epsilon_i^c + \frac{G(\epsilon_i^c)}{g(\epsilon_i^c)}$, the virtual cost of supplier i. Then, facing N_A category-A and N_B category-B suppliers, the buyer's expected procurement cost when auditing M_A category-A and M_B category-B suppliers is: $E[\text{cost}|M_A, M_B] = E[\min\{(\Lambda^{(A)} + J + \bar{\epsilon}^{\Delta})_{(1:N_A - M_A)}, (\Lambda^{(A)} + J + \epsilon^{\Delta})_{(1:M_A)}, (\Lambda^{(B)} + J + \bar{\epsilon}^{\Delta})_{(1:N_B - M_B)}, (\Lambda^{(B)} + J + \epsilon^{\Delta})_{(1:M_B)}, c_0\}]$.

PROPOSITION 15. *Suppose without loss of generality that the supplier categories are such that category-A has the lower total base-cost, i.e., $\Lambda^{(A)} < \Lambda^{(B)}$. The buyer would always prefer auditing a category-A supplier to a category-B supplier. Therefore, in her optimal auditing policy, she audits category-B suppliers only if she also audits all category-A suppliers.*

Facing two supplier pools (e.g., in two different geographic areas), a procurement manager may intuitively think her sustainability audits should focus on the supplier pool more associated with unsustainable characteristics. However, Proposition 15 shows that when deciding on which additional supplier to audit, the buyer should prioritize the supplier category with the lower total base-cost.

To understand total base-cost, let us consider our waste water treatment example. Suppose category-A suppliers have lower unsustainability cost (because they treat their waste water) but they also have higher production costs (for the same reason), and conversely for category B. Which supplier category (A or B) has a lower total base-cost is dependent on the functional forms of $c(\tau)$ and $\Delta(\tau)$. For example, per §5.2, if the buyer's sensitivity to sustainability is high enough, category-A suppliers would have a lower total base-cost than the suppliers in category-B, because the $\Delta(\tau)$ term would dominate the total base-cost. Conversely, if the buyer's sensitivity for sustainability is low enough, so $c(\tau)$ dominates the total base-cost, category-A suppliers would have a higher total base-cost than the suppliers in category-B. It follows that from an auditing policy perspective, the buyer first needs to understand her level of sensitivity (which is operationalized as a cost markup multiplier in §5.2) to sustainability issues in order to compare the total base-costs of the category-A and category-B suppliers, and then choose which supplier category to prioritize in audits accordingly.

In relation to the above observation, it is worth noting that Proposition 5 which analyzes the changes in *EAV* as the production cost and the unsustainability cost markup terms scale up (while ignoring the outside option) may no longer hold in this setting. For example, consider the case where the production cost multiplier is small, the categories A and B have similar total base-costs, and there is a benefit to auditing suppliers in both categories. As we increase the production cost multiplier (i.e., by scaling up the production costs), category-B suppliers (which have lower deterministic production cost terms) become far more preferred. As the the production cost multiplier keeps increasing, eventually auditing category-A suppliers would become unnecessary (as they would have a much higher total base-costs than category-B suppliers). Hence, the value of auditing all suppliers, *EAV*, may go down. A similar intuition holds for the unsustainability cost markup multiplier. Hence, when the buyer faces multiple categories of suppliers, increasing the importance of production cost or unsustainability cost may make one category dominant over another, diminishing the value of audits which are used to fine-tune the buyer's comparison of suppliers.

In the remainder, we will suppose that the buyer follows an optimal auditing policy per Proposition 15. Sequence the suppliers' indices such that the first N_A suppliers are category-A, and the next N_B suppliers are category-B, and let *EAV*$_{corr}(M)$ denote the expected audit value the buyer would derive from auditing the first M suppliers in this list (e.g., if $M \leq N_A$ the buyer audits only a category-A suppliers; if $M > N_A$, the buyer audits all category-A suppliers and $M - N_A$ category-B suppliers). In Proposition 16 and Proposition 17 we extend our earlier results to this setting.

PROPOSITION 16. *EAV*$_{corr}(M)$ *is piecewise concave (concave on* $[0, N_A]$ *and on* $[N_A, N_A + N_B]$*) and* N_A *is the vertex point where the two concave portions meet.*

PROPOSITION 17. *EAV*$_{corr}(M)$ *is non-monotonic in the outside option cost* c_0*. More specifically, there exist* $x_0^{(1)} < x_0^{(2)}$ *such that*
(i) EAV$_{corr}(M|M \leq N_A)$ *is increasing in* c_0 *when* $c_0 < x_0^{(1)}$*, and decreasing in* c_0 *when* $c_0 > x_0^{(1)}$*.*
(ii) EAV$_{corr}(M|N_A < M)$ *is increasing in* c_0 *when* $c_0 < x_0^{(1)}$*, and decreasing in* c_0 *when* $c_0 > x_0^{(2)}$*. EAV*$_{corr}(M|N_A < M)$ *can be increasing or decreasing in* c_0 *for* $x_0^{(1)} < c_0 < x_0^{(2)}$*.*

Proposition 16 shows that our earlier result on decreasing marginal returns from each additional audit is still valid. Further, Proposition 17 states that *EAV* may no longer be unimodal as before, as *EAV* may peak at different points depending on whether the buyer is committed to auditing one or multiple categories of suppliers.

6. Conclusion

Motivated by rising concerns about sustainable procurement and sustainable supplier selection, in this paper we investigate the value of sustainability audits as an information acquisition tool in a total-cost procurement auction setting. To this end, we model the buyer's decision on whether to utilize costly sustainability audits on her potential suppliers, given (*i*) uncertainty about the suppliers' sustainability levels, (*ii*) uncertainty about the suppliers' production costs, and (*iii*) the cost of the outside option that the buyer would have to turn to if she chooses not to award her business in the auction.

Sustainability audits are costly for the buyer, but the buyer has discretion over the auditing decisions. If she conducts the audits, she will be better informed, and will use her observations on the suppliers' unsustainability costs when making her procurement decision. We examine the buyer's "expected audit value" (*EAV*) and analyze how *EAV* changes with the underlying business environment, e.g., the cost of the buyer's outside option, and the uncertainty over suppliers' sustainability levels and production costs.

Addressing our research questions on when the buyer should use audits, we find that the answers depend on the underlying business factors in some surprising ways. For example, as the buyer's outside option cost increases she becomes more reliant on contracting with a supplier, hence one might expect the sustainability audits to become more valuable. However, we find that the opposite can happen and a higher outside option cost for the buyer can cause the value of audits to decrease. The reason behind this is that the audits do not only help the buyer to fine-tune her total-cost comparisons across suppliers, but also in deciding whether to use her outside option. Hence, when this outside option value is not overly high (nor overly low) the buyer actually gets more use out of the audits. The takeaway for a buyer is as follows: when allocating budgets for audits, the buyer should not necessarily focus only on those cases where she is most beholden to suppliers by virtue of not having an attractive outside option — in fact, audits are most valuable when the outside option is already fairly good.

One might expect that, since the audits are aimed at assessing the suppliers' unsustainability costs, greater uncertainty and a higher magnitude in the supplier unsustainability costs should make audits more valuable for the buyer. However, we find that this is not always the case. This is because, as unsustainability costs magnify, the outside option can become a relatively more attractive option, which also can diminish the value of audits when the outside option cost is relatively small. Intuitively, one would also expect that greater ex ante magnitude and variability in suppliers' production costs would diminish the effect of the sustainability levels in supplier selection and should lead to a decrease in the value of audits. However, we find that an increase in ex ante magnitude and variability in suppliers' production costs can actually lead to an increase in the value of audits. The managerial takeaway is as follows: A more sustainable supplier base with lower variability in sustainability levels or a supplier base with more variable and higher production costs does not necessarily mean that the audits will be less beneficial for the buyer.

Furthermore, currently industry practice, the buyers either audit all the competing suppliers or none of them. We find that the practice of "all or none" auditing policies can be improved upon by selectively auditing just a subset of suppliers, even if they are ex ante symmetric.

We extend our analysis to the case where the buyer's unsustainability cost function is non-linear, reflecting the situations where unsustainability can translate into high downside risks for buyers. We show that the above insights carry over. Additionally, we take a step back and consider a buyer who, even if she had audit data, might not have a good sense of how to operationalize this data by incorporating it into a total-cost formula for evaluating suppliers. To model this, we consider the case where the buyer faces a linear unsustainability cost function, but her unsustainability cost multiplier (the buyer's sensitivity to supplier unsustainability) is unknown to her. Intuitively, the buyer should invest in learning about her unsustainability cost multiplier (through market research, consumer surveys, etc.) when there is high variance in the possible unsustainability cost realizations. However, we find that the value of information on the unsustainability cost multiplier is not necessarily increasing in the ex ante variability of the unsustainability costs. In fact, the value of information on the unsustainability cost multiplier may decrease in the unsustainability cost variance beyond a threshold. This is because a high unsustainability cost variance dominates any effect that the price bids may have on the supplier selection process, and the unsustainability costs become the main driver of the buyer's supplier selection decision. We also study a setting where suppliers themselves have some partial information about their sustainability level. This leads to more information being readily available to the buyer, which the buyer can use to further fine-tune her auditing policy.

In our paper the buyer uses an optimal auction, and we showed that audits are always valuable for the buyer. The underlying reason is that greater dispersion in bidders' total costs enables the buyer to make a better (lower total-cost) supplier selection. Suppose that, instead, after conducting audits (if any), the buyer simply applies a cost markup $s_i = \delta_i$ for audited suppliers, and $s_i = \bar{\Delta}_i$ for un-audited suppliers, and runs a total-cost reverse English auction without a reserve price (e.g., akin to the open auction format studied in Kostamis et al. (2009), however in their setting the buyer is assumed to know all δ_i to start with). It is worth noting that in such cases with only two bidders (and no outside option) one can show that audits — which effectively create more dispersed supplier total costs — actually hurt the buyer. The reason is intuitive: The auction clears at the maximum of the two bidders' total costs, and this maximum increases in the dispersion. This unusual effect diminishes as the number of bidders increase beyond two, although due to the intractability of the underlying order statistics, exactly when this happens is difficult to characterize (see Board (2009) and Ganuza and Penalva (2010)). The takeaway for our audits setting is the following: If the buyer is in an unfavorable position with no option to use an optimal auction format, and she only has a few bidders, then she needs to carefully assess how to use the audits (i.e., how much information to acquire) in coordination with specific sub-optimal auction formats.

In brief, we provide an auditing policy analysis for a buyer firm facing ex ante uncertainty on her potential suppliers' non-price attributes in an optimal total-cost procurement auction setting. A supplier's sustainability level is one such attribute, and is our prime example. However, our results are generalizable to any differentiator-type attribute that the buyer may learn about using a TCO analysis. In fact, TCO analyses in competitive bid procurement are widely used in order to incorporate various non-biddable supplier attributes in the supplier selection decision. However, several steps in a TCO analysis (e.g., supplier audits, evaluations on supplier ratings and cost markup functions) are costly for the buyer. Hence, it is important to understand when and to what extent buyers should invest in such a costly information acquisition step for their various competitive bidding events. Surprisingly little research has addressed this question. To the best of our knowledge, this paper is the first to formally study this challenge faced in procurement. We hope that the results from this paper inspire additional research on this understudied topic.

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Appendix

Proof of Proposition 1 Without loss of generality, label the suppliers such that $i = \{1, \ldots, M\}$ are the audited suppliers (with known unsustainability cost δ_i), and $i = \{M+1, \ldots, N\}$ are the suppliers who have not been audited (with unknown unsustainability cost Δ_i). Let us denote by a the vector of observations that the buyer has on the suppliers' unsustainability costs. Note that $a_i = \delta_i$ if supplier is audited, and $a_i = \Delta_i$ if supplier i is not audited. As per the modeling assumption (and as is done in reality for transparency purposes), the buyer informs an audited supplier i about a_i, but the supplier i cannot observe a_{-i} (the set of observations on the other suppliers). Hence, an audited supplier can observe $a_i = \delta_i$, and the common unsustainability cost distribution F, however is uninformed on a_j $j \neq i$. Hence, in our problem the buyer (the mechanism designer) has private information that the suppliers cannot observe. One can imagine that the buyer can use this information to manipulate the suppliers' beliefs about each other by the choice of her information disclosure policy on the vector a. Skreta (2011) proves that in an independent private values setting such as ours, the informed auctioneer's choice of an information disclosure policy is irreverent in terms of her outcome from the optimal mechanism: i.e, there is no loss of optimality in treating the vector a as publicly announced (for more details, see Theorem 6 in Skreta (2011)). Hence, below in the optimal mechanism analysis we study the case where the information the buyer collects through the audits are publicly known by all parties.

Let us denote by (p,t) a direct revelation mechanism. We denote by $U_i(\hat{c}_i, \mathbf{c}_{-i})$ supplier i's expected payoff when he reveals $\hat{c}_i$ as his production cost. Without loss of generality, let us order the suppliers such that suppliers $i = \{1, \dots, M\}$ are the audited suppliers, and suppliers $i = \{M+1, \dots, N\}$ are the unaudited suppliers. The buyer needs to optimize the following objective function (2), over the set of feasible direct mechanisms satisfying incentive compatibility (3), participation (4), and allocation (5) & (6) constraints.

$$\min_{(p,t)} E_{\mathbf{c},\Delta}[\sum_{i=1}^{M}(p_i(\mathbf{c})\delta_i + t_i(\mathbf{c})) + \sum_{i=M+1}^{N}(p_i(\mathbf{c})\Delta_i + t_i(\mathbf{c})) + (1 - \sum_{i=1}^{N} p_i(\mathbf{c}))c_0] \tag{2}$$

subject to:

$$U_i(c_i, \mathbf{c}_{-i}) = E_{\mathbf{c}_{-i}}[t_i(\mathbf{c}) - p_i(\mathbf{c})c_i] \geq U_i(\hat{c}_i, \mathbf{c}_{-i}),\ \forall i, \hat{c}_i \tag{3}$$

$$U_i(c_i, \mathbf{c}_{-i}) \geq 0,\ \forall i, \tag{4}$$

$$\sum_{i}^{N} p_i(\mathbf{c}) \leq 1, \tag{5}$$

$$p_i(\mathbf{c}) \geq 0,\ \forall i. \tag{6}$$

Using the definition of i's utility function, the expected transfer to supplier i can be written as: $E_{\mathbf{c}_{-i}}[t_i(\mathbf{c})] = U_i(c_i, \mathbf{c}_{-i}) + E_{c_{-i}}[p_i(\mathbf{c})c_i]$. Then, $E_{\mathbf{c}}[t_i(\mathbf{c})] = \int_{c_{(l)}}^{c_{(u)}} U_i(t, \mathbf{c}_{-i})g(t)dt + E_c[p_i(\mathbf{c})c_i]$, where $g(c_i)$ is the pdf of the production cost. It follows that we can rewrite (2) as follows: $\min_{(p,t)} E_{\mathbf{c},\Delta}[\sum_{i=1}^{M} p_i(\mathbf{c})(\delta_i + c_i) + \sum_{i=M+1}^{N} p_i(\mathbf{c})(\Delta_i + c_i)] + \sum_i \int_{c_{(l)}}^{c_{(u)}} U_i(c_i)g(c_i)dc_i + (1 - \sum_{i=1}^{N} p_i(\mathbf{c}))c_0$. Let us denote $T_i(c_i) \triangleq E_{c_{-i}} t_i(\mathbf{c})$, and $P_i(c_i) \triangleq E_{c_{-i}} p_i(\mathbf{c})$. It follows that $U_i(\mathbf{c}) = T_i(\mathbf{c}) - P_i(\mathbf{c})c_i$. With standard manipulation (see for example Krishna (2010)), it can be shown that for an incentive compatible mechanism $U_i'(\mathbf{c}) = -P_i(\mathbf{c})$.

Now, let us consider $\int_{c_{(l)}}^{c_{(u)}} U_i(c_i)g(c_i)dc_i$. Integrating by parts, for an incentive compatible mechanism we get $\int_{c_{(l)}}^{c_{(u)}} U_i(c_i)g(c_i)dc_i = U_i(c_{(u)}) + E_{\mathbf{c}}[p_i(c_i, \mathbf{c}_{-i})\frac{G(c_i)}{g(c_i)}]$, so expected cost to the buyer is:

$$\begin{aligned} W &= E_{\mathbf{c},\Delta}[\sum_{i=1}^{M} p_i(\mathbf{c})(\delta_i + c_i) + \sum_{i=M+1}^{N} p_i(\mathbf{c})(\Delta_i + c_i) + \sum_i U_i(c_{(u)}) + \sum_i [p_i(c_i)\frac{G(c_i)}{g(c_i)}] + (1 - \sum_{i=1}^{N} p_i(\mathbf{c}))c_0], \\ &= E_{\mathbf{c},\Delta}[\sum_{i=1}^{M} p_i(\mathbf{c})(\delta_i + J(c_i)) + \sum_{i=M+1}^{N} p_i(\mathbf{c})(\Delta_i + J(c_i)) + (1 - \sum_{i=1}^{N} p_i(\mathbf{c}))c_0 + \sum_i U_i(c_{(u)})], \end{aligned}$$

$$= E_c[\sum_{i=1}^{M} p_i(\mathbf{c})(\delta_i + J(c_i)) + \sum_{i=M+1}^{N} p_i(\mathbf{c})(E_\Delta[\Delta] + J(c_i)) + (1 - \sum_{i=1}^{N} p_i(\mathbf{c}))c_0 + \sum_i U_i(c_{(u)})].$$

Hence we can replace the uncertain unsustainability cost term Δ_i with its expectation $E[\Delta] = \bar{\Delta}$ for the unaudited suppliers $i = \{M+1, \ldots, N\}$. We define a vector $\mathbf{s}$ such that the unsustainability cost markup $s_i = \delta_i$ if supplier i is audited, and the unsustainability cost markup $s_i = \bar{\Delta}$ if supplier i is not audited. Note that W is minimized for $U_i(c_{(u)}) = 0$, and by choosing an assignment rule favoring the supplier i with the lowest $s_i + J(c_i)$ if $s_i + J(c_i) \leq c_0$, and awarding the contract to the outside option if $s_i + J(c_i) > c_0$, $\forall i$. This is indeed the optimal assignment rule p_i^* given in the statement of the proposition. Characterizing the optimal transfer function t_i^* follows from $E_{\mathbf{c}_{-i}}[t_i(\mathbf{c})] = U_i(\mathbf{c}) + E_{c_{-i}}[p_i(\mathbf{c})c_i]$ and $U_i(c_{(u)}, \mathbf{c}_{-i}) = 0$. When writing the buyer's objective function as W above we assumed that we had an incentive compatible mechanism; one can verify that this indeed is true for (p^*, t^*), since $p^*(c_i, \mathbf{c}_{-i})$ is non-increasing in c_i. Finally, by using standard arguments (see for example Krishna (2010)) it is straightforward to show that, given a vector of unsustainability cost markups $\mathbf{s}$ prior to running the auction, the buyer's expected total cost from the optimal mechanism is $E_{\mathbf{c}}[\min\{s_1 + J(c_1), \ldots, s_N + J(c_N), c_0\}]$.

Proof of Proposition 2. Consider the sequence: $\lambda_M = \min\{\Delta_1 + J_1, \Delta_2 + J_2, \ldots, \Delta_M + J_M, \bar{\Delta} + J_{M+1}, \bar{\Delta} + J_{M+2}, \ldots, \bar{\Delta} + J_N\}$. Let us denote by $H_1(x) = Prob(\bar{\Delta} + J < x)$ and $H_2(x) = Prob(\Delta + J < x)$. Then, $Prob(\lambda_M \leq x) = 1 - (1 - H_1(x))^{N-M}(1 - H_2(x))^M$, $\forall M \leq N$, $M \in \mathbb{N}$. It follows that the buyer's expected total cost of procurement can be written as:

$$\begin{aligned} E[\text{Cost}|\text{Audit M suppliers}] &= \int_0^{c_0} 1 - \left(1 - (1 - H_1(x))^{N-M}(1 - H_2(x))^M\right) dx, \\ &= \int_0^{c_0} (1 - H_1(x))^{N-M}(1 - H_2(x))^M dx. \end{aligned}$$

Now, consider the change in the expected cost by auditing the $(M+1)$st supplier. Define

$$\begin{aligned} d_M^{(1)} &\triangleq E[\text{Cost}|\text{Audit M+1 suppliers}] - E[\text{Cost}|\text{Audit M suppliers}], \\ &= \int_0^{c_0} (1 - H_1(x))^{N-M-1}(1 - H_2(x))^{M+1} dx - \int_0^{c_0} (1 - H_1(x))^{N-M}(1 - H_2(x))^M dx, \\ &= \int_0^{c_0} (1 - H_1(x))^{N-M-1}(1 - H_2(x))^M (H_1(x) - H_2(x)) dx. \end{aligned}$$

We note that $(\Delta - \bar{\Delta})$ is a zero mean random variable, and the distribution of $(\Delta - \bar{\Delta}) + \bar{\Delta} + J$ is a mean-preserving spread of the distribution of $\bar{\Delta} + J$ (see Rothschild and Stiglitz (1970) for a definition of mean-preserving spread). Hence, the distribution of $\Delta + J$ (denoted by H_2) is a mean-preserving spread of the distribution of $\bar{\Delta} + J$ (denoted by H_1). This also means that H_1 second order stochastically dominates H_2, i.e., $\int_0^t (H_2(x) - H_1(x))\, dx \geq 0, \forall t \in \mathbb{R}^+$. Hence, $\int_0^{c_0} (H_1(x) - H_2(x))\, dx \leq 0$. Furthermore, there exists a point $x_0 \in \mathbb{R}^+$ such that $\forall x < (>) x_0$, $H_1(x) \leq (\geq) H_2(x)$ (Diamond and Stiglitz (1974); Müller and Stoyan (2002), Definition 1.5.25 and Theorem 1.5.26). Since $(1 - H_1(x))^M(1 - H_2(x))^{N-M-1}$ decreases in x,

$d_M^{(1)} \leq (1-H_1(x_0))^{N-M-1}(1-H_2(x_0))^M \int_0^{c_0}(H_1(x)-H_2(x))dx \leq 0$. Hence, the expected cost is decreasing in the number of audited suppliers. Consequently, *EAV* $= E[\text{Cost}|\text{Audit None}] - E[\text{Cost}|\text{Audit All}]$ is positive. Finally, note that

$$\begin{aligned}
EAV &= E[\min\{\bar{\Delta} + J_{1:N}, c_0\}] - E[\min\{(\Delta + J)_{1:N}, c_0\}],\\
&\leq E[\min\{\bar{\Delta} + J_{1:N}, c_0\}] - E[\min\{\Delta_{(l)} + J_{1:N}, c_0\}],\\
&= \bar{\Delta} + E[\min\{J_{1:N}, c_0 - \bar{\Delta}\}] - \Delta_{(l)} - E[\min\{J_{1:N}, c_0 - \Delta_{(l)}\}],\\
&\leq \bar{\Delta} + E[\min\{J_{1:N}, c_0 - \Delta_{(l)}\}] - \Delta_{(l)} - E[\min\{J_{1:N}, c_0 - \Delta_{(l)}\}] \;=\; \bar{\Delta} - \Delta_{(l)}.
\end{aligned}$$

Proof of Propositions 3 We can write the *EAV* in terms of the cumulative distribution functions H_1 and H_2 as follows: *EAV*$= \int_0^{c_0}(1-H_1(x))^N dx - \int_0^{c_0}(1-H_2(x))^N dx$. Then,

$$\begin{aligned}
\frac{\partial EAV}{\partial c_0} &= \frac{\partial}{\partial c_0}\int_0^{c_0}(1-H_1(x))^N dx - \frac{\partial}{\partial c_0}\int_0^{c_0}(1-H_2(x))^N dx,\\
&= (1-H_1(c_0))^N - (1-H_2(c_0))^N.
\end{aligned}$$

We note that by the proof of Proposition 2, there exists a x_0 such that $H_1(x) \leq H_2(x)$ for $x < x_0$, and $H_1(x) \geq H_2(x)$ for $x > x_0$. Hence, $(1-H_1(c_0))^N - (1-H_2(c_0))^N \geq (\leq) 0$ for $c_0 < (>) x_0$. Hence, *EAV* is increasing in c_0 when $c_0 < x_0$, and decreasing in c_0 when $c_0 > x_0$.

Proof of Proposition 4. Proposition 3 shows that *EAV* is unimodal in c_0. It follows that at x_0 (where *EAV* peaks), $\frac{\partial EAV}{\partial c_0}|_{x_0} = 0$, hence, $H_1(x_0) = H_2(x_0)$.

Proof of Proposition 5. Here, we ignore the outside option. First, consider multiplying Δ by a positive constant γ. For $M \leq N$, define

$$\begin{aligned}
EAV(M,\gamma) &\triangleq E[\min\{\gamma\bar{\Delta} + J_1, \ldots, \gamma\bar{\Delta} + J_N\}] - E[\min\{\gamma\Delta_1 + J_1, \ldots, \gamma\Delta_M + J_M, \gamma\bar{\Delta} + J_{M+1}, \ldots, \gamma\bar{\Delta} + J_N\}],\\
&= \gamma\bar{\Delta} + E[J_{1:N}] - \gamma\bar{\Delta} - E[\min\{\gamma(\Delta_1 - \bar{\Delta}) + J_1, \ldots, \gamma(\Delta_M - \bar{\Delta}) + J_M, J_{M+1}, \ldots, J_N\}],\\
&= E[J_{1:N}] - E[\min\{\gamma(\Delta_1 - \bar{\Delta}) + J_1, \ldots, \gamma(\Delta_M - \bar{\Delta}) + J_M, J_{M+1}, \ldots, J_N\}].
\end{aligned}$$

Let us denote by H_2^γ the distribution of $\gamma(\Delta_1 - \bar{\Delta}) + J$. Then,

$$\begin{aligned}
EAV(M,\gamma_2) - EAV(M,\gamma_1) &= E[\min\{\gamma_1(\Delta_1 - \bar{\Delta}) + J_1, \ldots, \gamma_1(\Delta_M - \bar{\Delta}) + J_M, J_{M+1}, \ldots, J_N\}]\\
&\quad - E[\min\{\gamma_2(\Delta_1 - \bar{\Delta}) + J_1, \ldots, \gamma_2(\Delta_M - \bar{\Delta}) + J_M, J_{M+1}, \ldots, J_N\}],\\
&= \int_0^\infty (1-\tilde{G}(x))^{N-M}((1-H_2^{\gamma_1}(x))^M - (1-H_2^{\gamma_2}(x))^M)dx.
\end{aligned}$$

We note that $(\Delta - \bar{\Delta})$ is a zero mean random variable, and the distribution of $\gamma_2(\Delta - \bar{\Delta}) + J$ is a mean-preserving spread of the distribution of $\gamma_1(\Delta - \bar{\Delta}) + J$ for $\gamma_2 \geq \gamma_1$. Furthermore, $(1-\tilde{G}(x))^{N-M}$ is a decreasing function in x. Hence, using a similar approach as used in the proof of Proposition 2, it can be shown that *EAV*$(M,\gamma_2) -$ *EAV*(M,γ_1) is positive.

Now, consider multiplying J by a positive constant κ. For $M \le N$, define

$$\begin{aligned}\textit{EAV}(M,\kappa) &\triangleq E[\min\{\bar{\Delta}+\kappa J_1,\ldots,\bar{\Delta}+\kappa J_N\}] - E[\min\{\Delta_1+\kappa J_1,\ldots,\Delta_M+\kappa J_M,\bar{\Delta}+\kappa J_{M+1},\ldots,\bar{\Delta}+\kappa J_N\}],\\ &= \bar{\Delta}+\kappa E[J_{1:N}] - E[\min\{\Delta_1+\kappa J_1,\ldots,\Delta_M+\kappa J_M,\bar{\Delta}+\kappa J_{M+1},\ldots,\bar{\Delta}+\kappa J_N\}].\end{aligned}$$

We will show that for $\kappa_2 \ge \kappa_1$, $\textit{EAV}(M,\kappa_1) - \textit{EAV}(M,\kappa_2)$ is positive. Note that

$$\begin{aligned}&E[\min\{\Delta_1+\kappa_2 J_1,\ldots,\Delta_M+\kappa_2 J_M,\bar{\Delta}+\kappa_2 J_{M+1},\ldots,\bar{\Delta}+\kappa_2 J_N\}]\\ &-E[\min\{\Delta_1+\kappa_1 J_1,\ldots,\Delta_M+\kappa_1 J_M,\bar{\Delta}+\kappa_1 J_{M+1},\ldots,\bar{\Delta}+\kappa_1 J_N\}],\\ &=\int_\Delta\int_J \big[\min\{\Delta_1+\kappa_2 J_1,\ldots,\Delta_M+\kappa_2 J_M,\bar{\Delta}+\kappa_2 J_{M+1},\ldots,\bar{\Delta}+\kappa_2 J_N\}\\ &-\min\{\Delta_1+\kappa_1 J_1,\ldots,\Delta_M+\kappa_1 J_M,\bar{\Delta}+\kappa_1 J_{M+1},\ldots,\bar{\Delta}+\kappa_1 J_N\}\big]dF(\Delta)d\tilde{G}(J),\\ &=\int_\Delta\int_J \big[\min\{\Delta_1+\kappa_1 J_1+(\kappa_2-\kappa_1)J_1,\ldots,\\ &\Delta_M+\kappa_1 J_M+(\kappa_2-\kappa_1)J_M,\bar{\Delta}+\kappa_1 J_{M+1}+(\kappa_2-\kappa_1)J_{M+1},\ldots,\bar{\Delta}+\kappa_1 J_N+(\kappa_2-\kappa_1)J_N\}\\ &-\min\{\Delta_1+\kappa_1 J_1,\ldots,\Delta_M+\kappa_1 J_M,\bar{\Delta}+\kappa_1 J_{M+1},\ldots,\bar{\Delta}+\kappa_1 J_N\}\big]dF(\Delta)d\tilde{G}(J)\\ &\ge\int_\Delta\int_J \big[\min\{\Delta_1+\kappa_1 J_1+(\kappa_2-\kappa_1)J_{1:N},\ldots,\\ &\Delta_M+\kappa_1 J_M+(\kappa_2-\kappa_1)J_{1:N},\bar{\Delta}+\kappa_1 J_{M+1}+(\kappa_2-\kappa_1)J_{1:N},\ldots,\bar{\Delta}+\kappa_1 J_N+(\kappa_2-\kappa_1)J_{1:N}\}\\ &-\min\{\Delta_1+\kappa_1 J_1,\ldots,\Delta_M+\kappa_1 J_M,\bar{\Delta}+\kappa_1 J_{M+1},\ldots,\bar{\Delta}+\kappa_1 J_N\}\big]dF(\Delta)d\tilde{G}(J),\\ &=\int_\Delta\int_J (\kappa_2-\kappa_1)J_{1:N}dF(\Delta)d\tilde{G}(J) \;=\; (\kappa_2-\kappa_1)E[J_{1:N}], \quad \text{which proves the result.}\end{aligned}$$

Finally, note that $\textit{EAV}(M) = E[\min\{\bar{\Delta}+J_{1:N}\}] - E[\min\{\Delta_1+J_1,\ldots,\Delta_M+J_M,\bar{\Delta}+J_{M+1},\ldots,\bar{\Delta}+J_N\}]$, hence adding a constant τ, to either random variable Δ or J, would not change *EAV(M)*.

Proof of Proposition 6. In the proof of Proposition 2 we have shown that $d_M^{(1)} < 0$ for $M \le N$. Let us now consider the second increment $d_M^{(2)} \triangleq d_{M+1}^{(1)} - d_M^{(1)}$:

$$\begin{aligned}d_M^{(2)} &= \int_0^{c_0}(1-H_1(x))^{N-M-1}(1-H_2(x))^{M+1}(H_1(x)-H_2(x)))dx\\ &-\int_0^{c_0}(1-H_1(x))^{N-M}(1-H_2(x))^{M}(H_1(x)-H_2(x)))dx,\\ &=\int_0^{c_0}(1-H_1(x))^{N-M-1}(1-H_2(x))^{M}(H_1(x)-H_2(x))^2dx.\end{aligned}$$

Since $d_M^{(2)} > 0$, we conclude that the sequence $E[\text{Cost}|\text{Audit M suppliers}]$ is convex decreasing in the number of audited suppliers, M. Equivalently, *EAV(M)* is concave increasing in M.

Proof of Proposition 7. The result follows from the proofs of Propositions 3-5.

Proof of Proposition 8. Suppose $\alpha_1 < \alpha_2$; we wish to show that $\textit{EAV}(\alpha_2, M) \ge \textit{EAV}(\alpha_1, M)$. To do so, we can simply apply the proof of Proposition 2, replacing H_1 with $H_2^{\alpha_1}$ and replacing H_2 with $H_2^{\alpha_2}$. Since $H_2^{\alpha_2}$ is a mean-preserving spread of $H_1^{\alpha_1}$, the result follows.

Proof of Proposition 9. The proofs of Propositions 2-7 hold as before, with H_2^α playing the role of H_2.

Proof of Proposition 10. Now we study the change in $EAV(M,\alpha)$ with simultaneous changes in α and c_0. $EAV(M,\alpha) = \int_0^{c_0}(1-H_1(x))^N dx - \int_0^{c_0}(1-H_1)^{N-M}(1-H_2^\alpha(x))^M dx$. From Proposition 8 $\frac{\partial EAV(M,\alpha)}{\partial \alpha} = \int_0^{c_0} M(1-H_1(x))^{N-M}(1-H_2^\alpha(x))^{M-1}\frac{\partial H_2^\alpha(x)}{\partial \alpha}dx > 0$. Then, $\frac{\partial^2 EAV(M,\alpha)}{\partial\alpha\partial c_0} = M(1-H_1(c_0))^{N-M}(1-H_2^\alpha(c_0))^{M-1}\frac{\partial H_2^\alpha(c_0)}{\partial\alpha}$. We note that $(1-H_1(c_0))^{N-M}(1-H_2^\alpha(c_0))^{M-1} > 0$ for finite c_0. Furthermore, $\frac{\partial H_2^\alpha(c_0)}{\partial\alpha} \geq (\leq) 0$ when $c_0 < (>) x_0$ where x_0 is the crossing point. Then, $\frac{\partial^2 EAV(M,\alpha)}{\partial\alpha\partial c_0} \geq (\leq) 0$ when $c_0 < (>) x_0$. Hence, the benefit from increased accuracy of audits can increase or decrease in the size of the outside option. In particular, it is increasing when $c_0 < x_0$, and decreasing when $c_0 > x_0$.

Proof of Proposition 11. Without loss of generality, we now study the change in expected cost from simultaneous increases in the auditing accuracies on supplier 1 (α_1) and supplier 2 (α_2). $E[\text{Cost}|\alpha_1,\alpha_2] = \int_0^{c_0}(1-H_2^{\alpha_1}(x))(1-H_2^{\alpha_2}(x))(1-H_2^{\alpha_3}(x))\dots(1-H_2^{\alpha_N}(x))dx$. It follows that $\frac{\partial^2 E[cost|\alpha_1,\alpha_2]}{\partial\alpha_1\partial\alpha_2} = \int_0^{c_0}\frac{\partial H_2^{\alpha_1}(x)}{\partial\alpha_1}\frac{\partial H_2^{\alpha_2}(x)}{\partial\alpha_2}(1-H_2^{\alpha_3}(x))\dots(1-H_2^{\alpha_N}(x))dx$.

By our modeling assumptions on the effect of more accurate audits, $\alpha_1 > \alpha_2$ implies that F_{α_1} is a mean-preserving spread of F_{α_2}, and $\frac{\partial H_2^\alpha(x)}{\partial\alpha} > 0$ for $x < x_0$ and $\frac{\partial H_2^\alpha(x)}{\partial\alpha} < 0$ for $x > x_0$, $\forall\alpha$. Therefore, $\frac{\partial H_2^{\alpha_1}(x)}{\partial\alpha_1}$ and $\frac{\partial H_2^{\alpha_2}(x)}{\partial\alpha_2}$ have the same sign $\forall x$. Hence, $\frac{\partial^2 E[cost]}{\alpha_i\alpha_j} \geq 0$, $i \neq j$.

Proof of Corollary 1. We note that the mean-preserving spread is a sufficient condition for Propositions 8-10 to continue to hold. Hence, given Lemma 1, these results are directly applicable under the non-linear unsustainability cost function case.

Proof of Proposition 12. Let us denote by L_1 the cumulative distribution function of $\bar{\Theta}\Delta + J$, and by L_2 the cumulative distribution function of $\Theta\Delta + J$. Then,

$$EVI = E_{\Delta,J}[\min\{(\bar{\Theta}\Delta+J)_{1:N}, c_0\}] - E_{\Delta,J,\Theta}[\min\{(\Theta\Delta+J)_{1:N}, c_0\}] = \int_0^{c_0}(1-L_1(x))^N dx - \int_0^{c_0}(1-L_2(x))^N dx$$

We note that $\Theta\Delta + J = (\Theta-\bar{\Theta})\Delta + \bar{\Theta}\Delta + J$. Hence, the distribution of $\Theta\Delta + J$, i.e., L_2, is a mean-preserving spread of the distribution of $\bar{\Theta}\Delta + J$, i.e., L_1, and $\int_0^t L_2(x) - L_1(x)dx \geq 0$, $\forall t \in \mathbb{R}^+$. It follows that *EVI* is positive for all distributions F, $\tilde{G}$, P defined on non-negative intervals.

Proof of Proposition 13. We now study the effect of changes of c_0 on *EVI*:

$$\frac{\partial EVI}{\partial c_0} = \frac{\partial}{\partial c_0}\int_0^{c_0}(1-L_1(x))^N dx - \int_0^{c_0}(1-L_2(x))^N dx = (1-L_1(c_0))^N - (1-L_2(c_0))^N$$

As given in the proof of Proposition 12, L_2 is a mean-preserving spread of L_1. Hence, there exists a point $x_0 \in \mathbb{R}^+$ where $L_1(x) \leq L_2(x)$ for $x < x_0$, and $L_1(x) \geq L_2(x)$ for $x > x_0$ (see Proposition 2). Then, $(1-L_1(c_0))^N - (1-L_2(c_0))^N \geq (\leq) 0$ for $c_0 < (>) x_0$. It follows that a larger outside option, c_0, can lead to an increase or decrease in *EVI*. In particular, for any F and $\tilde{G}$, there exists a x_0 such that *EVI* is increasing in c_0 for $c_0 < x_0$, and *EVI* is decreasing in c_0 for $c_0 > x_0$.

Proof of Proposition 14. First, consider $\sigma_\Delta = \sigma_J = \sigma$. Let us denote by $E[Z_{1:N}]$ the expected first order statistic from a standard normal distribution with N draws. The expected value of information on Θ is:

$$\begin{aligned}
\mathit{EVI}' &= E_{\Delta,J}[(\bar{\Theta}\Delta + J)_{1:N}] - E_{\Delta,J,\Theta}[(\Theta\Delta + J)_{1:N}],\\
&= E_{\Delta,J}[(\bar{\Theta}\Delta + J)_{1:N}] - \int_{\Theta_{(l)}}^{\Theta_{(u)}} E_{\Delta,J}(\theta\Delta + J)_{1:N}\rho(\theta)d\theta,\\
&= \sqrt{\bar{\Theta}^2\sigma_\Delta^2 + \sigma_J^2}E[Z_{1:N}] + \bar{\Theta}\bar{\Delta} + \bar{J} - \int_{\Theta_{(l)}}^{\Theta_{(u)}} (\sqrt{\theta^2\sigma_\Delta^2 + \sigma_J^2}E[Z_{1:N}] + \theta\bar{\Delta} + \bar{J})\rho(\theta)d\theta,\\
&= E[Z_{1:N}]\sigma(\sqrt{\bar{\Theta}^2 + 1} - \int_{\Theta_{(l)}}^{\Theta_{(u)}} \sqrt{\theta^2 + 1}\rho(\theta)d\theta).
\end{aligned}$$

Hence $\frac{\partial \mathit{EVI}'}{\partial\sigma} = E[Z_{1:N}](\sqrt{\bar{\Theta}^2+1} - E[\sqrt{\Theta^2+1}])$. $\sqrt{\Theta^2+1}$ is a convex and strictly increasing function of Θ, so by Jensen's inequality, $\sqrt{\bar{\Theta}^2+1} - E[\sqrt{\Theta^2+1}] \le 0$. Hence, since $E[Z_{1:N}] < 0$ for $N \ge 2$, $\frac{\partial \mathit{EVI}'}{\partial\sigma} \ge 0$.

Now, consider $\sigma_\Delta \ne \sigma_J$: $\mathit{EVI}' = E[Z_{1:N}](\sqrt{\bar{\Theta}^2\sigma_\Delta^2 + \sigma_J^2} - \int_{\Theta_{(l)}}^{\Theta_{(u)}} \sqrt{\theta^2\sigma_\Delta^2 + \sigma_J^2}\rho(\theta)d\theta)$.

Keeping everything else the same, consider multiplying Δ by a positive multiplier γ. It follows that $\mathit{EVI}'(\gamma) = E[Z_{1:N}](\sqrt{\bar{\Theta}^2\gamma^2\sigma_\Delta^2 + \sigma_J^2} - \int_{\Theta_{(l)}}^{\Theta_{(u)}} \sqrt{\theta^2\gamma^2\sigma_\Delta^2 + \sigma_J^2}\rho(\theta)d\theta)$. Then, $\frac{\partial \mathit{EVI}'}{\partial\gamma} = E[Z_{1:N}](\frac{\bar{\Theta}^2\gamma\sigma_\Delta^2}{\sqrt{\bar{\Theta}^2\gamma^2\sigma_\Delta^2+\sigma_J^2}} - E[\frac{\Theta^2\gamma\sigma_\Delta^2}{\sqrt{\Theta^2\gamma^2\sigma_\Delta^2+\sigma_J^2}}])$. We note that $\frac{\Theta^2\gamma\sigma_\Delta^2}{\sqrt{\Theta^2\gamma^2\sigma_\Delta^2+\sigma_J^2}}$ is a convex (concave) increasing function in Θ when $\frac{\sqrt{2}\sigma_J}{\Theta_{(u)}} > \gamma\sigma_\Delta$ ($\frac{\sqrt{2}\sigma_J}{\Theta_{(l)}} < \gamma\sigma_\Delta$). Then, by Jensen's inequality, $\frac{\bar{\Theta}^2\gamma\sigma_\Delta^2}{\sqrt{\bar{\Theta}^2\gamma^2\sigma_\Delta^2+\sigma_J^2}} - E[\frac{\Theta^2\gamma\sigma_\Delta^2}{\sqrt{\Theta^2\gamma^2\sigma_\Delta^2+\sigma_J^2}}] \le (\ge)0$, hence $\frac{\partial \mathit{EVI}'}{\partial\gamma} \ge (\le)0$ when $\frac{\sqrt{2}\sigma_J}{\Theta_{(u)}} > \gamma\sigma_\Delta$ ($\frac{\sqrt{2}\sigma_J}{\Theta_{(l)}} < \gamma\sigma_\Delta$).

Now, keeping everything else the same in the original EVI' expression, consider multiplying J by a positive multiplier κ. It follows that $\mathit{EVI}'(\kappa) = E[Z_{1:N}](\sqrt{\bar{\Theta}^2\sigma_\Delta^2 + \kappa^2\sigma_J^2} - \int_{\Theta_{(l)}}^{\Theta_{(u)}} \sqrt{\theta^2\sigma_\Delta^2 + \kappa^2\sigma_J^2}\rho(\theta)d\theta)$. Then, $\frac{\partial \mathit{EVI}'}{\partial\kappa} = E[Z_{1:N}](\frac{\kappa\sigma_J^2}{\sqrt{\bar{\Theta}^2\sigma_\Delta^2+\kappa^2\sigma_J^2}} - E[\frac{\kappa\sigma_J^2}{\sqrt{\Theta^2\sigma_\Delta^2+\kappa^2\sigma_J^2}}])$. We note that $\frac{\kappa\sigma_J^2}{\sqrt{\Theta^2\sigma_\Delta^2+\kappa^2\sigma_J^2}}$ is a convex (concave) decreasing function in Θ when $\sqrt{2}\Theta_{(l)}\sigma_\Delta > \kappa\sigma_J$ ($\sqrt{2}\Theta_{(u)}\sigma_\Delta < \kappa\sigma_J$). Then, by Jensen's inequality, $\frac{\kappa\sigma_J^2}{\sqrt{\bar{\Theta}^2\sigma_\Delta^2+\sigma_J^2}} - E[\frac{\kappa\sigma_J^2}{\sqrt{\Theta^2\sigma_\Delta^2+\sigma_J^2}}] \le (\ge)0$, hence $\frac{\partial \mathit{EVI}'}{\partial\kappa} \ge (\le)0$ when $\sqrt{2}\Theta_{(l)}\sigma_\Delta > \kappa\sigma_J$ ($\sqrt{2}\Theta_{(u)}\sigma_\Delta < \kappa\sigma_J$).

Proof of Proposition 15. E[cost|additional audit is on a category-B supplier] $= E[\min\{(\Lambda^{(A)} + J + \bar{\epsilon}^\Delta)_{(1:N_A-M_A-1)}, \Lambda^{(A)} + J + \bar{\epsilon}^\Delta, (\Lambda^{(A)} + J + \epsilon^\Delta)_{(1:M_A)}, (\Lambda^{(B)} + J + \bar{\epsilon}^\Delta)_{(1:N_B-M_B-1)}, (\Lambda^{(B)} + J + \epsilon^\Delta)_{(1:M_B)}, \Lambda^{(B)} + J + \epsilon^\Delta, c_0\}]$, and E[cost|additional audit is on a category-A supplier] $= E[\min\{(\Lambda^{(A)} + J + \bar{\epsilon}^\Delta)_{(1:N_A-M_A-1)}, (\Lambda^{(A)} + J + \epsilon^\Delta)_{(1:M_A)}, \Lambda^{(A)} + J + \epsilon^\Delta, (\Lambda^{(B)} + J + \bar{\epsilon}^\Delta)_{(1:N_B-M_B-1)}, \Lambda^{(B)} + J + \bar{\epsilon}^\Delta, (\Lambda^{(B)} + J + \epsilon^\Delta)_{(1:M_B)}, c_0\}]$. Note that E[cost|additional audit is on a category-B supplier] and E[cost|additional audit is on a category-A supplier] expressions are the same except for the following terms: $\min\{\Lambda^{(A)} + J + \bar{\epsilon}^\Delta, \Lambda^{(B)} + J + \epsilon^\Delta\}$ and $\min\{\Lambda^{(A)} + J + \epsilon^\Delta, \Lambda^{(B)} + J + \bar{\epsilon}^\Delta\}$. We have

$$\begin{aligned}
\min\{\Lambda^{(A)} + J + \bar{\epsilon}^\Delta, \Lambda^{(B)} + J + \epsilon^\Delta\} &= \epsilon^\Delta + \min\{\Lambda^{(A)} + J + \bar{\epsilon}^\Delta - \epsilon^\Delta, \Lambda^{(B)} + J\},\\
&= (\epsilon^\Delta - \bar{\epsilon}^\Delta) + \bar{\epsilon}^\Delta + \min\{\Lambda^{(A)} + J + \bar{\epsilon}^\Delta - \epsilon^\Delta, \Lambda^{(B)} + J\},
\end{aligned}$$

and, $\min\{\Lambda^{(A)} + J + \epsilon^\Delta, \Lambda^{(B)} + J + \bar{\epsilon}^\Delta\} = \bar{\epsilon}^\Delta + \min\{\Lambda^{(A)} + J + \epsilon^\Delta - \bar{\epsilon}^\Delta, \Lambda^{(B)} + J\}$.

Note that $\epsilon^\Delta - \bar{\epsilon}^\Delta$ and $\bar{\epsilon}^\Delta - \epsilon^\Delta$ are both zero-mean random variables and have the same distribution (due to the symmetric distribution of ϵ^Δ). Consequently, $(\epsilon^\Delta - \bar{\epsilon}^\Delta) + \bar{\epsilon}^\Delta + \min\{\Lambda^{(A)} + J + \bar{\epsilon}^\Delta - \epsilon^\Delta, \Lambda^{(B)} + J\}$ is second order stochastically smaller than $\bar{\epsilon}^\Delta + \min\{\Lambda^{(A)} + J + \epsilon^\Delta - \bar{\epsilon}^\Delta, \Lambda^{(B)} + J\}$.

Hence $E[\text{cost}|\text{additional audit is on a category-}A\text{ supplier}] \leq E[\text{cost}|\text{additional audit is on a category-}B\text{ supplier}]$. So, given two suppliers in different categories, the buyer prefers to audit whichever has the lowest base-cost.

Proof of Proposition 16. Here and in the proof of Proposition 17, H_1^A, H_2^A, H_1^B, and H_2^B denote, respectively, the cumulative distribution function of $\Lambda^{(A)} + J + \bar{\epsilon}^\Delta$, $\Lambda^{(A)} + J + \epsilon^\Delta$, $\Lambda^{(B)} + J + \bar{\epsilon}^\Delta$, and $\Lambda^{(B)} + J + \epsilon^\Delta$. As in the proof of Proposition 2, we first show that $d^{(1)}_{M|M<N_A} \triangleq E[\text{cost}|\text{audit } M+1] - E[\text{cost}|\text{audit } M]$ for $M < N_A$ is negative. Note that $E[\text{cost}|\text{audit } M, M \leq N_A] = \int_0^{c_0} (1 - H_2^A(x))^M (1 - H_1^A(x))^{N_A - M} (1 - H_1^B(x))^{N_B} dx$, so

$$d^{(1)}_{M|M<N_A} = \int_0^{c_0} (1 - H_2^A(x))^M (1 - H_1^A(x))^{N_A - M - 1} (1 - H_1^B(x))^{N_B} (H_1^A(x) - H_2^A(x)) dx.$$

Since H_1^A second order stochastically dominates H_2^A, the logic from the first part of Proposition 2 applies, so $\mathit{EAV}_{corr}(M)$ is increasing in M for $M \leq N_A$. Next consider $d^{(2)}_{M|M<N_A} \triangleq d^{(1)}_{M+1|M<N_A} - d^{(1)}_{M|M<N_A}$:

$$d^{(2)}_{M|M<N_A} = \int_0^{c_0} (1 - H_2^A(x))^M (1 - H_1^A(x))^{N_A - M - 2} (1 - H_1^B(x))^{N_B} (H_1^A(x) - H_2^A(x))^2 dx.$$

Since $d^{(2)}_{M|M<N_A}$ is positive, $\mathit{EAV}_{corr}(M|M < N_A)$ is concave increasing in M for $M \leq N_A$. The above analysis can easily be adapted for showing the concavity of $\mathit{EAV}_{corr}(M|N_A \leq M \leq N_A + N_B)$. Since the two concave portions of EAV_{corr} have different first and second increments, EAV_{corr} is piecewise-concave and N_A is the vertex point where the two concave portions meet.

Proof of Proposition 17. We first study the changes in $\mathit{EAV}_{corr}(M)$ for $M < N_A$ as c_0 changes.

$$\begin{aligned}\frac{\partial \mathit{EAV}_{corr}(M|M < N_A)}{\partial c_0} &= \frac{\partial}{\partial c_0} \int_0^{c_0} (1 - H_1^A(x))^{N_A - M} (1 - H_1^B(x))^{N_B} ((1 - H_1^A(x))^M - (1 - H_2^A(x))^M) dx, \\ &= (1 - H_1^A(c_0))^{N_A - M} (1 - H_1^B(c_0))^{N_B} ((1 - H_1^A(c_0))^M - (1 - H_2^A(c_0))^M).\end{aligned}$$

We note that $(1 - H_1^A(c_0))^{N_A - M} (1 - H_1^B(c_0))^{N_B}$ is always non-negative. Furthermore, H_2^A is a mean-preserving spread of H_1^B. Then, there exists a $x_0^{(1)}$ such that $H_2^A(x) \geq H_1^A(x)$ for $x < x_0^{(1)}$, and $H_2^A(x) \leq H_1^A(x)$ for $x > x_0^{(1)}$. Hence, $((1 - H_1^A(c_0))^M - (1 - H_2^A(c_0))^M) \geq (\leq) 0$ for $c_0 < (>) x_0^{(1)}$. It follows that, $\mathit{EAV}_{corr}(M|M < N_A)$ is increasing in c_0 when $c_0 < x_0^{(1)}$, and decreasing in c_0 when $c_0 > x_0^{(1)}$.

We now study the changes in $\mathit{EAV}_{corr}(M)$ for $N_A < M$ as the outside option c_0 changes.

$$\begin{aligned}\frac{\partial \mathit{EAV}_{corr}(M|N_A < M)}{\partial c_0} &= \frac{\partial}{\partial c_0} \int_0^{c_0} \big[(1 - H_1^A(x))^{N_A} (1 - H_1^B(x))^{N_B} \\ &\quad - (1 - H_2^A(x))^{N_A} (1 - H_1^B(x))^{N_B - (M - N_A)} (1 - H_2^B(x))^{M - N_A}\big] dx, \\ &= \frac{\partial}{\partial c_0} \int_0^{c_0} \big[(1 - H_1^B(x))^{N_B - (M - N_A)} ((1 - H_1^A(x))^{N_A} (1 - H_1^B(x))^{M - N_A} \\ &\quad - (1 - H_2^A(x))^{N_A} (1 - H_2^B(x))^{M - N_A})\big] dx,\end{aligned}$$

$$= (1 - H_1^B(c_0))^{N_B-(M-N_A)}((1 - H_1^A(c_0))^{N_A}(1 - H_1^B(c_0))^{M-N_A}$$
$$- (1 - H_2^A(c_0))^{N_A}(1 - H_2^B(c_0))^{M-N_A}).$$

Similar to above, it can be shown that $\exists x_0^{(1)}$ and $\exists x_0^{(2)}$ such that *EAV*$_{corr}(M|N_A < M)$ is increasing in c_0 when $c_0 < x_0^{(1)}$, and decreasing in c_0 when $c_0 > x_0^{(2)}$. However, depending on the distribution functions, M, and N_A, *EAV*$_{corr}(M|N_A < M)$ can be non-monotonic for c_0 between $x_0^{(1)}$ and $x_0^{(2)}$. H_1^B is identical to H_1^A except that it is shifted to the right by $\Lambda^{(B)} - \Lambda^{(A)}$>0, so H_2^B is also identical to H_2^A except that it is shifted to the right by $\Lambda^{(B)} - \Lambda^{(A)}$. Hence H_2^B and H_1^B cross at $x_0^{(1)} + \Lambda^{(B)} - \Lambda^{(A)}$, so $x_0^{(2)} > x_0^{(1)}$.