When does it pay to delay supplier qualification? Theory and experiments

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Abstract

We study a procurement setting in which the buyer seeks a low price but will not allocate the contract to a supplier who has not passed qualification screening. Qualification screening is costly for the buyer, involving product tests, site visits, and interviews. In addition to a qualified incumbent supplier, the buyer has an entrant of unknown qualification. The buyer wishes to run a price-only, open-descending reverse auction between the incumbent and the entrant, and faces a strategic choice about whether to perform qualification screening on the entrant before or after the auction. We analytically study the buyer's optimal strategy, accounting for the fact that under post-auction qualification the incumbent knows he could lose the auction but still win the contract. In our analysis we derive the incumbent's optimal bidding strategy under post-auction qualification and find that it follows a threshold structure in which high-cost incumbents hold back on bidding — or even boycott the auction — in order to preserve their profit margin, and only lower-cost incumbents bid to win. These results are strikingly different from the usual open-descending auction analysis where all bidders are fully qualified and bidding to win is always a dominant strategy. We test our analytical results in the laboratory, with human subjects. We find that qualitatively our theoretical predictions hold up quite well, although incumbent suppliers bid somewhat more aggressively than the theory predicts, making buyers more inclined to use post-auction qualification.

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1. Introduction

Procurement is important — the average US manufacturer spends roughly 60% of its revenue purchasing goods and services (U.S. Department of Commerce 2005). Competitive bidding (auctions) is a powerful tool that many buyers deploy to manage spend. Seeking price concessions from their incumbent suppliers, it is common for buyers to have a highly motivated new (entrant) supplier compete in a reverse auction against the incumbent supplier when the incumbent's contract is up for renewal.

While contract price is a concern, buyers generally will not switch the contract to an entrant unless the entrant has been verified to be fully *qualified* for the business. As is common in industry, we refer to performing *qualification screening* on a supplier as the act of verifying that the supplier is indeed able to comply with all the contract specifications (e.g., on product, delivery, packaging) with a reasonable degree of certainty. The process of screening an entrant's qualifications is costly for the buyer, and can include testing the entrant's products, visiting the entrant's production facilities, verifying the entrant's surge capacity availability, etc.

When it comes to screening an entrant's qualifications, the buyer has a strategic choice: attempt to qualify the entrant prior to the auction ("pre-qualification"); or delay qualification screening until after the auction ("post-qualification"). Both approaches are used in practice; for example, Beall et al. (2003) discusses cases where Bechtel (a large engineering firm) followed a pre-qualification approach, whereas Kulp and Randall (2005) discusses how a pharmaceutical company procuring industrial chemicals allowed bids by entrants whose qualifications had not yet been verified. Both approaches have their respective advantages. Under post-qualification the buyer attempts to screen entrants only if their eventual bid is sufficiently attractive. This allows the buyer to directly hold an auction and saves money that might have otherwise been wasted performing qualification screening on an entrant whose bid turns out not to be competitive relative to the incumbent's bid. On the other hand, if an auction occurs under pre-qualification, the buyer commits to awarding the contract to the lowest bidder, which can push the incumbent supplier to bid more aggressively.

Under post-qualification, the entrant supplier might win the auction but fail qualification screening afterwards, forcing the buyer to award the contract to the incumbent supplier (who lost the auction), paying a higher bid price. Standard auctions assume that the bid-taker commits to awarding the contract to the bidder who submitted the best bid, and most of the existing theoretical models of auctions also make this assumption, as do laboratory experiments that test these standard models. In contrast, procurement auctions in practice are oftentimes buyer-determined (such as with post-qualification), meaning that the buyer reserves the right to select the winner after the auction (see for example Jap 2002), so a supplier may lose the auction but win the contract. Buyer-determined, non-binding auctions have not yet been extensively studied (two exceptions, Engelbrecht-Wiggans et al. 2007 and Wan and Beil 2009, are discussed in the literature review). The question of the optimal bidding strategy in an open-bid auction where a bidder may lose the auction (by not submitting the lowest bid) but yet win the contract, is difficult analytically, and we are the first to tackle it. We derive new theoretical results for this setting, from both the buyer and bidder perspective, and experimentally test our theory's predictions using a controlled laboratory experiment with human subjects.

To enable both theoretical and experimental analyses, we study a stylized problem that captures the salient features of the buyer's pre- or post-qualification decision. In our model, the buyer has an expiring contract with her incumbent supplier, and, approached by a new entrant supplier, wishes to conduct an open-descending procurement auction between the incumbent and the entrant. The buyer can choose to screen the entrant using pre-qualification; if the entrant successfully passes it, the buyer can then hold a binding auction in which the low bid wins the contract. However, pre-qualification may backfire on the buyer if the entrant fails to be qualified — in such a case the buyer not only wastes the qualification cost but also loses the opportunity to run the auction, forcing her to renew the incumbent's contract without any reduction in price. Alternatively, the buyer can choose to use post-qualification screening on the entrant, and screen the entrant only if the entrant wins the auction. In this case the incumbent knows that he could lose the auction but still win the contract if the entrant fails post-qualification. We address the following questions:

Research Question 1: What is the incumbent's optimal bidding strategy under post-qualification?

Research Question 2: How does the answer to question 1 depend on the probability that the entrant is truly qualified, the buyer's cost of performing qualification screening on the entrant, and the auction reserve price?

Research Question 3: Under what circumstances will the buyer prefer to use post-qualification?

Research Question 4: To what extent does the theory we develop to answer the above questions predict subjects' behavior in controlled laboratory experiments?

We find the following. In the canonical open-descending auction analysis (e.g., Krishna 2002 Ch.2), there is no post-qualification stage and all bidders have a dominant strategy to bid down to their true cost. We find that this is no longer true under post-qualification, addressing *research question 1.* In fact, under post-qualification our theoretical analysis predicts that the incumbent will generally drop out of the auction before reaching its true cost, sometimes drastically (e.g., "boycott" the auction by dropping out at the reserve price). By abandoning the bidding effort *before* the price reaches his true cost — thus letting the entrant win the auction — the incumbent tries to preserve its profit margin and hopes the entrant fails post-qualification.

Addressing *research question 2*, we find that when it is costlier for the buyer to perform qualification screening on the entrant, the incumbent is more "advantaged" under post-qualification and consequently more likely to bid aggressively and win the auction outright. However, we also find that the incumbent, in seeking to retain a large profit, is more likely to drop out of the auction early when the reserve price is large; thus, high profit potential for the incumbent may, ironically, lead him to short-circuit the auction rather than compete harder to retain the contract. Our theory also predicts that the incumbent may bid less aggressively in response to a higher probability that the entrant would survive qualification screening, meaning that when the incumbent can lose the auction but win the contract, a tougher competitor — namely an entrant that is more likely to be qualified — might actually forestall competition. In spite of the fact that post-qualification can cause the incumbent to hold back on bidding, our theory predicts that the buyer's decision follows a threshold: If the cost of qualifying the entrant is large enough, the buyer prefers the post-qualification strategy, addressing *research question 3*.

To test our theory's detailed predictions in a controlled environment (*research question 4*), we employ laboratory experiments in which human subjects play the role of the incumbent supplier, the buyer, or both.[1] Consistent with theory, incumbent suppliers tend to drop out of the auction early in an attempt to preserve their profit margin when their costs are high, and bid to win the auction when their costs are low. We also find strong support in the data for how problem parameters qualitatively affect incumbent bidding. While incumbent suppliers overall bid more aggressively than the theory predicts, in a simplified setting they learn, over time, to bid closer to theoretical guidelines. Buyers, when faced with automated suppliers programmed to bid optimally, learn to time the qualification screening to minimize their total expected cost, matching the theory. But when buyers deal with human incumbents, the overly-aggressive human incumbent bidding makes post-auction qualification screening even more attractive, and consequently buyers tend to select it more often. Our main behavioral conclusion is that incumbents tend to bid more aggressively than the theory predicts, but the theory's qualitative predictions stand up well. The managerial implication is that post-auction qualification screening may even be more attractive in practice than predicted by theory alone.

The next section reviews related literature, followed by a discussion of the model in Section 3. Section 4 provides theoretical analyses, and Sections 5 and 6 describe our experimental design and results, respectively. Section 7 concludes.

2. Literature Review

Elmaghraby (2000) provides a detailed review of procurement work in the operations and economics literature. Many such papers, including ours, apply auctions as the means of price discovery during the procurement process. Krishna (2002) provides excellent treatments and references on auctions.

Our paper studies how supplier qualification screening manifests itself in the auction bidding behavior of entrants and incumbents. While supplier qualification is common in practice, surpris-

[1] We automated the entrant supplier, who always had a dominant strategy to bid down to its true cost.

ingly little has been written about it in the procurement auction literature. To our knowledge, only one other paper studies supplier qualification in the context of procurement auctions. That paper, Wan and Beil (2009), focuses on the buyer's optimal mechanism design problem for a setting with an unconstrained number of bidders whose qualification probabilities and costs are ex ante symmetric. Optimal mechanism design searches among all possible mechanisms and finds the one that maximizes the objective function of the designer. Bidder incentives are captured as constraints on an optimization problem, whose solution provides a theoretical benchmark. In the present paper our focus is different. We study a setting with two suppliers, an incumbent and entrant, who are ex ante asymmetric in cost and qualification probability. For this setting we focus on theoretical and experimental results. We do so for the reverse open-descending, price-only auction, where our reasons for examining this mechanism are twofold: First, reverse open-descending, price-only auctions are commonly used for procurement in practice, making it important to study such mechanisms. Second, the simplicity and practicality which make such mechanisms widely used in practice also make them amenable to study in the lab, a major purpose of the present paper.

In addition to providing the first equilibrium analyses for bidding behavior in reverse open-descending, price-only auctions with possibly unqualified bidders, in testing these predictions in the lab the present paper also is the first experimental study of auctions with possibly unqualified bidders. Laboratory experiments testing various aspects of auction theory go back to the early 1980's. Most early work focused on testing revenue equivalence among various auction formats and exploring possible explanations for its failure, as well as on investigating the winner's curse in common value auctions (see Kagel 1995 for a review of experimental auctions work prior to 1995 and Kagel and Levin 2002 for a review of lab experiments in the common and affiliated value settings). Experimental work on procurement auctions focused on settings where price is not the only attribute of interest (our paper falls into this category). Bichler (2000) was the first to test multi-attribute auctions in the laboratory. Chen-Ritzo et al. (2005) compare a multi-attribute procurement auction to a price-only auction and demonstrate that a multi-attribute auction can be more efficient. Engelbrecht-Wiggans et al. (2007) study a setting where suppliers have non-price attributes but the auction is conducted on price, and explain when the buyer is better off committing to award the contract on price alone, ignoring the non-price attributes. We analyze a different (and essential) non-price attribute, supplier qualification, where the buyer seeks to award the contract to the lowest-priced qualified supplier.

In the present paper, the incumbent, who is known to the buyer, has already passed qualification screening. The entrant, who is unknown to the buyer, has not yet been qualified. There is other theoretical work that model different features that make incumbent and entrant suppliers unalike. Zhou (2003) models informational differences between incumbents and entrants when the cost of the contract is highly uncertain, finding that the incumbent generally bids aggressively owing to its informational advantage. In contrast, we find that the incumbent's qualification "advantage" in our setting can cause it to bid much less aggressively. More generically, incumbent or entrant status can motivate studying auctions where bidders possess asymmetric cost distributions; for a review of this work, see Chapter 8 of Krishna (2002). In our analysis, in addition to the asymmetry over qualification, we allow the incumbent and entrant costs to follow different distributions.

A handful of papers have empirically studied incumbent and entrant bidding in auctions. Zhong (2007) examines incumbent and entrant behavior in multi-item procurement auctions held by a large high-tech company. Consistent with our model and experimental findings, her empirical data suggests incumbents seem to choose between timid testing and all-out competing for the contract, and that incumbents often win the contract without being the lowest bidder. De Silva et al. (2003) study sealed bids for road construction contracts for which all bidders are pre-qualified and the low bid always wins the contract. Finding that entrants bid more aggressively and win the auction with lower bids than incumbents, they explain this with a theoretical model in which entrants have more dispersed costs. Our model also predicts that entrants bid more aggressively than incumbents, but because the incumbent strategically holds back on bidding.

3. Model

We consider a procurement manager, or *buyer*, who seeks to award a single, indivisible contract for goods or services. The buyer already has a pre-existing *incumbent* supplier, denoted by i, who currently performs the contract. As is common in practice, we assume that the contract covers a finite period of time (e.g., one to two years), after which point it must be renegotiated. To this end we assume that the buyer becomes aware of an *entrant*, denoted by e, a new supplier who approaches the buyer seeking out new business. The buyer is interested in leveraging supply-side competition for the contract by conducting an auction in which she solicits competing bids from both the incumbent and the entrant. We let R denote the price the incumbent currently charges the buyer for the contract; thus, the incumbent's true cost to perform the contract, denoted by x_i, is assumed to be at most R. We assume that x_i is distributed according to a c.d.f. F_i (with p.d.f. $f_i > 0$) over $[l, R]$ where $l < 1 \leq R$, and that the entrant's true cost x_e follows a c.d.f F_e (with p.d.f. $f_e > 0$) over $[0, 1]$. We assume that $\frac{F_e}{f_e}$ is increasing,[2] x_i and x_e are privately-known and independently-distributed, and the distributions F_i and F_e are common knowledge.

We assume that both suppliers seek to maximize their expected utility. We let $U(\cdot)$ denote the incumbent's utility function. Thus, the utility of an incumbent with true cost x_i is $U(p - x_i)$ if he wins the contract and receives payment p from the buyer, or is $U(0)$ if he does not win the

[2]This assumption is satisfied, for example, when F_e is log-concave, including uniform, normal, logistic, and exponential distributions; see Bagnoli and Bergstrom (2005). This assumption is common in the auctions literature. Furthermore, we only need it for the effect of qualification cost in Theorem 2.

contract. We assume that $U(\cdot)$ is concave, i.e., the incumbent is risk-neutral or risk-averse. In our model setting, the entrant will have a dominant bidding strategy (see §4); thus, we do not explicitly specify the entrant's utility function. Our theoretical analyses of the suppliers' bidding behavior utilizes the Bayesian Nash equilibrium concept, which is standard in the auction literature.

Due to its incumbency status, the incumbent is already qualified for the contract; due to opaque requirements[3] set by the buyer, we assume that both the buyer and the suppliers only know that the probability that the entrant is indeed qualified equals $0 < \beta < 1$, the entrant's *qualification probability*. For instance, β close to one corresponds to very light qualification checks that any entrant supplier is very likely to pass, while β close to zero corresponds to very strict qualification requirements that relatively few entrant suppliers would be able to pass.[4] Qualification screening checks can be costly, involving tests of supplier products, trips to the supplier's production facilities, etc. We let $K \geq 0$ denote the *qualification cost*, that is, the cost that would be incurred by the buyer to verify whether the entrant is, or is not, qualified for the contract.

The buyer seeks to minimize her expected total procurement cost, that is, the contract price plus any supplier qualification costs. If the buyer holds an auction in which the entrant and incumbent compete, bidding occurs via a typical reverse clock auction (see Ausubel and Cramton 2006 for discussions about clock auctions in practice). In a clock auction, the calling price begins at an initial price, and then falls continuously until one of the two bidders drops out. The buyer faces a choice about the timing of qualification screening on the entrant: she can choose either "post-qualification" or "pre-qualification," as described next.

3.1 Post-qualification

Under post-qualification, the buyer directly conducts a price-based reverse clock auction between the entrant and incumbent, but *without attempting to qualify the entrant ahead of time.* For simplicity, we assume the auction kicks off with a calling price p equal to R (the reserve price) and the calling price p continuously drops as the auction progresses. The auction ends when either or both bidders drop out (ties are broken randomly). Suppose the auction ends at a calling price $p = b$. If it was the entrant that dropped out first, the incumbent wins the contract and gets paid b; otherwise, the buyer performs qualification screening on the entrant, and awards the contract to the entrant with a payment $b - \frac{K}{\beta}$ if the entrant passes, but contracts with the incumbent and pays

[3] Some aspects of qualification are straightforward such as having three satisfied references or passing product conformance tests, but others are more ambiguous. For instance, the buyer might have their internal production and design engineers conduct face-to-face meetings with the supplier's employees, to assess the "fit" between the buyer and supplier organizations on important issues such as lean principles and quality assurance, or to verify the supplier's in-house technical expertise on the product they are to produce. See Wan and Beil (2009).

[4] The strictness of the qualification screening performed by the buyer is typically commensurate with the buyer's perceived downside risk of supplier non-performance. While the buyer might be satisfied with only light qualification screening when purchasing indirect goods or services (such as office cleaning), when purchasing a critical direct input whose conformance to precise design specifications directly impacts the safety or performance of the buyer's product, the buyer's screening checks could be quite involved.

the incumbent b if the entrant fails.[5]

By subtracting $\frac{K}{\beta}$ when computing the entrant's contract payment, the buyer accounts for the need to post-qualify the entrant. Hence the buyer essentially runs a total-cost auction: She computes the total cost bid from a supplier to be the supplier's price offer plus a markup to account for qualification expenses. This markup is assumed to be zero for the incumbent, capturing the fact that the incumbent is already qualified for the contract. For the entrant, the markup is equal to $\frac{K}{\beta}$ which accounts for the fact that prior to accepting an entrant's bid, the buyer would have to perform K dollars worth of qualification checks on the entrant, which the entrant would pass with probability β.[6] One can regard markup $\frac{K}{\beta}$ as a "switching cost" related to the need to perform costly qualification screening on the entrant. Intuitively, as the cost of qualification (K) increases or the entrant's qualification probability (β) decreases, the entrant becomes less attractive to the buyer, which is reflected by a larger markup $\frac{K}{\beta}$. In effect, the markup shifts the entrant's cost distribution to the right, making the entrant less competitive. Because the entrant's true cost x_e is distributed between zero and one, the *effective cost* (i.e., true cost plus the markup) of the entrant is distributed between $[\frac{K}{\beta}, 1 + \frac{K}{\beta}]$. Of course, additional switching costs that are unrelated to qualification — such as the need to change order processing procedures — could also be incorporated into the model by simply shifting F_e to the right. We assume $R > \frac{K}{\beta}$; otherwise, no entrant cost type could earn a positive profit.

3.2 Pre-qualification

Under pre-qualification, the buyer pays K to screen the incumbent *before* the auction. With probability $1 - \beta$, the entrant is found to be unqualified and is discarded, and without any competitive threat to the incumbent the contract is defacto renewed with the incumbent at prevailing price R. This captures a situation in which the buyer must rely on supplier competition for price concessions. However, with probability β the pre-qualification establishes that the entrant is qualified, at which point an auction is conducted between the entrant and the incumbent. The auction details are exactly as before, save the need to use post-qualification: Whichever bidder drops out first loses (ties are broken randomly), and the other wins the contract and is paid the loser's dropout bid.

[5]The buyer can reasonably implement such a payment rule when she can commit to ignore renegotiation offers from suppliers after the auction. In other words, suppliers know their bids will be treated as binding. Given the auction rules, the suppliers plan accordingly.

[6]Note that the buyer would be indifferent between post-qualifying a price offer of $b - \frac{K}{\beta}$ from the entrant and directly accepting a price offer of b from the incumbent, since $(b - \frac{K}{\beta})\beta + b(1-\beta) + K = b$.

4. Theoretical Analyses

4.1 Incumbent bidding strategy under post-qualification

Standard open-descending auction analyses (e.g., Krishna 2002) conclude that a supplier who must win the auction to win the contract finds it a dominant strategy to stay in the auction until either their opponent drops out or the price drops so low that winning would be unprofitable, whichever happens first. The implication is that if the buyer chooses pre-qualification and the entrant passes pre-qualification, both suppliers would bid down to their true cost before dropping out in the ensuing auction. It also implies that under post-qualification, the entrant finds it optimal to bid down to its true effective cost ($x_e + \frac{K}{\beta}$). However, the standard theory does not specify how the incumbent should bid under post-qualification, because then the incumbent can lose the auction but still win the contract. The following theorem characterizes the incumbent's Bayesian Nash equilibrium bidding strategy under post-qualification, addressing *research question 1*.

Theorem 1 *Given any F_i, F_e, $U(\cdot)$, $\beta \in (0, 1)$, l and R, for an incumbent with cost x_i, there exists a static optimal bid-down-to level $\underline{p}(x_i)$ such that the incumbent should stay in the auction until the price reaches $\underline{p}(x_i)$ or the entrant drops out, whichever happens first. There exist two thresholds x_B and x_W, such that $x_W \leq x_B$, $x_B < R$, $x_W \leq \frac{K}{\beta}$, and*

(i) $\underline{p}(x_i) = R$ if and only if $x_i \geq x_B$,

(ii) $\underline{p}(x_i) = \frac{K}{\beta}$ if and only if $x_i \leq x_W$,

(iii) $x_i < \underline{p}(x_i)$, $\frac{K}{\beta} < \underline{p}(x_i) < R$, and $\underline{p}(x_i)$ strictly increases in x_i if $x_W < x_i < x_B$.

In words, Theorem 1 predicts that, depending on his true cost, x_i, the incumbent deploys one of three types of strategies: "boycott the auction," "bid-to-win," and "test-the-water." Under the boycott strategy, the incumbent drops out of the auction at the reserve price, and simply hopes that the entrant fails post-qualification. Part (i) predicts that this strategy is used when the incumbent knows he is unlikely to beat the entrant on price alone, i.e., when the incumbent's cost is quite high ($x_i \geq x_B$). In such a case the incumbent thinks it will likely be pointless to try and win on price, so he short-circuits the auction by dropping out immediately. Part (ii) predicts that only when his cost is very low ($x_i \leq x_W$) will the incumbent deploy the bid-to-win strategy in which he lowers his bid until the entrant is forced to drop out. In fact, since x_W is bounded by the lower bound on the entrant's effective cost distribution ($\frac{K}{\beta}$), the incumbent only bids this aggressively when he is absolutely certain he can beat the entrant on price alone. Part (iii) predicts that when the incumbent's cost is moderate he uses a mixture of these two extreme strategies, the test-the-water strategy: He bids against the entrant in the hopes of clinching the contract on price alone, but does so only half-heartedly — if the entrant stays in the auction long enough, eventually the incumbent will abandon the effort and drop out (at price $\underline{p}(x_i)$) before reaching his true cost. Surprisingly, this structure of the incumbent's bidding strategy (Theorem 1) is predicted regardless of the particular entrant and incumbent cost distributions F_e and F_i and the incumbent's utility function U (of course, since the entrant has a dominant strategy, the result also applies regardless of the entrant's utility function).

The key insight here is that buyers should not assume that her incumbent suppliers will bid aggressively to retain the contract just because she pits them against an entrant in an auction. The next result predicts how the incumbent's bidding behavior changes with the underlying procurement setting, addressing *research question 2*.

Theorem 2 *The optimal bid-down to level $\underline{p}(x_i)$ increases in R for all x_i. Furthermore, it decreases in K for all x_i such that $\underline{p}(x_i) > \frac{K}{\beta}$, and $\underline{p}(x_i)|_{K=\hat{K}} = \frac{\hat{K}}{\beta}$ implies $\underline{p}(x_i)|_{K=\tilde{K}} = \frac{\tilde{K}}{\beta}$ for all $\tilde{K} > \hat{K}$. Thus, the probability that the incumbent wins the auction outright (i.e., $\underline{p}(x_i) < x_e + \frac{K}{\beta}$) increases in K. However, $\underline{p}(x_i)$ is generally not monotone in β.*

Theorem 2 predicts that as it becomes costlier for the buyer to perform qualification screening on the entrant (as K increases), the incumbent is more likely to win the auction outright. This is because a higher qualification cost causes the buyer to add a higher "switching cost" ($\frac{K}{\beta}$) to the entrant's bid, making it easier for the incumbent to beat the entrant on price and thereby encouraging the incumbent to try to win the auction. However, Theorem 2 predicts that the incumbent is more likely to drop out of the auction early when the reserve price R is large. By dropping out early and letting the entrant win the auction, the incumbent loses the contract if the entrant survives post-qualification. This risk is more worthwhile for the incumbent if there is more profit that he is trying to preserve. Thus the theory predicts that high profit potential for the incumbent may lead him to short-circuit the competition rather than compete harder to retain the contract.

Theorem 2 also predicts that the incumbent may bid more or less aggressively in response to a higher probability (β) that the entrant would survive qualification screening. As one might intuitively expect, the incumbent is encouraged to try to win the auction when the entrant is more likely to survive post-qualification. But what is perhaps less initially obvious is that the buyer is more willing to post-qualify the entrant (who is more likely to survive the post-qualification), so the "switching cost" shrinks, scaring the incumbent away from trying to compete on price. In fact, the theory predicts that when the incumbent can lose the auction but win the contract, a tougher competitor — an entrant who is more likely to be qualified — might actually forestall competition.

4.2 Buyer's optimal qualification strategy

In the previous section we saw that the buyer should be careful not to assume that suppliers will bid aggressively simply because she pits them against each other in an auction. Pre-qualification is a tool available to the buyer to try and ensure suppliers do bid aggressively. However, it is not clear whether the buyer would always prefer to use pre-qualification, as we investigate now, addressing *research question 3*.

In comparing the post- and pre-qualification strategies, the buyer faces the following trade-offs. If she uses post-qualification, she avoids wasting money qualifying an entrant whose price in the auction might not turn out to be competitive, and also avoids losing the opportunity to run an auction in case the entrant is actually unqualified. However, post-qualification causes the incumbent to hold back on bidding in the auction. Pre-qualification can induce more aggressive bidding by the incumbent, but pre-qualification backfires on the buyer if the entrant fails pre-qualification and must be discarded. Thus, the buyer's decision depends on how the incumbent will bid in an auction with post-qualification. If the buyer thinks the incumbent will bid very aggressively even if the entrant might be unqualified, post-qualification can be an attractive strategy. On the other hand, if the incumbent will only bid aggressively if the buyer can tout the fact that the entrant is fully qualified and only the low bid will win the contract, the buyer may be forced to use pre-qualification.

As discussed in §3, under the pre-qualification strategy, the buyer spends qualification cost K on qualifying the entrant, which yields one of two outcomes: With probability β the entrant is found to be qualified and an auction is subsequently run with two fully qualified bidders, thus resulting in an expected contract payment of $E\max\{x_i, x_e\}$ (recall that $x_i \leq R$); and with probability $1-\beta$ the entrant is found to be unqualified and is discarded, and consequently the contract is renewed with the incumbent at price R. In summary, under pre-qualification the buyer's expected total (payment plus qualification) cost is

$$\beta E\max\{x_i, x_e\} + (1-\beta)R + K. \tag{1}$$

Per Theorem 1, if the buyer uses the post-qualification strategy, the incumbent's bidding strategy can be described as a bid-down-to level $\underline{p}(x_i)$. The entrant wins the auction if $\underline{p}(x_i) > \min\{x_e + \frac{K}{\beta}, R\}$; if so, the buyer incurs a qualification cost K to vet the entrant and pays $\underline{p}(x_i) - \frac{K}{\beta}$ to the entrant if the entrant survives post-qualification (which happens with probability β), but pays $\underline{p}(x_i)$ to the incumbent if the entrant fails post-qualification (which happens with probability $1-\beta$). Otherwise, if $\underline{p}(x_i) \leq \min\{x_e + \frac{K}{\beta}, R\}$, the incumbent wins the auction, and thus keeps the contract with a payment from the buyer equal to either the entrant's dropout bid or the reserve price (whichever is smaller), $\min\{x_e + \frac{K}{\beta}, R\}$. Therefore, under the post-qualification strategy, the buyer's expected total cost is

$$\begin{aligned} & E\max\{\min\{x_e + \frac{K}{\beta}, R\}, K + \beta[\underline{p}(x_i) - \frac{K}{\beta}] + (1-\beta)\underline{p}(x_i)\}, \\ = \; & E\max\{\min\{x_e + \frac{K}{\beta}, R\}, \underline{p}(x_i)\}. \end{aligned} \tag{2}$$

The buyer finds the optimal qualification strategy by comparing (1) with (2). The next proposition proves the buyer prefers post-qualification screening if the qualification cost K is large enough.

Theorem 3 *Given any F_i, F_e, $U(\cdot)$, $\beta \in (0,1)$, l and R, there exists a threshold $\underline{K}$ such that it is optimal for the buyer to choose post-qualification if $K > \underline{K}$.*

When the qualification cost is high enough, post-qualification screening is preferred because it helps the buyer avoid wasting money qualifying an entrant whose price in the auction might not turn out to be competitive. Although Theorem 3 predicts that the buyer's optimal strategy can be characterized by a threshold over qualification cost, there in general does not exist a similar threshold over qualification probability β; for example, a buyer can prefer post-qualification either when β is small enough or large enough, as we will show in §4.3. This is because the incumbent's optimal bidding behavior under post-qualification is in general not monotone in β (Theorem 2).

4.3 Theoretical predictions under uniform costs and risk-neutrality

Research question 4 asks to what extent the theory developed in this paper predicts subjects' behavior in laboratory experiments. These lab experiments will be discussed in §5. To facilitate that analysis, the present subsection provides specific predictions for the setting studied in the laboratory. In particular, while our analytical results assumed general entrant and incumbent cost distributions, for simplicity the lab experiments utilize uniform distributions. Thus, in this subsection we derive the incumbent's optimal bidding strategy when facing an entrant whose costs are uniformly distributed. Likewise, we also find the buyer's optimal decision between pre- and post-qualification when suppliers' costs are uniformly distributed. For simplicity, and as is common in the experimental auctions literature, our theoretical benchmarks employ the risk-neutral incumbent supplier model; that is, the theoretical results in this subsection are derived for linear $U(\cdot)$. In this subsection we call our results "propositions" (rather than theorems) to highlight the fact that they invoke the uniform distribution and linear utility assumptions which were not used in the general results earlier in the section.

Incumbent's optimal bidding function. For given K, β, and R, define $\hat{x}(K,\beta,R)$ to be the $x_i \in (-\infty, \frac{2\beta-1}{\beta} + \frac{K}{\beta})$ solving the following equation:[7]

$$0 = \begin{cases} \frac{(\beta x_i - K)^2}{2(2\beta-1)} - \beta x_i + \frac{K}{\beta} + \frac{1}{2} - (1-\beta)R, & \text{if } \frac{K}{\beta} < x_i \leq \frac{2\beta-1}{\beta} + \frac{K}{\beta}; \\ -\beta x_i + \frac{K}{\beta} + \frac{1}{2} - (1-\beta)R, & \text{if } x_i \leq \frac{K}{\beta}. \end{cases}$$

Proposition 1 *Assume $F_e \sim U[0,1]$. For a risk-neutral incumbent, the optimal bid-down-to level is $\underline{p}(x_i) = \frac{\beta x_i}{2\beta-1} - \frac{(1-\beta)K}{\beta(2\beta-1)}$ when $x_W < x_i < x_B$. The thresholds x_B and x_W depend on R, K, and*

[7]Note that $\hat{x}(K,\beta,R)$ is well-defined because the right hand side of the equation is continuous and convex on $(-\infty, \frac{2\beta-1}{\beta} + \frac{K}{\beta}]$, is negative at $x_i = \frac{2\beta-1}{\beta} + \frac{K}{\beta}$ and goes to positive infinity as x_i goes to negative infinity.

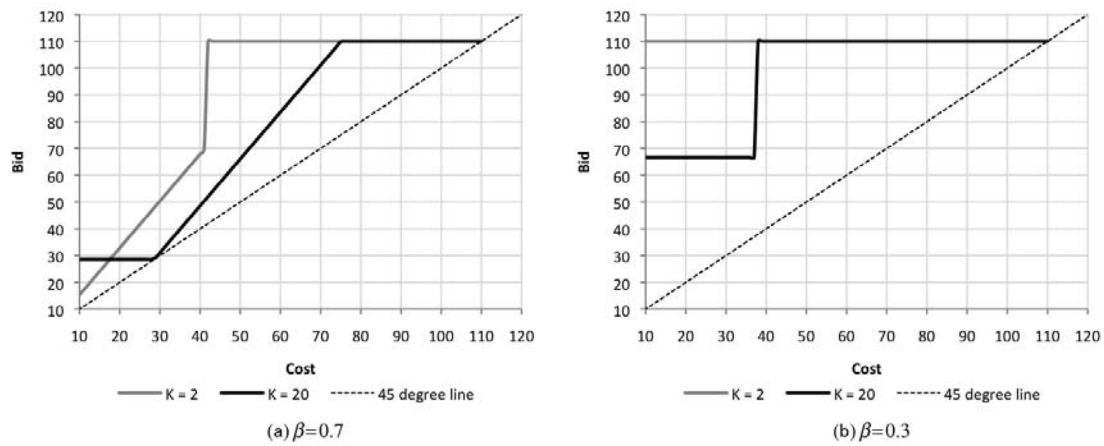


Figure 1: A risk-neutral incumbent's bidding functions when $K = 2$ and $K = 20$. Plots assume $F_i \sim U[10, 110]$, $F_e \sim U[0, 100]$, and $\beta = 0.7$ (left panel) or $\beta = 0.3$ (right panel).

β and are provided in the following table.

	$0 < \beta \leq \frac{1}{2}$		$\frac{1}{2} < \beta < 1$	
	$R \leq 1 + \frac{K}{\beta}$	$R > 1 + \frac{K}{\beta}$	$R \leq 1 + \frac{K}{\beta}$	$R > 1 + \frac{K}{\beta}$
x_B	$\frac{K}{2\beta^2} - \frac{R}{2\beta} + R$	$\frac{K}{\beta^2} + \frac{1}{2\beta} - \frac{R}{\beta} + R$	$\frac{2\beta-1}{\beta}R + \frac{(1-\beta)K}{\beta^2}$	$\hat{x}(K, \beta, R)$
x_W	$\frac{K}{2\beta^2} - \frac{R}{2\beta} + R$	$\frac{K}{\beta^2} + \frac{1}{2\beta} - \frac{R}{\beta} + R$	$\frac{K}{\beta}$	$\min\{\hat{x}(K, \beta, R), \frac{K}{\beta}\}$

Proposition 1 illustrates Theorem 1 by presenting closed-form expressions for the incumbent's optimal bidding function (bid-down-to level). Figure 1 plots this bidding function for the problem parameters used in our laboratory experiment.

Sensitivity of bidding behavior to problem parameters. A useful measure of the aggressiveness of the bidding behavior is the percentage of auctions incumbents win outright (as opposed to being awarded the contract after the entrant wins the auction but fails the qualification screening). Theorem 2 predicts that the probability of the incumbent winning the auction outright increases in the qualification cost K. The exact bidding functions furnished by Proposition 1 enable us to further predict qualitative effects of the entrant's qualification probability β on the aggressiveness of the incumbent's bidding behavior.

Proposition 2 *Assume $F_e \sim U[0, 1]$ and the incumbent is risk-neutral. The probability that the incumbent wins the auction outright is increasing in β if K is close to zero, and decreasing in β if K is large and close to βR.*

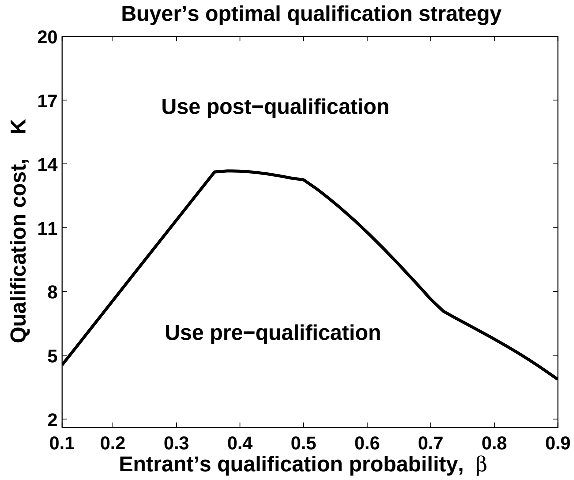


Figure 2: Buyer's optimal qualification strategy. Plot assumes $F_e \sim U[0, 100]$, $F_i \sim [10, 110]$, and that the incumbent is risk-neutral.

Proposition 2 predicts that β's effect on the incumbent's probability of winning the auction depends on the size of K. The intuition is that when K is small, the "switching cost" $\frac{K}{\beta}$ imposed on the entrant's bids is modest, and larger β scares the incumbent into bidding more aggressively to avoid losing the contract. However, when K is large, the switching cost imposed on the entrant is so significant that it becomes easy for the incumbent to clinch the contract by winning the auction. In such cases, larger β diminishes this switching cost and the incumbent bids less aggressively, anticipating that to win the auction he would have to sacrifice too much potential profit.

Illustration of buyer's optimal decision. Figure 2 shows the buyer's optimal decision for various qualification cost and probability pairs, (K, β). The hill-shaped line divides the plane: In the upper region the buyer finds it optimal to choose the post-qualification strategy, and in the lower region the buyer finds it optimal to use the pre-qualification strategy. Confirming Theorem 3, the buyer prefers pre-qualification only when the qualification cost K is relatively small (making pre-qualification cheap). Additionally, the figure illustrates that the size of the qualification probability β has a non-monotone effect on the buyer's decision: The buyer prefers pre-qualification when β is neither too small (in which case pre-qualification is very likely to disqualify the bidder), nor too large (in which case the incumbent is inclined to bid aggressively even under post-qualification because he knows the entrant would stand only a small chance of failing post-qualification).

5. Experimental Design and Research Hypotheses

5.1 Experimental design

We designed our experiments to test the research hypotheses in a way that gives the theory the best shot to work by simplifying the decision task for the participants and promoting learning. With this goal in mind, we start by analyzing the behavior of incumbent suppliers and buyers separately.

In all our laboratory settings, an incumbent supplier competes with an entrant for the right to provide a contract to the buyer. The entrant has a dominant strategy to bid truthfully, so we automated the entrant to bid according to this dominant strategy in all experiments. The incumbent's cost of providing the contract is $U[10, 110]$ and the entrant's cost is $U[0, 100]$. Both costs are rounded to the nearest integer. In our treatments we vary two factors at two levels. The buyer's cost of qualifying the entrant (K) is either low (2) or high (20), and the probability that the entrant will be qualified successfully (β) is either high (70%) or low (30%). In all treatments we set the reserve $R = 110$ at the top of the incumbent's cost distribution support. Under post-auction qualification screening the entrant's effective cost is increased by K/β, making the effective cost distribution $U[K/\beta, K/\beta + 100]$.

Figure 1 shows the incumbent's optimal bid function (bid-down-to level) with post-auction qualification screening in the four $K|\beta$ combinations of our experiments. When incumbent suppliers have to bid with post-auction qualification screening, their problem is especially complex: As can be seen from Figure 1, the bidding strategy can be radically different depending on the incumbent's cost x_i. Echoing Proposition 1, Figure 1 reveals, generally speaking, the following trend: When x_i is high enough, incumbents should boycott the auction by bidding $b = R$; when x_i is low enough, incumbents should bid to win; and when x_i is at intermediate levels, incumbents should bid above their cost.

It turns out that, for the four parameter configurations in our experiments, the benefit from the intermediate strategy (bidding between R and x_i) is quite small relative to the benefit from selecting the "right" extreme strategy (either boycott or bid-to-win). Figure 3 shows the benefit (measured in experimental currency units (ECU)) from boycotting relative to bidding to win. The benefit ranges from about plus or minus 25 ECU. In contrast, the benefit from using the intermediate (optimal) strategy relative to the better of the two extreme strategies is at most around 2 ECU in the $\beta = 0.7$ condition (we omit the plot verifying this to save space), and is always 0 for both treatments in the $\beta = 0.3$ condition (as can be seen from Figure 1(b), one of the extreme strategies is always optimal in that setting).

Keeping in mind the complexity of the incumbent's decision, we tested incumbents' bidding behavior in two parts. We designed the first experiment (called the *Incumbent Restricted* experiment) to test the extent to which suppliers are able to learn to recognize which of the two extreme strategies is better given a constant cost x_i. In this restricted setting, incumbents' cost could be either low $x_i = 10$ or high $x_i = 70$, and their bidding strategy was restricted to the two extremes: to either bid the reserve ($b = R = 110$) or to compete fully ($b = x_i$). In each session we kept the same K (either 2 or 20) and varied β within subjects. Each incumbent completed 50 rounds with $\beta = 0.3$ and 50 rounds with $\beta = 0.7$ (half of the participants had the high β first, and the rest had

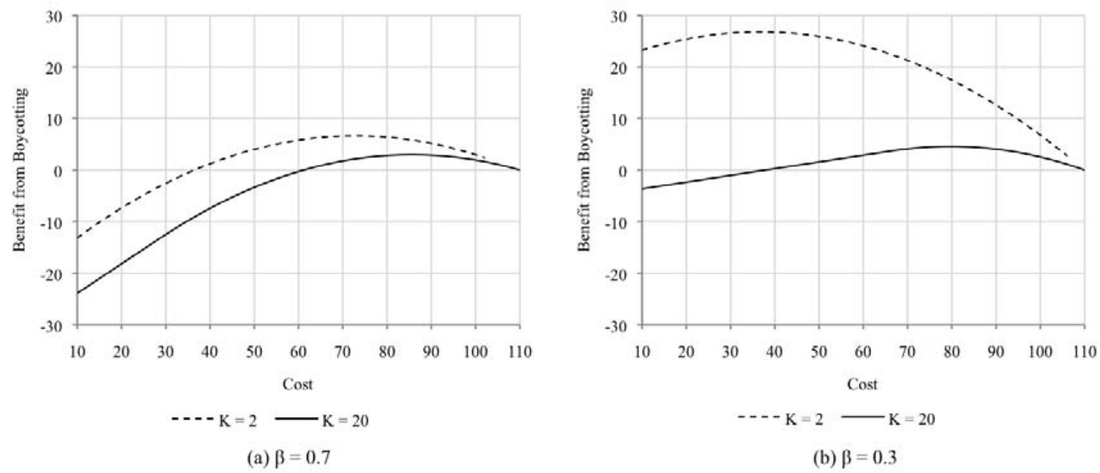


Figure 3: The benefit from boycotting relative to bidding to win.

the low β first). Within the 50 rounds with the same β, each participant completed 25 consecutive rounds with $x_i = 10$ and 25 consecutive rounds with $x_i = 70$. The order in which the two levels of x_i were presented also varied, with half of the participants completing the 25 rounds with $x_i = 10$ first, and the other half completed the 25 rounds with $x_i = 70$ first.[8]

We designed the second experiment (called the *Incumbent Full* experiment) to learn how incumbents bid in a more natural environment. In this experiment we kept the same β for the entire session and varied K after 50 rounds, but each round the incumbent received a new independent random draw of $x_i \sim U[10, 110]$, and was allowed to place any integer bid between x_i and $R = 110$.

We designed the third experiment (called the *Buyer* experiment) to test how human buyers select the timing of the auction qualification screening when faced with an automated incumbent and entrant, both programmed to bid optimally. In this experiment human subjects were in the buyer role, and completed 50 rounds. Buyers received a fixed revenue each round, which we set to be $120 + K$. Computerized suppliers received new independently-drawn costs each round, $x_i \sim U[10, 110]$ for the incumbent and $x_e \sim U[0, 100]$ for the entrant. The buyer had to decide on the pre- or post- auction qualification screening at the beginning of the round. If the buyer chose the pre-auction qualification screening, the entrant was either successfully qualified (with probability β) or not (with probability $1-\beta$). If the entrant failed the screening, the round ended with the contract price of 110 and the buyer earned 10 ($= 120 + K - K - 110$). If the entrant passed the screening, an auction was conducted in which the losing supplier bid down to his cost, the auction ended at price $P = \max\{x_e, x_i\}$, and the buyer earned $120 + K - K - P = 120 - P$. If the buyer chose the post-auction qualification screening, the auction was conducted immediately, and since the incumbent was programmed to bid down to $\underline{p}(x_i)$, the auction ended at price $P = \max\{x_e + K/\beta, \underline{p}(x_i)\}$.

[8]This *within subjects* design is used to control for individual differences. We tested for order effects and did not find any, and therefore we present analysis based on the pooled data.

Experiment	Buyer	Incumbent	x_i	b_i	K	β
Incumbent Restricted	Automated	Human	$\mathbf{\{10, 70\}}$	$\{x_i, R\}$	$\{2, 20\}$	$\mathbf{\{0.3, 0.7\}}$
Incumbent Full	Automated	Human	$\boldsymbol{U}[\mathbf{10, 110}]$	$\{x_i, x_i + 1, \ldots, R\}$	$\mathbf{\{2, 20\}}$	$\{0.3, 0.7\}$
Buyer	Human	Automated	$\boldsymbol{U}[\mathbf{10, 110}]$	$\{x_i, x_i + 1, \ldots, R\}$	$\{2, 20\}$	$\{0.3, 0.7\}$
Buyer-Incumbent	Human	Human	$\boldsymbol{U}[\mathbf{10, 110}]$	$\{x_i, x_i + 1, \ldots, R\}$	2	0.7

Table 1: Summary of the experimental design. Within-subjects parameters are in bold. In the experiments the human buyers chose between pre- and post-qualification, and the automated buyers used post-qualification.

At this point, if the incumbent won, the round ended and the buyer earned $120 + K - P$. If the entrant won, however, the qualification screening was conducted. If the entrant survived post-qualification (with probability β) he won the contract and was paid $P - K/\beta$; if the entrant failed post-qualification (with probability $1 - \beta$) the incumbent won the contract and was paid P. The buyer's average cost under post-qualification was thus $120 + K - P$. We varied the β and K parameters between subjects, so each participant faced only a single K and β combination.

The fourth and last experiment (called the *Buyer-Incumbent* experiment) involved human players in both roles (the buyer and the incumbent supplier). Each participant kept his or her role throughout the session and was randomly matched each round with another human participant with a different role. Incumbents' costs were randomly drawn each round according to $x_i \sim U[10, 110]$ and they could place any integer bid between x_i and R. Table 1 summarizes the experimental design. Sample sizes for each experiment are reported in the corresponding results section.[9]

We conducted all experimental sessions at the Laboratory for Economic Management and Auctions (LEMA) at the Smeal College of Business, Penn State University. Our participants were students, mostly undergraduates, from a variety of majors. We recruited them through the on-line recruitment system, offering earning cash as the only incentive to participate. Upon arrival at the laboratory the subjects were seated at computer terminals. We handed out written instructions (in the Online Appendix) to them and they read the instructions on their own. After all participants finished reading the instructions, we read the instructions to them aloud, to ensure common knowledge about the rules of the game. After we finished reading the instructions to the participants we started the actual game.

In each session each participant completed a number of rounds. Each individual always had the same role (buyer or supplier) in each round. We programmed the experimental interface using

[9]Experimental instructions are available from the authors upon request.

the zTree system (Fischbacher 2007). At the end of each session we computed cash earnings for each participant by multiplying the total earnings from all rounds by a pre-determined exchange rate and adding it to a $5 participation fee. Participants were paid their earnings in private and in cash, at the end of the session.

5.2 Theoretical Benchmarks and Research Hypotheses

As we can see from Figure 1, incumbent bid functions are quite complex, making it difficult to compare bidding behavior across treatments. Therefore we establish several metrics that allow us to compare behavior across combinations of K and β as well as to test whether behavior is qualitatively consistent with theoretical benchmarks. The metrics we use are (1) Boycotting Rates: proportion of $b = R$ bids; (2) Winning Rates: proportion of auctions the incumbent wins outright; and (3) Contract Prices: the average auction price P.

In the Incumbent Restricted treatments each player has only two actions, $b = x_i$ or $b = R$. The optimal actions for all parameter settings in those treatments are listed in Table 2. Since in the Incumbent Restricted treatments the actual bids participants are allowed to place are restricted to be either R or x_i, the *average proportion of* $b = R$ *bids* fully determines winning rates and contract prices.[10] For this reason, we focus on metric (1) in the Incumbent Restricted treatments, and formulate the following research hypothesis:

Hypothesis 1A: In the Incumbent Restricted treatments, the incumbent should boycott the auction in all $x_i = 70$ *conditions and in the* $x_i = 10$ *condition when* $K = 2$ *and* $\beta = 0.3$ *(see columns 2 and 3 of Table 2).*

In the Incumbent Full treatments, however, the proportion of $b = R$ bids is less meaningful because the number of possible bids incumbents can place is large. An alternative to $b = R$ may be any bid between x_i and R. If the optimal bid is R and the actual bid placed is slightly below R, for example, this deviation is likely to have a negligible effect on the auction outcome in terms of who wins and the resulting contract price. Therefore, when comparing different conditions in the Incumbent Full treatments we focus on metrics (2) and (3), the proportion of auctions incumbents win outright and the resulting contract prices, respectively. Table 2 shows optimal incumbent winning rates in columns 4 and 5, and contract prices in columns 6 and 7, for the Incumbent Full treatments. These optimal contract prices and winning rates are based on the actual realizations of x_i and x_e in the experiment. Standard deviations (in parentheses) take individual participants as the unit of analysis.

Hypothesis 1B: In the Incumbent Full treatments, average incumbent winning rates and contract

[10] Winning rates are lower and contract prices are higher than they should be whenever the $b = R$ action is taken in place of the $b = x_i$ action, and vice versa. We confirmed that this relationship is indeed what we see in the data, and do not report details in the interest of conserving space.

prices should not be different than the benchmarks in columns 4 through 7 of Table 2.

The next set of theoretical benchmarks and hypotheses follow from the predictions of Theorem 2 and Propositions 1–2 and deal with effects of K and β on the boycotting rates and contract prices.

Hypothesis 2A: In the Incumbent Restricted treatments, average boycott rates should not be affected by either K or β when $x_i = 70$. When $x_i = 10$ boycott rates decrease in β when $K = 2$ and decrease in K when $\beta = 0.3$ (refer to Table 2).

Hypothesis 2B: Average incumbent winning rates in the Incumbent Full treatments should increase as K increases, which implies that incumbents will win more often when $K = 20$ than when $K = 2$ for both levels of β.

Hypothesis 2C: In the Incumbent Full treatments, sometimes higher β causes incumbents to win more auctions outright, and sometimes fewer. When $K = 2$, incumbents should win more auctions when $\beta = 0.7$ than when $\beta = 0.3$. But when $K = 20$ the relationship is reversed, and incumbents should win more auctions when $\beta = 0.3$ than when $\beta = 0.7$.

Hypothesis 2D: Average contract prices in the Incumbent Full treatments should decrease in K and β.

The last set of benchmarks and hypotheses deals with the buyer's behavior. In both of our buyer experiments the main metric of interest is the proportion of post-auction qualification screening decisions made. Figure 2 illustrates the buyer's optimal decisions. A secondary metric is the average total cost incurred by the buyer (contract payment plus the cost of qualification screening). Theorem 3 shows that generally, and independently of β, there is a threshold in K such that when K is high enough, the buyer is better off always using post-auction qualification screening.

Hypothesis 3: (A) Buyers should select pre-auction qualification screening when $K = 2$ regardless of the β, and post-auction qualification screening when $K = 20$, also regardless of the β. (B) This behavior should not be affected by whether the incumbent suppliers are human subjects or automated agents programmed to follow the optimal bidding strategy.

6. Results

6.1 Do the theory's point predictions match the data?

To test Hypothesis 1A (point predictions for the Incumbent Restricted treatments) we compare the actual frequency of boycotting the auction to theoretical benchmarks. Table 3 shows the average boycott rates for the conditions in the restricted setting, their standard errors, and the results of hypothesis tests comparing them to theoretical benchmarks.[11] We are able to reject the

[11]For all one-sample tests we used Wilcoxon matched pairs signed rank tests. For all two-sample tests we used Wilcoxon rank-sum tests.

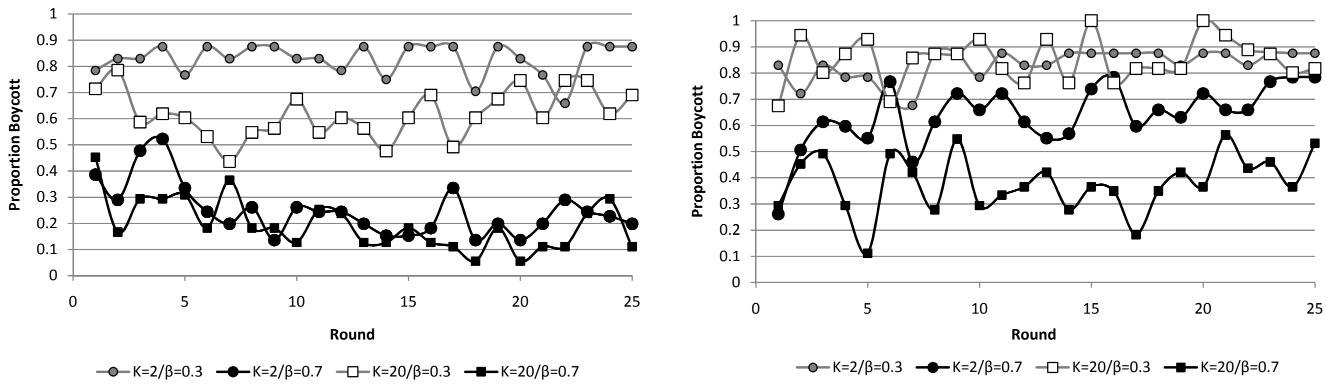
	Optimal Bids		Winning Rates		Contract Prices	
	$\beta = 0.3$	$\beta = 0.7$	$\beta = 0.3$	$\beta = 0.7$	$\beta = 0.3$	$\beta = 0.7$
$K = 2$	R, if $x_i = 10$	x_i, if $x_i = 10$	0.000	0.200	107.97	94.94
	R, if $x_i = 70$	R, if $x_i = 70$	(0.000)	(0.054)	(0.47)	(3.39)
$K = 20$	x_i, if $x_i = 10$	x_i, if $x_i = 10$	0.667	0.525	101.18	91.14
	R, if $x_i = 70$	R, if $x_i = 70$	(0.076)	(0.066)	(2.98)	(4.21)

Table 2: Theoretical benchmarks for the Incumbent Restricted treatments (columns 2 and 3) and the Incumbent Full treatments (columns 4 to 7)

	$\beta = 0.3$			$\beta = 0.7$			
	$K = 2$	$K = 20$	H_o : the effect of K	$K = 2$	$K = 20$	H_o : the effect of K	H_o : the effect of β
$x_i = 10$	0.846* (0.3070) $N = 19$	0.633** (0.3325) $N = 16$	$p = 0.0565$	0.280** (0.3101) $N = 19$	0.203** (0.2598) $N = 16$	$p = 0.5322$	$K = 2$: $p < 0.0001$ $K = 20$: $p = 0.0001$
$x_i = 70$	0.846* (0.3076) $N = 19$	0.850* (0.2089) $N = 16$	$p = 0.9678$	0.651* (0.3360) $N = 19$	0.395* (0.2755) $N = 19$	$p = 0.0332$	$K = 2$: $p = 0.0088$ $K = 20$: $p = 0.0002$
H_o : The effect of x_i	$p =$ 0.999	$p =$ 0.0384		$p <$ 0.0001	$p =$ 0.0006		

*H_o : Boycott rate > 0; $p < 0.05$ (one-sided).
**H_o : Boycott rate < 1; $p < 0.05$ (one-sided).

Table 3: Average boycotting rates, standard deviations (in parentheses) and sample sizes in the Incumbent Restricted treatments; results of the hypothesis test measuring the effect of x_i, β, and K.



(a) x_i = 10 (b) x_i = 70

Figure 4: Boycott rates over time

point predictions of the theory, because boycott rates are always significantly different from their theoretical benchmarks. However, it is not surprising that the observed averages do not confirm theory, since in this experiment there is only one possible direction for errors.

Perhaps more interesting is what happens to the average boycotting rates over time, which we plot in Figure 4. In the $x_i = 10$ condition we find that learning is significant only in the $K = 2|\beta = 0.7$ treatment, where boycott rates decrease significantly over time. Since boycotting is not the optimal option in this condition, participants in fact learn to select the optimal option more often. In contrast, in the $x_i = 70$ condition, boycott rates increase significantly over time in all but the $K = 20|\beta = 0.3$ condition, where they are already quite high at the start of the session.[12] Figure 4 makes it clear that to the extent there is learning, it is always in the direction consistent with the theory — participants learn to select the option that results in the higher expected profit.

Turning to Hypotheses 1B (point predictions for Incumbent Full treatments), Table 4 presents information about incumbent winning rates and contract prices. Contract prices in Table 4 give a sense of what the winning suppliers are paid on average. We first compare winning rates and contract prices to their theoretical benchmarks (Table 2). In the $K = 2$ condition winning rates are higher than they should be in theory, and in the $K = 20$ condition winning rates are not different from their theoretical benchmarks. Contract prices are significantly lower than the theory predicts, with the exception of the $K = 20|\beta = 0.7$ treatment. We can conclude that bidding behavior is, on average, more aggressive than the theory predicts. So, as in the Incumbent Restricted treatments, we can reject point predictions of Hypothesis 1.

[12]We measured the learning trends by estimating a logit model (with random effects for individuals), with the dependent variable 1 when the $b = R$ option was chosen, and independent variable round number. The sign and significance of the coefficient on the round number measure the learning trend.

	Winning Rates			Contract Prices		
	$\beta = 0.3$	$\beta = 0.7$	H_o : the effect of β	$\beta = 0.3$	$\beta = 0.7$	H_o : the effect of β
$K = 2$	0.096* (0.103)	0.298* (0.143)	$p < 0.0001$	105.69* (2.71)	90.71* (3.50)	$p < 0.0001$
$K = 20$	0.670 (0.148)	0.555 (0.139)	$p = 0.0114$	98.87* (4.45)	91.93 (5.00)	$p < 0.0001$
H_o : the effect of K	$N = 21$ $p < 0.0001$	$N = 22$ $p < 0.0001$		$p = 0.0002$	$p = 0.2310$	

*H_o : Realized amount = theoretical amount is rejected ($p < 0.01$).

Table 4: Realized winning rates and contracts prices in the Incumbent Full treatments, under post-auction qualification screening (standard errors in parentheses). Results of the hypothesis tests comparing the average costs.

6.2 Do the theory's qualitative predictions match the data?

Table 3 shows the results of hypothesis tests comparing the average boycott rates across Incumbent Restricted conditions. The effect of x_i is consistent with the theory, supporting Hypothesis 1A: Boycott rates are the same when $x_i = 10$ as when $x_i = 70$ in the $\beta = 0.3|K = 2$ condition, but in the other three cases boycott rates are significantly higher when $x_i = 70$ than they are when $x_i = 10$.

Turning to Hypothesis 2A, we do observe the shifts in the boycott rates that we should observe (the effect of K when $\beta = 0.3$ and $x_i = 10$ and the effect of β when $K = 2$ and $x_i = 10$) but the behavior appears more sensitive to K and β than our theory predicts; we observe several shifts in the boycott rates that are not predicted by our model (e.g., the boycott rates decrease in K when $x_i = 70$ and $\beta = 0.7$). These shifts in the bidding strategies can be traced to the magnitude of the strategies' profit differences; if subjects learn to select the more profitable strategy over time, based on the relative profitability of the different options they face, it is reasonable to assume that subjects learn faster to follow strategies when the benefit from doing so is large. (Refer to Figure 3 for the strategies' profit differences.) Overall, we find support for the trends that are predicted by Hypothesis 2A, and to the extent that unpredicted trends are also observed they all coincide with subjects finding the optimal decision more readily when the profit gain from doing so is larger.

Turning to the Incumbent Full treatments, Table 4 present results of hypotheses tests pertinent to Hypotheses 2B and 2C; we find support for both. Consistent with predictions of Hypothesis 2B, incumbent winning rates are higher when $K = 20$ than when $K = 2$ for both levels of β. Consistent with predictions of Hypothesis 2C, the incumbent's winning rates are higher when $\beta = 0.7$ than

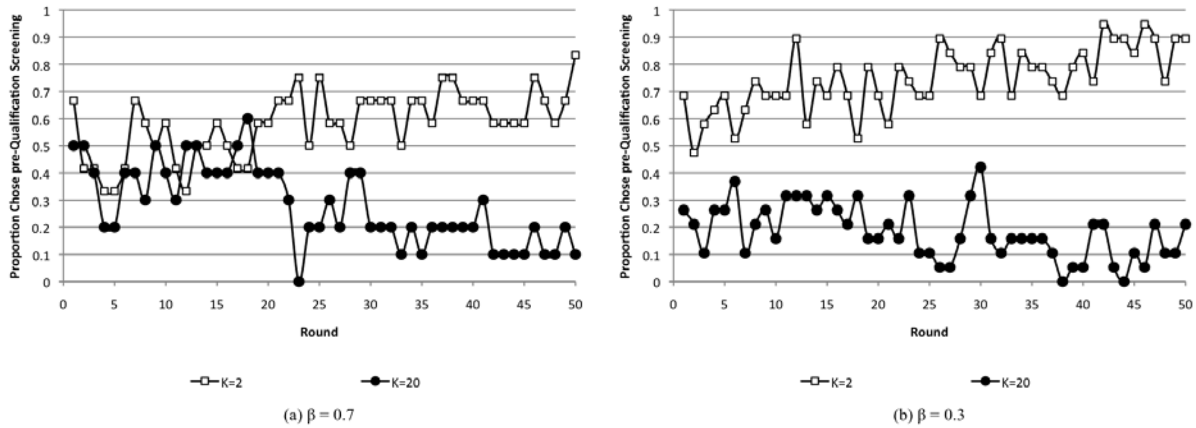


Figure 5: Proportion of pre-auction qualification screening choices over time.

when $\beta = 0.3$ in the $K = 2$ condition. Also consistent with Hypothesis 2C this relationship is reversed in the $K = 20$ condition, where incumbent winning rates are significantly lower when $\beta = 0.7$ than when $\beta = 0.3$.

Turning to Hypothesis 2D, contract prices are significantly higher when $\beta = 0.3$ than when $\beta = 0.7$ for both levels of K, which is consistent with Hypothesis 2D. For the β =0.3 condition contract prices under $K = 2$ are higher than under $K = 20$, which is again consistent with Hypothesis 2D. In the $\beta = 0.7$ condition, however, there is no statistically significant difference between contract prices for $K = 2$ and $K = 20$ which is contrary to Hypothesis 2D, which says that $K = 2$ prices should be higher than $K = 20$ prices.

6.3 Do buyers behave as predicted when facing automated incumbents?

Figure 5 shows the proportion of pre-auction qualification screening choices over time. It is clear from the figure that buyer behavior is qualitatively consistent with Hypothesis 3A. In the $\beta = 0.7$ condition there appears to be some uncertainty at the beginning of the session, but buyers learn over time to select the cost-minimizing option; by the end of the session, buyers in the $K = 2$ condition mostly select pre-auction qualification screening, while buyers in the $K = 20$ condition mostly select post-auction qualification screening. The results are even clearer in the $\beta = 0.3$ condition, where the behavior is already fairly consistent with theoretical benchmarks at the start of the session.[13]

We present a more formal analysis by fitting a logit model, with the dependent variable that is 1 when the pre-auction qualification screening is selected, and 0 when the post-auction qualification screening is selected. The dependent variables are listed in Table 5; $HighK = 1$ in the $K = 20$

[13] Incumbent bidding strategies are simpler in the $\beta = 0.3$ condition than they are in the $\beta = 0.7$ condition. For example, in the $\beta = 0.3|K = 2$ treatment, with post-qualification incumbents always bid 110, and this is what we told the buyer participants.

Independent Variables	$\beta = 0.7$	$\beta = 0.3$
Constant	-0.501 (0.5599)	0.367 (0.3084)
$(1 - HighK) \times Round$	0.031** (0.0072)	0.036** (0.0059)
$HighK$	0.151 (0.8361)	-1.662** (0.4499)
$HighK \times Round$	-0.048** (0.0087)	-0.028** (0.0066)
Log Likelihood	-546.73	-858.17

Table 5: Logit estimates of the buyer behavior; the dependent variable is 1 if pre-auction qualification screening was selected, and 0 if post-auction qualification screening was selected.

condition and 0 otherwise, and *Round* is simply the decision number 1 to 50.

This logit model tells the same story as Figure 5. In the $\beta = 0.7$ condition, buyers behave the same way for both levels of K at the start of the session, as evidenced by the insignificant $HighK$ coefficient. However, in the $K = 2$ condition buyers learn to select pre-auction qualification screening over time, as evidenced by the positive and significant $(1 - HighK) \times Round$ coefficient. Analogously, in the $K = 20$ condition buyers learn to select post-auction qualification screening over time, as evidenced by the negative and significant $HighK \times Round$ coefficient. Results are very similar in the $\beta = 0.3$ condition, except buyers in the $K = 20$ treatment already tend to select post-auction qualification screening more often even at the beginning of the session, as evidenced by the negative and significant $HighK$ coefficient.

Overall, we conclude that buyers, when faced with suppliers who consistently behave in a way that adheres to our model, also learn to behave consistently with our model's predictions; we find support for Hypothesis 3A.

6.4 Do buyers behave as predicted when facing human incumbents?

The Buyer-Incumbent experiment involves both human players, and we use it to test Hypothesis 3B. Participants played for multiple rounds but were in the same role, either the buyer or the incumbent supplier, for the entire session. We conducted this experiment in the $\beta = 0.7|K = 2$ setting only because that is the setting that was most challenging for incumbents (due to a complicated optimal bidding function) as well as buyers (due to the difficulty in responding to the complexity that resulted from the incumbents' complicated bidding function). We first analyze the behavior of

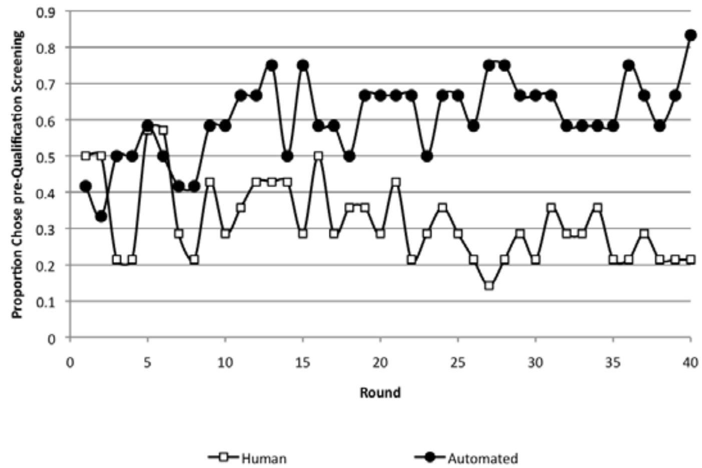


Figure 6: Comparison of buyer behavior when dealing with human and automated incumbents (entrants automated in all treatments). Proportion of pre-auction qualification screening selected over time in the $\beta = 0.7|K = 2$ condition.

human buyers. Figure 6 shows the proportion of buyers who selected the pre-auction qualification screening each round.[14] The figure also shows the same data from the $\beta = 0.7|K = 2$ condition with automated incumbents.

When incumbents bid optimally, buyers maximize their expected profits in the $\beta = 0.7|K = 2$ condition by using the pre-auction qualification screening, and we have already observed that when incumbent suppliers are automated, buyers learn, over time, to make this decision. However, in the treatments with human incumbents, the learning trend is the opposite: Over time, buyers learn to use the post-auction qualification screening. A logit model similar in structure to the models in Table 5 confirms what we clearly see in Figure 6.[15] The data clearly rejects Hypothesis 3B — when they deal with human incumbents, buyer behavior is different from what it is with automated incumbents.

To better understand the reasons for the different buyer behavior when faced with human vs. automated incumbents, we analyze the behavior of the human incumbents. For this purpose, it is useful to examine the total cost that buyers incur. Table 6 summarizes the average total costs borne by the buyer (payment to supplier plus any qualification screening cost) under the pre- and post-qualification screening strategies, for human and automated incumbents, their standard errors,

[14]Since each decision in the experiment with both human players took longer than each decision with automated incumbents, the all-human experiment lasted for 40 instead of 50 rounds. The figure shows all 40 rounds of the all-human experiment and the last 40 rounds of the experiment with automated incumbents.

[15]The dependent variable in the logit model is 1 if the pre-auction qualification screening is selected. The independent variables are as follows: the human incumbent indicator variable, which is not significant; the interaction variable between the human incumbent indication variable and round, which is negative and significant; and the interaction variable between the automated supplier and round, which is positive and significant.

Incumbent	Post-Auction Screening	Pre-Auction Screening	H_o : $TotalCost_{post} = TotalCost_{pre}$
Automated	97.63 (7.68)	91.63 (5.26)	$N = 12$ $p = 0.0261$
Human	92.05 (3.95)	87.431 (10.08)	$N = 14$ $p = 0.0621$
H_o : $TotalCost_{Automated} = TotalCost_{Human}$	$p = 0.0254$	$p = 0.2233$	

Table 6: Buyers' average total (payment plus qualification screening) cost, standard errors in parentheses, $\beta = 0.7|K = 2$.

and the results of hypothesis tests comparing average total costs.

The key observation is that while the buyer's average total cost under the pre-auction qualification screening strategy is not significantly different for human and automated incumbents (in other words, human incumbents' bidding behavior results in essentially the same buyer costs as the optimal incumbent behavior) the buyer's average total cost under the post-auction qualification screening is significantly lower with human incumbents than with automated incumbents (this is another piece of evidence against part of Hypothesis 1B). That is, human incumbents bid more aggressively than the optimal bidding strategy dictates. We already made the same observation when testing Hypothesis 1B in our analysis of human incumbent behavior under the post-auction qualification screening when faced with automated buyers (the Incumbent Full treatments) — average contract prices were consistently lower than they should have been in theory. These differences in suppliers' bidding behavior explain buyer behavior. When incumbents are automated, pre-auction qualification screening results in significantly higher profits, so buyers learn to use it. When incumbents are human, pre-auction qualification screening results in profits that are only slightly higher than profits from the post-auction qualification screening (and differences are only weakly significant), so there is no reason to expect buyers to strongly prefer one decision over the other.

6.5 Discussion of Experimental Results

We summarize our experimental results specifically as they pertain to our three research hypotheses. The relationship between cost x_i and bid is approximately consistent with the form of the optimal bid function — incumbents tend to boycott auctions when their costs are high and they tend to compete to win when their costs are low. They also learn to follow the optimal strategy better over time. Those findings support Hypothesis 1. However, our paper's main behavioral conclusion is that, contrary to Hypothesis 1, incumbents generally bid more aggressively than they should. We observe this overly aggressive behavior in the Incumbent Full setting, where incumbents tend to win more often than they should in theory, and contract prices tend to be lower than they should.

Overly-aggressive bidding is a well-known result in auction experiments, where it has been documented that bidders in sealed-bid first-price auctions with independent private values tend to bid above the risk-neutral Nash equilibrium (see Kagel 1995 for a thorough review). Exact reasons for the overly-aggressive bidding in sealed-bid first-price auctions are not well understood. One long-standing explanation that has been offered is risk aversion (Cox, Smith and Walker 1988) but several studies report results that run counter to the risk aversion explanation (Kagel and Levin 1993, Isaac and James 2000, Englebrecht-Wiggans and Katok 2009). Another explanation that seems to fit a wider range of settings is bidder aversion to regret (Feliz-Ozbay and Ozbay 2007, Engelbrecht-Wiggans and Katok 2007, 2008). While determining the exact reasons for overly-aggressive bidding is beyond the scope of our paper, we note that it is tenable that we would observe such behavior in our setting because the incumbent's problem under the post-auction qualification screening has the flavor of a sealed-bid auction, and consequently, overly aggressive bidding in our setting is likely due to the same reasons as overly aggressive bidding in sealed-bid first-price auctions. In spite of the overly-aggressive bidding by the incumbents, we find that qualitatively our theory works well; generally all parts of Hypotheses 2 are supported.

The buyer behavior is consistent with Hypothesis 3A — buyers tend to select pre-auction qualification screening when $K = 2$ and post-auction qualification screening when $K = 20$, and both of these tendencies are independent of β. We also find an interesting interaction between overly-aggressively bidding by incumbents and the decision to use post-auction qualification screening by the buyers. In a setting with $K = 2|\beta = 0.7$ buyers should select pre-auction qualification screening, and indeed, when incumbents are automated and programmed to bid optimally, buyers learn to do this. However, when incumbents are human, they bid more aggressively than they should under post-auction qualification screening, while under the pre-auction qualification screening they bid close to what the theory predicts. This causes post-auction qualification screening to be more attractive to buyers, and consequently the buyers (when facing human incumbents) tend to choose post-auction qualification screening more than they should.

7. Conclusions

Buyers often wish to leverage entrant suppliers against incumbent suppliers in order to reduce spend. However, incumbent suppliers might be reluctant — or even unwilling — to bid aggressively against an entrant if they suspect that the entrant is possibly unqualified for the contract. We analyze this with a two-pronged approach. First we model this phenomenon in the context of an open-descending price-only supply auction (which are commonly used in practice) and derive the incumbent's optimal bidding strategy. Next, we lab-test our model's theoretical predictions. In addition to providing the first equilibrium analyses for bidding behavior in reverse open-descending, price-only auctions with possibly unqualified bidders, in testing these predictions in the lab the present paper also is the first experimental study of auctions with possibly unqualified bidders.

Addressing *research question 1* (incumbent's optimal bidding strategy under post-qualification), we show that in an effort to preserve their profit margin, high- and medium-cost incumbents optimally let the entrant win the auction, sometimes even "boycotting" the auction by dropping out at the reserve price. This holds under quite general assumptions, and is strikingly different from the "bid-to-cost" strategy that would be dominant if all suppliers were fully qualified. The implication is that operational issues surrounding the auction (i.e., supplier qualification) can be as important for generating price concessions as the notion of an auction itself.

Addressing *research question 2* (sensitivity of the incumbent's bidding strategy), we find that the incumbent gains an advantage and is more likely to win the auction when it is costlier for the buyer to perform post-qualification screening on the entrant. However, the incumbent is more likely to drop out of the auction early when the reserve price is large; thus, high profit potential for the incumbent may lead him to short-circuit the auction rather than compete harder to retain the contract. Our theory also predicts that the incumbent may bid more or less aggressively in response to a higher probability that the entrant would survive qualification screening, meaning that a tougher competitor — namely an entrant that is more likely to be qualified — might actually forestall competition. In spite of the fact that post-qualification can cause the incumbent to hold back on bidding, if it is very expensive to qualify the entrant, the buyer prefers using post-qualification, addressing *research question 3* (buyer's preference between pre- and post-qualification).

Laboratory experiments testing our theory's predictions (*research question 4*) reveal that, consistent with our theoretical results, post-qualification does indeed cause incumbents to hold back on bidding and in general the theory's qualitative predictions (sensitivity of the incumbent's bidding strategy to contract and entrant characteristics) stand up quite well in the lab. Another important finding is that incumbents tend to bid more aggressively than theory predicts under post-qualification. A managerial implication is that this tendency may make buyers in practice more willing to go ahead an pit incumbents against possibly unqualified entrants, and indeed we found evidence of this in our experiments with human buyers and human incumbents: Human buyers reacted to overly aggressive human incumbents by choosing post-qualification more often than when they faced automated incumbents programmed to bid optimally.

Our analytical and experimental analyses were enabled by stylized modeling to capture the problem's salient features. Several avenues for future work are possible. One such avenue would include multiple entrant suppliers and multiple incumbent suppliers. While the analysis would become much more tedious due to complex information updating as the auction progresses, we suspect that our main insight would not change: Incumbents, in an attempt to retain their profit margin, would hold back on bidding to the extent that they perceive themselves as facing low-cost competitors who are likely unqualified. Since we focused this paper on the open-descending auction format, another extension would be to analyze the other format commonly used in practice, the first-price sealed-bid auction (Jap 2002). Our main insights extend to this setting: One can show that the incumbent's best response bid function follows a threshold structure in which low-cost incumbents bid aggressively but higher-cost incumbents hold back on bidding or even boycott. However, solving for the equilibrium bids would be much more challenging — bidder qualification issues aside, first-price sealed-bid equilibria with asymmetric bidders have been solved for just a few very special cases (see Maskin and Riley 2000). In summary, our paper reveals the importance of supplier qualification issues in industrial procurement auctions, and has the potential to help industrial buyers make better decisions about how to include entrants in procurement auctions.

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Appendix

A. Proof of Theorem 1

A.1 Existence of static bid-down-to level

During the auction, the price continuously decreases and the incumbent must dynamically decide whether or not to drop out. At the outset of the auction the incumbent could make the stay in/drop out decision he would make at any price, write down this strategy and simply follow it during the auction. Doing so does not sacrifice optimality, because all the actionable information revealed during the auction is subsumed by the price of the auction. Thus, the incumbent's optimal strategy is characterized by a static bid-down-to level chosen at the outset of the auction. Existence of an optimal bid-down-to level is guaranteed because the incumbent optimizes a continuous objective function over a compact set. In particular, the static optimal bid-down-to level $\underline{p}(x_i)$ is such that $\underline{p}(x_i) = \underset{t\in[\max\{x_i,\frac{K}{\beta}\},R]}{\arg\max} \Pi(t)$, where

$$\Pi(t) \triangleq \int_t^R U(y-x_i)f_e(y-\frac{K}{\beta})dy+F_e(t-\frac{K}{\beta})[(1-\beta)U(t-x_i)+\beta U(0)]+U(R-x_i)[1-F_e(R-\frac{K}{\beta})]. \quad (3)$$

The decision set is $t \in [\max\{x_i, \frac{K}{\beta}\}, R]$ because the auction price can never be outside the set $[\frac{K}{\beta}, R]$ and bidding below the true cost x_i can never be profitable for the incumbent. In the incumbent's expected utility as a function of the chosen bid-down-to level t, $\Pi(t)$, the first term $\int_t^R U(y - x_i)f_e(y - \frac{K}{\beta})dy$ corresponds to the cases in which the incumbent wins the auction outright because the entrant drops out at $y \in (t, R)$; the second term $F_e(t-\frac{K}{\beta})[(1-\beta)U(t-x_i)+\beta U(0)]$ corresponds to the cases in which the entrant wins the auction; and the last term $U(R - x_i)[1 - F_e(R - \frac{K}{\beta})]$ corresponds to the cases in which the entrant loses because his effective cost is above the reserve price.

A.2 Existence of thresholds x_W and x_B

First, we prove that $\underline{p}(x_i)$ increases in x_i; next, we show the existence and uniqueness of x_B and x_W; last, we show that $\underline{p}(x_i)$ strictly increases for $x_W < x_i < x_B$.

Monotonicity of $\underline{p}(x_i)$. Note that

$$\begin{aligned}\frac{d\Pi(t)}{dt} &= F_e(t-\frac{K}{\beta})(1-\beta)U'(t-x_i) - \beta f_e(t-\frac{K}{\beta})[U(t-x_i)-U(0)],\\ &= F_e(t-\frac{K}{\beta})(1-\beta)U'(t-x_i)\left[1-\frac{\beta}{1-\beta}\frac{f_e(t-\frac{K}{\beta})}{F_e(t-\frac{K}{\beta})}\frac{U(t-x_i)-U(0)}{U'(t-x_i)}\right], \quad (4)\end{aligned}$$

which strictly increases in x_i (i.e., we have $\frac{\partial^2\Pi(t)}{\partial x_i \partial t} > 0$) because $U'(t-x_i)$ increases in x_i (i.e., because $U(\cdot)$ is concave) and $[U(t - x_i) - U(0)]$ strictly decreases in x_i. Since $\frac{d\Pi(t)}{dt}$ strictly increases in x_i, we have that $\underline{p}(x_i)$ increases in x_i and, moreover, strictly increases when $\max\{x_i, \frac{K}{\beta}\} < \underline{p}(x_i) < R$. To see this, suppose $\underline{p}(x_i)$ does not increase in x_i; then there must exist $x_i^{(1)} < x_i^{(2)}$ such that $\underline{p}(x_i^{(1)}) > \underline{p}(x_i^{(2)})$. With a temporary abuse of notation, let $\Pi(t; x_i)$ denote the incumbent's expected utility if the incumbent's cost is x_i and it chooses t as the bid-down-to level. On one hand, by the definition of $\underline{p}(x_i^{(1)})$ and $\underline{p}(x_i^{(2)})$, we have

$$\Pi(\underline{p}(x_i^{(1)}); x_i^{(1)}) - \Pi(\underline{p}(x_i^{(2)}); x_i^{(1)}) \geq 0, \text{ and } \Pi(\underline{p}(x_i^{(2)}); x_i^{(2)}) - \Pi(\underline{p}(x_i^{(1)}); x_i^{(2)}) \geq 0. \tag{5}$$

On the other hand, we notice that

$$\begin{aligned} &\Pi(\underline{p}(x_i^{(1)}); x_i^{(1)}) - \Pi(\underline{p}(x_i^{(2)}); x_i^{(1)}) + \Pi(\underline{p}(x_i^{(2)}); x_i^{(2)}) - \Pi(\underline{p}(x_i^{(1)}); x_i^{(2)}) \\ &= \int_{\underline{p}(x_i^{(2)})}^{\underline{p}(x_i^{(1)})} \frac{d\Pi(t; x_i^{(1)})}{dt} dt - \int_{\underline{p}(x_i^{(2)})}^{\underline{p}(x_i^{(1)})} \frac{d\Pi(t; x_i^{(2)})}{dt} dt < 0 \end{aligned} \tag{6}$$

where the inequality holds because $\underline{p}(x_i^{(1)}) > \underline{p}(x_i^{(2)})$ and $\frac{d\Pi(t;x_i^{(2)})}{dt} > \frac{d\Pi(t;x_i^{(1)})}{dt}$. However, (5) contradicts (6), which implies that $\underline{p}(x_i)$ must increase in x_i. Moreover, if $\max\{x_i^{(1)}, \frac{K}{\beta}\} < \underline{p}(x_i^{(1)}) < R$, then it must be $\frac{d\Pi(t;x_i^{(1)})}{dt}|_{t=\underline{p}(x_i^{(1)})} = 0$; this implies that $\underline{p}(x_i^{(1)}) < \underline{p}(x_i^{(2)})$ — otherwise, if $\underline{p}(x_i^{(1)}) = \underline{p}(x_i^{(2)})$, we have $\frac{d\Pi(t;x_i^{(2)})}{dt}|_{t=\underline{p}(x_i^{(2)})} > \frac{d\Pi(t;x_i^{(1)})}{dt}|_{t=\underline{p}(x_i^{(2)})} = \frac{d\Pi(t;x_i^{(1)})}{dt}|_{t=\underline{p}(x_i^{(1)})} = 0$, which contradicts the optimality of $\underline{p}(x_i^{(2)})$.

Existence and uniqueness of x_B. Given any $x_i \in (\frac{K}{\beta}, R)$, we have $\frac{d\Pi(t)}{dt}|_{t\downarrow x_i} = F_e(x_i - \frac{K}{\beta})(1 - \beta)U'(0) > 0$, which has the following two implications.

Implication 1: $\underline{p}(x_i) > x_i$ for all $x_i \in (\frac{K}{\beta}, R)$.

Implication 2: There exists some $\delta_R > 0$ such that $\frac{d\Pi(t)}{dt} > 0$ for all $t \in [x_i, R)$ and all $x_i \in (R - \delta_R, R)$. To see Implication 2 is true, note that, for any $\delta > 0$, for all $x_i \in (R - \delta, R)$ and all $t \in [x_i, R)$, equation (4) and the fact that $U'(\cdot)$ decreases together imply that $\frac{d\Pi(t)}{dt}$ is greater than $F_e(R - \delta - \frac{K}{\beta})(1 - \beta)U'(\delta) - \beta f_e(t - \frac{K}{\beta})[U(\delta) - U(0)]$, which approaches $F_e(R - \frac{K}{\beta})(1 - \beta)U'(0) > 0$ as δ approaches zero.

Implication 2 and the fact that $\underline{p}(x_i)$ is increasing together imply that there exists a unique threshold $x_B < R$ such that $\underline{p}(x_i) = R$ if and only if $x_i \geq x_B$.

Existence and uniqueness of x_W. The fact that $U(\cdot)$ is concave implies that $\frac{U(t-x_i)-U(0)}{U'(t-x_i)}$ increases and goes to infinity as x_i decreases and goes to negative infinity, which in turn implies that (per (4)) there exists some $\delta_W > -\infty$ such that $\frac{d\Pi(t)}{dt} < 0$ for all $t \in [\frac{K}{\beta}, 1 + \frac{K}{\beta}]$ when $x_i < \delta_W$. Thus, when $R \leq 1 + \frac{K}{\beta}$, there exists a largest threshold x_W such that $\underline{p}(x_i) = \max\{x_i, \frac{K}{\beta}\}$ for all $x_i \leq x_W$. This result is also true when $R > 1 + \frac{K}{\beta}$. Note that (per (4)) $\frac{d\Pi(t)}{dt} > 0$ for all $t \in [1 + \frac{K}{\beta}, R]$ when $R > 1 + \frac{K}{\beta}$. Thus, either $\underline{p}(x_i) = \max\{x_i, \frac{K}{\beta}\}$, or $\underline{p}(x_i) = R$. It is easy to check that $\Pi(\max\{x_i, \frac{K}{\beta}\}) > \Pi(R)$ when x_i is small enough.

Implication 1 implies that $x_W \leq \frac{K}{\beta}$ and hence $\underline{p}(x_i) = \frac{K}{\beta}$ for all $x_i \leq x_W$. The fact that $\underline{p}(x_i)$ is increasing implies $x_W \leq x_R < R$ and that $\underline{p}(x_i) = \frac{K}{\beta}$ only if $x_i \leq x_W$, i.e., x_W is unique.

Property of $\underline{p}(x_i)$ for $x_W < x_i < x_B$. When $x_W < x_i < x_B$, we have $\frac{K}{\beta} < \underline{p}(x_i) < R$, which is implied by the existence and the uniqueness of the thresholds x_W and x_B. We have $\underline{p}(x_i) > x_i$, which holds for $x_i > \frac{K}{\beta}$ because of Implication 1 and holds for $x_W < x_i \leq \frac{K}{\beta}$ (when $x_W < \frac{K}{\beta}$) because $\underline{p}(x_i) > \frac{K}{\beta} \geq x_i$. Finally, that $\underline{p}(x_i)$ strictly increases when $x_W < x_i < x_B$ because we have proved that $\underline{p}(x_i)$ strictly increases when $\max\{x_i, \frac{K}{\beta}\} < \underline{p}(x_i) < R$.

B. Proof of Theorem 2

Effect of R. Consider any $R_1 < R_2$ and any $t_1 < t_2 < R_1$. For a given x_i, when $R = R_1$ we have $\Pi(t_2) - \Pi(t_1) = \int_{t_1}^{t_2} \frac{d\Pi(t)}{dt} dt$, which does not change as R_1 increases because $\frac{d\Pi(t)}{dt}$ does not change with R per (4). This implies that the optimal bid-down-to levels when $R = R_1$ and R_2, denoted by $\underline{p}(x_i)|_{R=R_1}$ and $\underline{p}(x_i)|_{R=R_2}$, respectively, should be such that either $\underline{p}(x_i)|_{R=R_2} = \underline{p}(x_i)|_{R=R_1}$ or $\underline{p}(x_i)|_{R=R_2} > R_1 > \underline{p}(x_i)|_{R=R_1}$. Namely, the optimal bid-down-to level increases in R.

Effect of K. We prove it by showing that i) $\underline{p}(x_i)$ decreases in K when $\underline{p}(x_i) > \frac{K}{\beta}$ and that ii) $\underline{p}(x_i)|_{K=\hat{K}} = \frac{\hat{K}}{\beta}$ implies $\underline{p}(x_i)|_K = \frac{K}{\beta}$ for all $K \geq \hat{K}$.

If i) is not true, then there must exist $K_1 < K_2$ and $t_1 < t_2$ such that either (or both) of the following two cases must be true.
Case i-1: $\frac{d\Pi(t;K_1)}{dt}|_{t=t_1} = 0$, $\frac{d\Pi(t;K_2)}{dt}|_{t=t_2} = 0$, $\Pi(t_1;K_1) > \Pi(t_2;K_1)$, and $\Pi(t_1;K_2) < \Pi(t_2;K_2)$;
Case i-2: $\frac{d\Pi(t;K_1)}{dt}|_{t=t_1} = 0$, $t_2 = R$, $\Pi(t_1;K_1) > \Pi(t_2;K_1)$, and $\Pi(t_1;K_2) < \Pi(t_2;K_2)$.

If ii) is not true, then there must exist $K_1 < K_2$ and $t_1 < t_2$ such that either (or both) of the following two cases must be true.
Case ii-1: $t_1 = \frac{K_1}{\beta}$, $\frac{d\Pi(t;K_2)}{dt}|_{t=t_2} = 0$, $\Pi(t_1;K_1) > \Pi(t_2;K_1)$, and $\Pi(t_1;K_2) < \Pi(t_2;K_2)$;
Case ii-2: $t_1 = \frac{K_1}{\beta}$, $t_2 = R$, $\Pi(t_1;K_1) > \Pi(t_2;K_1)$, and $\Pi(t_1;K_2) < \Pi(t_2;K_2)$.

Note that any of the four cases must imply that

$$[\Pi(t_1;K_2) - \Pi(t_1;K_1)] - [\Pi(t_2;K_2) - \Pi(t_2;K_1)] < 0. \tag{7}$$

Since the left hand side of (7) equals $\int_{K_1}^{K_2} [\frac{\partial\Pi(t_1;K)}{\partial K} - \frac{\partial\Pi(t_2;K)}{\partial K}] dK$, it suffices to draw a contradiction to prove i) and ii) by showing that $\frac{\partial\Pi(t_1;K)}{\partial K} \geq \frac{\partial\Pi(t_2;K)}{\partial K}$ for all $K \in [K_1, K_2]$ under all the four cases. We can rewrite (3) as $\Pi(t) = [1 - F_e(R - \frac{K}{\beta})]U(R - x_i) + \int_{t-\frac{K}{\beta}}^{R-\frac{K}{\beta}} U(z + \frac{K}{\beta} - x_i) f_e(z) dz + F_e(t - \frac{K}{\beta})[(1-\beta)U(t - x_i) + \beta U(0)]$, and thus we have

$$\frac{\partial\Pi(t;K)}{\partial K} = f_e(t - \frac{K}{\beta})[U(t - x_i) - U(0)] + \frac{1}{\beta}\int_{t-\frac{K}{\beta}}^{R-\frac{K}{\beta}} U'(z + \frac{K}{\beta} - x_i) f_e(z) dz. \tag{8}$$

To prove $\frac{\partial \Pi(t_1;K)}{\partial K} \geq \frac{\partial \Pi(t_2;K)}{\partial K}$ for all $K \in [K_1, K_2]$ under all the four cases, we show that

$$
\begin{aligned}
& \frac{\partial \Pi(t_1;K)}{\partial K} - \frac{\partial \Pi(t_2;K)}{\partial K} \\
= \; & f_e(t_1 - \frac{K}{\beta})[U(t_1 - x_i) - U(0)] - f_e(t_2 - \frac{K}{\beta})[U(t_2 - x_i) - U(0)] + \frac{1}{\beta}\int_{t_1-\frac{K}{\beta}}^{t_2-\frac{K}{\beta}} U'(z + \frac{K}{\beta} - x_i) f_e(z) dz \\
\geq \; & \frac{1-\beta}{\beta}[F_e(t_1 - \frac{K}{\beta})U'(t_1 - x_i) - F_e(t_2 - \frac{K}{\beta})U'(t_2 - x_i)] + \frac{1}{\beta}\int_{t_1-\frac{K}{\beta}}^{t_2-\frac{K}{\beta}} U'(z + \frac{K}{\beta} - x_i) f_e(z) dz \\
\geq \; & -\frac{1-\beta}{\beta}\int_{t_1-\frac{K}{\beta}}^{t_2-\frac{K}{\beta}} U'(z + \frac{K}{\beta} - x_i) f_e(z) dz + \frac{1}{\beta}\int_{t_1-\frac{K}{\beta}}^{t_2-\frac{K}{\beta}} U'(z + \frac{K}{\beta} - x_i) f_e(z) dz \geq 0.
\end{aligned}
$$

The first equality is by (8). The second inequality is by integration by parts and concavity of $U(\cdot)$. We explain the first inequality for case i-1; the other cases are similar. Note that equation (4) and the fact that $\frac{d\Pi(t;K_1)}{dt}|_{t=t_1} = 0$ implies that $f_e(t_1 - \frac{K_1}{\beta})[U(t_1 - x_i) - U(0)] = \frac{1-\beta}{\beta}F_e(t_1 - \frac{K_1}{\beta})U'(t_1 - x_i)$, which, together with the assumption that $\frac{F_e(x_e)}{f_e(x_e)}$ increases in x_e, implies that $f_e(t_1 - \frac{K}{\beta})[U(t_1 - x_i) - U(0)] \geq \frac{1-\beta}{\beta}F_e(t_1 - \frac{K}{\beta})U'(t_1 - x_i)$ for all $K \in [K_1, K_2]$. Similarly, equation (4), $\frac{d\Pi(t;K_2)}{dt}|_{t=t_2} = 0$, and $\frac{F_e(x_e)}{f_e(x_e)}$ increasing in x_e together imply $f_e(t_2 - \frac{K}{\beta})[U(t_2 - x_i) - U(0)] \leq \frac{1-\beta}{\beta}F_e(t_2 - \frac{K}{\beta})U'(t_2 - x_i)$ for all $K \in [K_1, K_2]$.

Effect of β. One can numerically check that the optimal bid-down-to level is not monotone in β, for example, when $l = 0$, $R = 1$, $K = 0.24$, $\beta \in [0.4, 0.8]$, $F_e \sim U[0, 1]$, and the incumbent is risk-neutral.

C. Proof of Theorem 3

For any $0 < \beta < 1$, the expected total cost under pre-qualification linearly increases in qualification cost K (per (1)), whereas the expected total cost under post-qualification is bounded by R from above (per (2)). Thus, post-qualification yields a lower expected total cost when K is large enough; namely, it must exist a $\underline{K} \geq 0$ such that the buyer prefers post-qualification whenever $K > \underline{K}$.

D. Proof of Proposition 1

When $U(t - x_i) = t - x_i$ and $F_e \sim U[0, 1]$, per (3) and (4), we have for $t \in [\frac{K}{\beta}, R]$,

$$
\Pi(t) = \begin{cases} \int_t^R (y - x_i)dy + (t - \frac{K}{\beta})(1-\beta)(t - x_i) + (R - x_i)(1 + \frac{K}{\beta} - R), & \text{if } 1 + \frac{K}{\beta} \geq R, \\ \int_t^{1+\frac{K}{\beta}} (y - x_i)dy + (t - \frac{K}{\beta})(1-\beta)(t - x_i), & \text{if } 1 + \frac{K}{\beta} < R \text{ and } t \leq 1 + \frac{K}{\beta}, \\ (1-\beta)(t - x_i), & \text{if } 1 + \frac{K}{\beta} < R \text{ and } t > 1 + \frac{K}{\beta}, \end{cases}
$$

and
$$
\frac{d\Pi(t)}{dt} = \begin{cases} \beta x_i + (1-2\beta)t - \frac{K(1-\beta)}{\beta}, & \text{if } 1 + \frac{K}{\beta} \geq R, \text{ or if } 1 + \frac{K}{\beta} < R \text{ and } t \leq 1 + \frac{K}{\beta}, \\ 1 - \beta, & \text{if } 1 + \frac{K}{\beta} < R \text{ and } t > 1 + \frac{K}{\beta}. \end{cases}
$$

Let $t^*(x_i) \equiv \frac{\beta x_i}{2\beta - 1} - \frac{(1-\beta)K}{\beta(2\beta-1)}$, that is, $\frac{d\Pi(t)}{dt}|_{t=t^*(x_i)} = 0$ if $1 + \frac{K}{\beta} \geq R$, or if $1 + \frac{K}{\beta} < R$ and $t \leq 1 + \frac{K}{\beta}$.

Cases with $R \leq 1 + \frac{K}{\beta}$. Note that $\Pi(t)$ is convex in t when $0 < \beta < \frac{1}{2}$, is linear in t when $\beta = \frac{1}{2}$, and is concave in t when $\frac{1}{2} < \beta < 1$.

- When $0 < \beta \leq \frac{1}{2}$, the convexity (actually, linearity when $\beta = 1/2$) of $\Pi(t)$ implies that the optimal solution $\underline{p}(x_i)$ equals either $t = \max\{x_i, \frac{K}{\beta}\}$ or $t = R$; this together with the fact that $\underline{p}(x_i) > x_i$ for all $x_i \in (\frac{K}{\beta}, R)$ (which was proved in the proof of Theorem 1) implies that the optimal solution $\underline{p}(x_i)$ equals either $t = \frac{K}{\beta}$ or $t = R$. Note that $\Pi(R) = (R - \frac{K}{\beta})(1-\beta)(R - x_i) + (R - x_i)(1 + \frac{K}{\beta} - R)$ and $\Pi(\frac{K}{\beta}) = \int_{\frac{K}{\beta}}^{R} (y - x_i)dy + (R - x_i)(1 + \frac{K}{\beta} - R)$. It is easy to check that $\Pi(R) > \Pi(\frac{K}{\beta})$ if and only if $x_i > \frac{K}{2\beta^2} - \frac{R}{2\beta} + R$. Namely, $x_W = x_B = \frac{K}{2\beta^2} - \frac{R}{2\beta} + R$.

- When $\frac{1}{2} < \beta < 1$, the concavity of $\Pi(t)$ implies that $\underline{p}(x_i) = \max\{\frac{K}{\beta}, \min\{t^*(x_i), R\}\}$. That is, $\underline{p}(x_i) = \frac{K}{\beta}$ if $x_i \leq \frac{K}{\beta}$ (because $t^*(x_i) \leq \frac{K}{\beta}$ when $x_i \leq \frac{K}{\beta}$), $\underline{p}(x_i) = R$ if $x_i \geq \frac{2\beta-1}{\beta}R + \frac{(1-\beta)K}{\beta^2}$ (because $t^*(x_i) \geq R$ when $x_i \geq \frac{2\beta-1}{\beta}R + \frac{(1-\beta)K}{\beta^2}$), and $\underline{p}(x_i) = t^*(x_i)$ if $\frac{K}{\beta} < x_i < \frac{2\beta-1}{\beta}R + \frac{(1-\beta)K}{\beta^2}$. Namely, $x_W = \frac{K}{\beta}$ and $x_B = \frac{2\beta-1}{\beta}R + \frac{(1-\beta)K}{\beta^2}$.

Cases with $R > 1 + \frac{K}{\beta}$.

- When $0 < \beta \leq \frac{1}{2}$, the convexity of $\Pi(t)$ over $t \in [\frac{K}{\beta}, 1 + \frac{K}{\beta}]$ and the fact that $\Pi(t)$ increases when $t \in [1 + \frac{K}{\beta}, R]$ together imply that $\Pi(t)$ is quasiconvex, and hence imply that the optimal solution $\underline{p}(x_i)$ equals either $t = \max\{x_i, \frac{K}{\beta}\}$ or $t = R$. Again, the fact that $\underline{p}(x_i) > x_i$ for all $x_i \in (\frac{K}{\beta}, R)$ (which was proved in the proof of Theorem 1) further implies that the optimal solution $\underline{p}(x_i)$ equals either $t = \frac{K}{\beta}$ or $t = R$. It is easy to check that $\Pi(R) = (1-\beta)(R - x_i)$ and $\Pi(\frac{K}{\beta}) = \frac{1}{2} + \frac{K}{\beta} - x_i$, and that $\Pi(R) > \Pi(\frac{K}{\beta})$ if and only if $x_i > \frac{K}{2\beta^2} + \frac{1}{2\beta} - \frac{R}{\beta} + R$. Namely, $x_W = x_B = \frac{K}{2\beta^2} + \frac{1}{2\beta} - \frac{R}{\beta} + R$.

- When $\frac{1}{2} < \beta < 1$, the concavity of $\Pi(t)$ over $t \in [\frac{K}{\beta}, 1 + \frac{K}{\beta}]$ and the fact that $\Pi(t)$ increases when $t \in [1 + \frac{K}{\beta}, R]$ together imply that $\underline{p}(x_i)$ equals either R or $\hat{t}(x_i) \equiv \max\{\frac{K}{\beta}, \min\{t^*(x_i), 1 + \frac{K}{\beta}\}\}$. Note that $\Pi(R) = (1-\beta)(R - x_i)$ and $\Pi(\hat{t}(x_i)) = \int_{\hat{t}(x_i)}^{1+\frac{K}{\beta}} (y - x_i)dy + [\hat{t}(x_i) - \frac{K}{\beta}](1 - \beta)[\hat{t}(x_i) - x_i]$. Thus, for $\frac{2\beta-1}{\beta} + \frac{K}{\beta} \leq x_i < R$, we have $t^*(x_i) \geq 1 + \frac{K}{\beta}$, thus $\hat{t}(x_i) = 1 + \frac{K}{\beta}$ and hence $\Pi(\hat{t}(x_i)) = (1-\beta)[1 + \frac{K}{\beta} - x_i] < \Pi(R)$; for $x_i < \frac{K}{\beta}$, we have $t^*(x_i) < \frac{K}{\beta}$, thus $\hat{t}(x_i) = \frac{K}{\beta}$ and hence $\Pi(\hat{t}(x_i)) = \int_{\frac{K}{\beta}}^{1+\frac{K}{\beta}} (y - x_i)dy = \frac{1}{2} + \frac{K}{\beta} - x_i$, which is less than $\Pi(R)$ for x_i small enough. Therefore, the continuity of $\Pi(\hat{t}(x_i))$ implies that there exists a threshold $\hat{x}_i < \frac{2\beta-1}{\beta} + \frac{K}{\beta}$ such that $\Pi(\hat{t}(\hat{x}_i)) = \Pi(R)$ and $\Pi(\hat{t}(x_i)) < \Pi(R)$ if and only if $x_i < \hat{x}_i$; namely, $x_B = \hat{x}_i$. In particular, $\hat{x}_i$ solves

$$\int_{\hat{t}(\hat{x}_i)}^{1+\frac{K}{\beta}} (y - \hat{x}_i)dy + [\hat{t}(\hat{x}_i) - \frac{K}{\beta}](1-\beta)[\hat{t}(\hat{x}_i) - \hat{x}_i] = \Pi(\hat{t}(\hat{x}_i)) = \Pi(R) = (1-\beta)(R - \hat{x}_i);$$

after simplification, we have for given K, β, and R, $x_B = \hat{x}(K, \beta, R)$ is the $x_i \in (-\infty, \frac{2\beta-1}{\beta} +$

$\frac{K}{\beta}$) solving the following equation:

$$0 = \begin{cases} \frac{(\beta x_i - K)^2}{2(2\beta-1)} - \beta x_i + \frac{K}{\beta} + \frac{1}{2} - (1-\beta)R & \text{, if } \frac{K}{\beta} < \ x_i \leq \frac{2\beta-1}{\beta} + \frac{K}{\beta}; \\ -\beta x_i + \frac{K}{\beta} + \frac{1}{2} - (1-\beta)R & \text{, if } x_i \leq \frac{K}{\beta}. \end{cases}$$

Finally, if $\hat{x}(K, \beta, R) > \frac{K}{\beta}$, we have $\underline{p}(x_i) = \hat{t}(x_i)$ for $x_i < \hat{x}(K, \beta, R)$ (i.e., $\underline{p}(x_i) = t^*(x_i)$ for $\frac{K}{\beta} < x_i < \hat{x}(K, \beta, R)$ and $\underline{p}(x_i) = t^*(x_i)$ for $x_i \leq \frac{K}{\beta}$), which implies that $x_W = \frac{K}{\beta}$; if $\hat{x}(K, \beta, R) \leq \frac{K}{\beta}$, we have $\underline{p}(x_i) = \frac{K}{\beta}$, which implies that $x_W = x_B$. To summarize, $x_W = \min\{\frac{K}{\beta}, x_B\}$.

E. Proof of Proposition 2

The probability that the incumbent wins the auction outright equals

$$1 - Prob\left[x_e + \frac{K}{\beta} \leq \underline{p}(x_i)\right] = \int_l^R \max\{1 + \frac{K}{\beta} - \underline{p}(x_i), 0\} dx_i.$$

We first consider the case with $K = 0$.

- When $0 < \beta \leq \frac{1}{2}$, $Prob\left[x_e + \frac{K}{\beta} \leq \underline{p}(x_i)\right]$ equals $\frac{(R-x_B)\min\{R,1\}}{R-l}$. It decreases in β because x_B increases in β. To see this, $\frac{dx_B}{d\beta} = \frac{R}{2\beta^2}$ (if $R \leq 1$) or $-\frac{1}{2\beta^2} + \frac{R}{\beta^2}$ (if $R > 1$), which is positive. Thus, the winning probability increases in β.

- When $\frac{1}{2} < \beta < 1$, $R \leq 1$ and $l < 0$, $Prob\left[x_e + \frac{K}{\beta} \leq \underline{p}(x_i)\right]$ equals $\frac{R}{2(R-l)}(R + R - x_B)$, which equals $\frac{R^2}{2(R-l)\beta}$ (because $x_B = (2 - \frac{1}{\beta})R$) and decreases in β. Thus, the winning probability increases in β.

- When $\frac{1}{2} < \beta < 1$, $R \leq 1$ and $0 < l \leq x_B$, $Prob\left[x_e + \frac{K}{\beta} \leq \underline{p}(x_i)\right]$ equals $[\frac{R}{2}(R + R - x_B) - \frac{l^2}{2}\frac{\beta}{2\beta-1}]\frac{1}{R-l} = [\frac{R^2}{2\beta} - \frac{l^2\beta}{2(2\beta-1)}]\frac{1}{R-l}$, which decreases in β, because its derivative with respect to β equals $[-\frac{R^2}{2\beta^2} + \frac{l^2}{2(2\beta-1)^2}]\frac{1}{R-l}$, which is non-positive because $l \leq x_B = \frac{2\beta-1}{\beta}R$. Thus, the winning probability increases in β.

- When $\frac{1}{2} < \beta < 1$, $R \leq 1$ and $l > x_B$, $Prob\left[x_e + \frac{K}{\beta} \leq \underline{p}(x_i)\right]$ equals R. It is constant as β changes.

- When $\frac{1}{2} < \beta < 1$, $R > 1$, $x_B \leq 0$, the winning probability equals $\max\{x_B - l, 0\}\frac{1}{R-l}$, which increases in β because x_B increases in β. To see that, per Proposition 1 we know if $x_B \leq 0$ then $x_B = \frac{1}{2\beta} - \frac{R}{\beta} + R$, with its first order derivative with respect to β equal to $\frac{R-0.5}{\beta^2} > 0$.

- When $\frac{1}{2} < \beta < 1$, $R > 1$, $x_B > 0$ and $l \leq 0$, the winning probability equals $[(x_B - l) - (x_B)^2\frac{\beta}{2(2\beta-1)}]\frac{1}{R-l}$, equal to $[\frac{1}{2\beta} - \frac{R}{\beta} + R - l]\frac{1}{R-l}$ per Proposition 1, which increases in β.

- When $\frac{1}{2} < \beta < 1$, $R > 1$, $x_B > 0$ and $0 < l \leq x_B$, the winning probability equals $[(x_B - l) - (x_B)^2 \frac{\beta}{2(2\beta-1)} - \frac{l}{2}(1+1-\frac{\beta l}{2\beta-1})]\frac{1}{R-l}$, equal to $[\frac{1}{2\beta} - \frac{R}{\beta} + R - l + \frac{\beta l^2}{2(2\beta-1)}]\frac{1}{R-l}$ (per Proposition 1), with the first order derivative with respect to β equal to $[\frac{2R-1}{2\beta^2} - \frac{l^2}{2(2\beta-1)^2}]\frac{1}{R-l}$, which is positive because $l \leq x_B < \frac{2\beta-1}{\beta}$ per Proposition 1. Hence, the probability increases in β.

- When $\frac{1}{2} < \beta < 1$, $R > 1$, $x_B > 0$ and $l > x_B$, the winning probability equals zero. Thus, the winning probability is constant in β.

We next consider the winning probability as K approaches βR.

- When $0 < \beta \leq \frac{1}{2}$, since $R \leq 1+\frac{K}{\beta}$ we have $x_B = x_W = \frac{K}{2\beta^2} - \frac{R}{2\beta} + R$ and $Prob\left[x_e + \frac{K}{\beta} \leq \underline{p}(x_i)\right] = (R - \frac{K}{\beta})(R - x_B)\frac{1}{R-l}$, which increases in β because $\frac{d(K/\beta)}{d\beta} < 0$ and $\frac{dx_B}{d\beta} = \frac{1}{\beta^2}(\frac{R}{2} - \frac{K}{\beta}) < 0$ when K approaches βR. Thus, the winning probability decreases in β.

- When $\frac{1}{2} < \beta < 1$, since $R \leq 1 + \frac{K}{\beta}$ and $l < \frac{K}{\beta}$ we have $Prob\left[x_e + \frac{K}{\beta} \leq \underline{p}(x_i)\right] = (R - \frac{K}{\beta})^2 \frac{1}{2\beta}\frac{1}{R-l}$, which increases in β when K is close to βR, because its derivative with respect to β equals $\frac{R-\frac{K}{\beta}}{\beta^2}(\frac{K}{\beta} - \frac{1}{2}(R - \frac{K}{\beta}))\frac{1}{R-l}$, which is positive if K is close enough to βR. Thus, the winning probability decreases in β as K approaches βR.