Split-Award Auctions: Insights from Theory and Experiments

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We investigate procurement in a setting in which the buyer is bound by sourcing rules. Sourcing rules may limit the minimum and maximum amounts of business that can be awarded to a single supplier or dictate the minimum number of suppliers who are awarded business, thus necessitating split awards. The buyer announces the splits before the auction, and suppliers bid accordingly. We consider two auction formats: the sealed-bid first price auction, and the open-bid descending price auction. We characterize the suppliers' symmetric equilibrium bidding strategy for both formats and find that the two formats yield the same expected buyer's cost. We characterize the cost of multi-sourcing, showing among other things that it is always costly for the buyer to split its award among more suppliers if the suppliers' costs are regularly distributed, but that doing so can actually reduce the buyer's expected auction payment if the suppliers' costs are not regularly distributed. The results from controlled laboratory experiments, involving human subjects, indicate that expected cost equivalence fails when costs are regularly distributed because suppliers bid more aggressively in the sealed-bid auction. However, with non-regularly distributed costs the sealed-bid prices are actually higher than predicted by theory. Moreover, experimental results indicate that the model does a good job of predicting the relationship between the buyer's average cost and the award splits as well as the cost of multi-sourcing. Importantly, the experiments confirm that when suppliers' costs come from a non-regular distribution, it may be to the buyer's advantage to diversify the supply base more than is strictly necessitated by sourcing rules.
Key words : Auctions, Sourcing rules, Experiments, Multi-sourcing

1. Introduction and Literature Review

Getting a low cost of input is often thought of as the primary goal of a procurement auction. However, in reality the quest for low input costs must be tempered by other, both short- and long-range considerations of the firm. Rather than setting its procurement managers loose to minimize purchase costs, many firms establish guidelines for procurement managers that ensure such short- and long-run considerations are addressed. These "sourcing rules" establish things like the maximum order size that can be allocated to any one supplier (for resilience to supply disruptions), and

the minimum number of suppliers who must be awarded business (to foster long-run health of the suppliers and ensure viable sourcing options in the future). But multisourcing also increases the total purchase cost, as well as administrative costs associated with managing multiple suppliers. Therefore, sourcing rules often also specify a minimum amount of business that can be awarded to any active supplier. Hohner et al. (2003) and Bichler et al. (2006) discuss these and other sourcing rules used in practice for the procurement of materials at Mars incoporated.

One straightforward and transparent approach that buyers often use in practice is to announce, prior to the procurement auction, that the best bidder will be allocated the highest share $x\%$ of the contract, the second-best bidder will be allocated the second-highest share $y\%$ of the contract, and so on. (Of course, the allocation the buyer announces must be consistent with the firm's sourcing rules.) Such split-award auctions is the focus of our paper. They are different from standard winner-take-all auctions in terms of both the bidding strategies of the suppliers, and the auction design perspective of the buyer—the two issues that we examine in this paper.

We consider two auction design decisions that buyers make for split-award auctions: the bidding format (sealed-bid vs. open-bid) and the allocation rules. We investigate an open-bid format that can be used in a split-award setting that is a generalization of the simple open-descending format for the winner-take-all auction (see Chen et al. (2014)). Likewise, the sealed-bid format we analyze is a generalization of the canonical sealed-bid winner-take-all auction; each supplier submits its bid, and the lowest bid receives the largest share, the second-lowest bid receives the second-largest share, etc. We characterize the bidding equilibrium for both formats, and note that in equilibrium the two formats are equivalent in terms of the expected buyer cost. We then use these insights to analyze the problem procurement managers face in deriving the allocation rule to announce that minimizes the expected procurement cost while satisfying the sourcing rules.

A natural conjecture is that since allocating business to high bid suppliers is costly, the buyer would minimize her expected cost by allocating as much as possible (given the sourcing rules) to the supplier with the lowest bid, and then as much as possible to the supplier with the second lowest bid, and so on. We show that such a *greedy* allocation is optimal when the cost distribution is well behaved (i.e., is regular).[1] But surprisingly, we also show that sometimes the optimal allocation is not greedy, for instance, the buyer might want to allocate its business to more than the minimum number of suppliers required by the sourcing rules. In other words, sometimes the buyer can minimize its payment costs by splitting its contract among more suppliers.

We test several of our theoretical predictions using a controlled laboratory experiment with human subjects incentivized with money. We find that bidding behavior in open-bid auctions is

[1] Specifically, a regular distribution is defined as a continuous distribution for which $c + F(c)/f(c)$ is increasing in c.

quite close to theoretical predictions, but in sealed-bid auctions, participants bid more aggressively than they should under the risk-neutral Nash equilibrium for regular cost distributions. As a result, contrary to theoretical predictions, the two bidding formats are not equivalent in terms of their expected cost—the average buyer cost is significantly lower under the sealed-bid format for regularly distributed costs. By contrast, when the cost distribution is non-regular, the participants in the sealed-bid auction bid above the theoretical predictions of the risk-neutral Nash equilibrium. Thus unlike the overly-aggressive bidding that has been repeatedly observed in sealed-bid auctions for regular cost distributions (see Kagel 1995), we find the opposite with a non-regular cost distribution in our experiments. Consistent with the theory, when the cost distribution is well-behaved, the buyer's average cost monotonically increases as the allocation becomes less weighted towards lower-cost bids. We also find, consistent with the theory, that the buyer's average cost could be lower with non-greedy split as compared to greedy split when cost distribution is not well behaved. Overall, we find that the model does a good job of predicting the cost of multi-sourcing for the buyer. Because sourcing rules have to balance the costs and benefits of multi-sourcing, having an analytical model that is able to accurately predict the cost of multi-sourcing can help buyers design effective sourcing rules.

The existing literature has investigated the use of multi-sourcing in auctions for reducing the total procurement cost for the buyer due to several factors such as entry cost, convex production costs, supply risk or supply base maintenance. Klotz and Chatterjee (1995) show that when suppliers face entry costs, a two-way split may outperform a winner-take-all auction for the buyer if the suppliers are extremely risk averse. We analyze arbitrary allocation rules, not just a two-way split, and we show that non-greedy splits can be optimal even without appealing to entry costs and risk aversion.

Dasgupta and Spulber (1990) characterize the buyer's theoretically optimal mechanism when suppliers' costs are increasing and convex in production volume, and show how the buyer optimally uses split awards to minimize her purchase costs. In our setting, suppliers face constant marginal production costs, and the buyer uses split awards in order to satisfy sourcing rules, subject to which she chooses the cost minimizing allocation. Interestingly, we show that greater splits can also be used as a tool to reduce purchase costs even in our setting, without appealing to convexity in supplier production costs. Thus we contribute to the extant split-award literature by showing that greater splits can reduce the buyer's sourcing costs, without invoking entry costs, risk aversion, supply risks, or cost convexity.

Tunca and Wu (2009) analyze a two-stage procurement event in which the buyer evenly divides the business amongst the k lowest bidders in the first stage and then negotiates with them in the second stage. Their focus is on bounding the optimality loss imparted by using this two-stage

procurement process versus a single-stage optimal mechanism (that takes into account supplier production cost convexity). We on the other hand focus on a single-stage auction event where the award splits are chosen by the buyer subject to sourcing rules, and characterize the effect of multi-sourcing on the buyer's expected cost.

Chaturvedi and Martínez-de-Albéniz (2011) investigate the optimal mechanism that multi-sources to diversify away supply risk and Chaturvedi et al. (2013) investigate the optimal mechanism design that multi-sources to retain the suppliers in the supply base. Here, we abstract away from these factors for multi-sourcing by taking them into account through the buyer's sourcing rules. Subject to these sourcing rules, we then analyze the optimal allocation splits for sealed-bid and open-bid auction formats.

Also related to our work is the literature that has investigated the bidding equilibrium in split-award auctions when the splits are exogenously specified. Anton and Yao (1992) characterize the bidding equilibrium when the two participating suppliers can submit multiple bids, on the complete business and on each of the splits. The buyer then decides to either single-source or multi-source, depending on the bids submitted by the two suppliers. They find that such an auction format results in coordinated bids, i.e., each of the suppliers can unilaterally veto any split. This happens because the buyer is auctioning a two-way split among just two bidders. We too characterize the bidding equilibrium for given splits, however, we consider n bidders facing splits announced up front as part of the auction format, and we then go on to show how the buyer can determine these splits given the allocation rules it faces.

Bichler et al. (2014) analyze a setting in which the buyer splits the contracts into two parts, and compare bidding behavior and the resulting performance under two auction formats—the sealed-bid first price (which they call Yankee) auction, and a format under which each bidder can place a separate sealed bid for each of the two parts of the contract. The focus of their experimental study is on separating various behavioral causes for why bidders deviate from the risk-neutral Bayes-Nash Equilibrium. They find, like other experimental studies before them, that human subject bidders underbid, as compared to the predictions of Bayes-Nash equilibrium and they go on to conjecture that risk aversion is a major driver for underbidding. By contrast, we analyze an arbitrary split, not just two-way splits. Moreover, as explained above, we study the buyer's problem in terms of how to design allocation rules given sourcing constraints and test the model that helps estimate the cost of multi-sourcing. Finally, we test whether bidders always underbid and find that they might overbid (above the Nash-equilibrium predictions) when costs are not regularly distributed, which is inconsistent with the risk aversion explanation.

Testing auction theory using controlled laboratory experiments has a long tradition (see Kagel (1995) for an overview of early work and Kagel and Levin (2010) for an overview of more recent

work in economics). Much of the early work focused on testing revenue equivalence among the four basic forward auction formats (the sealed-bid first price, Dutch, English, and the sealed-bid second price). The findings are that the bidding in the sealed-bid first price auction is more aggressive than the risk-neutral Nash equilibrium, and the bidding in the Dutch auction is not independent of the speed of the Dutch clock (see Katok and Kwasnica (2008)). Thus, generally revenue equivalence (in our case cost equivalence) fails in the laboratory. Elmaghraby, Katok and Santamaria (2012) report a similar finding in open bid reverse auctions with rank feedback, and Haruvy and Katok (2013) observe the same thing in buyer-determined reverse auctions. To the best of our knowledge, all previous experimental studies of auctions used a regular cost distribution (usually Uniform). Our paper is the first to consider a non-regular cost distribution and to investigate the effect of multi-sourcing with more than two splits in the laboratory, while comparing sealed- and open-bid formats in that setting.

In the next section we explain the basic set-up for the model of a split-award auction with sourcing rules. In §3.1 we analyze the bidding equilibrium in the sealed-bid first price auction and then characterize corresponding expected cost for the buyer. In §3.2 we formally explain the open descending format and then evaluate the buyer's expected cost. In §4 we analyze the buyer's problem of deciding the optimal allocations, subject to sourcing constraints. In §5 we present the details and results of the laboratory experiments. We conclude the paper in §6 by summarizing our results and discussing managerial implications of our work. All the proofs and derivations are presented in the Appendix.

2. Model

We model a buyer that faces a unit (normalized) one-time demand for a standardized homogeneous and divisible product. It can buy this quantity from the $n \geq 2$ qualified suppliers in its supply base. In order to discover the best available price for the required product, the buyer invites the suppliers to competitively bid for its business. As is common in the literature (e.g., Chen (2007)), for each supplier i the cost to produce q units is given by $q \cdot c_i$, where c_i is the supplier's privately known (known only to the supplier) constant per-unit cost of production. This captures situations where variable costs are the dominant cost drivers. This arises in a variety of settings, e.g., plastic injection molding (where buyers typically purchase the tooling, so machine time, resin, and electricity are the primary cost drivers at the supplier), or labor-intensive work like simple assembly. With this setup we will show that greater split awards can actually help the buyer reduce costs, without appealing to notions of cost convexity which clearly favor the use of split awards (e.g., Dasgupta and Spulber (1990)). We let $\mathbf{c} = (c_1, c_2, ..., c_n)$ denote the vector of the suppliers' per-unit cost of production. We assume that the costs are identically, independently and continuously distributed

in the interval $[\underline{c}, \bar{c}]$, according to cumulative distribution F (with density f and $\bar{F} = 1 - F$). Finally, we assume that F is common knowledge, the suppliers are risk neutral profit maximizers, and that the buyer is risk neutral and seeks to minimize its expected cost subject to sourcing rules.

The buyer faces *sourcing rules* that address multiple operational concerns in procurement, such as supply risk, maintaining the supply base or controlling the administrative cost of purchasing from multiple suppliers. To avoid being too reliant on any one supplier the buyer might cap the maximum business that any one supplier can win (this reduces the buyer's exposure to supply risk, by diversifying). Moreover, recognizing that suppliers who do not win any business may disengage from the supply base, buyers wishing to maintain competition for future bidding events may require that at least a handful of suppliers win business in any given bidding event, to keep the suppliers from abandoning the supply base in search of greener pastures. At the same time, to avoid the hassle of managing and administering many small contracts, the buyer firm may specify that no contract should be below a certain size. These sourcing rules can be formally characterized as follows:

1. No one supplier can win more than fraction $0 \leq A \leq 1$ of the business, to avoid too much dependence on any one supplier.
2. There must be a minimum number of suppliers, $M \leq n$, that are awarded business, for supply base maintenance purposes.
3. Any supplier awarded business should win at least $0 < B \leq A$ of the business, to avoid administrative inefficiencies of working with very small contracts.

To leverage supplier bid competition and also ensure that its sourcing rules are followed, the buyer organizes a split-award auction. As is common in practice, to keep the auction procedure transparent and straightforward the buyer announces, before the auction, the percentage of its business that it would allocate to the suppliers as a function of the rank of their bid. We let Q_1 denote the highest fraction of the total business that the buyer would award to the lowest bid, Q_2 denotes the second-highest fraction of business that would be awarded to the second-lowest bid, and so on up to Q_n. We denote by $\mathbf{Q} = (Q_1, Q_2, \ldots, Q_n)$, the vector of these allocations such that $Q_i \geq Q_j$ for any $i < j$ and $\sum_{i=1}^{n} Q_i = 1$. Thus, if the buyer announces that it will award 70% of its business to the lowest bidder and 30% to the second-lowest then $Q_1 = 0.7$, $Q_2 = 0.3$, and $Q_3 = \ldots = Q_n = 0$.

The buyer decides to announce the vector of allocations $\mathbf{Q}$ that minimizes its expected purchase cost such that $\mathbf{Q}$ is consistent with the sourcing rules that the buyer faces. We let $C_{buyer}(\mathbf{Q})$ denote the expected purchase cost of the buyer if it announces the vector of allocations $\mathbf{Q}$.[2] Overall, the buyer's problem of deciding the optimal $\mathbf{Q}$, given the sourcing rules, can be characterized as follows:

[2] In §3 we explicitly characterize $C_{buyer}(\mathbf{Q})$ for the sealed-bid and open-bid auctions.

$$\min_{\mathbf{Q}} C_{buyer}(\mathbf{Q}) \tag{1a}$$

$$\begin{aligned} \text{s.t. } & Q_i \geq Q_j,\ \forall i < j, \\ & \sum_{i=1}^{n} Q_i = 1, \\ & Q_i \in \{0, [B, A]\},\ \forall i, \\ \sum_{i=1}^{n} z_i \geq M,\ & \text{ s.t. } z_i = 1 \text{ if } Q_i > 0 \text{ else } z_i = 0,\ \forall i. \end{aligned} \tag{1b}$$

Objective function (1a) characterizes the buyer's objective whereas constraints (1b) characterize the sourcing constraints that $\mathbf{Q}$ must satisfy. The sourcing constraints (1b) are *consistent*, i.e., give a non-empty set of feasible $\mathbf{Q}$s if the following assumptions are satisfied:

$$\begin{aligned} & n \cdot A \geq 1, \\ & B \cdot \max(M, \lceil 1/A \rceil) \leq 1. \end{aligned} \tag{2}$$

Note that $\lceil 1/A \rceil$ gives the nearest integer greater than $1/A$. The first condition ensures that the constraint on the maximum fraction of business A that can be given to a supplier does not restrain the buyer from satisfying its unit demand. The second condition ensures that the constraint on the minimum amount of business given to a supplier (that gets a non-zero allocation) does not result in the buyer having to procure more than its demand. As an example, values of $A = 40\%$ and $B = 35\%$ would imply that 3 suppliers would each get at least 35% of the buyer's business, which is not consistent. In this paper we assume that the conditions (2) are always satisfied.

We will analyze two types of auctions: the sealed-bid first price and the open-bid formats. First, in §3 we investigate the bidding equilibrium in both the auction formats for a given vector of allocations $\mathbf{Q}$. We assume that the buyer must transact for her full quantity, so her reserve price is set to the upper bound of the cost distribution, $\bar{c}$. For both auction formats, §4 characterizes the buyer's problem of deciding the optimal splits, $\mathbf{Q}$, that minimizes the buyer's expected cost, C_{buyer}, subject to sourcing constraints (1b).

3. Analysis of Sealed and Open Split-award Auctions

In this section we analyze the suppliers' bidding equilibrium in both the sealed-bid and the open auction formats for a given allocation structure $\mathbf{Q}$ and formulate the corresponding expected cost for the buyer. We first analyze the sealed-bid auction and then the open-bid auction.

3.1 Sealed-Bid Auction

The sealed-bid auction is implemented in the following way: the buyer announces the allocation vector, $\mathbf{Q} = Q_1, \ldots, Q_n$, where Q_i represents the allocation to the ith-ranked bid. If two or more suppliers bid the same, then each of the tied suppliers is awarded the average of the allocations

associated with the tied ranks. The buyer also announces that the price paid per unit of allocation would be the bid quoted by the supplier. Each supplier then submits its bid to the buyer. After collecting all the bids the buyer makes the allocations and payments. The expected payoff function for supplier i when it bids b_i can be expressed as:

$$\Pi_i(b_i, c_i, \mathbf{Q}) = (b_i - c_i)H(b_i, \mathbf{b_{-i}}, \mathbf{Q}),$$

where $H(b_i, \mathbf{b_{-i}}, \mathbf{Q})$ represents the expected allocation to supplier i and $\mathbf{b_{-i}}$ represents the vector of bids submitted by all the other suppliers. To find the equilibrium bidding strategy we adapt the canonical, winner-take-all sealed-bid auction equilibrium analysis, e.g., Krishna (2010). Assume that the bidding strategy of all suppliers, except for supplier i, is $b_j = \beta(c_j, \mathbf{Q})$ defined on the domain $[\underline{c}, \overline{c}]$. For now, we assume that $\beta(c, \mathbf{Q})$ is a continuously differentiable and increasing function of c. We later verify that these assumptions are indeed true in equilibrium. Let $C_{m:n}$ represent the mth-lowest per-unit production cost out of n suppliers. Since β is an increasing function, one can represent the likelihood of supplier i being the lowest bidder as $\mathbb{P}(b_i < \beta(C_{1:n-1}, \cdot)) = \mathbb{P}(\beta^{-1}(b_i, \cdot) < C_{1:n-1})$, where $\beta^{-1}(\beta(c, \mathbf{Q}), \mathbf{Q}) = c$. Similarly, the likelihood of supplier i being the mth-lowest bidder, for an $m > 1$, can be expressed as $\mathbb{P}(\beta(C_{m-1:n-1}, \cdot) < b_i < \beta(C_{m:n-1}, \cdot)) = \mathbb{P}(C_{m-1:n-1} < \beta^{-1}(b_i, \cdot) < C_{m:n-1})$. Using the bidding strategy β, one can then express supplier i's expected allocation only as a function of $\beta^{-1}(b_i, \cdot)$, i.e.,

$$H(b_i, \mathbf{b_{-i}}, \mathbf{Q}) = H(\beta^{-1}(b_i, \mathbf{Q}), \mathbf{Q}) = \mathbb{E}_{\mathbf{c}} \begin{pmatrix} Q_1\mathbb{P}(\beta^{-1}(b_i, \cdot) < C_{1:n-1}) + \delta_1\mathbb{P}(C_{1:n-1} = \beta^{-1}(b_i, \cdot)) + \dots \\ +Q_2\mathbb{P}(C_{1:n-1} \leq \beta^{-1}(b_i, \cdot) < C_{2:n-1}) + \delta_2\mathbb{P}(C_{2:n-1} = \beta^{-1}(b_i, \cdot)) \\ +Q_n\mathbb{P}(C_{n-1:n-1} \leq \beta^{-1}(b_i, \cdot)) \\ +\delta_{n-1}\mathbb{P}(C_{n-1:n-1} = \beta^{-1}(b_i, \cdot)) + \Delta \end{pmatrix}, \tag{3}$$

where $\delta_m = (Q_m - Q_{m+1})/2$ for any $1 \leq m \leq n-1$ and Δ is the expected allocation under all events whereby two or more consecutive order statistics take the same value of $\beta^{-1}(b_i, \cdot)$. Indeed, the assumption of a continuous cost distribution implies that $\mathbb{P}(C_{m:n-1} = \beta^{-1}(b_i, \cdot))$ and Δ are negligibly small for any $m = 1, \dots, n-1$ and any b_i and hence the related terms can be ignored. Define

$$H(x, \mathbf{Q}) \equiv \left\langle \begin{matrix} Q_1\bar{F}(x)^{n-1} + Q_2\binom{n-1}{1}\bar{F}(x)^{n-2}F(x) + \dots \\ +Q_{n-1}\binom{n-1}{n-2}\bar{F}(x)F^{n-2}(x) + Q_nF^{n-1}(x) \end{matrix} \right\rangle. \tag{4}$$

Then supplier i's expected payoff can be characterized as

$$\Pi_i(b_i, c_i, \mathbf{Q}) = (b_i - c_i)H(\beta^{-1}(b_i, \mathbf{Q}), \mathbf{Q}). \tag{5}$$

Thus, one can differentiate equation (5) with respect to b_i to characterize the best response of supplier i given that all other suppliers use a symmetric bidding strategy, β. Then, by assuming

that supplier i's best response is also β, one can characterize the symmetric equilibrium strategy β. If the strategy β satisfies all the assumptions, i.e., it is continuously differentiable, increasing and is the best response of supplier i given all other suppliers' best response is β, then it does indeed formulate the symmetric bidding equilibrium strategy. In the proposition below we formalize this argument.

PROPOSITION 1. *For any allocation $Q_1 \geq Q_2 \geq \ldots \geq Q_n$, the symmetric equilibrium bid function in a sealed first price auction is given by*

$$\beta(c, \mathbf{Q}) = \bar{c} \cdot \frac{H(\bar{c}, \mathbf{Q})}{H(c, \mathbf{Q})} - \frac{1}{H(c, \mathbf{Q})} \int_{x=c}^{\bar{c}} x dH(x, \mathbf{Q}). \tag{6}$$

Note that for a winner-take-all auction, the allocation vector would be $Q_1 = 1$ and $Q_2 = \ldots = Q_n = 0$ and accordingly the equilibrium bid function (6) gives $\beta(c) = \frac{1}{\bar{F}(c)^{n-1}} \int_{x=c}^{\bar{c}} x(n-1)\bar{F}(x)^{n-2} f(x)dx$, which is indeed the equilibrium bid function for a sealed-bid winner-take-all auction, i.e. each supplier, conditional on it being the lowest-cost supplier, bids the expected cost of the lowest-cost supplier among the other $n-1$ suppliers (see Krishna(2010)).

Using the equilibrium bids we can characterize the buyer's expected cost in the sealed-bid auction. Even though the equilibrium bid function of the suppliers appears complicated, it turns out that the buyer's expected cost can be simplified to a rather clean expression. The proposition below does exactly that. We define $\mu_m \equiv E_c[C_{m:n}]$, the expected m^{th} order statistic from n draws. For notational convenience, for any $m > n$ we take $\mu_m = \bar{c}$.

PROPOSITION 2. *The expected buyer's cost in the sealed-bid auction is*

$$C_{buyer}(\mathbf{Q}) = \begin{array}{c} \mu_2 Q_1 + (2\mu_3 - \mu_2)Q_2 + \ldots + (m\mu_{m+1} - (m-1)\mu_m)Q_m + \ldots \\ +((n-1)\mu_n - (n-2)\mu_{n-1})Q_{n-1} + (n\bar{c} - (n-1)\mu_n)Q_n. \end{array} \tag{7}$$

Thus the buyer's problem for deciding the optimal $\mathbf{Q}$ for the sealed-bid split-award auction can be characterized as: $\min_{\mathbf{Q}} C_{buyer}(\mathbf{Q})$ such that $\mathbf{Q}$ satisfies the constraints (1b). In §4 we find the optimal allocation vector $\mathbf{Q}$ for the sealed-bid auction by solving this problem.

But before that, we first re-arrange the terms of the expected buyer's cost in equation (7) as follows:

$$C_{buyer}(\mathbf{Q}) = \begin{array}{c} nQ_n\bar{c} + (n-1)(Q_{n-1} - Q_n)\mu_n + \ldots \\ +m(Q_m - Q_{m+1})\mu_{m+1} + \ldots + (Q_1 - Q_2)\mu_2. \end{array} \tag{8}$$

Equation (8) implies that if the buyer gave Q_n amount of business to each of the n bidders at price $\bar{c}$ and then gave $Q_{n-1} - Q_n$ amount of business to $n-1$ bidders at a price equal to the cost of the highest bidder and so on, then the buyer's expected cost would match equation (7). Below we describe how this outcome can be implemented through an open-descending price-clock auction.

3.2 Open Descending Auction

For any allocation structure $\mathbf{Q}$, we implement the open auction as a descending price-clock auction. In this auction format the price clock starts at price $\bar{c}$. At the start of the price clock the buyer allocates Q_n amount of business to each of the n suppliers, and for this quantity pays them a per-unit price of $\bar{c}$. The price clock then begins to move down. The suppliers can drop out of the auction at any time. Dropping out of the auction does not give any additional allocation or payment to the supplier who drops out, beyond what it has already received. However, a supplier that drops out does result in each of the suppliers who remain in the auction getting allocated some non-negative quantity for a per-unit payment equal to the price at which the supplier dropped out of the auction. Specifically, the first supplier to drop out results in each of the remaining $n-1$ suppliers getting an allocation of $Q_{n-1} - Q_n$ at a per-unit payment equal to the price at which that first supplier dropped out. More generally, the dropping-out of the $(n-m+1)^{th}$ supplier (for $2 \leq m \leq n$) results in the remaining $m-1$ suppliers getting allocated $Q_{m-1} - Q_m$ amount of business at the auction price at which the $(n-m+1)^{th}$ supplier dropped out. The auction stops when the second-to-last (the $(n-1)^{th}$) supplier drops out of the auction. Note that the overall allocation awarded to the $(n-m+1)^{th}$ supplier to drop-out is Q_m and the last remaining supplier gets an overall allocation of Q_1.

In such an auction, for any allocation $\mathbf{Q}$ such that $Q_1 \geq Q_2 \geq \ldots \geq Q_n$, a supplier i finds it optimal to drop out when the price clock reaches its marginal cost c_i. If supplier i drops out any sooner (at $b > c_i$) then it only loses the opportunity to get an allocation at a profitable price had some other supplier dropped out between b and c_i. If supplier i drops out later (at $b < c_i$), then it only increases the likelihood of getting an allocation at a loss-making price which happens if some other supplier drops out at a price between c_i and b. Thus in an open-descending auction, the equilibrium strategy for all suppliers (except the lowest-cost, since the auction stops when the second-to-last supplier drops out) is to drop out when the price clock reaches their respective per-unit cost.[3]

Since bidders find it optimal to drop out at their true costs, it is easy to see that the above auction results in the same expected cost as the sealed-bid format from the previous section. This follows naturally given the well known revenue equivalence theorem; indeed, Wambach (2002) formally extends the notion of revenue equivalence (in a forward auction context) to split-award auctions that award the largest share to the bidder with the highest bid, the second-largest share to the bidder with the second-highest bid and so on. For us, the implication of cost equivalence is that the buyer's problem of deciding the optimal splits $\mathbf{Q}$ for the open descending split-award auction also remains the same as for the sealed-bid auction.

[3] Note that this descending price-clock auction is similar in spirit to the *clinching* auction mechanism used for the auctioning of multiple units in an ascending price-clock auction (as discussed in Ausubel(2004)).

4. Optimal Splits

In this section we solve for the buyer's problem of deciding the optimal splits, $\mathbf{Q}$, for both the sealed-bid and open-descending auction formats. Namely, we embed the buyer's expected cost, $C_{buyer}(\mathbf{Q})$, as characterized by equation (7), into the objective (1a). The decision variables $Q_1, \ldots, Q_n$ are real numbers not less than 0, and $C_{buyer}(\mathbf{Q})$ in equation (7) is linear in $Q_1, \ldots, Q_n$, hence the math program (1) formulates a constrained fractional knapsack problem and thus would give corner solutions. However it is not obvious whether the solution is also greedy. We define a greedy allocation as one that satisfies the sourcing constraints (1b) and for which a positive quantity can not be transferred from the allocation of a higher-bidding supplier to the allocation of a lower-bidding supplier without violating the sourcing constraints (1b). Intuitively, in a greedy allocation, the buyer allocates the maximum possible business sequentially, starting from the lowest-bidding supplier and moving towards higher-bidding suppliers, such that the sourcing constraints (1b) are satisfied. As an example, for parameter values $B = 10\%$, $M = 3$ and $A = 50\%$, a greedy allocation would imply allocating 50%, 40%, 10% of the business to the lowest, second-lowest and third-lowest-bidding suppliers respectively and giving a zero allocation to all of the remaining suppliers. In the Lemma below we show that allocating greedily is similar to solving an optimization problem.

Lemma 1. *Maximizing* $\sum_{i=1}^{n} Q_i^2$ *such that the sourcing constraints* (1b) *are satisfied would give a unique solution, namely the greedy allocation.*

Note that $\sum_{i=1}^{n} Q_i^2$ is the same as the Herfindahl-Hirschman index (HHI) used to describe market concentration. Thus allocating greedily is the same as maximizing the HHI of the allocations such that sourcing constraints (1b) are satisfied.

Indeed, a greedy allocation is always (i.e., for all sourcing constraints) optimal if and only if the coefficients of the objective function given in (7) are increasing, i.e., the coefficient of Q_1 is less than that of Q_2 and so on. The Lemma below characterizes the necessary and sufficient conditions for the greedy allocation to be optimal.

Lemma 2. *Allocating greedily to the lowest-bidding supplier is always (i.e., for all sourcing constraints) optimal. Allocating greedily to all the other suppliers is always (i.e., for all sourcing constraints) optimal if and only if*

$$m\mu_{m+1} - (m-1)\mu_m \geq (m-1)\mu_m - (m-2)\mu_{m-1}, \forall\, 2 < m < n, \tag{9a}$$

$$\textit{and} \quad n\bar{c} - (n-1)\mu_n \geq (n-1)\mu_n - (n-2)\mu_{n-1}. \tag{9b}$$

Lemma 2 implies that the buyer would never find it optimal, as an example, to go from a $80-20$ split to a $70-30$ split, provided that the sourcing constraints are met in both the cases.

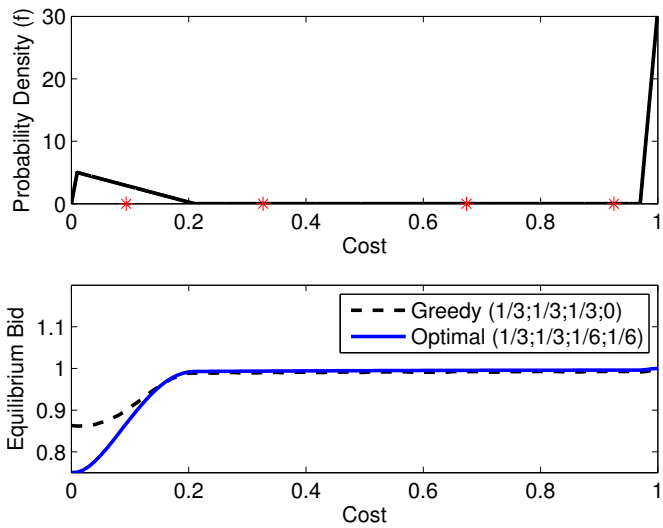


Figure 1 Top figure shows the pdf $f(x)$ characterized by equation (10) and the bottom figure shows the equilibrium bid for greedy and non-greedy allocations when cost are distributed according to Equation (10). The first four expected order statistics are marked with an asterisk.

Similarly, the buyer would never benefit from splitting a sole-award into more splits, provided that sole-sourcing does not violate the sourcing constraints. However, what about splitting the award fraction amongst more suppliers who do not bid the lowest, when the sourcing constraints require the buyer to procure from at least two suppliers? In the presence of such sourcing constraints, the optimality of the greedy allocation would depend on whether the conditions (9) holds or not.

Let us build some intuition into why conditions (9) might not hold (we will use an example that we will return to later in the experiments). Suppose the buyer has 4 bidders, and is comparing $Q_1 = 1/3, Q_2 = 1/3, Q_3 = 1/3, Q_4 = 0$ split versus $Q_1 = 1/3, Q_2 = 1/3, Q_3 = 1/6, Q_4 = 1/6$ split. Compared to the former greedy allocation, the latter non-greedy allocation has advantages: It encourages competition among the lowest-cost suppliers. However, it has disadvantages in that it sacrifices competition among the higher-cost suppliers — now even the worst-cost supplier receives some allocation. Interestingly, it turns out that the advantage of non-greedy can outweigh the disadvantage. To see how this might happen, consider the following bi-modal probability density function of suppliers' cost (in the top part of Figure 1 we show the p.d.f. characterized below):

$$f(x) = \begin{cases} 500x \text{ if } 0 \le x \le 0.01 \\ 5 - 24.95(x - 0.01) \text{ if } 0.01 \le x \le 0.21 \\ 0.01 \text{ if } 0.21 \le x \le 0.97 \\ 0.01 + 1035.78(x - 0.97) \text{ if } 0.97 \le x \le 1 \\ 0 \text{ otherwise.} \end{cases} \tag{10}$$

For $n = 4$ suppliers and sourcing constraint $A = 1/3$, the optimal allocation according to math program (1) would be non-greedy, i.e., $Q_1 = Q_2 = 1/3$ and $Q_3 = Q_4 = 1/6$. On the other hand the

greedy allocation would be $Q_1 = Q_2 = Q_3 = 1/3$ and $Q_4 = 0$. The intuition is that with the non-greedy allocation, competition at the low-end of the cost distribution is more fierce, and this is not offset by less competition at the high-end of the cost distribution. The bottom part of Figure 1 depicts the equilibrium bids for both optimal and greedy splits when the cost distribution is bi-modal (according to equation (10)). One can see that for the non-greedy allocation the low-cost bidders bid significantly lower than they would under the greedy allocation. The reason is that under the greedy allocation the low-cost bidders simply want to avoid coming in last, but they have no incentive to be the first-best or second-best bidder like they would with the non-greedy allocation. The resulting difference in the buyer's expected cost between the greedy allocation and the optimal allocation would be 3.81%. Moreover, we see that in presence of sourcing constraints, it might be optimal for the buyer to source from 4 suppliers instead of the minimum required 3 suppliers, i.e., the buyer can reduce its expected purchasing cost by diversifying more. Indeed, other distributions can be thought of for which a non-greedy allocation is optimal. In the Appendix we show another such example in which the buyer can reduce its expected cost by splitting its business amongst four suppliers, even though the sourcing constraints require awarding business to just two suppliers (e.g., when $A = 55\%$, $B = 0$ and $M = 0$).

The above discussion provided intuition as to why non-greedy can outperform greedy. What matters is how competition heats up among lower-cost bidders when moving to a non-greedy allocation, and whether or not this offsets allocating more units to higher-cost bidders. The importance of Lemma 2 is underscored by this discussion, precisely because it helps us understand the conditions for when greedy will still be optimal. What matters is that the gaps between the order statistics of the underlying cost distribution are well behaved. It turns out that a familiar condition is all we need to guarantee that the gaps between the order statistics are well-behaved, and thus allocating greedily is optimal. In particular, all we need is that the underlying distribution is regular. The next theorem formalizes this result.

Theorem 1. *Shifting a positive amount from a low bidder's allocation to a high bidder's allocation (i.e. shifting an $\epsilon > 0$ from Q_i to Q_j for any $i < j$), with everything else being the same, decreases the Herfindahl-Hirschman index (HHI) of the allocations, and for any regular distribution (a continuous distribution for which $c + \frac{F(c)}{f(c)}$ is increasing) will increase the buyer's expected cost.*

Distributions that have a log-concave density satisfy the regularity condition, including Uniform, Exponential, Normal and Power-function distributions (see Bagnoli and Bergstrom (2005)).

By definition an allocation is greedy if a positive quantity can not be transferred from Q_j to Q_i for any $j > i$ without violating the sourcing constraints (1b). The next Corollary follows from Theorem 1.

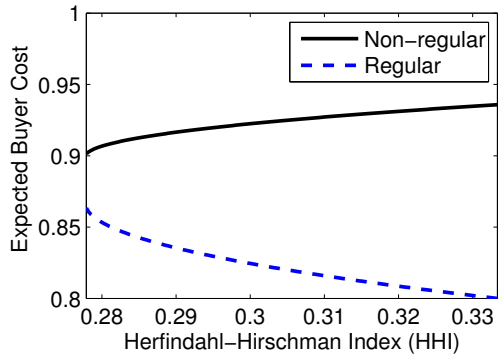


Figure 2 Expected buyer cost as allocation is transferred from lower-cost supplier to higher-cost supplier. $n=4$, $Q_1=1/3$, $Q_2=1/3$, and Q_3 and Q_4 are changed from $1/3$ to $1/6$ and 0 to $1/6$ respectively. Costs are uniformly distributed in unit interval for the regular distribution and are distributed according to Equation (10) for the non-regular distribution.

Corollary 1. *For any regular cost distribution the buyer finds it optimal to announce a greedy allocation, i.e., maximize the Herfindahl-Hirschman index (HHI) of the allocations such that the sourcing constraints* (1b) *are satisfied.*

In Figure 2 we evaluate the buyer's expected cost with 4 participating suppliers when it allocates $1/3^{rd}$ of its business to the two lowest-cost suppliers and progressively changes the allocation to the third-lowest- and the fourth-lowest-cost suppliers from 1/3 to 1/6 and 0 to 1/6 respectively. Thus we evaluate the buyer's expected cost as it diversifies more (and hence decreases the HHI of allocations). Consistent with Theorem 1, we find that the buyer's expected cost increases as it diversifies more when the underlying costs are uniformly distributed. However, we see that the buyer's expected cost decreases as is diversifies more when the underlying costs follow a non-regular distribution (characterized by equation (10)). Thus, we see that a buyer might decrease its expected purchasing cost by diversifying more than what is strictly required by its sourcing rules when the underlying cost distribution is not regularly distributed.

Moreover, note that Theorem 1 only provides a sufficient condition for optimality of the greedy allocation. Thus cost distributions that do not satisfy the regularity condition of Theorem 1 can still result in optimality of the greedy allocation. As an example consider an Arc-Sine cost distribution that has a density function defined as $f(c)=\dfrac{1}{\pi\sqrt{c(1-c)}}$ and c.d.f. $F(c)=\dfrac{2}{\pi}\sin^{-1}(c)$ in the interval $[0,1]$. It can be easily established that this distribution does not satisfy the regularity condition. However, for $n=3$ suppliers this distribution does satisfy conditions (9), thus resulting in the greedy allocation being optimal.[4]

[4] For $n=3$ an arc-sine distribution gives $\mu_1=0.3618$, $\mu_2=0.6644$ and $\mu_3=0.8836$. Thus the coefficients of Q_3, Q_2 in equation (7) are $3-2\mu_3=1.2327$ and $2\mu_3-\mu_2=1.1028$ respectively.

Sensitivity of Buyer's Cost

The results of Theorem 1 and Lemma 1 also allow us to investigate the sensitivity of the buyer's expected cost as the sourcing rules (parameters A, B, M defined in §2) are relaxed. For that we define the buyer's expected cost given the sourcing constraints (1b) as

$$C^*_{buyer}(A, B, M) = \begin{cases} \min\limits_{\mathbf{Q}} C_{buyer}(\mathbf{Q}) \\ \text{s.t. the sourcing constraints (1b) are satisfied.} \end{cases} \tag{11}$$

The next result characterizes the sensitivity of the buyer's expected cost with respect to the sourcing rules.

Proposition 3. *For a regular cost distribution the buyer's expected cost, $C^*_{buyer}(A, B, M)$ is convex decreasing in A, and convex increasing in B and M.*

Note that with respect to M we are using discrete convexity since M takes integer values. Decreasing A, or increasing B or M manifests as splitting the allocation more evenly, i.e., increasing diversification. Intuitively, the sourcing constraints are substitutable levers that the buyer can use for changing diversification levels. Proposition 3 implies that marginally increasing diversification at low levels of diversification will cost the buyer less as compared to marginally increasing diversification at higher levels of diversification. In the Appendix we provide exact expressions for the rate of change in the buyer's expected cost (C^*_{buyer}) as the sourcing rules (A, B and M) are changed (in equations (17), (18) and (19) of the Appendix). Thus a buyer could compute how much additional diversification would cost as it changes the sourcing rules.

5. Experimental Design, Research Hypotheses, and Results

In this section we experimentally test the theoretical predictions that we developed about split-award auctions in the preceding sections. For this we conduct split-award auctions in a controlled laboratory environment and compare the results obtained in these experiments to those predicted by the theory. We begin by testing the cost equivalence between sealed- and open-bid auctions. We then proceed to test our main theoretical result: the prediction about optimality of the greedy allocation for regular and non-regular cost distributions. We also present results about suppliers' bidding behavior when confronted with different splits and with a regular vs. non-regular cost distribution. Lastly, we test the model's prediction about efficiency and the cost of multisourcing. We begin by discussing the design of the experiments and its general setup. We then present our hypotheses and the experimental results.

Table 1 Treatment descriptions, predicted buyer's cost, and sample sizes

Treatment Number	Split Format	Auction	HHI	Buyer's Cost Prediction	Cost Distribution	Sample Size (Sessions)
1	40-35-25	Open Bid	34.5%	75.27	Regular	48(4)
2	40-35-25	Sealed Bid	34.5%	75.27	Regular	60(5)
3	50-25-25	Sealed Bid	37.5%	71.42	Regular	48(4)
4	50-35-15	Sealed Bid	39.5%	67.62	Regular	32(4)
5	50-50-0	Sealed Bid	50.0%	61.82	Regular	24(3)
6	80-15-5	Sealed Bid	66.5%	51.38	Regular	60(5)
7	100-0-0	Sealed Bid	100.0%	41.13	Regular	48(4)
8	34-34-17-17	Sealed Bid	25.5%	88.75	Non-Regular	48(4)
9	34-34-34	Sealed Bid	34.0%	92.73	Non-Regular	48(4)

5.1 Experimental Design, Implementation and Protocol

Our study includes nine experimental treatments—all between subjects. In all treatments, $n = 4$ suppliers compete for a contract to provide units of a commodity to a computerized buyer seeking 100 units in total (in treatments 8 and 9 the number of units is 102 to make all splits integer). Suppliers' costs are privately known. In treatments $1-7$, we use a regular cost distribution; costs are distributed according to the uniform distribution from 0 to 100, $c_i \sim U(0, 100)$. In treatments 8 and 9 we use a non-regular cost distribution with the PDF described by equation (10), scaled to the interval $[0, 100]$.

For the regular distribution treatments, we considered six split awards that vary in their market concentration (Herfindahl-Hirschman) index, going from 100% to 34.5%. In treatment 1, our only open-bid treatment, we use the $40-35-25$ split. For the non-regular distribution treatments, we compare the $34-34-34$ (greedy) and the $34-34-17-17$ (non-greedy) splits. We list all treatments, including the auction format used (sealed- and open-bid) their splits, Herfindahl-Hirschman indices, the kind of distribution, predicted average buyer's costs, and sample sizes, in Table 1.

In total, 416 participants were included in our study. We randomly assigned participants to treatments. Each human subject participated in one treatment only. We conducted all sessions at a public university in the United States, in a computer laboratory dedicated to research. Our participants were students, mostly master-level, primarily business and engineering majors. We recruited them through ORSEE, which is an online recruitment system (Greiner 2004), offering earning cash as the only incentive to participate.

Upon arrival at the laboratory the subjects were seated at computer terminals in isolated cubicles. We handed out written instructions (see the Online Appendix for samples) to participants. After they read the instructions, to ensure common knowledge about the game's rules we then read the instructions aloud before starting the actual game. Each session included $8-12$ participants who

competed in a series of 40 auctions. For each auction, we randomly re-matched participants in each session, into two-three groups of four bidders each. Typically at least 24 participants were in the laboratory at the same time, and participants did not know the session size. In sealed-bid format treatments, each participant placed a single per-unit bid. After all the bids were received, market shares were allocated according to the splits in the treatment.

In the open-bid format treatment 1 (which included the $40-35-25$ split), the per-unit price started at 100 and automatically decreased. Bidders could drop out of the auction by clicking a button on their screens. After the first bidder dropped out of the auction, each of the remaining three bidders were allocated 25 units at the price at which the first bidder dropped out. After the second bidder dropped out of the auction, the remaining two bidders were allocated an additional 10 units at the price at which the second bidder dropped out. And finally, after the third bidder dropped out of the auction, the remaining bidder was allocated the additional 5 units at the price at which the third bidder dropped out.

We programmed the experimental interface using the zTree system (Fischbacher 2007). At the end of each session we computed cash earnings for each participant by multiplying the total earnings from all rounds by a pre-determined exchange rate and adding it to a \$5 participation fee. Participants were paid their earnings in private and in cash, at the end of the session. Average earnings, including the show-up fee, were \$25.

5.2 Research Hypotheses

The first hypothesis deals with treatments 1 and 2: testing the buyer cost equivalence between the sealed-bid and the open-bid formats.

HYPOTHESIS 1 *(Buyer Cost Equivalence). Average buyer's cost using the* $40-35-25$ *split (a) should not be significantly different from* 75.27*, and (b) should not be different under the open bid and sealed bid formats.*

We formulate the next hypothesis to test the predictions in Proposition 2 and Proposition 1, regarding the average cost of the buyer and auction efficiency, respectively.[5]

HYPOTHESIS 2 *(Buyer's cost and auction efficiency): The average cost of the buyer in all treatments (a) will not be significantly different from the Buyer's Cost Prediction amounts in Table 1, (b) the auction will be* 100% *efficient.*

Part (a) is a strong test of Proposition 1 because it requires the buyer's cost to match the predictions of the risk-neutral Nash equilibrium (RNNE) (see the Appendix for the sealed first-price bidding equilibrium function for uniformly distributed costs). Part (b) tests the efficiency

[5] We define an efficient allocation as one that allocates at least as much market share to a lower-cost supplier as to a higher-cost supplier. Any allocation that violates this definition for any of the splits is labeled as inefficient.

of allocation, because it is possible for the bidding behavior to be different from Proposition 1, rejecting H2(a), while the auction still remains efficient.

Next, we test the prediction of Theorem 1. This is a qualitative test rather than a test about point predictions. According to Theorem 1, shifting some positive amount from the allocation of a low bidder to the allocation of a high bidder, with everything else remaining unchanged, would decrease the Herfindahl-Hirschman index (HHI) of allocation, and for any regular cost distribution should increase the average buyer's cost. In treatments $7-3$, each allocation is obtained from the allocation of the preceding treatment by shifting a positive amount from the allocation of a high bidder to the allocation of a low bidder.

HYPOTHESIS 3 *(Optimality of Greedy Allocation for Regular Distribution): The average cost of the buyer in all regular distribution treatments will increases as the Herfidahl-Hirschman index of allocations decreases.*

The next hypothesis deals with the two treatments with non-regular distribution. In this case, the non-greedy allocation should result in lower buyer's cost than the greedy allocation.

HYPOTHESIS 4 *(Optimality of Non-Greedy Allocation for Non-Regular Distribution): The average cost of the buyer with the* $34-34-34$ *split will be higher than the average cost of the buyer with the* $34-34-17-17$ *split.*

A useful feature of our model is that it predicts the cost of multi-sourcing. For example, we can compute the predicted cost of multi-sourcing in our study by comparing the predicted buyer's costs in Table 1. More specifically, the cost of multi-sourcing is the cost that the buyer incurs by spreading its award more. We formulate our final hypothesis to test the cost of multi-sourcing predicted by the theory.

HYPOTHESIS 5 *(The cost of multi-sourcing). Pair-wise differences in average buyer cost will not be different from those predicted in Table 1.*

Proposition 3 characterizes how the buyer's cost changes with respect to the sourcing constraints. Since a change in the sourcing constraints manifests as a change in the buyer's splits, Hypothesis 5 provides an indirect test of Proposition 3.

5.3 Results

5.3.1 Buyer Cost Equivalence (Hypothesis 1): We test buyer cost equivalence using data from treatments 1 and 2. Figure 3 summarizes the test of buyer cost equivalence between the open-bid and the sealed-bid formats for the $40-35-25$ split (H1). Here and in the rest of the results section we use session average as the unit of analysis and report two-sided p-values from a t-test.

Under the sealed-bid format, the average buyer's cost for the $40-35-25$ split is significantly below the RNNE, but for the same split, the average buyer's cost is only weakly different from the

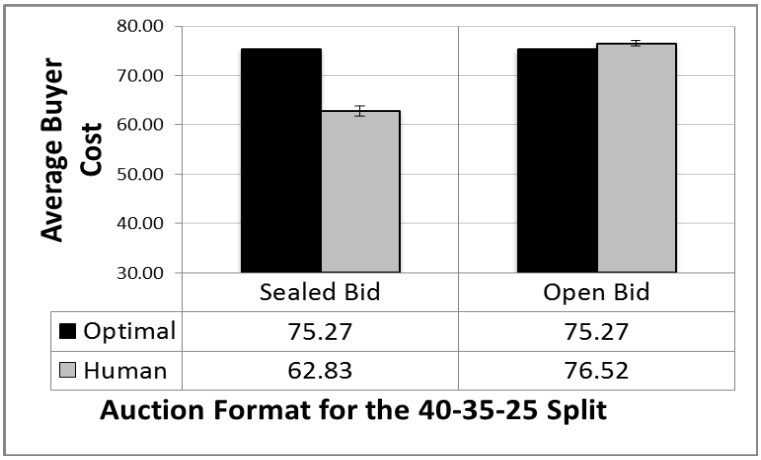


Figure 3 **Comparison of average buyer cost for the $40-35-25$ split under the sealed-bid and open-bid formats, and the RNNE benchmark.**

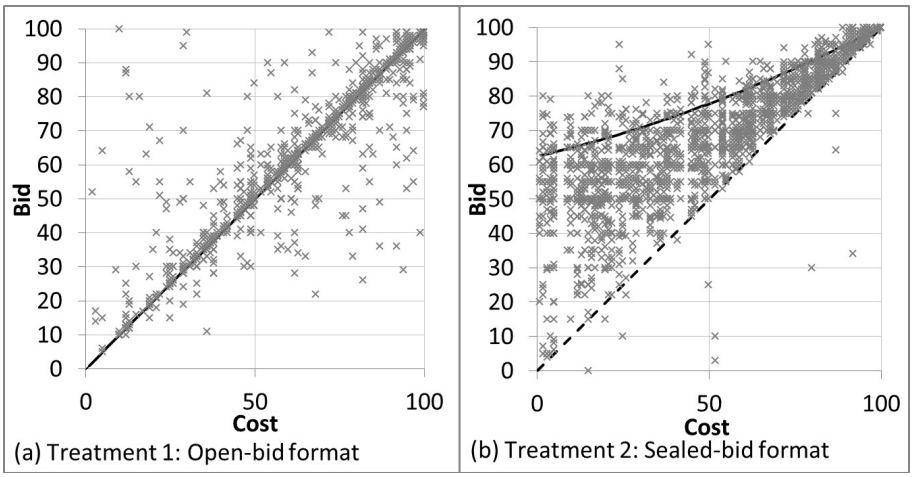
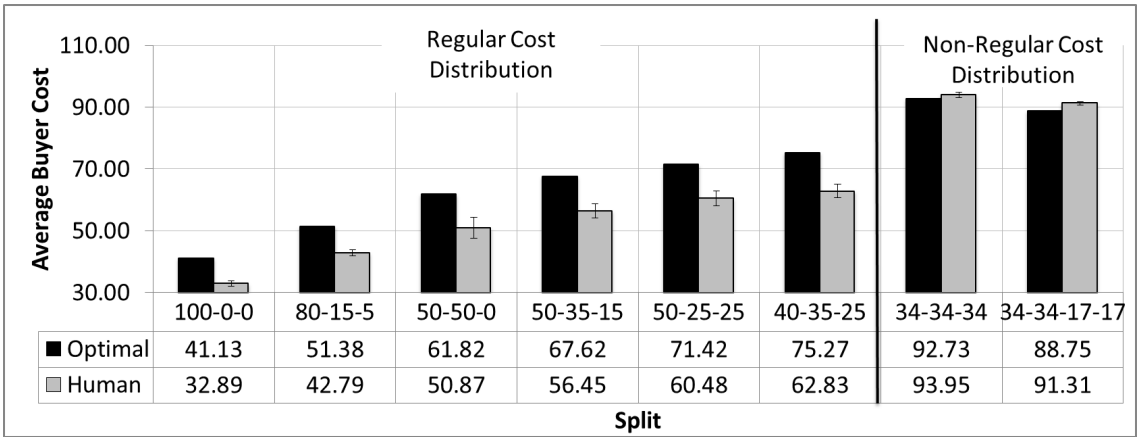


Figure 4 **Bids as a function of cost for treatments with $40-35-25$ split. RNNE is marked by a solid line, and the 45° line is marked by a dashed line.**

RNNE prediction when the open-bid format is used ($p = 0.0887$), and directionally, the average buyer's cost is slightly above predicted, rather than below.

We plot individual bids as a function of cost in the open-bid treatment in Figure 4(a) and sealed-bid treatment in Figure 4(b). Comparing the two treatments we can see that overall bidding is much closer to equilibrium under the open-bid than under the sealed-bid format. So the average buyer's cost is significantly lower under the sealed-bid than under the open-bid-format ($p < 0.001$) contrary to H1.

5.3.2 Buyer Cost, Efficiency and Individual Bidding Behavior (Hypothesis 2): In Figure 5 we display data from all sealed-bid treatments, comparing average buyer costs to their theoretical benchmarks. The black bars show RNNE predictions, while the gray bars show average buyer costs. The vertical tines indicate ± 1 standard error; we computed standard errors using



	100-0-0	80-15-5	50-50-0	50-35-15	50-25-25	40-35-25	34-34-34	34-34-17-17
Optimal	41.13	51.38	61.82	67.62	71.42	75.27	92.73	88.75
Human	32.89	42.79	50.87	56.45	60.48	62.83	93.95	91.31

Figure 5 **Comparison of average buyer cost under the sealed-bid format and the RNNE benchmarks.**

session average as a unit of analysis. Average buyer cost is below the RNNE benchmark for the six regular distribution treatments, and the differences are statistically significant ($p < 0.05$ using a two-sided t-test). But at the same time, for the two non-regular distribution treatments, the average buyer cost is above the RNNE benchmark ($p = 0.214$ for the $34-34-17-17$ split and $p = 0.018$ for the $34-34-34$ split). Thus, we can reject H2(a). It is worth noting that we are the first to document bidding behavior with non-regular cost distribution, and point out that unlike the overly-aggressive bidding that has been repeatedly observed in sealed-bid auctions with regular cost distributions (see Kagel 1995), we document the opposite with non-regular cost distribution.

H2 depends on the extent of the individual bidding behavior matching the equilibrium prediction, so in Figure 6 we plot, separately for each split that uses the regular cost distribution, individual bids as a function of the cost. The figure also indicates the optimal bid with a solid line. We do the same for the two treatments with non-regular cost distribution in Figure 7.

We clearly see in Figure 6 that in regular distribution treatments most bids are between the cost and the equilibrium bid. This is in contrast to non-regular distribution treatments in Figure 7, in which most of low-cost bidders bid above the equilibrium bid. Another point to note is that the equilibrium bid functions are structured in a way such that even the lowest-cost bidders should bid above a certain level. This level depends on the split and on the cost distribution, and in Figure 6 and Figure 7 we indicate it with a dotted horizontal line. The prediction is that there should not be any bids observed below the horizontal line, however, we clearly observed in Figure 6 a good deal of bidding activity in this range for the regular distribution treatments. In contrast, for non-regular distribution treatments in Figure 7, there is much less bidding activity in this region.

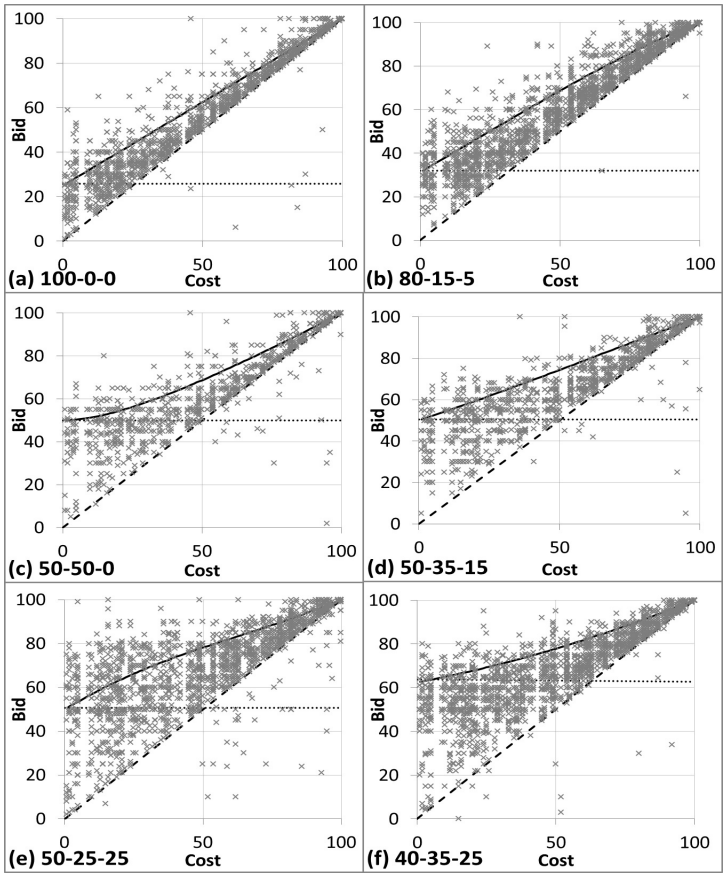


Figure 6 **Bids as a function of cost for sealed-bid treatments with regular distribution. RNNE is marked by a solid line, and the $45°$ line is marked by a dashed line.**

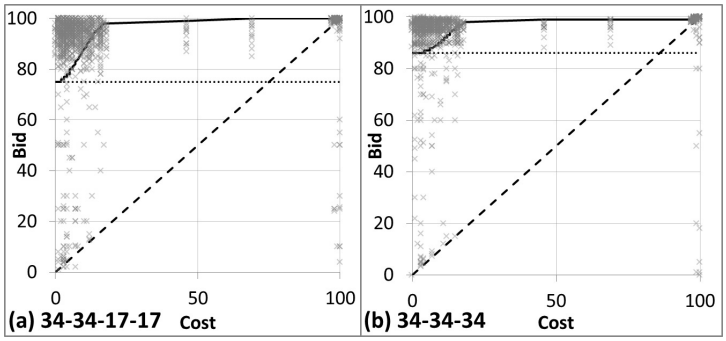


Figure 7 **Bids as a function of cost for sealed-bid treatments with non-regular distribution. RNNE is marked by a solid line, and the $45°$ line is marked by a dashed line.**

We formally analyze bidding behavior and compare it to RNNE by fitting a tobit regression for each split, with the dependent variable bid and independent variable cost.[6] We then compare coefficients for the cost and the constant term to corresponding coefficients for the linear approx-

[6] We use the tobit model with the upper limit at 100 and the lower limit at cost, because the bids are clearly censored at those levels. If we estimate uncensored model, results are virtually unchanged. We use random effects for individuals.

Table 2 Estimates of linear approximation of bid functions.

	Regular Cost Distribution						Non-Regular Cost Distribution	
Independent Variable	100− 0 − 0	80− 15 − 5	50− 50 − 0	50− 35 − 15	50− 25 − 25	40− 35 − 25	34 − 34− 17 − 17	34− 34 − 34
Constant	15.34* (0.455) [25.00]	22.46* (0.508) [32.83]	29.38* (0.978) [43.78]	36.30* (0.907) [49.30]	41.39* (1.125) [53.76]	42.00* (0.6645) [60.08]	86.20 (1.596) [85.43]	90.64 (1.046) [90.58]
Cost	0.840* (0.005) [0.750]	0.777* (0.005) [0.699]	0.660* (0.010) [0.536]	0.599* (0.008) [0.506]	0.552* (0.008) [0.468]	0.542* (0.006) [0.378]	0.398 (0.129) [0.410]	0.230 (0.098) [0.258]

Note: Standard errors are in parenthesis. Estimates of linear approximations of RNNE bid functions are in square brackets; * $p < 0.001$ for comparing the coefficient to the corresponding RNNE linear approximation coefficient in square brackets.

imation of the RNNE bid function, for the entire cost range for regular distribution treatments, and for costs below 25 for the two non-regular distribution treatments.[7] [8] We summarize regression results in Table 2.

This regression analysis shows formally that the main reason average buyer costs are lower than they should be in equilibrium in regular distribution treatments is that, independently of their costs, bidders bid too low. For treatments with regular cost distributions, the estimated constant terms are significantly lower than the corresponding constant term should be for a linear approximation of the RNNE bid function (all p-values are below 0.001). In those treatments, bidders make up some ground because cost coefficients are significantly above the RNNE linear approximation (all p-values are below 0.001), but on balance the bids are too low due to the too-low constant term. For the two non-regular distribution treatments, neither the constant terms, nor the cost coefficients are significantly different from RNNE linear approximations.

We measure efficiency in three ways. Allocational efficiency is the proportion of efficient allocation. To measure allocational efficiency, we code an allocation as efficient whenever a bidder with lower cost is not allocated a market share that is lower than the market share allocated to any bidder with a higher cost. To measure bidding efficiency, we code allocation as efficient only when the order of bids matches the order of costs. Bidding efficiency is a much stricter measure than allocational efficiency whenever an allocation awards the same market share to bids of different rank.

[7] In the two non-regular distribution treatments, equilibrium bids for costs above 25 jumps to very close to 100, so while the equilibrium bid function for the cost range of 0-25 can be reasonably approximated by a linear function, the equilibrium bid function for the entire 0-100 range is clearly non-linear.

[8] Although optimal functions are non-linear (with the exception of the 100-0-0 split) non-linearities are not very strong, and the linear approximation captures the effect of the intercept and the cost, providing a fair comparison for the estimates.

Table 3 Proportion of efficient allocations in the sealed-bid treatments.

Treatment (Splits)	Allocation Efficiency (Standard Error)	Bidding Efficiency (Standard Error)	Cost Efficiency (Standard Error)
$100-0-0$	$0.865^{**}(0.012)$	$0.731^{**}(0.017)$	$1.480^{**}(0.086)$
$80-15-5$	$0.615^{**}(0.033)$	$0.615^{**}(0.033)$	$1.251^{**}(0.035)$
$50-50-0$	$0.890^{**}(0.011)$	$0.558^{**}(0.023)$	$1.287(0.106)$
$50-35-15$	$0.503^{**}(0.029)$	$0.503^{**}(0.029)$	$1.226^{**}(0.022)$
$50-25-25$	$0.729^{**}(0.020)$	$0.377^{**}(0.035)$	$1.230^{**}(0.064)$
$40-35-25$	$0.438^{**}(0.030)$	$0.438^{**}(0.030)$	$1.228^{**}(0.023)$
$34-34-34$	$0.602^{**}(0.013)$	$0.267^{**}(0.016)$	$0.999(0.012)$
$34-34-17-17$	$0.702^{**}(0.020)$	$0.235^{**}(0.026)$	$1.000(0.013)$

Note: standard errors are in parenthesis. Testing H_o that the proportion $= 100\%$, $*p < 0.05; **p < 0.01$.

Finally, we measure cost efficiency as the buyer's cost that would have been achieved under the RNNE bidding, divided by the observed buyer's cost. Table 3 reports all three efficiency measures in the sealed-bid treatments. Allocational and bidding efficiency is generally below 100% allowing us to reject H1(b). Interestingly, cost efficiency is significantly above 100% in most of the regular distribution treatments, due to overly aggressive bidding, and not significantly different from 100% in the non-regular distribution treatments.

5.3.3 Optimality of Greedy Allocation (Hypothesis 3): In this sub-section we test H3 by comparing average buyer cost for each split to the average buyer cost for the split with the next lowest Herfindahl-Hirschman index. Average buyer cost significantly increases when we move from $100-0-0$ to $80-15-5$ split ($p = 0.0001$) and when we move from $80-15-5$ to $50-50-0$ split ($p = 0.0267$). Moving to less competitive splits causes average buyer cost to decrease, in line with H1(b), but not significantly so (p-values > 0.1).

To better understand the effect of reward spreads on the buyer's cost we estimate a regression model using treatments $2-7$ only, in which the dependent variable is the buyer's observed cost and independent variables are the buyer's RNNE equilibrium cost, and two variables that measure the reward spread: $\Delta_1 = Q_1 - Q_2$, and $\Delta_2 = Q_2 - Q_3$.

Table 4 shows estimates for the model with the equilibrium buyer cost alone (Model 1) and compares it to estimates for the model with Δ_1 and Δ_2 (Model 2). We can see from Table 4 that Δ_1 has explanatory power. When multi-sourcing, larger reward spread between the highest and the second-highest market share decreases buyer cost by about 0.06 ECU per unit in spread. So we conclude that our data is consistent with H3.

Table 4 Dependent variable is Buyer Cost.

Independent Variables	Model 1	Model 2
Constant	-11.27^{**} (1.103)	-15.12^{**} (1.60)
Equilibrium Cost	1.017^{**} (0.015)	1.032^{**} (0.016)
Δ_1		-0.063^{**} (0.021)
Δ_2		-0.045 (0.049)
R^2	0.758	0.764
Observations (Groups)	2720(25)	2720(25)

Note: Regression was estimated with random effects for individuals. Standard errors are in parenthesis; ** indicates $p < 0.01$.

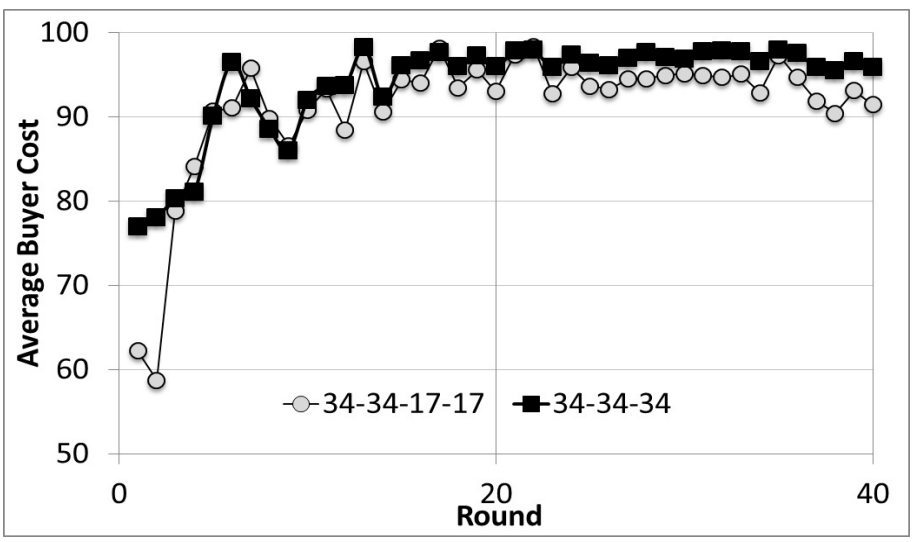


Figure 8 Average buyer cost over rounds for treatments 8 $(34-34-34)$ **and** 9 $(34-34-17-17)$**.**

5.3.4 Optimality of Non-Greedy Allocation for Non-Regular Distributions (Hypothesis 4): In this sub-section we test H4 by comparing average buyer cost in treatments 8 $(34-34-34)$ and 9 $(34-34-17-17)$. Average buyer cost in the $34-34-34$ treatment — that is, the treatment that uses greedy allocation — is 93.95 (std. error is 0.78) and it is significantly higher than the average buyer cost of 91.31 (std. error is 0.54) in the $34-34-17-17$ treatment — the treatment with non-greedy allocation ($p = 0.0318$). In Figure 8 we plot average buyer cost in those two treatments over time.

Table 5 Pair-Wise Differences in Average Buyer Cost, and their Standard Errors.

Treatment	$80-15-5$	$50-50-0$	$50-35-15$	$50-25-25$	$40-35-25$
$100-0-0$	9.90 (0.626) [10.65]	17.98 (2.000) [21.09]	23.94* (0.645) [26.89]	27.59 (1.332) [30.69]	29.93* (0.641) [34.54]
$80-15-5$		8.08* (1.993) [20.05]	14.05* (0.622) [16.24]	17.69 (1.321) [20.05]	20.04* (0.617) [23.89]
$50-50-0$			5.97 (1.999) [5.80]	9.61 (2.314) [9.60]	11.96 (1.997) [13.44]
$50-35-15$				3.64 (1.330) [3.80]	5.99 (0.637) [7.65]
$50-25-25$					2.35 (1.328) [3.84]

Note: Comparison between actual and predicted pair-wise differences. Standard errors are in parenthesis and predicted differences (from Table 1) are in square brackets. $*p < 0.05$.

The figure suggests a good deal of learning in several initial rounds, as low-cost bidders learn. In the last 20 rounds, in which learning appears to have stopped, the average buyer cost in the $34-34-34$ treatment is 96.95 (std. error is 0.34), and is again, significantly higher than the average buyer cost of 94.33 (std. error is 0.45) in the $34-34-17-17$ treatment ($p = 0.003$). We conclude that our data is consistent with H4.

5.3.5 The Effect of Multi-Sourcing on the Buyer's Cost (Hypothesis 5): To directly test H5 (the extent to which the model is able to accurately predict the effect of multi-sourcing on the buyer's cost) we summarize pair-wise differences in average buyer cost, and their standard errors, for regular distribution treatments, in Table 5. For comparison we also include in square brackets pair-wise differences predicted by the model. Average pair-wise differences are generally smaller than predicted, although most differences are not statistically significant. For the two non-regular distribution treatments, the difference in average buyer's cost is 2.64, which is marginally smaller than the predicted difference of 3.98 ($p = 0.048$). So although we can reject H5, we note that the model is fairly accurate overall in predicting the effect of multi-sourcing on the buyer's cost.

6. Conclusion

In this paper we study the use of auctions by a buyer who is faced with sourcing rules that may dictate the minimum or maximum amount of business that can be awarded to any given supplier,

or specify the number of suppliers in the supply base. Some of the reasons for these kinds of sourcing rules include reducing supply risk associated with a single supplier, maintaining appropriate supplier relationships, and controlling sourcing-related administrative costs. To incorporate these sourcing rules into the auction, the buyer announces upfront, the percentage of business that it would allocate to each supplier, depending on the rank of this supplier's bid. We study two types of these split-award auctions: the sealed-bid first-price auction, and the open-descending price auction. We characterize the bidding equilibrium for both auction formats and find that in theory the two auctions result in the same expected cost for the buyer.

We then analyze the optimal splits that the buyer should announce prior to the auction, given the sourcing rules. We find that for regular cost distributions, an allocation scheme that awards greedily is optimal, i.e., an allocation scheme in which no positive amount can be transferred from a higher cost supplier to a lower cost supplier without violating the sourcing constraints. However, we find that allocating greedily might not always be optimal when the underlying distribution of suppliers' cost is non-regular. Thus we find that a buyer's cost reduction and diversification goals might be aligned when the cost distribution is non-regular. We then carry out sensitivity analyses of the buyer's expected cost with respect to the sourcing constraints in order to characterize the cost of multi-sourcing. We find that for regular cost distributions the marginal cost for the buyer is increasing and convex as the sourcing constraints force the buyer to split the award more.

We test the theoretical predictions of our model in the laboratory. We find that when bidders' costs come from a regular distribution (we use the uniform distribution for these laboratory experiments), the buyer's average cost in the sealed-bid first-price auction experiments is significantly lower than predicted by theory because bidders bid more aggressively than under the risk-neutral Nash equilibrium. However, in treatments where the cost distribution is non-regular the buyer's average cost is actually higher than predicted. Thus unlike the overly-aggressive bidding that has been repeatedly observed in sealed-bid auctions for regular cost distributions (see Kagel (1995)), we find the opposite with a non-regular cost distribution. Moreover, the buyer's expected cost in open-bid auction experiments is quite close to the one predicted by theory. Thus the cost equivalence between sealed-bid and open-bid auctions does not hold in the laboratory. We also find that for a regular cost distribution, the buyer's cost increases as it splits its business more evenly, while for a non-regular cost distribution we used in our experiment, the buyer's cost was significantly lower in the treatment with more even splits. Both of those observations are consistent with our theoretical results. Finally, we compare the predicted and observed effect of multi-sourcing on the buyer's cost, i.e., the difference in the cost that the buyer incurs by splitting the awards. This information can help buyers appropriately balance diversification concerns with cost concerns. We find that the model does a good job of predicting these differences, and that — surprisingly — cost and diversification concerns can be aligned when supplier costs follow a non-regular distribution.

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Appendix: Another Example for Optimality of Non-Greedy Allocation

Consider $n = 4$ suppliers and the following sourcing rules: $A = 55\%$, $B = 0$ and $M = 0$. The suppliers' costs are distributed according to the following non-regular density function:

$$f(x) = \begin{cases} 0.35 \text{ if } 0 \leq x \leq 1 \\ 0.35 + 199650(x-1) \text{ if } 1 \leq x \leq 1.001 \\ 200 \text{ if } 1.001 \leq x \leq 1.0037 \\ 0 \text{ otherwise.} \end{cases} \tag{12}$$

In this scenario the optimal splits will be $Q_1 = 0.55, Q_2 = 0.45/3$, $Q_3 = 0.45/3$ and $Q_4 = 0.45/3$. Compared to the optimal splits the non-optimal (greedy) splits of $Q_1 = 0.55, Q_2 = 0.45$, $Q_3 = 0$ and $Q_4 = 0$ would result in an increase of about 2% in the expected buyer cost.

Appendix: Sample Instructions to Participants (Sealed bid, split-award $50-35-15$)

You are about to participate in an experiment in the economics of decision-making. If you follow these instructions carefully and make good decisions, you will earn money that will be paid to you in cash at the end of the session. If you have a question at any time, please raise your hand and the experimenter will come to your station and answer it. We ask that you not talk with one another for the duration of the experiment.

The session consists of 40 rounds. The unit of exchange in all the transactions is called experimental currency unit (ECU). At the end of the session, your earnings in ECUs will be converted to US dollars at a pre-specified rate, and paid to you in cash.

In this experiment, you will be in the role of a Supplier who participates in a Reverse Auction, competing against three competitors in this room.

How you earn money

You will bid in 40 auctions for the right to provide 100 units of a commodity to a computerized buyer. In each auction the number of units awarded to each bidder are determined as follows:

- The bidder with the lowest bid will provide 50 units,
- The bidder with the second lowest bid will provide 35 units,
- The bidder with the third lowest bid will provide 15 units,
- The bidder with the highest bid will provide 0 units.

Your per-unit cost, as well as the costs of the three competitors in each auction is an integer from 0 to 100, with each integer in that range being equally likely. There is no relationship between your cost and any of the other bidders' costs, or between costs in different auctions (all costs are independent).

You will see your own cost at the start of each auction. Each bidder, including you, only knows their own cost but not the cost of any other bidders.

After you observe your cost you will place a bid in the bid box on your computer screen and click the "Submit" button. Your bid can be any number, with at most two decimal places, from 0 to 100.

You earn money by being awarded units at a good price:

Your Earnings = (Your Bid - Your Cost) x (Number of Units Awarded to You),

where the number of units awarded to you is determined in the manner stated above. Please note that if your bid is below your cost, and you are awarded any units, you will lose money on each unit, so bid carefully.

Information you will see at the end of each auction

At the end of each auction you will see the following information:

- Your own cost and bid in this auction
- The bids placed by competitors
- The number of units each of the bidders were awarded
- Your earnings from the auction.

You will also have access to this information for all past auctions.

How you will be paid

At the end of the session you will see a final screen summarizing your earnings for the session. This screen will calculate your net profits from 40 auctions, convert them to US dollars at the rate of 3200 ECU per \$1, and add them to your \$5 participation fee.

Please use this information to fill out your check-out form and wait quietly until the monitor calls you to come to the front of the room and be paid your earnings in private and in cash. After you have been paid, you will be free to leave the laboratory.

Appendix: Equilibrium bid function in sealed-bid first-price auction with uniform costs and $n = 4$ suppliers

For $n = 4$ suppliers invited to bid, the equilibrium expected allocation in equation (4) can be written as

$$H(x) = \begin{pmatrix} (1-F(x))^3 \cdot Q_1 + \binom{3}{1} \cdot F(x) \cdot (1-F(x))^2 \cdot Q_2 + \\ \binom{3}{2} \cdot F^2(x) \cdot (1-F(x)) \cdot Q_3 + \\ \binom{3}{3} \cdot F^3(x) \cdot Q_4 \end{pmatrix}.$$

For uniformly distributed cost, substituting the values $F(x) = \dfrac{x-\underline{c}}{\bar{c}-\underline{c}}$ and $1-F(x) = \dfrac{\bar{c}-x}{\bar{c}-\underline{c}}$ in the above equation, we get

$$H(x) = \begin{pmatrix} (\bar{c}-x)^3 \cdot Q_1 + 3 \cdot (x-\underline{c}) \cdot (\bar{c}-x)^2 \cdot Q_2 + \\ 3 \cdot (x-\underline{c})^2 \cdot (\bar{c}-x) \cdot Q_3 + (x-\underline{c})^3 \cdot Q_4 \end{pmatrix} \cdot \left(\frac{1}{\bar{c}-\underline{c}}\right)^3.$$

Therefore,

$$-\int_{x=c}^{\bar{c}} x dH(x) =$$

$$\frac{3}{(\bar{c}-\underline{c})^3} \cdot \int_{x=c}^{\bar{c}} \left(x(\bar{c}-x)^2(Q_1-Q_2) + 2x(\bar{c}-x)(x-\underline{c})(Q_2-Q_3) + x(x-\underline{c})^2(Q_3-Q_4)\right) dx.$$

Evaluating the above integral, we get

$$-\int_{x=c}^{\bar{c}} x dH(x) = \frac{1}{(\bar{c}-\underline{c})^3} \cdot \left(\begin{array}{r} \left((3c+\bar{c})\cdot(\bar{c}-c)^3\right)\cdot\dfrac{Q_1-Q_2}{4} \\ +\left(6\cdot(c^2-cs)\cdot(\bar{c}-c)^2+(3c-2\underline{c}+\bar{c})\cdot(\bar{c}-c)^3\right)\cdot\dfrac{Q_2-Q_3}{2} \\ +\left((3\bar{c}+\underline{c})\cdot(\bar{c}-\underline{c})^3-(\underline{c}+3c)(c-\underline{c})^3\right)\cdot\dfrac{Q_3-Q_4}{4} \end{array} \right).$$

Substituting these values in Equation (6) gives the equilibrium bidding function of a supplier with cost c.

$$\beta(c) = \frac{\left(\begin{array}{r} \dfrac{(\bar{c}-c)^3}{4}\cdot(3c+\bar{c})\cdot Q_1 \\ +(\bar{c}-c)^2\cdot\left(2c^2-2c\underline{c}+c\bar{c}-\bar{c}\underline{c}+\dfrac{(\bar{c}-c)^2}{4}\right)\cdot Q_2 \\ +\left((c-\underline{c})^2\cdot\left(2c\bar{c}-2c^2-c\underline{c}+\bar{c}\underline{c}-\dfrac{(c-\underline{c})^2}{4}\right)+\dfrac{(\bar{c}-\underline{c})^4}{4}\right)Q_3 \\ +\left(c(c-\underline{c})^3+\dfrac{(\bar{c}-\underline{c})^4}{4}-\dfrac{(c-\underline{c})^4}{4}\right)Q_4 \end{array} \right)}{\left((\bar{c}-c)^3\cdot Q_1+3\cdot(c-\underline{c})\cdot(\bar{c}-c)^2\cdot Q_2+3\cdot(c-\underline{c})^2\cdot(\bar{c}-c)\cdot Q_3+(c-\underline{c})^3\cdot Q_4\right)}.$$

The above expression can be cross-checked by taking $\underline{c}=0$, $\bar{c}=1$, $Q_1=1$ and $Q_2=Q_3=Q_4=0$. We then get $\beta(c)=(3c+1)/4$ which is precisely expected cost of the second-lowest cost supplier conditional upon the cost of the lowest-cost supplier being c. ■

Appendix: Proofs

Proof of Proposition 1: To find the optimal bid b_i of supplier i, given that all other suppliers are bidding with strategy β, we first differentiate equation (5) with respect to b_i.

$$\frac{\partial\Pi(b_i,c_i,\cdot)}{\partial b_i} = H(\beta^{-1}(b_i,\cdot),\cdot)+(b_i-c_i)\frac{\partial H(\beta^{-1}(b_i,\cdot),\cdot)}{\partial b_i}. \tag{13}$$

Since $\dfrac{\partial\Pi(b_i,c_i,\cdot)}{\partial b_i}>0$ at $b_i=c_i$ for any $\underline{c}\le c_i<\bar{c}$, any unique solution to the first order condition: $\partial\Pi(b_i,c_i,\cdot)/\partial b_i=0$ would necessarily represent the bid b_i that maximizes the supplier's surplus.

We now assume that a symmetric equilibrium exists, i.e., if all other suppliers bid with strategy β then supplier i also uses strategy β. This assumption implies that the solution to the first order condition would give the bidding function $\beta(c_i,\cdot)$, i.e., $b_i=\beta(c_i,\cdot)$ (after characterizing β we show that β is indeed a symmetric equilibrium strategy, i.e., if all suppliers bid with strategy β then it is in the best interest of supplier i to also use β as its bidding strategy). Making the change of variables $b_i=\beta(c_i,\cdot)$ in the first order condition we get

$$H(c_i,\cdot) = -(\beta(c_i,\cdot)-c_i)\frac{\partial H(c_i,\cdot)}{\partial c_i}\cdot\frac{1}{\frac{\partial\beta(c_i,\cdot)}{\partial c_i}}.$$

We can therefore characterize the bidding function, given that a symmetric equilibrium exists, through the following differential equation

$$\frac{\partial \beta(c_i,\cdot)}{\partial c_i}\cdot H(c_i,\cdot)+(\beta(c_i,\cdot)-c_i)\frac{\partial H(c_i,\cdot)}{\partial c_i}=0. \tag{14}$$

We can re-write equation (14) as

$$\frac{\partial (H(c_i,\cdot)\cdot\beta(c_i,\cdot))}{\partial c_i}=c_i\frac{\partial H(c_i,\cdot)}{\partial c_i}. \tag{15}$$

At $c_i=\overline{c}$ it is an equilibrium strategy for supplier i to bid $\overline{c}$, i.e., $\beta(\overline{c},\cdot)=\overline{c}$. Thus, integrating equation (15) in the limits c_i to $\overline{c}$ gives us the solution

$$\beta(c_i,\cdot)=\overline{c}\cdot\frac{H(\overline{c},\cdot)}{H(c_i,\cdot)}-\frac{1}{H(c_i,\cdot)}\int_{x=c_i}^{\overline{c}} x\cdot\frac{\partial H(x,\cdot)}{\partial x}\cdot dx. \tag{16}$$

To find the optimal bidding strategy in equation (16) we had assumed that $\beta(c,\cdot)$ is continuously differentiable, increasing in c and that $\beta(c,\cdot)$ forms a symmetric equilibrium. Indeed, $\beta(c,\cdot)$ as characterized by equation (16) is continuously differentiable. Next, we show that $\beta(c,\cdot)$ in equation (16) also satisfies the other two assumptions and therefore formulates the equilibrium bidding strategy in the first price auction.

Differentiating $\beta(c,\cdot)$ with respect to c in equation (16) we get

$$\frac{\partial \beta(c,\cdot)}{\partial c}=-\frac{\beta(c,\cdot)}{H(c,\cdot)}\cdot\frac{\partial H(c,\cdot)}{\partial c}+\frac{c}{H(c,\cdot)}\cdot\frac{\partial H(c,\cdot)}{\partial c}=-\frac{\beta(c,\cdot)-c}{H(c,\cdot)}\cdot\frac{\partial H(c,\cdot)}{\partial c}.$$

To determine the sign of $\frac{\partial \beta(c,\cdot)}{\partial c}$ we need to determine the sign of $\frac{\partial H(c,\cdot)}{\partial c}$. For this we use the expression of $H(x,\cdot)$ from equation (4). Thus

$$\begin{aligned}\frac{\partial H(c,\mathbf{Q})}{\partial c}=&-\binom{n-1}{1}\bar{F}(c)^{n-2}f(c)\cdot(Q_1-Q_2)\\&-2\binom{n-1}{2}\bar{F}(c)^{n-3}F(c)f(c)\cdot(Q_2-Q_3)\\&-3\binom{n-1}{3}\bar{F}(c)^{n-4}F(c)^2f(c)\cdot(Q_3-Q_4)-\\&\ldots-(m-1)\binom{n-1}{m-1}\bar{F}(c)^{n-m}F(c)^{m-2}f(c)\cdot(Q_{m-1}-Q_m)-\\&\ldots-(n-1)\binom{n-1}{n-1}F(c)^{n-2}\cdot(Q_{n-1}-Q_n).\end{aligned}$$

For $Q_1\geq Q_2\geq\ldots\geq Q_n$ we get $\frac{\partial H(c,\cdot)}{\partial c}\leq 0$ and therefore $\beta(c,\cdot)$ is increasing in c.

We next show that $\beta(c,\cdot)$ forms a symmetric equilibrium, i.e., if all other $n-1$ suppliers bid with strategy β then supplier i would also bid with the same strategy. Suppose, that supplier i with marginal cost c chooses to bid $b=\beta(z,\cdot)$, i.e., it emulates a supplier with marginal cost z. Then its payoff, from equation (5), is $\Pi(b,c,\cdot)=(\beta(z,\cdot)-c)H(z,\cdot)$. Substituting the value $\beta(z,\cdot)$ we get

the supplier's payoff if it emulates a supplier of cost z as $\Pi(b,c,\cdot)=\bar{c}H(\bar{c},\cdot)-\int_{x=z}^{\bar{c}} x(\partial H(x,\cdot)/\partial x)\cdot dx - cH(z,\cdot)=(z-c)H(z,\cdot)+\int_{x=z}^{\bar{c}} H(x,\cdot)dx$. If, instead the supplier with cost c does bid $\beta(c,\cdot)$ then its payoff is $\Pi(\beta(c,\cdot),c,\cdot)=\int_{x=c}^{\bar{c}} H(x,\cdot)dx$. Since $H(c,\cdot)$ decreases in c, for any $z\neq c$, we get $\Pi(\beta(c,\cdot),c,\cdot)-\Pi(\beta(z,\cdot),c,\cdot)=(c-z)H(z,\cdot)+\int_{x=c}^{z} H(x,\cdot)dx\geq 0$. Therefore, β is indeed a symmetric equilibrium bidding strategy.■

Proof of Proposition 2: The buyer's expected cost can be characterized as

$$C_{buyer}=\begin{pmatrix} Q_1\int_{x=0}^{\bar{c}}\beta(x,\mathbf{Q})n\bar{F}(x)^{n-1}f(x)dx \\ +Q_2\int_{x=0}^{\bar{c}}\beta(x,\mathbf{Q})n\binom{n-1}{1}\bar{F}(x)^{n-2}F(x)f(x)dx \\ +\ldots+Q_n\int_{x=0}^{\bar{c}}\beta(x,\mathbf{Q})nF(x)^{n-1}f(x)dx \end{pmatrix}=n\cdot\int_{x=0}^{\bar{c}}\beta(x,\mathbf{Q})\cdot H(x,\mathbf{Q})f(x)dx.$$

Substituting the value of β from equation (6) we get $C_{buyer}=n\bar{c}Q_n-n\int_{x=0}^{\bar{c}}\int_{y=x}^{\bar{c}} ydH(y,\mathbf{Q})f(x)dx$. Integrating by parts, we get $C_{buyer}=n\int_{x=0}^{\bar{c}}(xH(x,\mathbf{Q})+\int_{y=x}^{\bar{c}}H(y,\mathbf{Q})dy)\cdot f(x)dx$. Now, $n\int_{x=0}^{\bar{c}}xH(x,\mathbf{Q})f(x)dx=Q_1\mu_1+Q_2\mu_2+\ldots+Q_n\mu_n$. We next resolve $n\int_{x=0}^{\bar{c}}\int_{y=x}^{\bar{c}}H(y,\mathbf{Q})dyf(x)dx$ term by term. For this take any m^{th} term between 1 and $n-1$ of $H(y,\mathbf{Q})$, i.e., take $Q_m\binom{n-1}{m-1}\bar{F}^{n-m}(y)F^{m-1}(y)$. We then characterize

$$n\cdot\binom{n-1}{m-1}\int_{x=0}^{\bar{c}}\int_{y=x}^{\bar{c}}\bar{F}^{n-m}(y)F^{m-1}(y)dyf(x)dx=m\cdot\binom{n}{m}\int_{x=0}^{\bar{c}}\int_{y=x}^{\bar{c}}\bar{F}^{n-m}(y)F^{m-1}(y)dyf(x)dx$$

$=m\cdot\binom{n}{m}\int_{x=0}^{\bar{c}}\bar{F}^{n-m}(x)F^{m}(x)dx$ (upon integrating by parts). Pearson (1902) showed that $\mu_{m+1}-\mu_m=\binom{n}{m}\int_{x=0}^{\bar{c}}\bar{F}^{n-m}(x)F^{m}(x)dx$ for any $m=1,\ldots,n-1$. For $m=n$ we get $\bar{c}-\mu_n=\int_{x=0}^{\bar{c}}F^n(x)dx$. Thus we have that

$$n\int_{x=0}^{\bar{c}}\int_{y=x}^{\bar{c}}H(y,\mathbf{Q})dyf(x)dx=\begin{array}{r}Q_1(\mu_2-\mu_1)+2Q_2(\mu_3-\mu_2)+\ldots+m(\mu_{m+1}-\mu_m)Q_m+\ldots\\ +(n-1)Q_{n-1}(\mu_n-\mu_{n-1})+nQ_n(\bar{c}-\mu_n).\end{array}$$

Finally adding this term back, we get

$$C_{buyer}=\begin{array}{r}\mu_2Q_1+(2\mu_3-\mu_2)Q_2+\ldots+(m\mu_{m+1}-(m-1)\mu_m)Q_m+\ldots\\ +((n-1)\mu_n-(n-2)\mu_{n-1})Q_{n-1}+(n\bar{c}-(n-1)\mu_n)Q_n.\end{array}$$

■

Proof of Lemma 1: We first show that maximizing $\sum Q_i^2$ such that sourcing constraints (1b) are satisfied would give a unique solution. We show this result by contradiction. Consider any two sets of allocations $\mathbf{Q}$ and $\mathbf{Q}'$ which maximize $\sum Q_i^2$ and satisfy the sourcing constraints (1b). Since one of the constraint is $\sum Q_i=\sum Q_i'=1$, one can always find some $1\leq l<m\leq n$ such that $Q_l>Q_l'$

and $Q_m^{'} > Q_m$ (or the reverse, i.e., $Q_l < Q_l^{'}$ and $Q_m^{'} < Q_m$). Assuming the former, i.e, $Q_l > Q_l^{'}$ and $Q_m^{'} > Q_m$ implies that an $0 < \epsilon < \min(Q_l - Q_l^{'}, Q_m^{'} - Q_m)$ can be found such that $Q_l > Q_l^{'} + \epsilon$ and $Q_m^{'} - \epsilon > Q_m$. Moreover, $Q_l^{'} + \epsilon$ and $Q_m^{'} - \epsilon$ would also satisfy the sourcing constraints. Now, $(Q_l^{'} + \epsilon)^2 + (Q_m^{'} - \epsilon)^2 = Q_l^{'2} + Q_m^{'2} + 2\epsilon(\epsilon + Q_l^{'} - Q_m^{'}) > Q_l^{'2} + Q_m^{'2}$. Thus $\mathbf{Q}^{'}$ can not be an optimal solution. Following the same steps one can show that for $Q_l < Q_l^{'}$ and $Q_m^{'} < Q_m$, $\mathbf{Q}$ can not be optimal. Hence maximizing $\sum Q_i^2$ such that constraints of equation (1b) are satisfied would give a unique solution. Next, we show that this unique solution, denoted by $\mathbf{Q}$, is also greedy. We show this result also by contradiction, i.e., if $\mathbf{Q}$ is not greedy then an allocation $\mathbf{Q}^{'}$ can be found that satisfies the constraints of equation (1b) and $\sum Q_i^{'2} \geq \sum Q_i^2$. By definition of greedy allocation, if $\mathbf{Q}$ is not greedy then for some $\epsilon > 0$ and some $1 \leq l < m \leq n$, the allocation vector $\mathbf{Q}^{'}$ would satisfy sourcing constraints if $Q_l^{'} = Q_l + \epsilon$, $Q_m{}' = Q_m - \epsilon$ and $Q_j{}' = Q_j$ for all $j = 1, \ldots, n$ excluding $j = l$ and excluding $j = m$. Now, $\sum Q_i^{'2} = \sum Q_i^2 + 2\epsilon(\epsilon + Q_l - Q_m) > \sum Q_i^2$. ■

Proof of Lemma 2: Since the decision variables $Q_1, \ldots, Q_n$ are real numbers not less than 0, and C_{buyer} in equation (7) is linear in $Q_1, \ldots, Q_n$, hence the math program (1) formulates a constrained fractional knapsack problem. Therefore, the optimal allocations are always greedy (for all sourcing constraints) if and only if the coefficients of Q_i's in equation (7) are increasing in i. This implies that the optimal allocation is always greedy to all but the lowest-bidding supplier if and only if $m\mu_{m+1} - (m-1)\mu_m \geq (m-1)\mu_m - (m-2)\mu_{m-1}$ is true for all $m > 2$ and if $n\bar{c} - (n-1)\mu_n \geq (n-1)\mu_n - (n-2)\mu_{n-1}$. For the lowest-bidding supplier it can be shown that $\mu_2 \leq m\mu_{m+1} - (m-1)\mu_m$ for all $m \geq 2$ since $\mu_2 - \mu_{m+1} \leq 0 \leq (m-1)(\mu_{m+1} - \mu_m)$ for all $m \geq 2$. Also $\mu_2 \leq n\bar{c} - (n-1)\mu_n$ since $\mu_2 - \bar{c} \leq 0 \leq (n-1)(\bar{c} - \mu_n)$ for any $n > 1$ suppliers participating in the auction. Hence allocating greedily to the lowest-bidding supplier is always optimal. ■

Proof of Theorem 1: Rearranging the terms of equation (7) gives: $C_{buyer}(\mathbf{Q}) = \sum_{j=1}^{n}(\mu_j Q_j + \sum_{i=j}^{n}(\mu_{i+1} - \mu_i)Q_i)$. For a given vector of costs $\mathbf{c} = c_1, \ldots, c_n$, we define

$$q(x, \mathbf{c}) \equiv \begin{cases} Q_i \text{ if } C_{i:n} \leq x \leq C_{i+1:n} \text{ for all } i = 1, \ldots, n-1, \\ Q_n \text{ if } C_{n:n} \leq x \leq \bar{c}, \end{cases}$$

where $C_{j:n}$ represents the jth-lowest cost from the sample $\mathbf{c}$. One can then express, $C_{buyer}(\mathbf{Q}) = \mathbb{E}_{\mathbf{c}}(\sum_{j=1}^{n}(C_{j:n}Q_j + \int_{x=C_{j:n}}^{\bar{c}} q(x, \mathbf{c})dx)) = \mathbb{E}_{\mathbf{c}}(\sum_{i=1}^{n}(c_i q(c_i, \mathbf{c}) + \int_{x=c_i}^{\bar{c}} q(x, \mathbf{c})dx))$. Integrating by parts gives $\mathbb{E}_{\mathbf{c}}(\int_{x=c_i}^{\bar{c}} q(x, \mathbf{c})dx) = \mathbb{E}_{\mathbf{c}}(q(c_i, \mathbf{c})F(c_i)/f(c_i))$. Thus, $C_{buyer}(\mathbf{Q}) = \mathbb{E}_{\mathbf{c}}(\sum_{i=1}^{n}(c_i + F(c_i)/f(c_i))q(c_i, \mathbf{c}))$. Thus, for a regular distribution (i.e., $c + F(c)/f(c)$ increasing in c) shifting an $\epsilon > 0$ from Q_i to Q_j for any $i < j$, with everything else being the same, would increase $C_{buyer}(\mathbf{Q})$. Finally, shifting $\epsilon > 0$ from Q_i to Q_j for any $i < j$, with everything else being the same, would increase the Herfindahl-Hirshman index.■

Proof of Proposition 3: We show three different proofs of this proposition for A, B and C respectively. In all these proofs, we denote by z the number of suppliers that get a strictly positive allocation. If the allocations are greedy then z can be characterized as

$$z = \max(M, \lceil 1/A \rceil).$$

<u>**Constraint A**</u>: Let $z_A \geq 0$ denote the number of suppliers that get allocated A under the optimal allocation. Since the greedy allocation is optimal for regular cost distributions, z_A can be characterized as

$$z_A = \left\lfloor \frac{1 - B \cdot z}{A - B} \right\rfloor.$$

Indeed, $dz_A/dA \leq 0$. We next consider cases for which $dz_A/dA = 0$, i.e., changing A to $A + \epsilon$ for an $\epsilon > 0$ does not change z_A. For $z_A \geq 1$, changing A to $A + \epsilon$ (such that $dz_A/dA = 0$) would transfer ϵ amount to each of the allocations $Q_1, \ldots, Q_{z_A}$ from the allocation Q_{z_A+1} (since allocating greedily is optimal). From equation (7) we see that this would result in a change in C^*_{buyer} of $z_A \epsilon \mu_{z_A+1} - z_A(z_A+1)\epsilon\mu_{z_A+2} + z_A^2 \epsilon \mu_{z_A+1} = -z_A(z_A+1)\epsilon(\mu_{z_A+2} - \mu_{z_A+1})$. Indeed, for $z_A = 0$ and $dz_A/dA = 0$, the constraint A is non-binding and therefore there would be no change in C^*_{buyer}. Hence

$$dC^*_{buyer}/dA = -z_A(z_A+1)(\mu_{z_A+2} - \mu_{z_A+1}) \leq 0 \text{ for } dz_A/dA = 0. \tag{17}$$

This implies that C^*_{buyer} is piecewise linear and decreasing in A. Moreover, from Theorem 1 we know that for regularly distributed costs $(z_A+1)\mu_{z_A+2} - z_A\mu_{z_A+1} \geq z_A\mu_{z_A+1} - (z_A-1)\mu_{z_A}$ for all $z_A \geq 1$. Multiplying both sides of the inequality by z_A and then subtracting $z_A\mu_{z_A+1}$ from both sides of this inequality gives $z_A(z_A+1)(\mu_{z_A+2} - \mu_{z_A+1}) \geq (z_A-1)z_A(\mu_{z_A+1} - \mu_{z_A})$. Therefore, dC^*_{buyer}/dA is non-decreasing in A.

$\mathbf{d^2C^*_{buyer}/dAdB \geq 0}$: Finally, from equation (17) we see that dC^*_{buyer}/dA is non-increasing in z_A, for regularly distributed costs. Moreover, z_A is non-increasing in B. Hence dC^*_{buyer}/dA is non-decreasing in B.

<u>**Constraint B**</u>: Let z_B denote the number of suppliers that get allocated B. Since the greedy allocation is optimal for regular distributions, z_B can be characterized as

$$z_B = z - \left\lceil \frac{1 - B \cdot z}{A - B} \right\rceil.$$

Indeed, $dz_B/dB \geq 0$. Since greedy is optimal, for $z_B \geq 1$, the allocations $Q_{z-z_B+1} = \ldots = Q_z = B$. For $z_B \geq 1$, changing B to $B + \epsilon$ such that $dz_B/dB = 0$ for an $\epsilon > 0$ would result in a transfer of ϵ to each $Q_{z-z_B+1}, \ldots, Q_{z_B}$. Since the greedy allocation is optimal, $z_B \cdot \epsilon$ (the total allocation

transferred) would be transferred from Q_{z-z_B}. From equation (7) we see that this would result in a change in C^*_{buyer} of

$$C^*_{buyer}(A,B+\epsilon,M)-C^*_{buyer}(A,B,M)=\left\{\begin{array}{l}-\langle(z-z_B)\mu_{z-z_B+1}-(z-z_B-1)\mu_{z-z_B}\rangle z_B\cdot\epsilon\\+\langle(z-z_B+1)\mu_{z-z_B+2}-(z-z_B)\mu_{z-z_B+1}\rangle\cdot\epsilon+\dots\\+\langle(z\mu_{z+1}-(z-1)\mu_z\rangle\cdot\epsilon\end{array}\right\}.$$

Thus for $dz_B/dB=0$ we can write

$$dC^*_{buyer}(A,B,M)/dB=\left\{\begin{array}{l}-\langle(z-z_B)\mu_{z-z_B+1}-(z-z_B-1)\mu_{z-z_B}\rangle\cdot z_B\\+\langle(z-z_B+1)\mu_{z-z_B+2}-(z-z_B)\mu_{z-z_B+1}\rangle+\dots\\+\langle(z\mu_{z+1}-(z-1)\mu_z\rangle\end{array}\right\}.$$

For regular distributions we know from Theorem 1 that $(z-z_B+m+1)\mu_{z-z_B+m+2}-(z-z_B+m)\mu_{z-z_B+m+1}\geq(z-z_B)\mu_{z-z_B+1}-(z-z_B-1)\mu_{z-z_B}$ for any $m\geq-1$. Therefore $dC^*_{buyer}/dB\geq 0$. Indeed, for $z_B=0$, there would be no change in C^*_{buyer} if B is changed, as long as $dz_B/dB=0$. Next, we show how dC^*_{buyer}/dB changes as one changes z_B. For this we increase B to $B^{'}$ such that $z^{'}_B=z_B+1$. We can then write

$$dC^*_{buyer}(A,B^{'},M)/dB=\left\{\begin{array}{l}-\langle(z-z^{'}_B)\mu_{z-z^{'}_B+1}-(z-z^{'}_B-1)\mu_{z-z^{'}_B}\rangle\cdot z^{'}_B\\+\langle(z-z^{'}_B+1)\mu_{z-z^{'}_B+2}-(z-z^{'}_B)\mu_{z-z^{'}_B+1}\rangle+\dots\\+\langle(z\mu_{z+1}-(z-1)\mu_z\rangle\end{array}\right\}.$$

Note that z remains unaffected from changes in B. Substituting $z^{'}_B=z_B+1$ in the above equation gives

$$dC^*_{buyer}(A,B^{'},M)/dB=\left\{\begin{array}{l}-\langle(z-z_B-1)\mu_{z-z_B}-(z-z_B-2)\mu_{z-z_B-1}\rangle\cdot(z_B+1)\\+\langle(z-z_B)\mu_{z-z_B+1}-(z-z_B-1)\mu_{z-z_B}\rangle+\dots\\+\langle(z\mu_{z+1}-(z-1)\mu_z\rangle\end{array}\right\}.$$

Again, for regular distributions we know that $(z-z_B-1)\mu_{z-z_B}-(z-z_B-2)\mu_{z-z_B-1}\leq(z-z_B)\mu_{z-z_B+1}-(z-z_B-1)\mu_{z-z_B}$. Hence $dC^*_{buyer}(A,B^{'},M)/dB\geq dC^*_{buyer}(A,B,M)/dB$. Therefore, dC^*_{buyer}/dB is non-decreasing in B. Finally, we can simplify the expression for dC^*_{buyer}/dB to

$$\begin{array}{c}dC^*_{buyer}/dB=-z_B(\mu_{z-z_B+1}-\mu_{z-z_B})(z-z_B-1)+z(\mu_{z+1}-\mu_{z-z_B+1})\\ \text{for } dz_B/dB=0 \text{ and for } z_B\geq 1 \text{ and } dC^*_{buyer}/dB=0 \text{ for } z_B=0.\end{array}\tag{18}$$

<u>Constraint M</u>: Note that changing M would effect the parameters z and z_A defined above. We denote these parameters as functions of M, i.e., as $z(M)$ and $z_A(M)$. Increasing M to $M+1$ would increase C^*_{buyer} only if an additional bidder gets an allocation of B. Thus $C^*_{buyer}(M+1)-C^*_{buyer}(M)\neq 0$ only if $z(M+1)-z(M)>0$, i.e., if $z(M)=M$. For $z(M)=M$, increasing M to $M+1$ implies that the $(M+1)^{th}$ lowest cost supplier now gets allocated B instead of 0. Since allocations are greedy, the allocation B to $(M+1)^{th}$ ranked supplier is transferred from the allocation of $(z_A(M)+1)^{th}$ and higher ranked (lower cost) suppliers (such that the sourcing constraints are not violated). A maximum allocation of $Q_{z_A(M)+1}-B$ can be transferred from the $(z_A(M)+1)^{th}$

ranked supplier. The remaining $2B - Q_{z_A(M)+1}$ has to be transferred from higher ranked (lower cost) suppliers. A maximum of $A-B$ can be transferred from each of the higher ranked suppliers, therefore $A-B$ is transferred from

$$z_M = \max(\lfloor \frac{2B - Q_{z_A(M)+1}}{A-B} \rfloor, 0)$$

suppliers. To sum up, a total of B is transferred to Q_{M+1} as follows: $Q_{z_A(M)+1} - B$ from $Q_{z_A(M)+1}$, $A-B$ each from $Q_{z_A(M)}, \ldots, Q_{z_A(M)-z_M+1}$ and $(2B-(A-B)z_M - Q_{z_A(M)+1})^+$ from $Q_{z_A(M)-z_M}$. Thus the net change in C^*_{buyer} can be found from equation (7) as

$$\begin{aligned}
&C^*_{buyer}(A,B,M+1) - C^*_{buyer}(A,B,M) = B((M+1)\mu_{M+2} - M\mu_{M+1}) \\
&-\langle (z_A(M)+1)\mu_{z_A(M)+2} - z_A(M)\mu_{z_A(M)+1}\rangle (Q_{z_A(M)+1} - B) \\
&-z_A(M)\mu_{z_A(M)+1}(A-B) + (z_A(M) - z_M)\mu_{z_A(M)-z_M+1}(A-B) \\
&-\langle (z_A(M) - z_M)\mu_{z_A(M)-z_M+1} - (z_A(M) - z_M - 1)\mu_{z_A(M)-z_M}\rangle \cdot (2B - (A-B)z_M + Q_{z_A(M)+1})^+, \\
&\text{for } z(M) = M \\
&\text{and } C^*_{buyer}(A,B,M+1) - C^*_{buyer}(A,B,M) = 0 \text{ otherwise,}
\end{aligned} \tag{19}$$

where $(x-a)^+ \equiv \max(x-a, 0)$. Since $M \geq z_A(M)$ and since for regular distributions conditions of Lemma 2 hold, therefore $C^*_{buyer}(M+1) - C^*_{buyer}(M) \geq 0$. Also $z_A(M+1) \leq z_A(M) - z_M$, since $A-B$ was transferred from $Q_{z_A(M)}, \ldots, Q_{z_A(M)-z_M+1}$ ranked suppliers. Hence increasing $M+1$ to $M+2$ would transfer B to the $(M+2)^{th}$ ranked supplier from suppliers ranked $z_A - z_M$ and higher (i.e. lower cost). Hence $C^*_{buyer}(M+2) - C^*_{buyer}(M+1) \geq C^*_{buyer}(M+1) - C^*_{buyer}(M)$.■