

Mathematics for Elementary and Middle School Teachers

MTH 485 Summer 2006

Course Instructors:

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Meeting Dates and Times:

Monday, July 17: 1:10pm – 4:00pm
Wednesday, July 19: 1:10pm – 4:00pm
Monday, July 24: 9:10am – 12:00pm
Wednesday, July 26: 9:10am – 12:00pm
Monday, July 31: 9:10am – 12:00pm
Tuesday, August 1: 9:10am – 12:00pm
Thursday, August 3: 9:10am – 12:00pm
Monday, August 7: 9:10am – 12:00pm
Tuesday, August 8: 9:10am – 12:00pm
Thursday, August 10: 9:10am – 12:00pm
Monday, August 14: 9:10am – 12:00pm
Tuesday, August 15: 9:10am – 12:00pm

Location: Morning classes are in 2310; afternoon classes are in 2229
School of Education Building

Class email list: elmac9a@umich.edu

We will use email extensively to communicate with you; we encourage you to do the same with us, and with others in the class. Please check your email regularly.

Course website: <http://www-personal.umich.edu/~sleepl/math485elmac9a.htm>

We will use our course website to post assignments and documents from class. On it you will also find links to some interesting mathematics teaching resources. Be sure to let us know if internet access will be a problem for you during the course.

Study hall: To be determined based on our survey of your availability and preferences.

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A. Course Focus:

This course is intended for students who are preparing to teach mathematics at the elementary and middle school level. The course focuses on the *mathematical* resources needed for teaching mathematics. Together with ED 518 (Teaching Children Mathematics), which emphasizes more explicitly the *instructional* aspects of mathematics teaching, this course is designed to prepare you with the mathematical skills, knowledge, and ways of reasoning that you need for teaching mathematics. The goal across the two courses is to help you develop a sense of yourselves as professionals who are capable of teaching mathematics well. It is helpful to think of this as a continuum, moving from simply knowing mathematics as an educated adult, to knowing mathematics as a teacher, to knowing about ways to help others learn mathematics. The two courses are designed to help you move along that continuum for a variety of mathematical topics and skills.

This is a mathematics course designed especially for people who seek to become teachers of mathematics. Teaching mathematics requires a great deal of specialized mathematical knowledge - knowledge that is different from what it takes to do well in a math course as a student yourself or to be good at other jobs that require mathematics. Your own understanding, fluency, and comfort with mathematics will be important to your effectiveness as a teacher. In preparing to teach, you will have to determine the mathematical goals of activities, anticipate the varied ways students might respond, and prepare mathematically for what might happen as a lesson unfolds. You will need to prepare good questions to ask. You will have to generate easier as well as harder versions of problems, either as back-up plans or as ways to focus or extend students' work. As a teacher, you will need a keen sense of the complexity of particular mathematical ideas, and ways they can be scaffolded for students' learning. When your students have trouble, or get answers wrong, you will need to do more than know that they are confused, or that they have incorrect solutions: You will need to be able to figure out what they are doing mathematically, and whether it makes sense.

One thing to notice about all these examples in the previous paragraph is that they are all instances of mathematical thinking that teachers have to do in the course of their work. They are not all things that other adults have to be able to do with mathematics. This is what we mean when we say that teaching requires *specialized mathematical knowledge*, and the course is designed to help you revisit and extend your own mathematical knowledge and build it into this specialized kind of knowledge you will need as a teacher.

B. Structure of the Course:

The course is structured around two main strands: learning mathematics for *yourself as a teacher* (mathematical content for teaching), and connecting the mathematics we will be working on with the *work of teaching* (mathematical applications for teaching). The goal is to use these strands as sites to learn the mathematics in ways you will need it in your future mathematics teaching.

- Mathematical content for teaching: Learning mathematics for *yourself as a teacher*. This course is about providing you with resources necessary to *understand* the mathematics you will need for your work as a teacher of mathematics. In other words, the course is designed for you as a *learner* of mathematics who, specifically, is becoming a *teacher* of mathematics.

While working on the mathematics in MTH 485, we will not yet focus on how children would understand certain mathematical ideas or explanations. Although we may occasionally discuss some points about children or teaching, we will not this term systematically take up how to represent the ideas to make them accessible to children, how children typically understand certain ideas, or how to teach particular content. This is not because these issues are not important. On the contrary, these issues are right at the heart of the work you will be doing as a teacher and, for this reason, will be the focus of your fall term work in ED 518. In this course, we will focus primarily on the necessary foundation for this work by concentrating on the

mathematical thinking and reasoning that teachers have to do in order to teach students mathematics.

- Mathematical applications for teaching: Connecting the mathematics we will be working on with the *work of teaching*. In this course, we will make an explicit attempt to connect the mathematical ideas we will be working on to the *work of teaching*. This strand aims to achieve this goal and distinguishes it clearly from courses not designed for people who plan to become teachers of mathematics.

The applications to the work of teaching will take many different forms, all of them associated with *mathematical* activities in which *teachers* frequently engage. Examples of such mathematical activities include: analyzing students' work; interpreting student ideas; giving and evaluating mathematical explanations; appraising unexpected claims, solutions, and methods; analyzing mathematically curriculum materials and resizing mathematical problems; and examining correspondences among representations and solutions. The teachers who engage in these activities need to have a specialized kind of mathematical knowledge different from that needed by other professionals. For example, why would an engineer need to know whether or not dividing fractions can be done by dividing numerators and denominators? The "invert and multiply" procedure would be enough for the needs of the engineer's work. A teacher, however, needs to be able to decide accurately and quickly about this problem to respond to a child who brought this up or to organize instruction around this idea so that students' understanding of operations with fractions will deepen.

In service of this strand of the course, we will make available to our work *records of practice* (e.g., videotapes of classrooms and student interviews, transcripts, student work, curriculum materials). These resources will help us get more of a feel of what the *mathematical* work of teaching is and will allow us to look up close at the *mathematics* that arises in teaching: What do students think about and do mathematically? And therefore: What do teachers have to manage mathematically?

C. Goals of the Course:

In this course, we will work on developing mathematics knowledge that is useful for teaching. The mathematics that we have selected has two major strands, one that is typically thought of as "content" - topics, ideas, procedures in specific mathematical domains, and a second strand that centers more on the particular mathematical thinking, skills, and reasoning involved in the mathematical tasks that teachers do.

- Unpacking and development of flexible understanding of *important mathematical ideas and processes* within the areas of number theory (e.g., factors and multiples; even, odd, prime, composite, and square numbers; divisibility tests); connections between algebra and geometry; fractions (different meanings and representations, e.g., fractions as parts of wholes, as portions of discrete sets, as division, as points on a number line).
- Development of the ability to engage in important *mathematical practices* that are central to teaching, that is, important activities that successful teachers do and that are mathematical in nature. Particular emphasis will be placed on the following mathematical practices:
 - Explanation: *Explanation* refers to the forms of expression that justify mathematically and help others understand why a mathematical claim is true. We will explore central features of an adequate mathematical explanation; purposes of explanation; the social nature of explanation (difference between explaining to oneself and explaining to

others); providing adequate explanations; listening to and appraising other's explanations.

- Representation and recording: Choosing and using representations (verbal, symbolic, visual, material, manipulative, technological); examining correspondences and equivalences among representations; attending to and making sense of representations different from one's own.
- Understanding and reacting to mathematical thinking that is different from your own: Making sense of and responding to others people's ideas. As a teacher, you will need to understand, evaluate, and react to your students' explanations and representations. In preparing to do this well, it is important that you get accustomed to thinking in multiple ways and to identifying correspondences among alternative explanations and representations. A good way to start preparing for this kind of work is to pay particular attention to other people's thinking in our class. Can you make sense of it? Is it mathematically valid? How would you respond to it?
- Talking mathematically: Communicating your thinking orally in a clear, convincing, and accurate way, and making use of appropriate representations when applicable. Teachers do a lot of talking. The opportunities you will have in this class to practice this skill will not only help you prepare to become a teacher of mathematics, but will also help you learn things better. Being able to express well your mathematical ideas orally is both a prerequisite for, and an indication of, deep understanding.

These practices were selected because of their centrality in learning and doing mathematics, as well as their crucial role in teaching mathematics. Teachers must explain mathematics, and must size up students' explanations. Classrooms are settings where mathematical ideas are constantly being represented and recorded, in public and in individual work. And since talk is the medium of teaching and learning in any view of teaching, teachers' care with language matters to their clarity, their capacity to help students understand, and their ability to consider seriously how to manage the connections and conflicts between everyday and mathematical language.

D. Course Requirements:

Attendance and class participation: Your participation in our class activities and discussions is important not only for your own learning but also the learning of others. Sharing your solutions, undeveloped ideas, and puzzlements with the group, as well as responding to classmates' ideas, statements, and questions, are critical to our work together. As a teacher, you need to do more than understand your own thinking, or your own solutions to math problems—you have to be able to track on others' thinking, figure out what others are saying, and determine whether and how they make sense mathematically. In our class, the "others" will be your classmates; but in the future, they will be your students. So listening to and interacting with others in our class is explicitly to help you develop mathematical dispositions and skills that matter for teaching.

We expect you to attend every class, to arrive on time for a prompt start, and to participate in and contribute to class. If circumstances prevent you from attending class, please send us an email or call in advance.

Notebook: Another form of participation in the course will be to keep mathematical and other drawings, notes, and reflections in a special mathematics notebook. The work you do in your notebook is a central part of the course. You will use the notebook to create a record of your own work and thinking, as well as our class' accumulating understandings, investigations, conjectures, arguments,

and solutions. Your notebook will be a place for you to track and record your thinking about issues and ideas. It will also provide an additional avenue of communication between us. You will use your notebook to record the work we do in class including solving mathematics problems and analyzing the work of teaching. Some out-of-class assignments will also be completed in your notebook, as described below.

You will have daily assignments for each class. These will include mathematical problems that help you develop mathematical knowledge for teaching, as well as reflections on or extensions of the work we have done in class. Daily assignments will be posted on our course website after each class and should be completed in your notebook before our next class meeting. These assignments are designed to provide additional practice or to give you an opportunity to prepare for upcoming mathematical work. They will be considered part of your notebook grade.

This notebook may not be like other journals you have kept in the past. Think of it like a sort of laboratory notebook where one keeps systematic records of experiments, or the notebooks that photographers keep about lighting, lenses, and settings. Bird-watchers keep notebooks; so do cooks and architects. These records all help these professionals keep track of, study, and improve their practice. You are developing a notebook to record and support your learning as a teacher of mathematics.

We will collect your notebook at the end of each class (except for the first week), and it will be available for you to pick up later that day. We will read your notebooks to help us better understand what you are learning in the course, how you are engaging with class work and assignments, the issues and insights you are keeping track of, and the questions you have, as well as to help us plan for our next class together. In general we won't write individual comments in your notebook, but we will often share observations with the class, including suggestions for amplifying your use of this tool to foster your own learning.

To earn full credit for the notebook portion of your final grade, you need to make records of your in-class work and thoughtfully complete all assigned tasks. In addition, you should actively try to make connections among the various aspects of our work. Notebooks that are chronically thin, haphazard, inaccurate, and/or missing more than a very small amount of what is expected will be considered unsatisfactory and graded accordingly. We will notify you right away if your notebook is unsatisfactory and suggest a course of action to help you improve your work. Therefore, unless you hear otherwise, you can assume your notebook meets the current expectations for full credit. One way to do particularly productive work is to keep track of other's solutions, annotate and reflect on your work, use multiple representations. We will talk more about ways to work in your notebook in class, too.

Partner assignments: In addition to the daily assignments described above, you will have three partner assignments to be done over a weekend, one assigned at the end of each week except the week of the midterm and the last week. These assignments will provide opportunities for practice and review, as well as provide opportunities to extend and apply the mathematical work we have done in class. You will complete these activities with an assigned partner and turn in a joint write-up. Partner assignments will not be completed in your notebook, but you are encouraged to paste your assignments in your notebook after you have gotten them back so that your notebook is a good record of your thinking and your work.

We will give you regular feedback on your assignments and on the quality of your work. Our goal is to help you make good use of this course's opportunities to learn mathematics for teaching.

Examinations: There will be two examinations: a take-home midterm and an in-class final. The midterm will be distributed at the end of class on Tuesday, August 1, and will be due at the beginning of class on Thursday, August 3. The final exam will be held in class on Tuesday, August 15.

Consistent with the course goals of developing your mathematical knowledge for teaching, these exams will ask you to demonstrate your performance in the main areas of the course. The tasks and problems you will be asked to do will be consistent with those you encountered during the course. You will be able to prepare in advance for the performances that the exam will ask of you, working with others if you choose. Although the exams must be completed individually, you may use your notebook, and other preparation, during the exams.

E. Course Materials:

All materials needed for the course, including your notebook, will be provided. We will also have colored pencils, fine-line markers, scissors, glue sticks, and rulers available for work in your notebook; however, you may wish to bring anything you especially like to use.

F. Grading and Evaluation:

Everyone is expected to complete all course requirements. The effort, detail, and thoughtfulness you put into your work should reflect the standards of performance you will be expected to meet as a teacher: meticulous preparation; use and application of mathematical knowledge; careful consideration of alternatives; genuine curiosity about ideas (including those of others); collegial work; analysis and reflectiveness; clear expression, with respect for the place and value of precision; organization; and timeliness. Please let us know if you have special needs so that we can best accommodate you as a learner.

Your final grade will be composed as follows based on your performance of each of the course requirements. Specific details about grading criteria will accompany assignments and exams.

Attendance and participation in class	15%
Notebook (including daily assignments)	15%
Partner assignments	20%
Midterm examination	25%
Final examination	25%