Teaching Children Mathematics
ED 411
Fall 2006

Course Instructors:

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Class meetings: Tuesdays
4:10 pm – 7:00 pm
Room 4212

Exceptions: We will not have class on Tuesday, November 21 (the week of Thanksgiving). We will make up this class the week of Fall Break, on Thursday, October 19, from 4:10 – 7:00 pm in Room 2228.

Our in-class final exam will be held on Tuesday, December 12, from 4:10 – 7:00 pm.

Office hours: By appointment

Class email list: 2005cohort2@umich.edu

Course website: http://www-personal.umich.edu/~sleepl/ed411fall2006.htm

We will use email extensively to communicate with you; we encourage you to do the same with us, and with others in the class. We will use our course website to post assignments and documents from class. On it you will also find links to some interesting mathematics teaching resources. Be sure to let us know if internet access will be a problem for you during the course.

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Course Goals and Themes

This course focuses directly on four domains of teachers' work crucial to becoming a skillful beginning teacher of mathematics:[1]

1. Leading a whole class discussion about mathematics;
2. Representing mathematical ideas;
3. Assessing students' mathematical knowledge, skills, and dispositions; and
4. Planning mathematics lessons.

[1] There is substantial evidence to support the importance of acquiring professional knowledge and skills in these particular practice domains, grounded in the principles formulated in the next section. Further information about and support for these practices and principles can be located in professional standards articulated in the following resources: Interstate New Teacher Assessment and Support Consortium (INTASC) Core Standards; National Board for Professional Teaching Standards (NBPTS) Core Propositions; Professional Teaching Standards from the National Council of Teachers of Mathematics (NCTM); University of Michigan Student Teaching Handbook; State of Michigan Standards for Ensuring Excellent Educators; and Mathematical Education of Teachers (MET) from Conference Board of the Mathematics Sciences (CBMS). There is also research focusing on teacher learning, development, and the knowledge/skill set needed by beginning teachers that supports work on the principles and practices articulated in this syllabus (e.g. Feiman-Nemser, 2000; Reynolds, 1992).

You will learn specific teaching techniques in each domain, as well as to appraise the effectiveness of your mathematics teaching. The practices on which this course focuses are important for good mathematics teaching. Interestingly, they are also central to the discipline of mathematics. Acquiring professional knowledge and skills in these four domains will support your daily work as a teacher.

Domain 1: Leading a whole class discussion about mathematics
Discussion can play a central role in the learning and teaching of mathematics. Productive mathematics discussions can be held in any classroom setting, across a wide variety of mathematical topics, and with most curriculum materials. However, productive mathematical discussions do not happen by chance. During discussions, teachers pose questions to elicit, engage, and challenge students' thinking. Teachers support students' mathematical reasoning and explanations, and decide when to add clarification or make connections. They monitor students' participation, supporting the engagement of all students and encouraging students to listen and respond to each other's contributions. By the end of this course, you will be able to use and analyze your use of specific techniques to conduct a mathematics discussion.

Domain 2: Representing mathematical ideas
Mathematical ideas and processes can be represented in many ways. Having a wide repertoire of mathematical representations and knowing how to establish the equivalence of different representations are important not only in doing mathematics but also in teaching it. For example, with a given task or explanation, one representation of the mathematics may be better suited than another to solve the problem or to make an explanation clear. By the end of this course, you will be able to use representational resources in several mathematical domains, particularly numbers (whole numbers, integers, fractions, decimals) and the operations of arithmetic (addition, subtractions, multiplication, division). You will be able to construct careful correspondences between physical and pictorial representations and some of the key ideas and procedures of the K – 8 curriculum. You will also have developed more general skills for representing mathematical ideas in domains beyond those in this course.

Domain 3: Assessing students' mathematical knowledge, skill, and dispositions
Finding out what students know, how they work on mathematical tasks, and their dispositions toward the subject is essential for responsible instruction. Teachers use a variety of assessment practices to improve their teaching, to document their students' achievement, and to inform students, parents, and other educational stakeholders. Assessment encompasses much more than grading and testing. It includes interacting with students as they are learning, pausing to document what students are saying, and noticing patterns in students' work. By the end of this course, you will be able to use multiple assessment techniques and make use of the information you gain.

Domain 4: Planning mathematics lessons
Good mathematics teaching does not happen by chance. Effectively connecting students with mathematics requires *planning* — deliberate design and preparation. Furthermore, planning matters more than ever in an environment where there are so many demands on teachers. Good teaching depends on being able to manage these multiple demands. To provide high-quality mathematics instruction, teachers need to know in detail both the mathematics and the students they are teaching. Instruction must focus on essential content and skills, be coherent and engaging, and support diverse learners. When planning lessons, teachers consider whether the tasks and examples appropriately represent and engage students in the central mathematical ideas of the lesson. Teachers also think about what students will need to know in order to do the work, anticipate likely misconceptions or confusions, and figure out ways to support students' learning. They consider whether the specific features of the lesson, such as the context or language, are accessible to and support the mathematics learning of all students. Even when teachers have well-designed instructional materials, they still need to plan their lessons to adapt them to district priorities and resources and to specific learner needs. By the end of this course, you will have developed skills with planning that allow you to take account of both your students and the mathematics as you prepare for day-to-day instruction.

By developing skills in these four domains, you will be learning to *teach mathematics effectively to all your students*. When we say "effectively," we mean that, because the goal of teaching is students' learning, successful teaching is measured by whether or not students actually learn. And when we say "all your students," we signal that good mathematics teaching is teaching that works for each of your students, promoting each student's equitable access to and learning of mathematics. Learning to attend to your students' learning, and by all your students, is essential. In addition, we aim to help you develop a greater sense of yourself as a professional, as someone joining a community of practice that shares norms, specialized knowledge, and ethical commitments.

Principled Practice

Knowing and being skillful with particular teaching practices is only part of what it takes to teach well. Proficient teaching also requires the professional judgment needed to make decisions about what to do in specific contexts, with particular students and content. Professional practice is guided by *principles.* Principles are overarching professional commitments, drawn from the values and wisdom of the teaching profession, academic disciplines, and society. Principles are not only useful in guiding interpretation and judgment, but also for supporting action in novel situations, adding new practices to your repertoire, and fine-tuning your skills. We have synthesized numerous professional standards, University of Michigan and governmental policy documents, and research to arrive at the following four principles as foundations for skillful teaching of mathematics:[1]

1. Attending to the integrity of the mathematics presented to students;
2. Committing to the learning and achievement of all students;
3. Establishing and managing a productive learning environment; and
4. Learning from and systematically improving practice.

These are not separate considerations, but rather an integrated foundation for our work. Our attention to these principles cuts across the four domains of practice as represented in the following graphic:

Guiding **Principles** / Domains of Teaching **Practice**	Attending to the **integrity of the mathematics**	Committing to the learning and achievement of **all students**	Establishing and managing a **productive learning environment**	Learning from and systematically **improving practice**
Leading a whole class discussion about mathematics				
Representing mathematical ideas				
Assessing students' knowledge, skill, and disposition				
Planning mathematics lessons				

In this table, particular examples of principled work of teaching can be located in the cells. For instance, a teacher who is leading a discussion considers issues about the integrity of the mathematics being discussed, concerns herself with the participation of all of her students, attends carefully to the environment, and considers afterwards how well the discussion worked to help students learn. You can also see that your concern for creating a productive learning environment is manifest in each of the practice domains.

Principle 1: Attending to the integrity of the mathematics
Good teaching depends on the substantive quality of the mathematics and the intellectual honesty of its presentation. The content presented to students should accurately, clearly, and accessibly represent the mathematical ideas and practices that students are learning and should support their future learning.

A primary consideration of good teaching is attention to the quality of the mathematical content and its treatment. This means that the mathematics that students encounter should be "honest" and should not distort mathematical ideas for the sake of convenience (e.g., to make the ideas easier to remember in the moment). Of course the ideas must also be comprehensible to students. Being able to manage these twin imperatives — integrity of the math and accessibility to students — depends on your understanding of and ability to use mathematics in ways needed for the work of teaching.

Teaching mathematics requires specialized mathematical knowledge and reasoning. Your own understanding, fluency, and comfort with mathematics will be important to your effectiveness as a teacher. Still, teaching depends on kinds of mathematical understanding and skill different from what it takes for you as a student to do well in a math course, or to be good at other jobs that require mathematics. For example, in preparing to teach, teachers need to determine the mathematical goals of activities, anticipate the varied ways students might respond, and prepare for what might happen as the lesson unfolds. Teachers also choose good examples and ask strategic questions. These are but some of the examples of important *mathematical* work that teachers do.

We will work this semester on developing mathematics knowledge that is useful for teaching and on learning to use mathematics as teachers. We will focus on the following mathematical content: place value with whole numbers and decimals; the operations of addition, subtraction, multiplication, and division; and procedures, meanings, and representations for whole and decimal number computation. Within these topics, we will also work on some particular mathematical practices: using mathematical language, representing mathematical ideas, and reasoning mathematically. We selected these mathematical topics and practices because they are a core part of the K-8 curriculum, because they are difficult for many students, and because teaching them well is not easy. Your fluency in these areas of mathematics will give you significant leverage as a beginning teacher.

Principle 2: Committing to the learning and achievement of all students
Good mathematics teaching embodies high expectations for all students, carried out with explicit support for each student's mathematical learning. These expectations are informed by national, state, and local goals. Attending to each student's attainment of these expectations demands active efforts to counteract inequities in opportunity and outcomes.

A teacher's commitment to promote equity in student learning and achievement is essential. "Promoting equity" means teaching in ways that actively support the learning of every student and that do not inadvertently reproduce inequality across social groups. The fact is that our system continues to produce dramatically uneven results for different groups of students. While a small fraction of students do become mathematically proficient in school, most do not. And those who do not are disproportionately students of color and students living in poverty. These differences in achievement are due not to innate deficiencies in the students, but in the quality of the opportunities and instruction provided them. Teaching can shape students' experiences, their sense of themselves as mathematics learners, and the development of their mathematical capacities. We ask: What can teachers do to promote equity in their mathematics teaching? Your work with your students can develop mathematical knowledge and practices that help them be successful with mathematics. It can capitalize and account for the different cultural and linguistic resources that students bring to instruction. Teachers need more than awareness and commitment; they also need skills and knowledge in order to *act effectively on their commitments to high quality equitable mathematics instruction for all students.* In this course, you will learn more about the students you will teach and will develop teaching practices that are sensitive to and respectful of differences. These practices will enable you to help each of your students become successful mathematics learners.

In order to do these things well, teachers need to appreciate what kinds of understanding and skill they are aiming at with their students. So we ask: What is involved in knowing and doing mathematics proficiently? This course is designed to help you consider what it means to be capable with mathematics. We begin with a provisional definition: that mathematical proficiency involves conceptual understanding, procedural fluency and skill, and the use of a variety of mathematical practices — skills, tools, and habits of mind and action — important to learning, doing, and using mathematics.[2] Examples include using mathematical language, justifying claims and solutions, and developing and comparing representations. Being mathematically proficient also includes confidence in one's own capacity and a sense of how to work productively. We will use our work together over the semester and your work in your classroom to elaborate this definition of mathematical proficiency so that it is useful as a guide for your goals with your students.

Principle 3: Establishing and managing a productive learning environment
Good mathematics teaching depends on creating a productive learning environment with the social and material resources to foster the development of each student's mathematical knowledge, skills, and dispositions. Productive learning environments are explicitly inclusive and equitable and sensitive to culture and language.

[2] Adapted, in part, from Kilpatrick, Swafford, and Findell (2001), *Adding it Up: How Children Learn Mathematics*. Washington, DC: National Research Council.

A productive classroom environment is important for students to be successful in math. Teachers can create productive environments by establishing appropriate mathematical norms, by respecting and using students' ideas and ways of thinking, and by attending to the cultural diversity of the class. How error is treated, what counts as "competence," or how to disagree are examples of elements that matter for such an environment. Establishing a productive learning environment begins in the very first interactions among students, mathematics, and the teacher and requires care throughout the school year. Productive learning environments depend on thoughtful use of the physical space, material resources, social resources, and practices, as well as sensitivity to and use of students' cultural resources and differences. Our focus this term will be on learning how teachers can manage particular situations, such as conducting whole group discussions or moving around the room while students work independently on mathematical tasks. We will also attend to how teachers consider and analyze how particular learning environments support different students' learning of mathematics.

Principle 4: Learning from and systematically improving practice
Teachers improve their practice over time by learning from experience, interacting with colleagues, and using professional resources.

Professionals systematically examine their ideas and practices to improve their effectiveness with their clients —— in the case of teachers, with their students. When we consider the teaching of mathematics, practice is strengthened by personal reflection, but also by consulting with colleagues and referring to research, standards, curricula, mathematics as a discipline, and other professional resources. We have designed your work this term to help you learn how to learn in and from your teaching of mathematics. This involves learning how to study and examine practice, developing criteria for judging alternative instructional decisions and moves, and learning to evaluate the effects of one's own teaching.

How We Will Work Together

Our work will be "practice-based" in four senses of the phrase:

1. *Participating in a common practice*: Our class activities, discussions, and interactions offer us opportunities to study practice from the inside. What we do with you is *teaching*, and you and your classmates are directly engaged in *learning*. There will be things to learn from reflections on our interactions together. We will work on all the strands of the course, developing insights and knowledge from the work and activities we do together.
2. *Learning from records of real classroom practices*: We will study records of practice to learn the work of teaching. These records —— videotapes of lessons, students' work, and teacher's plans and reflections —— make it possible for us to study classroom mathematics, the work of teaching, and students.
3. *Practicing specific techniques*: We will engage in cycles of learning focused on techniques from each of the four domains. We will rehearse and critique the use of specific teaching techniques to improve enactment and deepen understanding.
4. *Performing in practice settings*: The major assignments and assessments will involve performances set in teaching contexts. Expectations for performance will reflect skill and knowledge levels necessary for engagement as a "competent beginning teacher."

Course Requirements and Grading

Attendance and class participation: Your participation in our class activities and discussions is important not only for your own learning but also the learning of others. Sharing your ideas and questions with the group, as well as responding to those of your classmates, are critical to our work together. As a teacher, you need to do more than understand your own thinking —— you have to be able to track on others' thinking, figure out what others are saying, and determine whether and how they make sense. In our class, the "others" will be your classmates. But in the field and in the future, they will be your students, and sometimes your fellow teachers. So listening to and interacting with others in our class is explicitly to help you develop dispositions and skills that matter for teaching. Talking in class is also crucial —— as a goal, not a means. As a teacher, you will have to talk mathematics all the time. This course provides you the opportunity to learn to speak more clearly, with an attentive focus on your listener.

We expect you to attend every class, to arrive on time for a prompt start, and to participate in and contribute to class. If circumstances prevent you from attending class, we ask that you call or send an email in advance and that you make plans for how you will make up the work you will miss.

Notebook: You will keep mathematical and other drawings, writings, and reflections in a special mathematics notebook. The work you do in your notebook is a central part of the course. You will use your notebook to create a record of your own work and thinking, as well as our class' accumulating understandings, investigations, conjectures, arguments, and solutions. Your notebook will be a place for you to track and record your thinking about issues and ideas. You will use your notebook to record the work we do in class, including solving mathematics problems and analyzing the work of teaching. Some out-of-class assignments will also be completed in your notebook.

This notebook may not be like other journals you have kept in the past. Think of it like a sort of laboratory notebook where one keeps systematic records of experiments, or like the notebooks that photographers keep about lighting, lenses, and settings. Bird-watchers keep notebooks; so do cooks and architects. Such records help these professionals keep track of, study, and improve their practice. You are developing a notebook to record and support your learning as a teacher of mathematics.

We will collect your notebooks after each class, and they will be available for pick up on Thursday mornings. Each week we will read segments of your notebooks to help us get a better sense of how you are engaging with class work and assignments, the issues and insights you are keeping track of, and the questions you have. If there is something in particular you would like us to look at, please flag it for us. In general, we will not write individual comments in your notebook, but will often share observations with the class, including suggestions for amplifying your use of this medium for your own learning.

To earn full credit for the notebook portion of your final grade, you need to make records of your in-class work and thoughtfully complete assigned tasks and reflections. In addition, you should actively try to make connections among the various aspects of our work, keep track of other's ideas, and annotate and reflect on your work. Notebooks that are chronically thin, haphazard, inaccurate, and/or missing more than a very small amount of what is expected will be considered unsatisfactory and graded accordingly. We will notify you right away if your notebook is unsatisfactory and suggest a course of action to help you improve your work. Therefore, unless you hear otherwise, you can assume your notebook meets the current expectations for full credit.

Weekly assignments: You will have assignments for each class. Assignments will be posted on our course website after class and should be completed by the specified due date.

Some assignments will be completed in your notebook and will be considered part of your notebook grade. These will typically include reflections on what you read, mathematical problems that help you learn to use mathematics for teaching, and tasks related to the core practices of teaching that you are developing this term. Assignments might be designed to provide additional practice or to give you an opportunity to prepare for upcoming work in class.

You will be reading a variety of articles and other materials for this class. Some of the readings will be discussed explicitly in class, some you will comment on in your notebook, and others will simply be used in the context of our work. We expect you to be able to bring these readings to bear in assignments and discussions.

Other assignments will be opportunities to try out and get feedback on the core practices of teaching that we are working on this term. These assignments will ask you to carry out specific tasks of teaching using the skills we have developed in class and then turn in a write-up or other record of your practice. These professional practice pieces will be graded apart from your notebook, and we will provide detailed feedback aimed at helping you improve your practice. The point is to learn to do teaching, not only analyze it.

Three of these will take place in your field placement classroom:

- *Teaching a "Mini-problem"* – To develop your skills and moves for leading a whole class discussion, you will lead a 10-minute discussion about a short mathematics problem. The problem could occur at the

beginning or end of the mathematics period or during a transition in another part of the day. You will confer with your cooperating teacher about which problem would be most appropriate for your class.

- *Student Thinking Interview* – To help you develop your skills at eliciting, interpreting, and assessing student thinking, you will meet with one student for about 30 minutes to do some one-on-one work investigating how the student understands and can use place value concepts to count, represent numbers, and use algorithms to compute.
- *Teaching While Students are Working Independently* – To build on and further develop the skills you worked on during the student thinking interview, you will record and analyze your interactions with students while circulating during the independent work portion of a mathematics lesson.

Culminating performance assessments: Throughout the term, you will be learning and developing skills and techniques in the four focal practice domains. To assess your progress and ability to perform these practices, you will complete four culminating performance assessments, one in each domain:

Teaching Practice Domain	Culminating Performance Assessment	Setting
Leading a whole class discussion about mathematics	Teach a mathematics lesson that includes a whole class discussion	In your field placement classroom
Representing mathematical ideas	Use manipulatives to model one of the standard computational algorithms	Oral performance question during final examination
Assessing students' mathematical knowledge, skills, and dispositions	Design and implement a short prompt to use with students as a quick method of assessment at the end of a lesson, and then analyze what students produce	In your field placement classroom
Planning mathematics lessons	Meet with your methods instructor to describe and discuss your analysis of and plans for the lesson you will be teaching	By appointment, at least two days before you teach your lesson

As you can see, the culminating performance assessments will occur in a variety of settings — for example, in the field or as part of the final exam. Some of the assessments will also include a written analysis of your practice. Details about the culminating performance assessments will be given later in the term.

In the context of completing these culminating performance assessments, you will plan, teach, and analyze one complete mathematics lesson in your field placement. You will apply what you have been learning in the course, and to bring together and further refine the skills you have developed throughout the term.

Final examination: The course will conclude with a final examination designed to focus on your knowledge and skills for teaching mathematics. Consistent with the course goals of developing your proficiency as a beginning teacher of mathematics, this exam will ask you to demonstrate your performance in the main areas of the course. The tasks you will be asked to do will be consistent with those you will be practicing all term. You will be able to prepare in advance for the performances (written and oral) that the exam will ask of you. At least one week before the final examination, we will distribute a "study guide" listing possible exam problems and tasks. You will be able to prepare for each, working with others if you choose. You may make notes, gather ideas, and practice skills. The actual examination will consist of a subset of those tasks and problems.

Grading: This is a professional course. The standards of performance are tied to your knowledge and enactment of principled practice as described in this document *and* to standards you will be expected to meet as a beginning teacher:

- meticulous preparation,
- appropriate use of professional knowledge,
- careful consideration of alternatives,
- genuine curiosity about ideas and about learners,
- exercise of professional judgment,
- collegial work on teaching,
- analysis and reflectiveness,
- skills of ongoing professional learning,

- clear expression,
- organization,
- timeliness.

Specific criteria for grading will accompany all professional practice pieces, culminating performance assessments, and the final. There will be a marked emphasis upon performance, as well as collecting and thoughtfully analyzing records of your performances.

Your final grade will be composed as follows, based on your performance of each of the course requirements:

Attendance and class participation	10%
Notebooks (class work and notebook-based assignments)	10%
Professional practice pieces	25%
Culminating performance assessments (10% each)	40%
Final exam	15%

If you have special needs for which accommodations may be needed, please inform us as soon as possible.

Course Materials

Required

1. The following two books are required. You can purchase them at Ulrich's or order them online.
   - Lampert, M. (2001). *Teaching problems and the problems of teaching*. New Haven: Yale University Press.
   - Chapin, S., O'Connor, C., & Anderson, N. (2003). *Classroom discussions: Using math talk to help students learn, Grades 1-6.* Sausalito, CA: Math Solutions Publications.

2. Additional readings are available online at http://www.lib.umich.edu/reserves/. Once you are at this site, you will see the "Items on Reserve" heading on the left side -- click on "Browse Electronic Reserves." After you have logged in, click on "Fall 2006 Course Reserves." You will be redirected to a list of courses. Scroll down to the School of Education and you will find a link to "School of Education 411…Instructor: Laurie Sleep." Click on this, and then you should be able to download pdfs of the readings.

3. Register for a trial membership to the National Council of Teachers of Mathematics at http://standardstrial.nctm.org/triallogin.asp.

4. Digital voice recorder: You will need to make audio records of your practice throughout the course. You do not need to purchase a digital recorder; what is required is that you be able to reliably use a digital recorder this semester. However, because making records of practice is an important way for you to learn from your own teaching, we recommend purchasing your own. We do not have a preference for the type of recorder you use; however here are a few specifications:
   - It must be a digital voice recorder (not microcassette, or other tape method).
   - It must allow recorded files to be downloaded to a computer. (Be sure to check about support for different operating systems, i.e., Macintosh or Windows.)
   - Recommended file formats: AAC, m4a or mp3, WMA, or DSS.

Recommended

1. Van de Walle, J. (2006). *Elementary and middle school mathematics: Teaching developmentally (6th ed.)*. Boston, MA: Allyn & Bacon.

2. National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

Teaching Activities to be Scheduled in Your Field Placement

Below is a tentative schedule of the major field-based teaching activities for the course. It is up to you to arrange time for these with your cooperating teacher. Please be considerate in talking with your cooperating teacher about making times for you to do this work.

October 2 – 6	Teach a "Mini-problem"
October 23 – 27	Meet with a student for the Student Thinking Interview
November 13 – 17	Make an audio record of interactions with students while circulating during the independent work portion of a mathematics lesson
November 20 – December 8	Teach a mathematics lesson that includes a whole class discussion and end of lesson assessment