What is *Really* Involved in Getting All Students to Succeed in Math?

Mathematics Teaching Pro®

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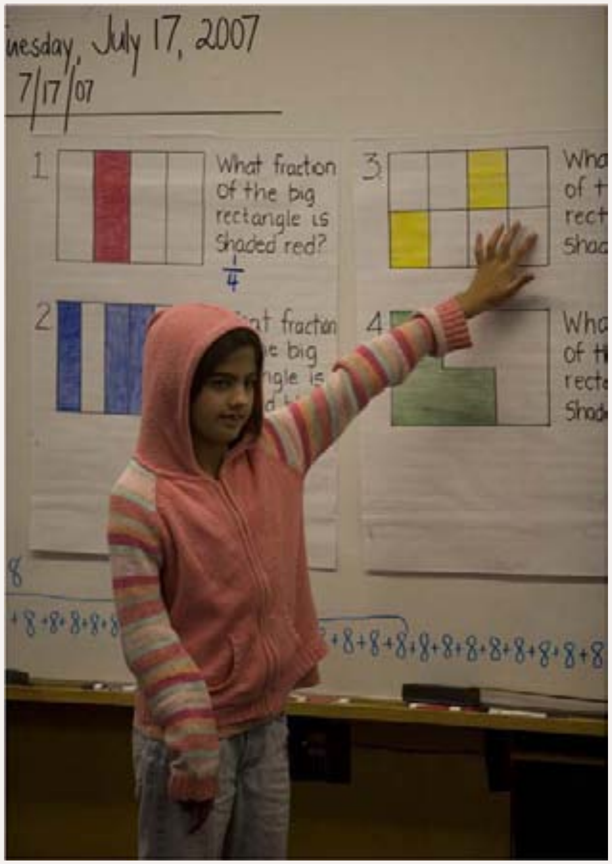
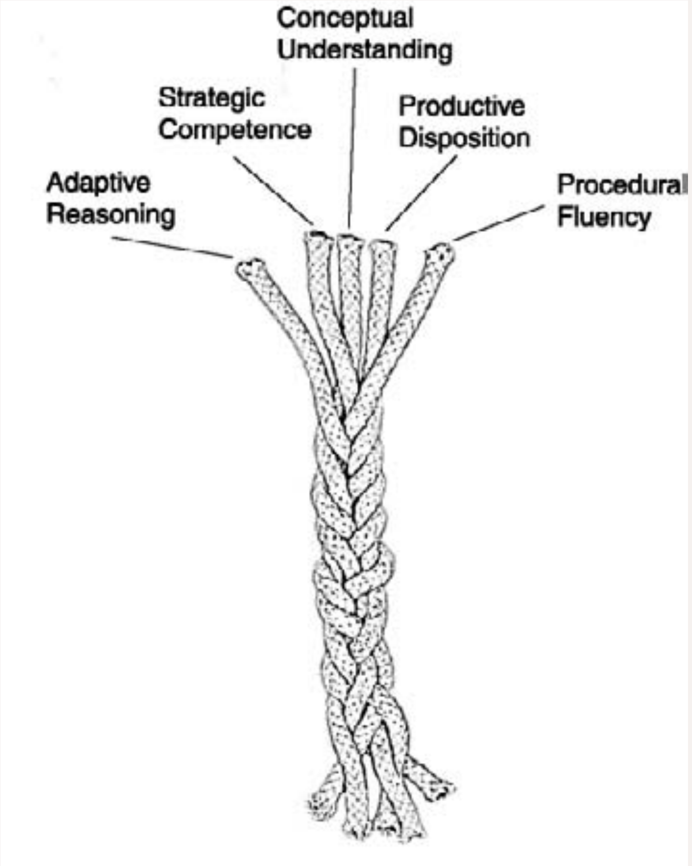
SCHOOL OF EDUCATION UNIVERSITY OF MICHIGAN

An urgent problem, and a new risk

1. Enormous gaps in learning opportunities and disparities in achievement (within U.S. and in international comparisons)
2. Rapidly changing school population
3. Higher, more complex academic goals
4. High expectations for <u>all</u> students



Strands of mathematical proficiency



- ***Conceptual understanding*** - comprehension of mathematical concepts, operations, and relations
- ***Procedural fluency*** - skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- ***Strategic competence*** - ability to formulate, represent, and solve mathematical problems
- ***Adaptive reasoning*** - capacity for logical thought, reflection, explanation, and justification
- ***Productive disposition*** - habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy

Kilpatrick, J., J. Swafford, and B. Findell. (2001). *Adding It Up: How Children Learn Mathematics*. Washington, DC: National Academy Press.

The practice of teaching[1]

- Ubiquitous “natural” learning versus deliberately guided learning
- Distinguishing between “natural” and “professional” teaching; professional teaching is not a natural proficiency.
- Professional teaching is deliberate practice aimed at specific learning.
- Pervasive evidence that professional teachers have significant effects on student learning.

[1]David Cohen, *Teaching Practice and Its Predicaments*, to appear in 2010, Harvard University Press)

What makes teaching professionally specialized work?

1. Knowing content for teaching
2. Being able to hear and see content from others' perspectives, and to learn and know students
3. Using high-leverage practices to enable students to succeed with the content

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Overview of tonight's session

1. Exploring what it means to say that teaching is professionally specialized work
2. Investigating specific high leverage practices of mathematics teaching
3. Developing professional skill as teachers: How can we keep improving our special expertise to be able reach all our students?

1. What makes teaching professionally specialized work?

- Knowing content for teaching
- Being able to hear and see content from others' perspectives, and to learn and know students
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❖ Knowing content for teaching

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Knowing how to read words

Read these words:

oleander

bead

beak

led

lead

break

read

head

dear

Knowing reading for teaching it

What do you notice about these words?

beak bead head

break led lead oleander read

dear

What is this word? **read** Or this one? **lead**

How could you explain how to know which one is which?

How many different sounds can the letters "**ea**" make?

Knowing mathematics

Calculate:

$$\frac{5}{6} \div \frac{1}{3}$$

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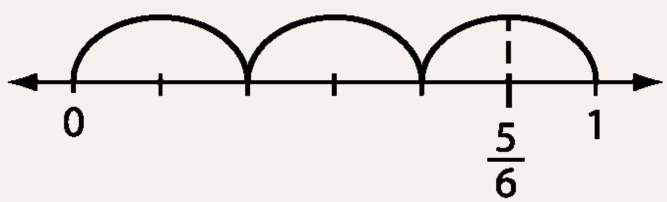
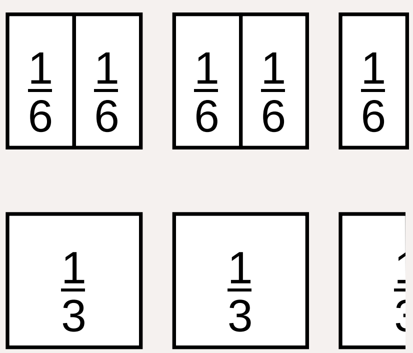
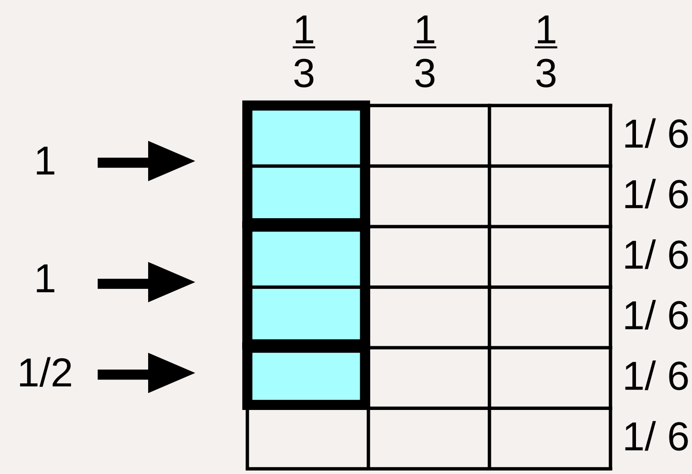
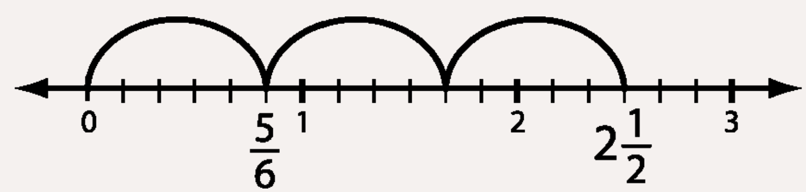
Knowing mathematics for teaching it

Which of these can be used to represent $\frac{5}{6} \div \frac{1}{3}$?









❖ Being able to hear and see content from others' perspectives, and to learn and know one's students

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Teaching multi-digit multiplication

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline \end{array}$$

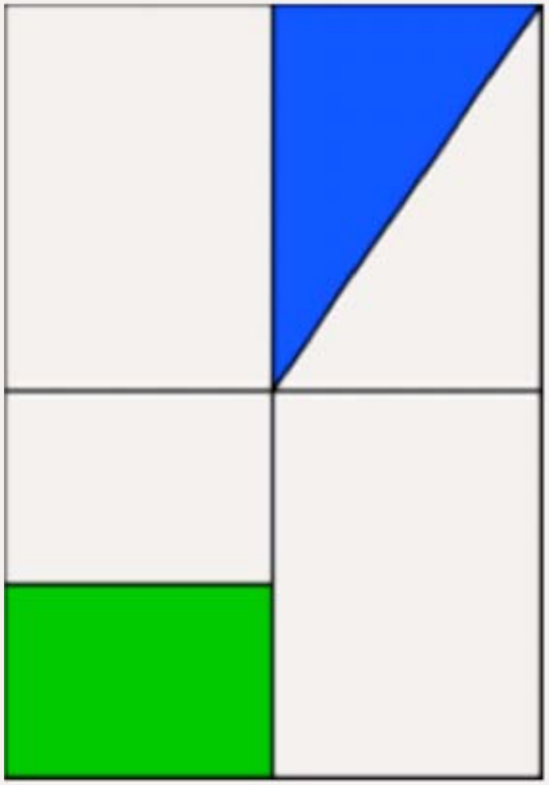
Seeing multiplication from the learner's perspective

(a)	(b)	(c)
49	49	49
x 25	x 25	x 25
405	225	1250
108	100	25
1485	325	1275

What might students have done to produce these answers?

- Using high-leverage practices to enable students to succeed with the content

The math problem



What fraction of the big rectangle is shaded blue?

What fraction of the big rectangle is shaded green?

What fraction of the big rectangle is shaded altogether?

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What practices of teaching do you see?

1. Selecting/designing tasks
2. Identifying and working toward the mathematical goal of the lesson
3. Listening to and interpreting students' responses
4. Teaching students what counts as "mathematics" and mathematical practice
5. Making error a fruitful site for mathematical work
6. Attending to ambiguity of language ("big rectangle")
7. Deciding what to clarify, what to make more precise, what to leave in student's own language

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2. Exploring “high leverage” practices of teaching

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What characterizes “high leverage” practices?

- Central to building bridges between students and content
- Crucial to improve the learning and achievement of all students
- Address inequities that can arise based on diversity of opportunity and experience
- Highly useful and used frequently in teaching
- Not natural to do; improve upon normal help

(Ball, Sleep, Boerst, & Bass, 2009; Grossman & McDonald, 2008; Grossman, Compton, Igra, Ronfeldt, & Shahan, 2009; Lampert & Graziani, 2009)

Examples of high-leverage practices

- Choosing and using mathematical tasks
- Using representations skillfully, mapping clearly across contexts
- Choosing examples (Rowland)
- Using the whiteboard carefully across a lesson and a discussion (Yoshida, Takahashi, Suzuka & Boerst)
- Using student error
- Broadening what it means to be "good at math" in school (Boaler, Cohen, Lotan)
- Attending to issues of mathematical language (Schleppegrell)
- Teaching to the mathematical point (Sleep)
- Designing and getting students to do useful homework, with attention to issues of equity
- Building and maintaining a mathematical and intellectually serious culture

Enacting high expectations in mathematics instruction

1. Broaden what it means to be successful in math class
2. Make mathematical practices explicit
3. Support students' mathematical work both publicly and privately
   - Listen carefully to students' talk
   - Notice and improve ambiguous talk
   - Ask mathematical questions
4. Teaching students to be "people who study mathematics"

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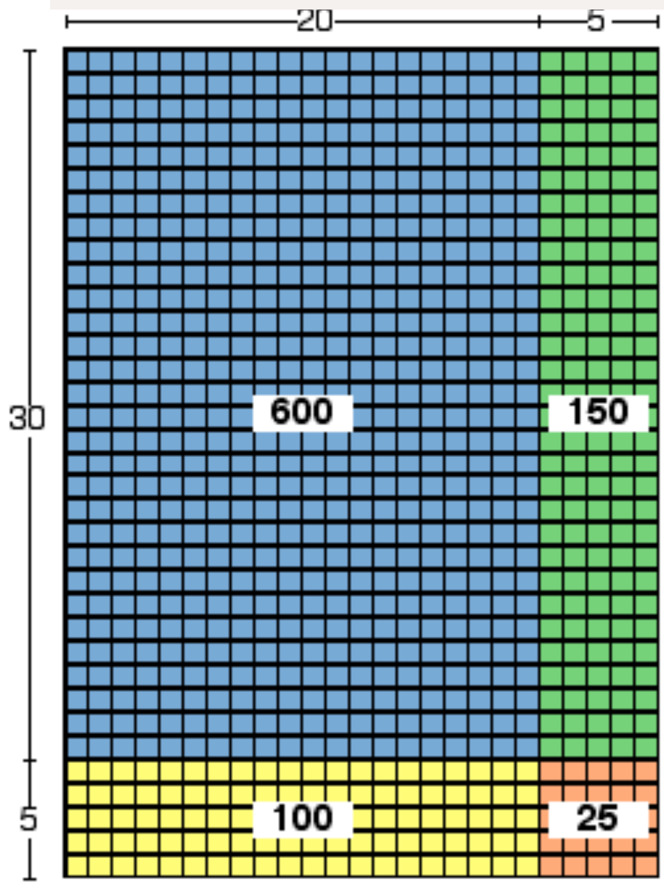
Using representations and mapping across representations

$$\begin{array}{r} 35 \\ \times\ 25 \\ \hline \end{array}$$

- Calculate the answer
- Show as an area with base ten blocks or on large grid paper
- Explain the correspondence between the area and the written procedure

Modeling multiplication as area



A

```
  35
x 25
----
 125
+ 75
----
 875
```

B

```
  35
x 25
----
 175
+700
----
 875
```

C

```
  35
x 25
----
  25
 100
 150
+600
----
 875
```

Choosing mathematical tasks

Use the numbers 9, 1, and 2 once on each line to make 6 different numbers. The first one is done for you:

1 9 2   _ _ _

_ _ _   _ _ _

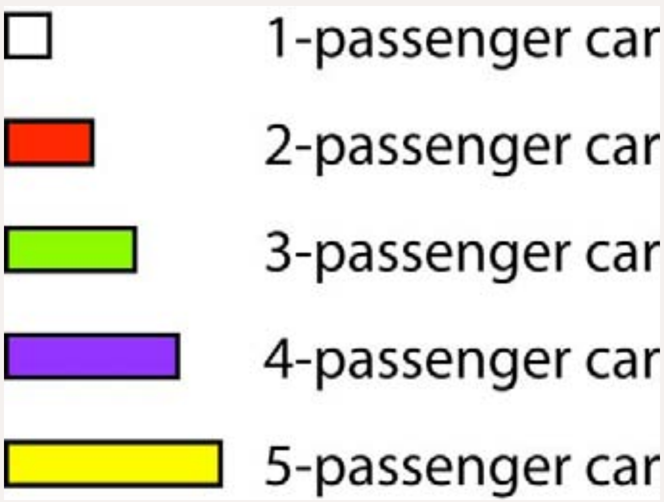
_ _ _   _ _ _

Today's date is **9/12.**

- How many three-digit numbers can you make using 9, 1, and 2?
- How can you be sure that you have all the possibilities?

What are the key similarities and differences between these two tasks?

The Train Problem



A special Customer wants to order a special five-car train that uses one of each of the different-sized cars. She wants to be able to break apart her 5-car train to form smaller trains that hold exactly 1 to 15 people. In addition, she wants to be able to form these smaller trains using cars that are next to each other in the larger train.

Can the Train Company fill the Customer's order? Explain how you know.

What are the conditions of the Train Problem?

Anticipating students' work on the Train Problem

- What might be difficulties for your students?
- How might you scaffold their work?
- What contingency plans do you need?

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3. Developing professional skill: How can we keep improving our special expertise to be able reach all our students?

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A provocation

- The professional plateau: Studies show that there is little gain in teachers' capacity to help students learn after the fourth year
- What is your explanation of this?
  1. Maybe teachers hit the pinnacle of professional practice quickly
  2. Maybe the differences between beginning and accomplished teachers is more subtle than these measures
  3. Premature leveling off: Lack of a system of professional support to help teachers develop

Learning in and from practice[1]

1. Studying content and doing mathematics as a teacher
2. Studying curriculum and specific tasks
3. Studying student work
4. Creating and examining one's own and others' records of practice, e.g.:
   - Videotape of classroom discussions
   - Photographs of the whiteboard and other public records of class mathematical work

[1]Ball & Cohen (1999), *Developing practice, developing practitioners.*

1. Learning math as a teacher

Doing mathematical work of teaching:

1. Construct a definition of "even number" that is mathematically correct and usable/comprehensible by upper elementary students.
2. Examine definitions of "even number" from different textbooks; determine which are mathematically correct and usable; fix ones that are not.

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Do these correctly define "even number"?

Is 7 even?

1. A number that can be divided in two equal parts with nothing left over is even.
2. A whole number is even if it can be divided into groups of 2 with nothing left over.
3. A number with 0, 2, 4, 6, or 8 in the ones place is even.

Is 32.7 even?

2. Studying curriculum and specific tasks

- Comparing fractions on the number line
- Select a set of 5 numbers for pupils to place on the line and explain why you chose that set of numbers. What will each provide in terms of opportunity to learn?

Choosing examples

Which of the following is best for setting up a discussion about different solution paths for simplifying radical expressions?

(a) $\sqrt{54}$

(b) $\sqrt{156}$

(c) $\sqrt{128}$

3. Studying student work

1

35
x 25
175
70
245

2

1
2
35
x 25
255
80
1055

3

35
x 25
625
85
105
60025

- What is the error?
- What produces this error, and what does that show about the student's understanding?

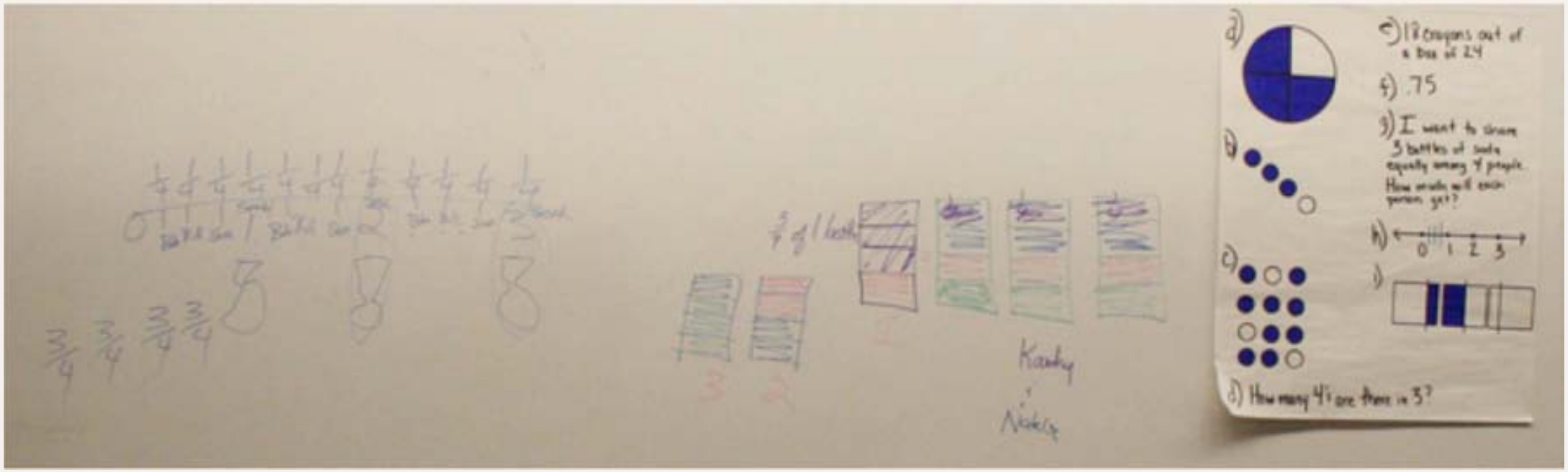
Taking next steps

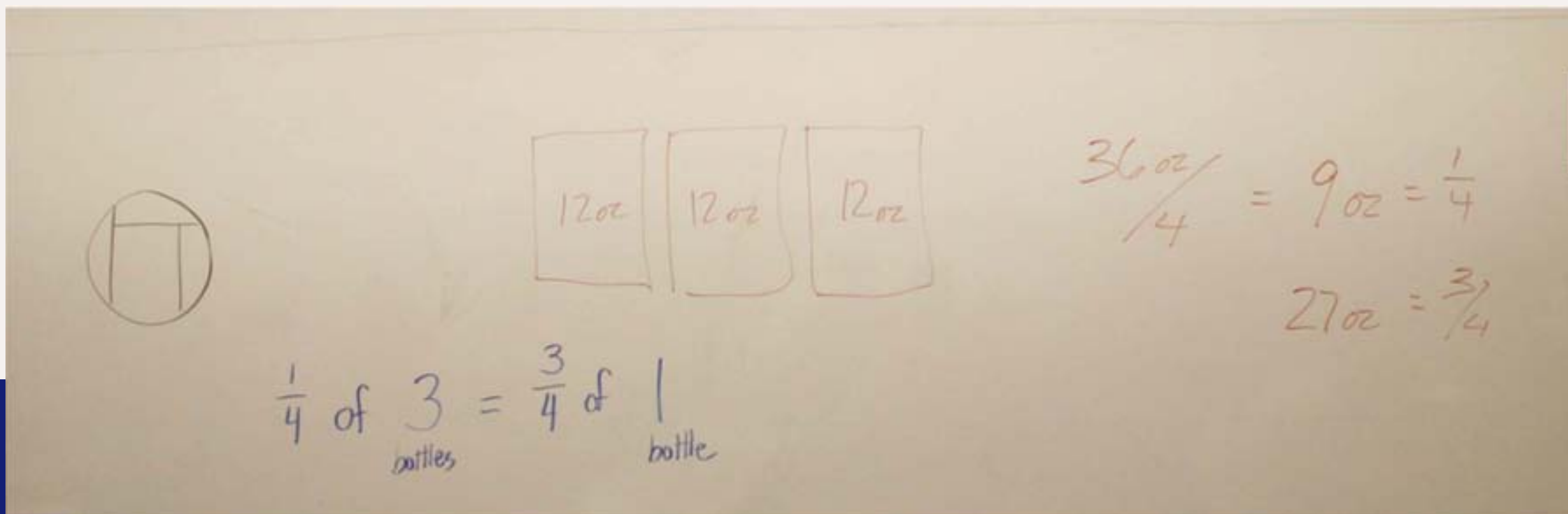
Consider this error:

```
 1
  2
   35
x  25
-----
  255
  80
-----
 1055
```

- What are three different things that might have led to this error? What could you do to find out?
- What would you do to help learners understand the mistake, and what would you try to remedy it?

4. Creating and studying records of one's own and others' practice





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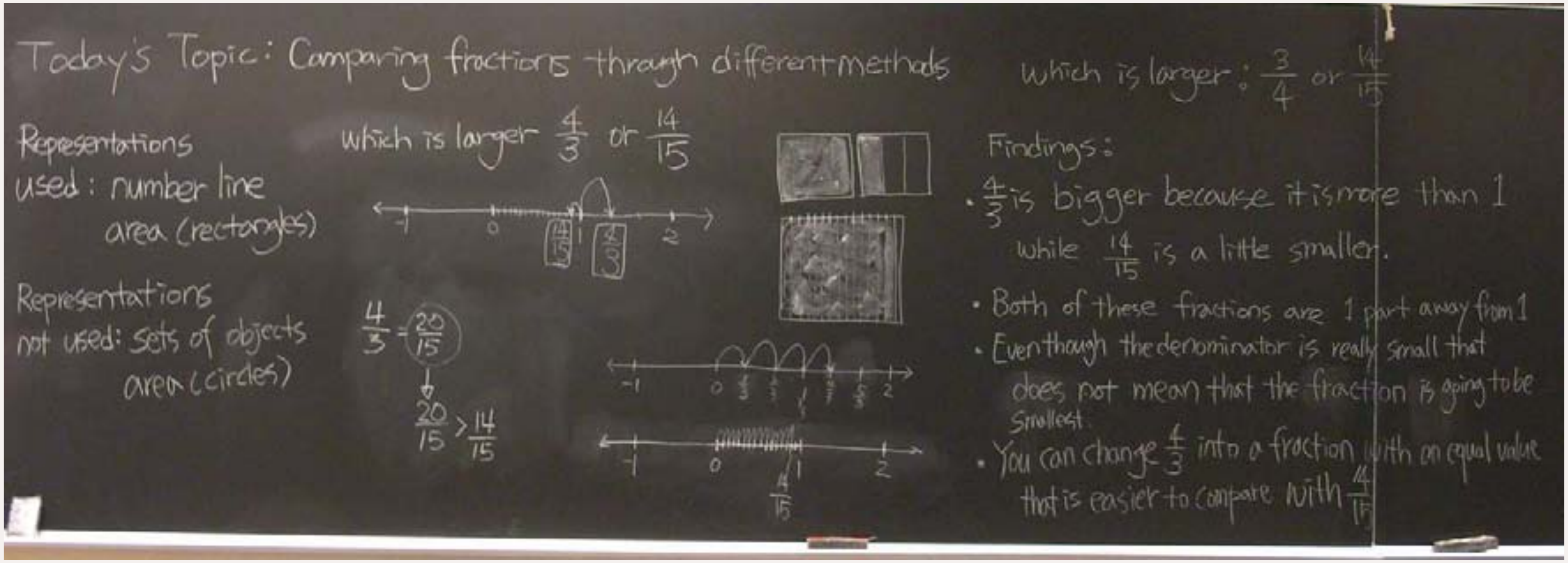
Today's Topic: Comparing fractions through different methods
Representations used: number line
area (rectangles)
Representations not used: sets of objects
area (circles)
Which is larger 4/3 or 14/15
4/3 = 20/15
20/15 > 14/15
Which is larger: 3/4 or 14/15
Findings:
• 4/3 is bigger because it is more than 1 while 14/15 is a little smaller.
• Both of these fractions are 1 part away from 1
• Even though the denominator is really small that does not mean that the fraction is going to be smallest.
• You can change 4/3 into a fraction with an equal value that is easier to compare with 14/15

Challenges of learning in and from practice

1. Lack of an adequate knowledge base about teaching practice
   - Inadequate language (in English)
   - Difficulty parsing the work into basic elements
2. Problem of expertise and tacit knowledge
3. Widely held view of teaching as uncertain, artistic, and unable to be specified
   - Resistance to seeing teaching as high-precision work, requiring high levels of skill
   - View of detail as "prescriptive" and as de-skilling professional work
4. Lack of people prepared to teach practice
   - Unspecified professional group, with no preparation for the work
5. No common K-12 curriculum in the U.S. and lack of agreement about what to make core

In celebration of professional teaching

- We urgently need to improve all students' opportunities and learning.
- This requires skillful professional teaching.
- Teaching is intricate work, and not natural, and needs to be learned and, hence, studied and continuously improved.
- Seeing teaching as skilled, high-precision work, that is not a matter of personal style and preference, is to acknowledge its professional nature, not to repudiate its "creativity."

Thank you!

Slides will be available on my website.

(Search for “Deborah Ball” for link.)