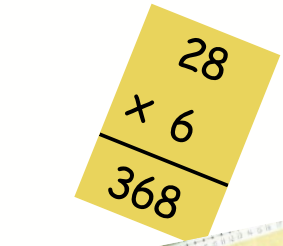
# Meeting the Mathematical Challenges of Making Connections


## Deborah Loewenberg Ball and Jennifer Lewis

Hyman Bass, Merrie Blunk, Heather Hill, Laurie Sleep, Mark Hoover Thames

Seán Delaney, Imani Goffney, Deborah Zopf

University of Michigan


California Mathematics Council

November 3, 2006



UNIVERSITY OF MICHIGAN
SCHOOL OF EDUCATION

# Overview

1. What problem are we trying to solve?
2. Connection #1: What does it mean to "know" mathematics for teaching?
3. Connection #2: Developing opportunities for teachers to learn mathematical knowledge for teaching
4. Connection #3: Assessing PD and teacher learning in and for practice (instruction and achievement)

# Clarifying the Problem

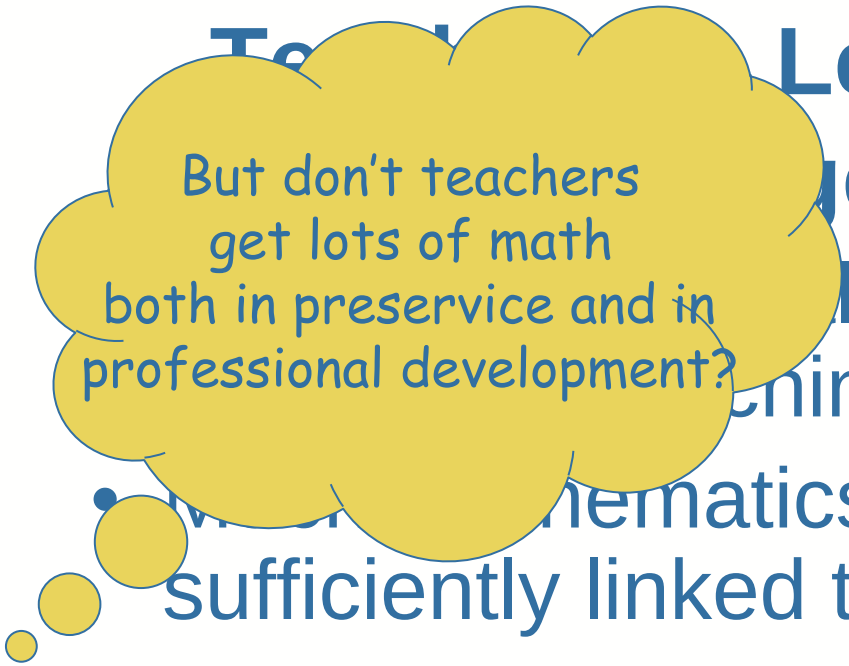
**The quality of mathematics teaching and learning**

**Teachers' knowledge of mathematics and their ability to use it in practice**

# A New Problem: How to Create Opportunities for Teachers to Learn Mathematical Knowledge for Teaching

- Much professional development (PD) that focuses on teaching practice is content-free
- Much mathematics-focused PD is not sufficiently linked to practice
- Materials are scarce for learning this sort of mathematical knowledge, skills, and problem solving special to teaching

# A New Problem: How to Create Opportunities for Te[illegible] Learn Mathematical [illegible]ge for Teaching



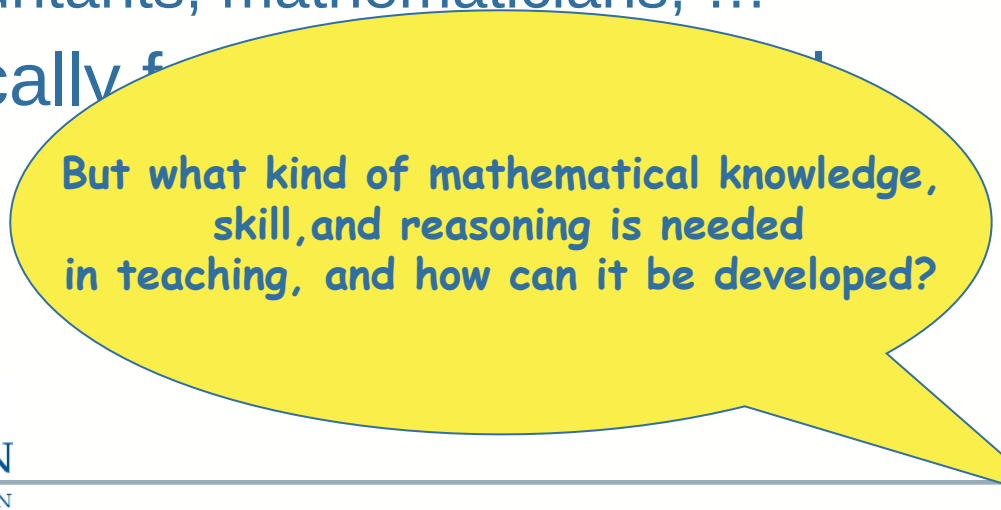

- [illegible] development (PD) that [illegible]ching practice is content-free
- [illegible]thematics-focused PD is not sufficiently linked to practice
- Materials are scarce for learning this sort of mathematical knowledge, skills, and problem solving special to teaching

# Some Approaches to Solving the Problem, and the Unresolved Question

- Require more mathematics for certification
  - More mathematics courses
  - A major (or minor) in mathematics
  - Mathematics test
- Recruit mathematically trained people into teaching
  - Engineers, accountants, mathematicians, …
- Fund mathematically focused professional development

# Some Approaches to Solving the Problem, and the Unresolved Question

- Require more mathematics for certification
  - More mathematics courses
  - A major (or minor) in mathematics
  - Mathematics test
- Recruit mathematically trained people into teaching
  - Engineers, accountants, mathematicians, …
- Fund mathematically f… development



# Toward A Practice-Based Theory of Mathematical Knowledge for Teaching

1. **Study instruction**, and identify the mathematical work of teaching
2. Analyze what **mathematical knowledge is needed to do that work** effectively, and how it must be understood to be useful for the work
3. Develop and evaluate approaches to **helping teachers learn mathematical knowledge for teaching (MKT)**
4. Develop, test, and refine **measures of MKT** using multiple methods as a means to evaluate professional education, investigate effects on students' learning, and improve theory

# Mathematical Knowledge for Teaching

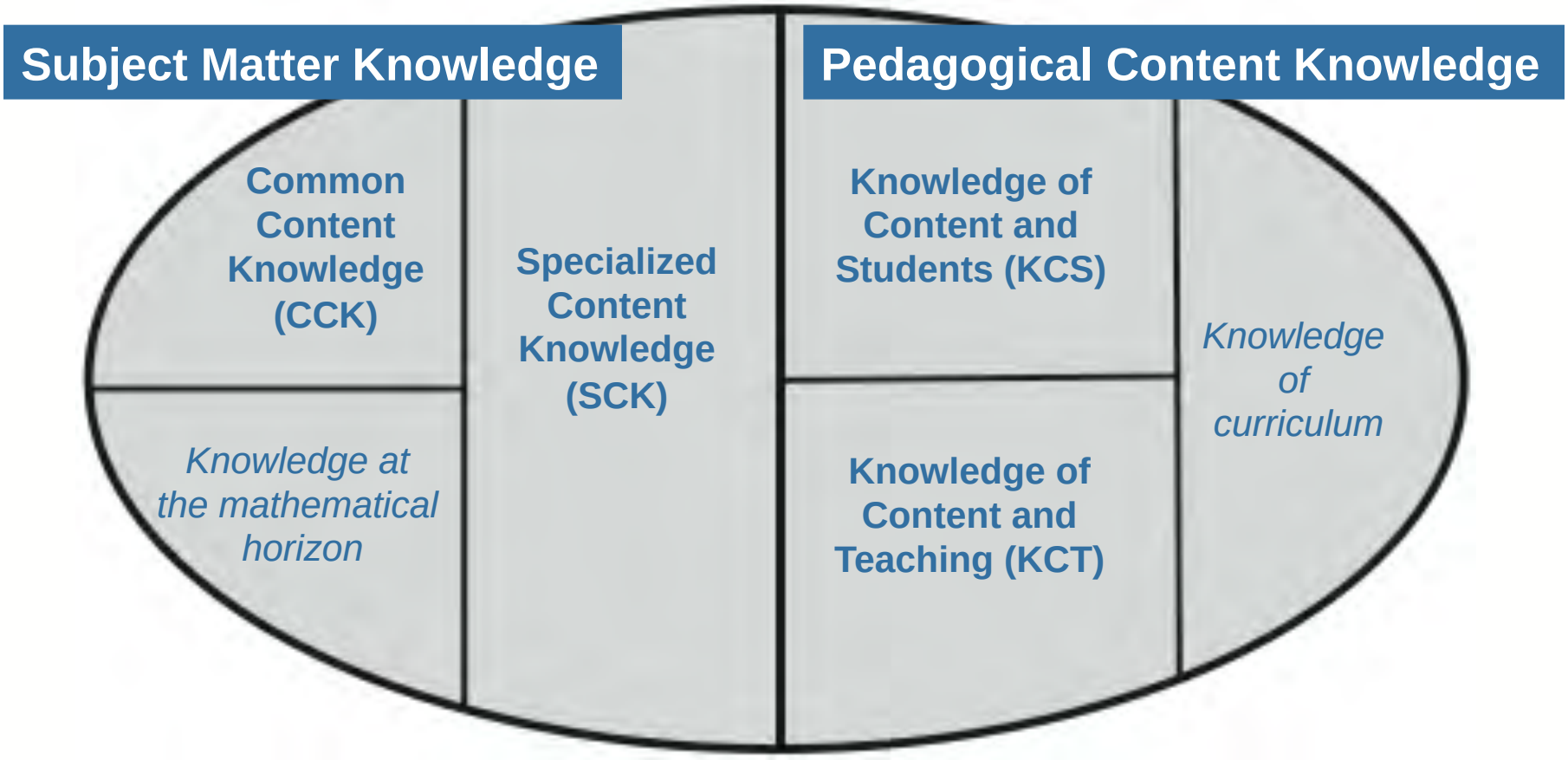
What do we mean when we use this term, "**mathematical knowledge for teaching (MKT)**"?

- Mathematical knowledge, skill, habits of mind that are entailed by the work of teaching

What do we mean by the "**work of teaching**"?

- The tasks in which teachers engage, and the responsibilities they have, to teach mathematics, both inside and outside of the classroom

# Shulman's Original Category Scheme (I985) Compared with Ours



# Knowing Multiplication

Which is more?

$$\begin{array}{r} 54 \\ \times\ 100 \\ \hline \end{array} \qquad \begin{array}{r} 540 \\ \times\ 10 \\ \hline \end{array}$$

# Knowing Multiplication for Teaching

100 is only 90 more than 10,
but 540 is 486 more than 54,
So 540 x 10 is more than
54 x 100.

| | |
|---|---|
| 540 | 54 |
| x 10 | x 100 |

What is going on here?
Could that be right?
I don't think it is, but what is wrong with it?
What could I ask her or show her?

# Other Work of Teaching

- Using and defining terms
- Using notation
- Explaining and guiding explanation
- Interpreting and evaluating alternative solutions and thinking
- Choosing contexts with care for mathematical integrity, equity, and transparency for learning
- Writing a quiz
- Explaining students' progress to parent

# Findings from Our Research

- Mathematical knowledge for teaching is the knowledge needed for the work of teaching.
- A kind of content knowledge distinct from that required by other mathematically-intensive professions
- Some particular specialized uses:
  - interpreting and evaluating alternative mathematical statements or solutions
  - choosing and using representations and examples
  - producing and evaluating mathematical explanations
- Mathematical knowledge for teaching linked to student achievement gains
- Importance of building this theory *in* practice

# Connecting Opportunities for Learning MKT Closer to Practice

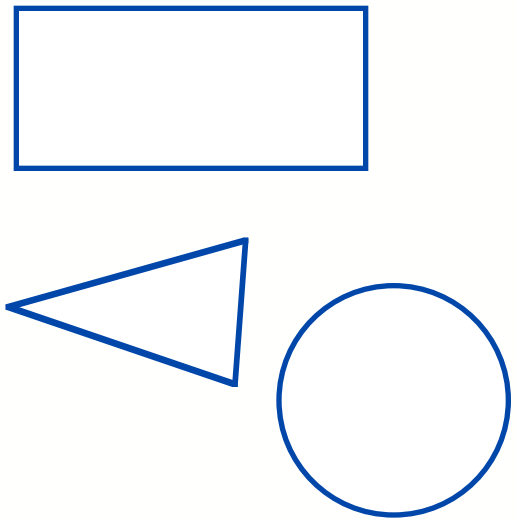
- Many projects already attempting this
- Need for more conceptual structure to enable collective discussion and design
- Need for better ways to assess outcomes that examine teachers' learning for practice and its relationship to students' learning

# Learning Mathematics as a Teacher: Connecting to the Work of Teaching

- Focus professional development on mathematical content, and build explicit **connections** to the work of teaching
  - Study content and do applications that apply that knowledge to problems of teaching
  - Examine student work and analyze it mathematically
  - Write worksheets and quizzes
  - Analyze a textbook lesson for its mathematical purposes, connect to grade level standards
  - Analyze and write precise and usable mathematical definitions
  - Keep a list of mathematical terms that have multiple uses (in and outside of mathematics); learn to distinguish which ones require careful attention, and how
  - Analyze assessment items for their mathematical language demands
- Provide opportunities to teachers to develop fluency with mathematical knowledge for teaching
- Connect direct work on achievement disparities with mathematical resources
  - Mathematical language
  - Care with use of contexts
  - Opportunities to broaden what it means to be “good at” mathematics

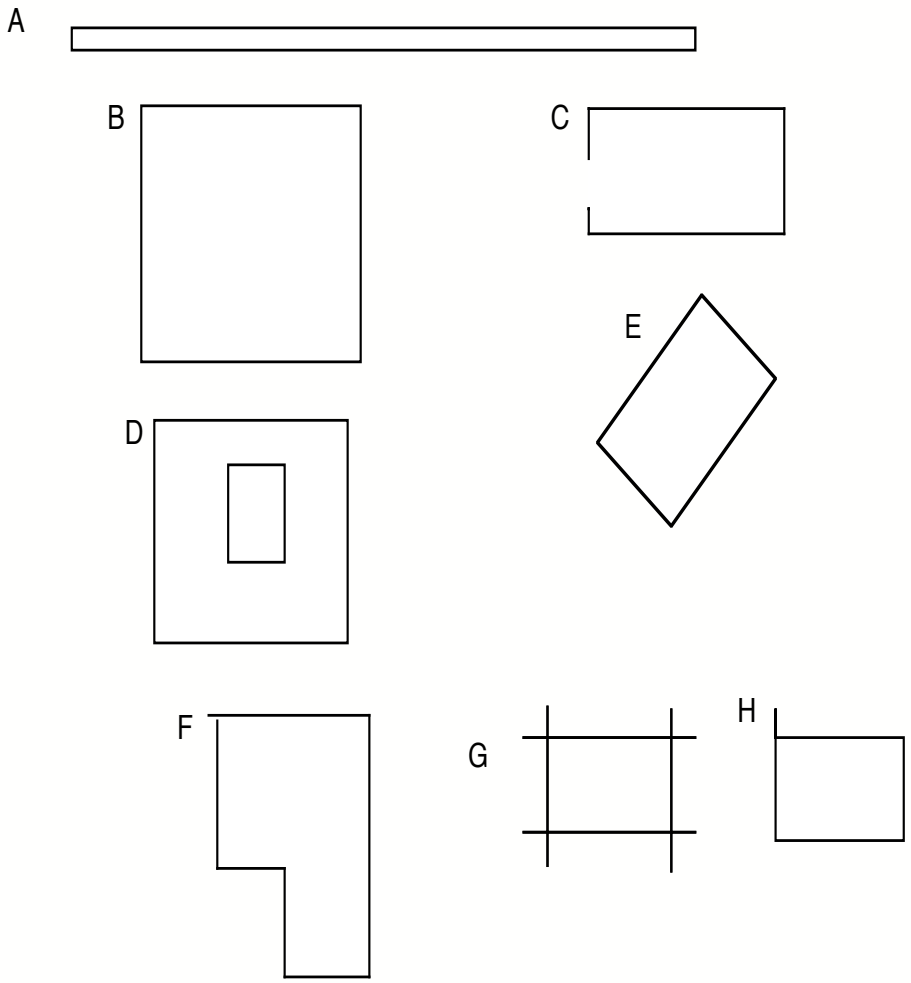
# Knowing Geometric Figures

What are these figures?



What are mathematical definitions for each?

# Choosing Examples to Develop Concepts




What is a reason for using each of the figures on the left?

Do the same for *circles*.

# Defining Terms and Appraising Curriculum Materials

## 1. Defining terms

- with mathematical precision
- in ways that are usable
- in ways that extend as the mathematical domain expands

Define *rectangle* and *circle* for fourth graders.

## 2. Examining materials

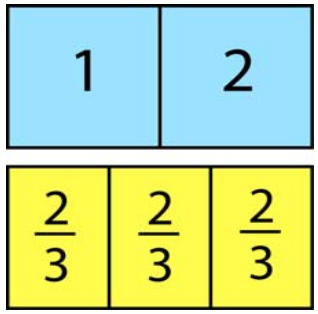
How do your curriculum materials or resources define —

- rectangle
- circle

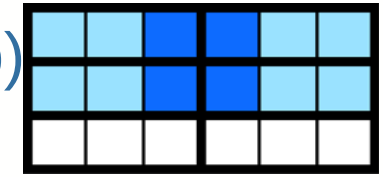
# Choosing and Using Representations

**Which of the following can be used to represent 2 ÷ 2/3?**

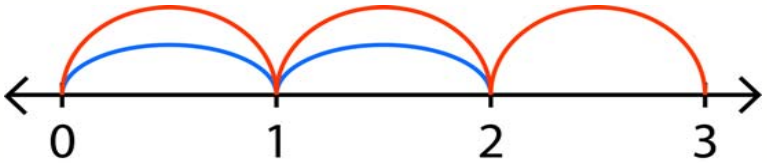
(a)



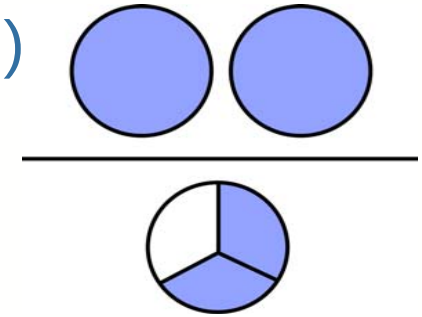

(b)



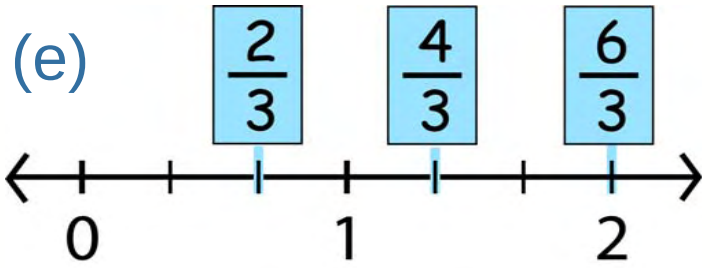
(c)




(d)



(e)



# Other Sites for Learning and Practicing MKT

- Analyzing and writing tests or quizzes
- Making mathematical posters
- Writing newsletters or letters to parents
- Selecting examples
- Examining the language demands of textbook math problems
- Talking mathematics

# Connecting Assessments of Professional Development to Practice

- Not just how well teachers know mathematics, or how much they liked the PD, but —
- How well can teachers do the mathematical work of teaching?
- How is their work effective for student learning?

# Knowing Radical Expressions

Simplify: $\sqrt{150}$

# Knowing Radical Expressions for Teaching: Choosing Numerical Examples

Which of the following is best for setting up a discussion about different solution paths for simplifying radical expressions?

(a) $\sqrt{54}$

(b) $\sqrt{156}$

(c) $\sqrt{128}$

(d) These examples all work equally well.

# Other Means to Assess Teachers MKT in Practice

- Our validation work
- Observing teachers teaching mathematics
- Interviewing teachers with mathematical problems of teaching (produce definitions for particular ages, selecting examples, using representations with mathematical care)
- Computer-adaptive testing
- Using tasks linked to student achievement

# Final Comments

1. Mathematical knowledge is crucial to teaching effectiveness.
2. The mathematical knowledge needed is specialized and is more than common content knowledge known by any well-educated adult.
3. Teachers need opportunities to learn mathematics for teaching and to practice using it.
4. We need to design professional development to connect teachers, content and pedagogy.
5. Professional development should be evaluated by its effects on
   - instruction
   - student learning (and we need new methods for doing this sensibly)

Slides will be available at

**http://www-personal.umich.edu/~dball/**

# Knowing Multiplication for Teaching: Analyzing Student Errors

**What mathematical steps produced each of these answers?**

(a)

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline 405 \\ 108 \\ \hline 1485 \end{array}$$

(b)

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline 225 \\ 100 \\ \hline 325 \end{array}$$

(c)

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline 1250 \\ 25 \\ \hline 1275 \end{array}$$

# Linking Teacher Knowledge and Student Achievement

- Questionnaire consisting of 30 items (scale reliability .88)
- Model: Student Terra Nova gains predicted by:
  - Student descriptors (family SES, absence rate)
  - Teacher characteristics (math methods/content, content knowledge)
- Teacher content knowledge significant
  - Small effect (< 1/10 standard deviation): 2 - 3 weeks of instruction
  - But student SES is also about the same size effect on achievement

    (Hill, Rowan, and Ball, AERJ, 2005)

# Teaching MKT: Goals andChallenges

- Tackling issues of “coverage” and scope
  - Choosing and justifying mathematical content (topics and practices) for **in-depth work**
  - Choosing and justifying content for **brief treatment**
  - Choosing and justifying content **to omit**
- Understanding and developing mathematical fluency for teaching
- Developing tasks and pedagogy appropriate for MKT: Courses/ workshops should not necessarily be taught as teachers would teach their own students
- Developing performance assessments: knowing and using mathematics (e.g., explaining, representing)
- Rethinking the intersection of “content” and “methods”
- The professional development of professional developers