Improving U.S. Mathematics Education: History, Myths, and Strategies

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College of Sciences, San Diego State University • June 5, 2008

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Inside cover

- Is there something we can glean from the National Math Panel report, and is it any different from the shelf-full of other reports?
- What have we learned from experience, from learning about other educational systems, and from research on interventions?

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This afternoon

1. A short pre-test
2. The urgency of framing and understanding "the problem"
3. What can we "harvest" from the National Mathematics Advisory Panel report?
4. A missing piece: Intervening in schools
5. Post-test: Checking your answers

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A short pre-test

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1. Which are myths? Which are true?

Mark M (myth) or T (true).

a. The number of mathematics courses that a teacher has taken is a good predictor of how effective he or she will be.
b. Most high-performing countries use math specialist teachers in the upper primary grades.
c. Creating professional learning communities is at the heart of effective professional development.
d. Individual teachers can have a significant impact on students' learning.

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2. Do you YOU know math well enough to teach third grade?

$38 \div 4$

Write **four distinctly different** word problems that correspond to this division expression, representing different interpretations of the meaning of division, and with different possible numerical answers, depending on the context.

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3. Which is most likely to improve mathematics learning?

Choose the best single answer.

a. Install a more challenging mathematics curriculum.

b. Increase instructional time spent on mathematics and reduce class size.

c. Coordinate professional education, curriculum, assessment, and policy signals.

d. Recruit more mathematically-trained people to be teachers.

e. Lower the barriers for entry to teaching.

f. Pay teachers more to raise performance.

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The urgency of framing and understanding the problem

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What is "the problem"?

1. Too many students not learning mathematics well enough
   - For citizenship, continued mathematics study, global competitiveness
   - Social acceptance of not being good at math
2. Pervasive inequality
   - Unequal distribution of mathematical success by race, social class
3. Weak capacity for improvement
   - Public understanding, support, investment
   - Teacher shortages, weak interventions, thin professional knowledge base
   - Teacher educator development and resources

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How do U.S. youth perform?

- U.S. 15-year-olds were outperformed in math by students in 23 of 38 other countries
- They were outperformed in science by students in 18 of 38 countries

- In 2005, 69% of 8th graders scored above "basic" level on National Assessment of Educational Progress (NAEP) math
- Significant differences by race: 42% of African American, 52% of Latino, and 80% of White 8th graders score at this level
- And by income level: 51% of low income students scored above "basic"; 79% of others
- Similar results in reading.

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Another look

Nine year-olds growing up in low-income communities—

- are on average three grade levels behind their peers in high-income communities.
- have a 50% chance of graduating from high school.
- who graduate from high school read and do basic math at an 8th grade level.

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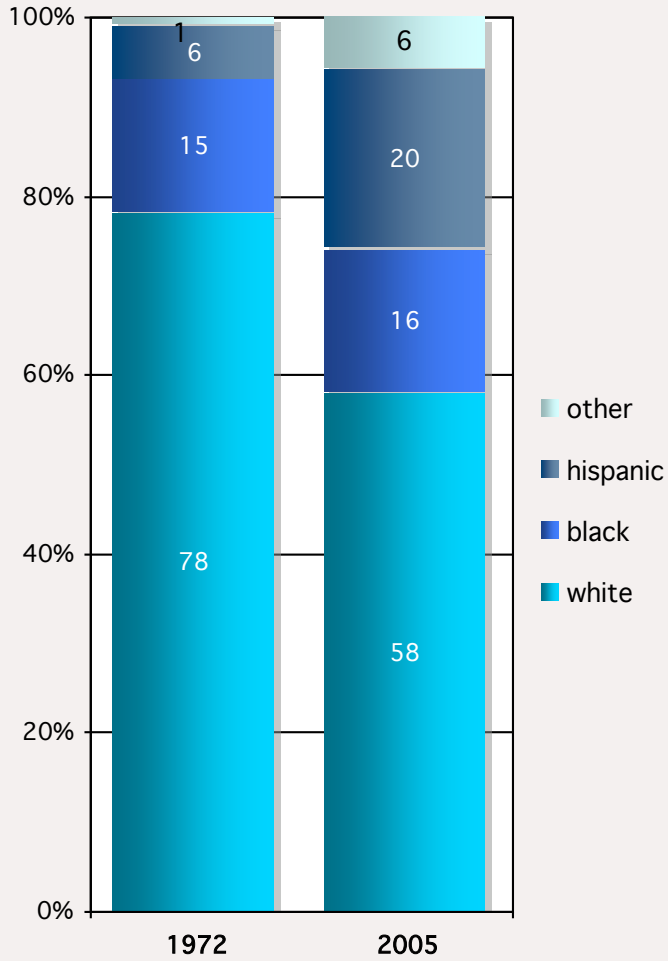
U.S. school-age population: Changing demographics



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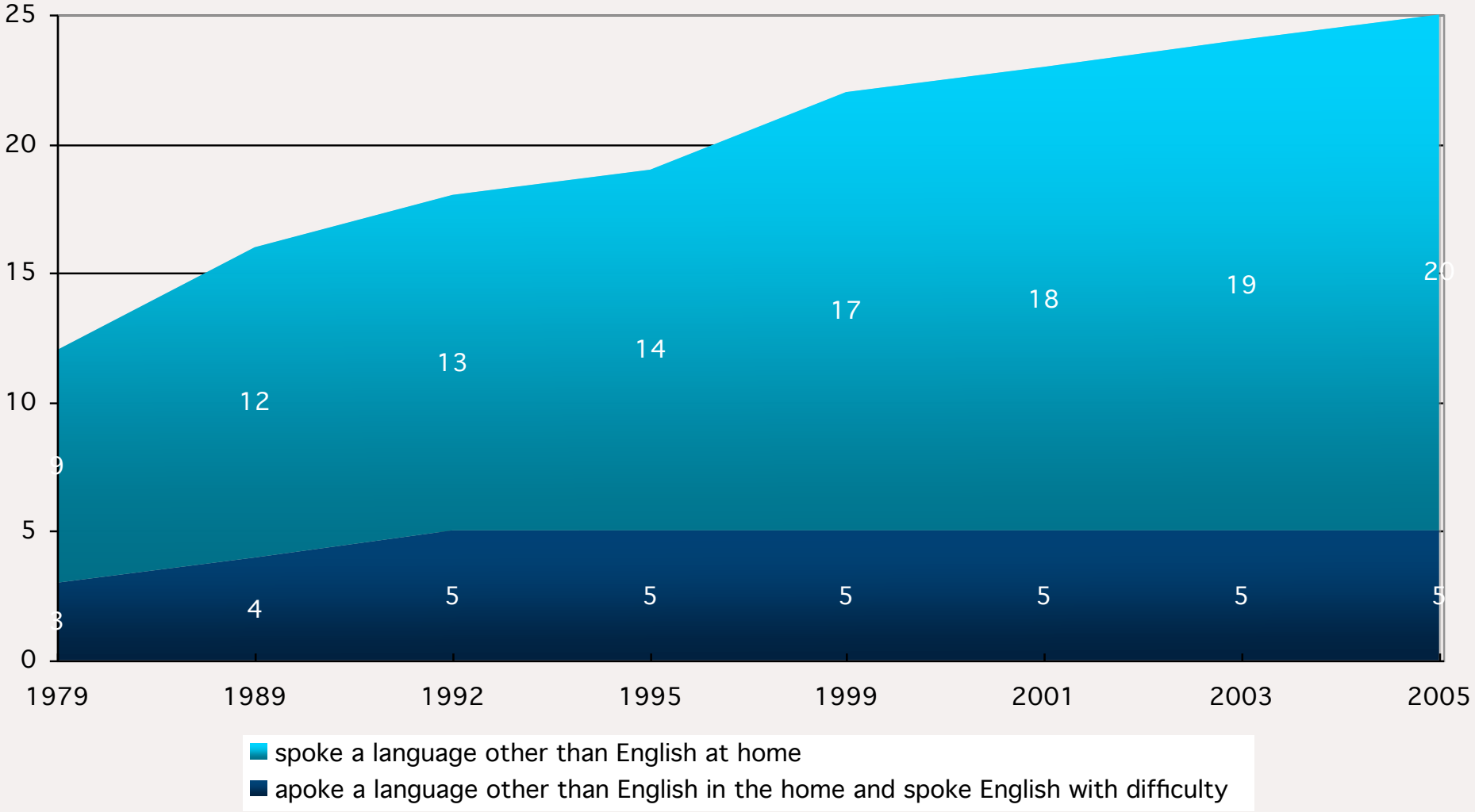
Pupils' home language



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We want a lot of our schools

- Reduce disparities in opportunities and achievement
- Aim for higher and more complex outcomes for more students than ever before

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- We've been here many times before (e.g., the "new math," back to basics, A Nation at Risk, the Glenn Commission report, *A Gathering Storm*).
- We keep gravitating to the same strategies.
- Have we learned anything that can help change the pattern? Can we avoid assembling the same NMP report 10 years from now?

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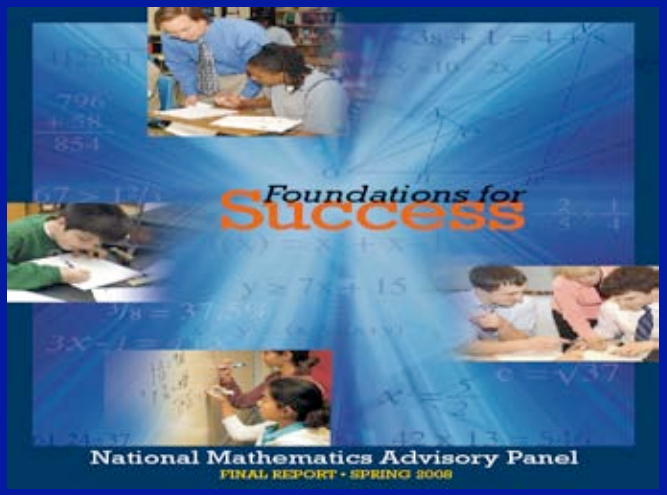

What can we "harvest" from the National Mathematics Advisory Panel report?

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Overview

- Panel charge, process, and structure
- Analysis of Panel product
- Summary of report's main points and scope
- Commentary

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Presidential Executive Order April 2006

- The Panel will advise the President and the Secretary of Education on the best use of scientifically based research to advance the teaching and learning of mathematics, with a specific focus on preparation for and success in algebra.

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Elements of the report

1. Conceptual knowledge and skills
2. Learning processes
3. Instructional practices
4. Teachers and teacher education
5. Assessment
6. Standards of evidence
7. Survey of algebra teachers
8. Instructional materials

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Headlines

1. Substantial agreement about what students should learn to be successful in algebra (algebra as gate)—foundations, benchmarks, critical topics. Neither inclusive nor comprehensive.
2. Conceptual understanding, procedural skill, reasoning, use of language and representations are intertwined in mathematical competence.
3. Early opportunities to learn mathematics can capitalize on and extend young learners' resources; this is a particularly important intervention to narrow opportunity gaps.
4. Automaticity and fluency are important, to free attention; achieving this takes practice.
5. Instruction cannot be reduced to oversimplified dichotomies (e.g., teacher-directed versus student-centered).
6. Teachers' mathematical knowledge matters; strongest signal with content measured close to its use in practice.
7. Lack of evidence to identify effective teacher education at any stage of teacher development.
8. U.S. textbooks tend to be long and cluttered.

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Rationale for what is included

1. Directly related to the Panel's charge
2. Sufficient evidence that could responsibly support policy recommendations
3. Adequate time and resources to pursue

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What's missing in the report, and why?

Not part of NMP charge or scope

- Teaching and learning mathematical topics and skills beyond algebra and its key foundations
- School working conditions
- Role of students' out-of-school experiences and other influences
- Parental and public attitudes toward mathematics
- What it takes to intervene successfully at scale

Insufficient evidence for policy

- Effects of particular curricula
- Instructional practices that focus on and redress inequities
- Effective forms or practices of teacher education
- Specifics about the mathematical preparation of teachers

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What's missing because of lack of resources, time, or priority?

- Mathematical practices as core content (e.g., reasoning, proving, representing, defining)
- Effects of other instructional practices (e.g., homework, writing, individualized instruction)
- Teachers' knowledge, skills, and dispositions other than mathematical knowledge
- Designs for school improvement and how they work

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Content Knowledge and Skills

- Follow a coherent progression, with emphasis on mastery of key topics
- Focus on the critical foundations for algebra
  - Proficiency with whole numbers
  - Proficiency with fractions
  - Particular aspects of geometry and measurement

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Critical Foundations

- The Critical Foundations *are not meant to comprise a complete preschool-to-algebra curriculum.*
- "Critical" for the study of algebra, whether as a part of a dedicated algebra course in Grade 7, 8, or 9, or within an integrated mathematics sequence in the middle and high school grades.
- Deserve ample time in any mathematics curriculum.

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Curricular Content

The Major Topics of School Algebra

- Symbols and expressions
- Linear equations
- Quadratic equations
- Functions
- Algebra of polynomials
- Combinatorics and finite probability

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Learning Processes

- Children develop considerable knowledge of mathematics before they begin kindergarten; early programs are important to help all students enter school with key foundations.
- Students' goals and beliefs about learning are related to their mathematics performance; students' beliefs about the importance of effort and ability can be changed.

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Learning Processes

- Learners must simultaneously develop conceptual understanding, computational fluency, factual knowledge and problem solving skills.
- Limitations in the ability to keep many things in mind (working-memory) can hinder mathematics performance.
  - Practice can offset this through automatic recall, which results in less information to keep in mind and frees attention for problem solving.
  - Learning is most effective when practice is combined with instruction on related concepts.
  - Conceptual understanding promotes transfer of learning to new problems and better long-term retention.

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Instructional Practices

1. Formative assessment
2. Uses of technology
3. Effective practices for students with learning difficulties
4. Mathematically-precocious students
5. Teacher-directed versus student-centered instruction
6. "Real-world" problems

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Instructional Materials

- U. S. mathematics textbooks are far too long -- often 700-1000 pages.
- Mathematics textbooks are much smaller in many nations with higher mathematics achievement than the U.S.
- Excessive length makes our books unnecessarily expensive and tends to undermine coherence and focus.
- Publishers must ensure the mathematical accuracy of their materials.

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Assessment

- NAEP and state tests must focus on the mathematics that students should learn, with scores reported and tracked over time.

States and NAEP need to develop better quality control and oversight procedures to ensure that test items:

- Are of the highest quality.
- Measure what is intended.
- Do not include design or wording problems that provide unintended sources of difficulties.

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Teachers

- Persistent evidence that a large proportion of the variability in student achievement gains is due to who the teacher is
- Less clear from the evidence exactly what it is about particular teachers that makes them more effective
- Need to know how more effective teachers differ from less effective ones and how to measure this

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Teachers' mathematical knowledge

- Overall signal: teachers' content knowledge is a positive factor in students' achievement.
- Number of courses, degree, or certification in math do not predict student achievement gains at K-8 and very inconsistently at high school level
- Closer measures (tests of relevant knowledge) show stronger signal

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Teacher education

Lack of evidence overall about:

- Features of teacher preparation or professional development produce changes in teachers' knowledge or their students' learning
- What features of teacher preparation or professional development produce changes in teachers' knowledge or their students' learning

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Teacher pay

- Salary differential between teaching and other technical fields is large.
- Location-based pay can keep experienced teachers in high-need schools.
- Performance pay for teachers can enhance students' achievement.
- Lack of evidence overall on:
  - How to best design teacher pay schemes to enhance student achievement (e.g., individual or school; competitive or not; levels of compensation)
  - Whether and how location-based pay helps to attract teachers to high-need areas

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"Mathematics specialists"

- Different models of "math specialists": lead teacher, elementary math teachers, math coaches
- Promising to explore the use of full-time mathematics teachers in elementary schools.
- Lack of evidence overall on whether math specialists (any model) lead to greater gains in student achievement

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Disappointments

- The treatment and lack of treatment of equity
- All the things about which there simply wasn't adequate evidence to make claims about their effectiveness—
  - Teacher education and professional development
  - Instructional approaches
  - Curriculum designs and sequences
  - What mathematical knowledge, skills, habits of mind matter for teachers

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Next steps: Using the NMP

1. Focus on areas of progress and substantial agreement; build on and extend those
2. Develop theories of learning connected to instruction
3. Articulate and study instructional practice, investigating how it is affected by who the students are and what the setting is
4. Strengthen teachers' opportunities to learn mathematics for teaching practice
5. Develop measures of that knowledge that can be related to instruction and student learning
6. Use strong research designs to study alternatives in professional training (preservice, early career support, professional education) for their impact on teachers' effectiveness

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- http://www.ed.gov/about/bdscomm/list/mathpanel/index.html

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A missing piece: Intervening in schools

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A sprawling "system"

- 50 million students in K-12 schools
- 90,000 schools
- 15,000 school districts
- 3.6 million teachers
- Per-pupil spending by state varies from about $6K - $14K

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The U.S. has a long history of trying to fix schools

— and an equally long history of failing to do so.

What are the key levers for educational improvement?

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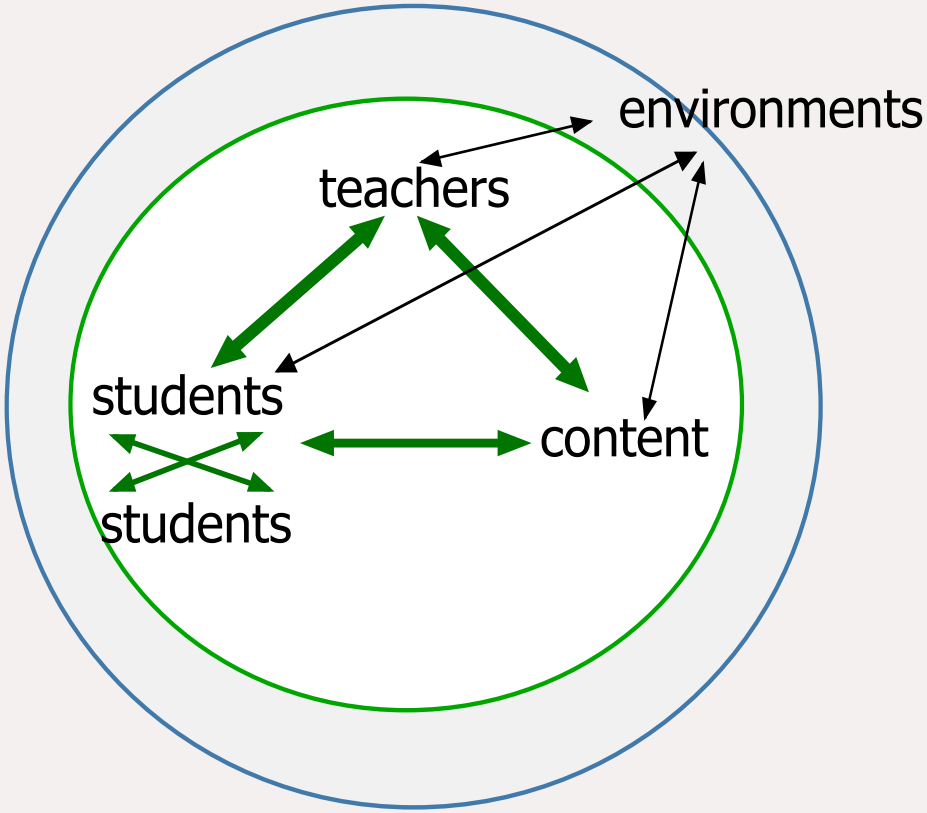
Instruction as interaction



- Instruction as the "black box" of interactions among students, teachers, content
- (Cohen) Teaching is a thoughtful human construction designed to improve learning
- Teaching practice is deliberate and attentive.
- Practicing teachers seek to connect their teaching to students' learning.
- Coordinating, over time, and with groups of students to accomplish specific goals

Cohen, Raudenbush, & Ball (2003). Resources, instruction, and research. EEPA.

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Interventions

- Comprehensiveness
- Design
  - Elaboration
  - Development
- Coordinate instruction
- Manage incentives
- Use resources
- Manage the environment

Cohen & Ball (1999, 2007); Cohen, Raudenbush, & Ball (2003)

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The post-test: Checking your answers

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① Ricardo has 38 cookies. If he packages them in bags with 4 cookies in each bag, how many bags can he fill? (measurement, 9)

② Ricardo has 38 cookies. If he distributes them equally among 4 of his colleagues, how much will each person get? (partitive, 9½, or 9 r2)

③ Ricardo has 38 pupils. If he wants to arrange the classroom into tables that seat 4 people, how many tables does he need? (measurement, 10)

④ Ricardo has 38 commemorative Red Wings pencils and wants to make 4 equal-sized packs of pencils. How many pencils will be in each pack? (partitive, 9, with 2 pencils left over)

⑤ Ricardo has 38 cm of licorice. He wants to cut the licorice into 4-cm. lengths. How many lengths will he able to make? (measurement, 9 ½)

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Lessons from success

1. Teaching is a professional practice that must be learned, and continuously developed. (e.g., Japan, China)
2. Improving students' learning requires a focus on instruction:
   - Provide sufficient detail
   - Provide for professional learning (and "unlearning") about content for teaching, about good student work and how to produce it with actual kids
   - Involve multiple stakeholders
   - Attend to coordination and incentives

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Where should we place our bets, given what we know (and what we don't)?

- Build a usable knowledge base for instruction
- Provide much more detailed instructional guidance
- Provide systematic professional training for successful practice
- Learn how other professions prepare practitioners (e.g., Grossman)
- Install sustained professional opportunities for continuous improvement, connected to the actual tasks of practice in settings
- Coordinate policy signals, curriculum, assessment, professional education

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