



Making Mathematics Learnable in School:

What is the Work of Teaching Mathematics?

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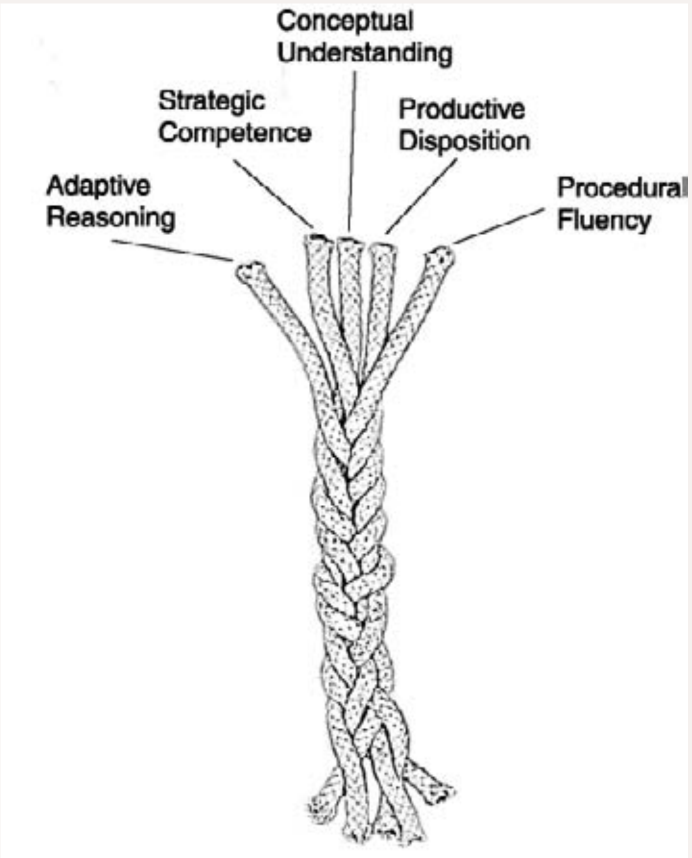
Following the two Davids

The need to

1. Shift pedagogy to more complex academic and social outcomes
2. Manage a policy environment that can narrow what gets taught and what gets learned

Strands of mathematical proficiency



- ***Conceptual understanding*** - comprehension of mathematical concepts, operations, and relations
- ***Procedural fluency*** - skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- ***Strategic competence*** - ability to formulate, represent, and solve mathematical problems
- ***Adaptive reasoning*** - capacity for logical thought, reflection, explanation, and justification
- ***Productive disposition*** - habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy

Kilpatrick, J., J. Swafford, and B. Findell. (2001). *Adding It Up: How Children Learn Mathematics*. Washington, DC: National Academy Press.

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The main argument of my talk

1. Teachers as professionals are key to the need to more complex learning in school, and to managing the environments of schooling.
2. Building this capacity depends in significant part on truly professional education, centered in the work of teaching.

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Overview

1. Knowing mathematics — and helping others know mathematics
2. The work of teaching mathematics
3. Learning to do the work of teaching mathematics
4. Centering professional education in practice: What are the resources — and the challenges?

1. Knowing mathematics — and helping others know mathematics

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Knowing mathematics . . .

Can the list of numbers 1, 2, 3, 4, and 5 be re-arranged so that all sums from 1 – 15 are possible by adding together sets of adjacent numbers?

52
-39
13

Is 1 a prime number?

What is 7 ÷ 0 ?

Is this a polygon?

. . . involves learning and doing for <u>oneself</u> . . .

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Teaching mathematics . . .

. . . involves getting <u>others</u> to learn and do mathematics

A fundamental paradox

- Good teachers must know and love the domains they teach.
- But to be good teachers, they must be fascinated by other people's thinking, learning, and work in the domain, not only their own.
- But being fascinated depends on knowing the domain really well.

Imagine . . .

- A piano teacher whose students mostly watch her play
- A Chinese teacher whose students mostly listen to her speak
- A mathematics teacher who stands at the board talking and doing mathematics for most of each class

What are the tacit assumptions underlying “performance pedagogy”?

1. That observing expert performance can enable learners to perform a practice
2. That experts can explicate what they are doing that must be learned
3. That learners’ learning trajectories mirror experts’ accomplished practice

What is the work of getting <u>others</u> to learn and do?

Building bridges between

learners' experience and ways of thinking

and

domain-specific skills, knowledge, dispositions, and capacities

What does this “bridge building” entail?

- Understanding the domain in multiple ways
- Explicit knowledge of tacit aspects of the domain
- Seeing the domain from others’ viewpoints

A simple example

Calculate: $2 \div \frac{2}{3}$

Answer: $2 \div \frac{2}{3} = 3$

What are different ways to solve $2 \div \frac{2}{3}$?

① $2 \bullet \frac{3}{2} = \frac{6}{2} = 3$

② How many $\frac{2}{3}$ are there in 2? (3)

③ If 2 is $\frac{2}{3}$ of some whole, what is the whole? $2 = \frac{2}{3}x$; $x = 2 \bullet \frac{3}{2}$; $x = 3$

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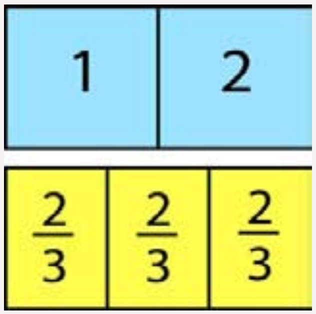
Analyzing representations: Which of the following can be interpreted to represent $2 \div \frac{2}{3}$?

(a)



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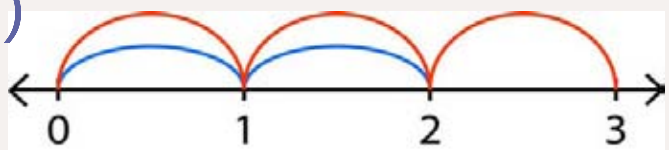

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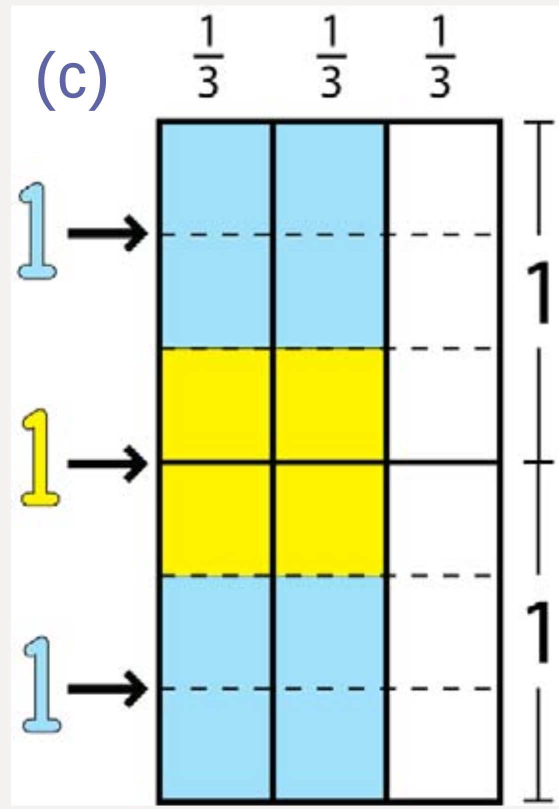

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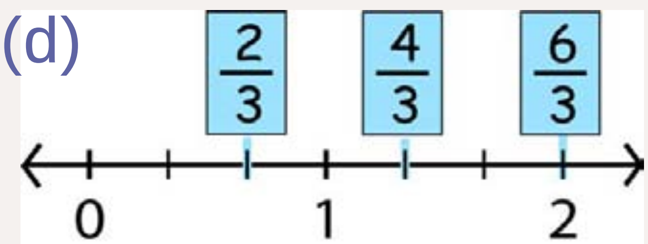

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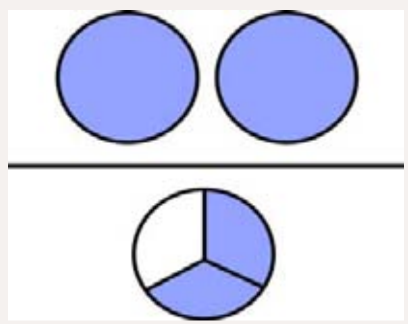


Interpreting others' reasoning

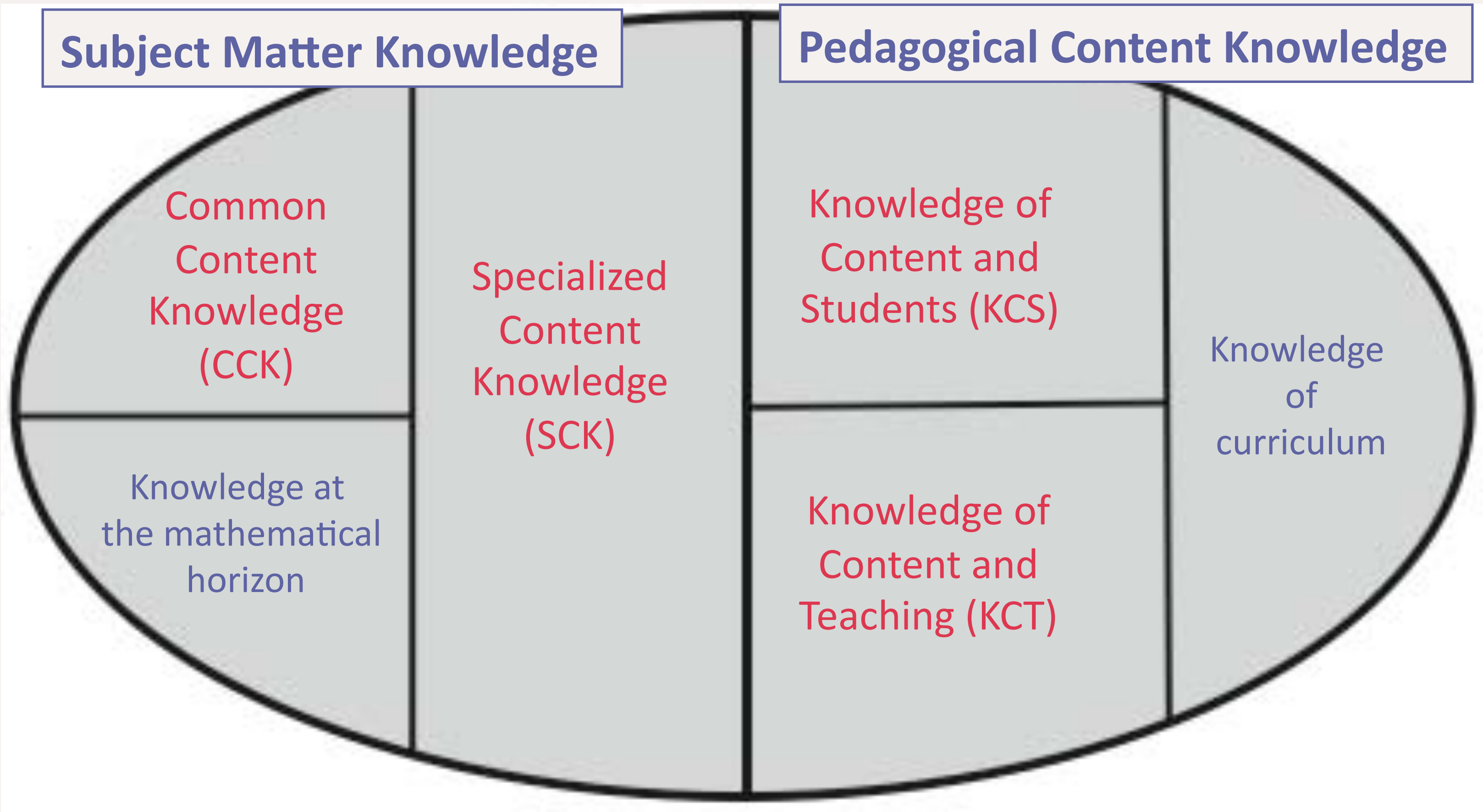
$$2 \div \frac{2}{3} = \frac{2}{1} \div \frac{2}{3} = \frac{6}{3} \div \frac{2}{3} = \frac{3}{1} = 3$$

What is going on here?

Then why do we use 'invert and multiply'? Or is this a fluke?

Does this work for all division of fractions problems?

Mathematical Knowledge for Teaching (MKT)



Ball, Thames, and Phelps (2008), Journal of Teacher Education

2. The work of teaching mathematics

The work of teaching mathematics: One look

1. "Off camera": Before this episode
2. During these 6 minutes

What are specific instances of the work of teaching in this segment?

Teaching fractions in third grade

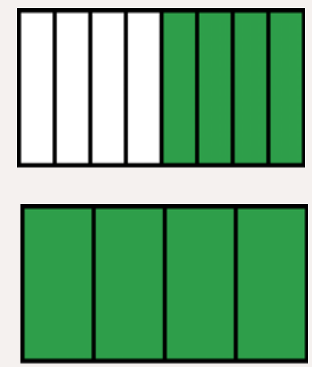
Comparing fractions

Multiple representations:
area model, number line

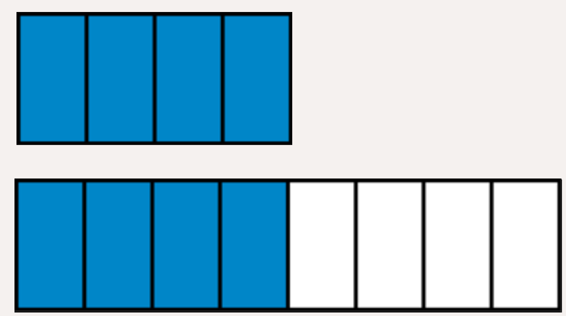
Which is more—

$$\frac{4}{4} \text{ or } \frac{4}{8} \text{ ?}$$

Lin's explanation:



Kevin's error:



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Homework
Dismissal
PM 1:00:25

The work of teaching mathematics: One look

"Off camera": Before this episode

1. Learn about individual children and what they know, care about, are worried about, can do, etc.
2. Establish the environment to manage behavior
3. Teach intellectual habits (e.g., drawing, speaking to peers, knowing and being able to choose and make different kinds of mathematical moves)
4. Choose the specific problem: Which is more 4/4 or 4/8? Why that question? Why those numbers? What's a similar or better choice?

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During these 6 minutes

12:58:35—Open the discussion: use specific tone, body movement around the room, choose whom to call on, and call on that child

12:58:38—Watch students while walking around; figure out who is drifting and encourage students' attention; maintain tenor of class while Lin draws

1:00:58—Lin completes drawing. Decide what to do about "I took four out of it"; direct her to repeat, "more loudly"; ask others to comment; work to get other students to comment besides Bernadette

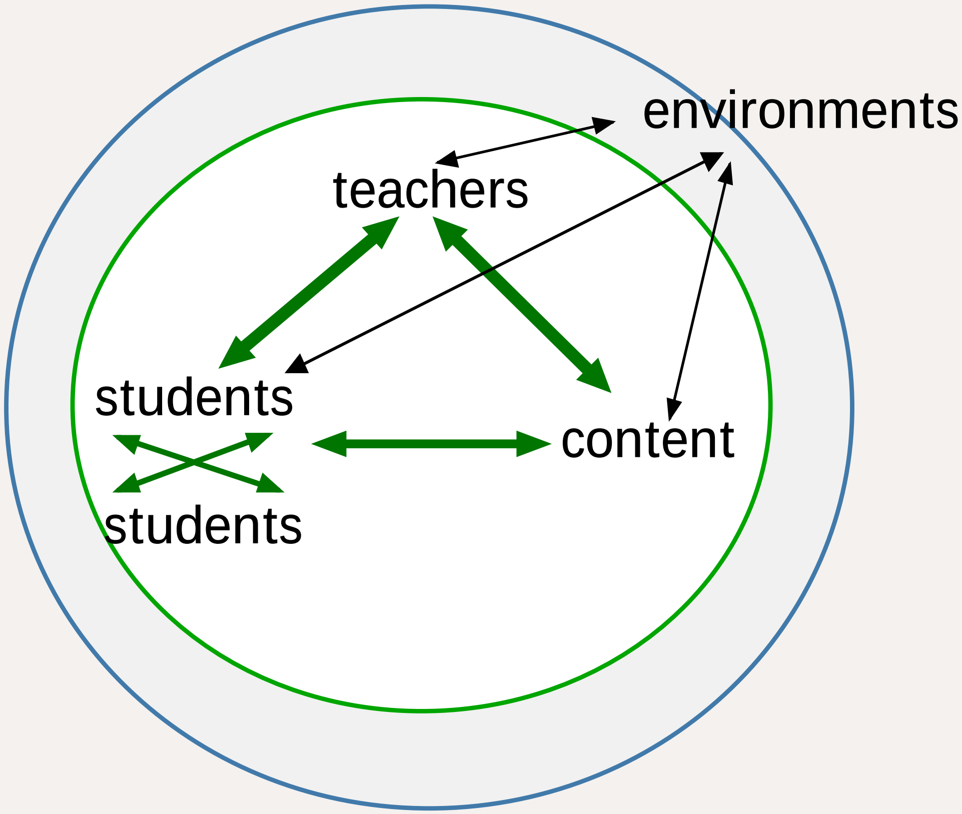
1:01:28—David comments. Work to understand; manage risk of losing class; decide not to take up; close interchange with David kindly.

1:02:40—Bernadette suggests the number line. Decide to have her work on the side; make her a number line to work on.

1:02:57—Kevin agrees and says first he did something else that was wrong. Decide to probe and to take this up; highlight for others; amplify by drawing incorrect picture on board

1:24:57—Pose question to assess students' understanding; make up specific question; decide how to take up answers

Teaching mathematics as intricate work



- Coordinating, over time, and with groups of students to accomplish specific goals
- The role of the teacher in deliberately improving the interactions to increase the probability that students learn what they need to learn
- Managing the environment

Cohen, Raudenbush, & Ball (2003); Lampert (2001); Lee (2007).

3. Learning to do the work of teaching mathematics

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An additive view

Knowledge of teaching practices

Knowledge of students

Knowledge of mathematics

A closer view: Teaching as mathematical work

- Using and analyzing representations, and mapping across different kinds of representations
- Defining terms and attending closely to language
- Using and inventing notation
- Producing and analyzing explanations
- Generating simpler and more complex versions of a problem
- Asking mathematical questions
  - Why does this work? Does this work in all cases? Do we have all the solutions? How are these two representations related?
- Thinking of special cases
  - Boundary cases, or examples that might push an initial idea

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How sensible is this view?

Consider some obvious mathematical demands of teaching. What do teachers do?

- Use textbooks
- Present content (either from the textbook or by one's own design)
- Show students how to solve problems
- Answer students' questions
- Assess students' work (responses in class, homework, tests)

These all require 'knowing' mathematics.

Teaching as mathematically 'natural' work, and the limits of this perspective

- Some aspects of teaching depend on mathematical instincts, habits of mind, practices
- So the additive view of learning to teach may make sense — add other knowledge to mathematical knowledge and habits

but —

- Teaching mathematics also involves doing things that are mathematically *unnatural*

Teaching as unnatural work*

Common ways of being

- Asking questions to which you do not know the answers
- Telling and showing others, doing things for people
- Assuming you know what others mean
- Correcting and smoothing over mistakes
- Assuming others experience things as you do
- Liking/disliking people
- Being "yourself"

Ways of being in teaching

- Asking questions to which you often do know (at least part of) the answer
- Listening and watching others, help others do
- Probing others' ideas
- Provoking disequilibrium and error
- Not presuming shared identity; seeking to learn others' experiences and perspectives
- Seeing people more descriptively
- Being in professional role

*Jackson, 1986; Murray, 1999

The specific case of mathematics teaching as mathematically unnatural work

- Unpacking mathematical ideas
- Listening to mathematically imprecise language
- Not automatically affirming correct statements
- Hearing what others say, not what you think
- Surfacing “error”

Given the fact that teaching math is both unnatural and intricate work . .

. . . and that we want more complex outcomes from mathematics education than many teachers had themselves,

and we have a policy environment that threatens that aspiration,

then equipping all students with good teachers of mathematics is a problem of effective *professional* education.

3. Learning to do the work of teaching mathematics

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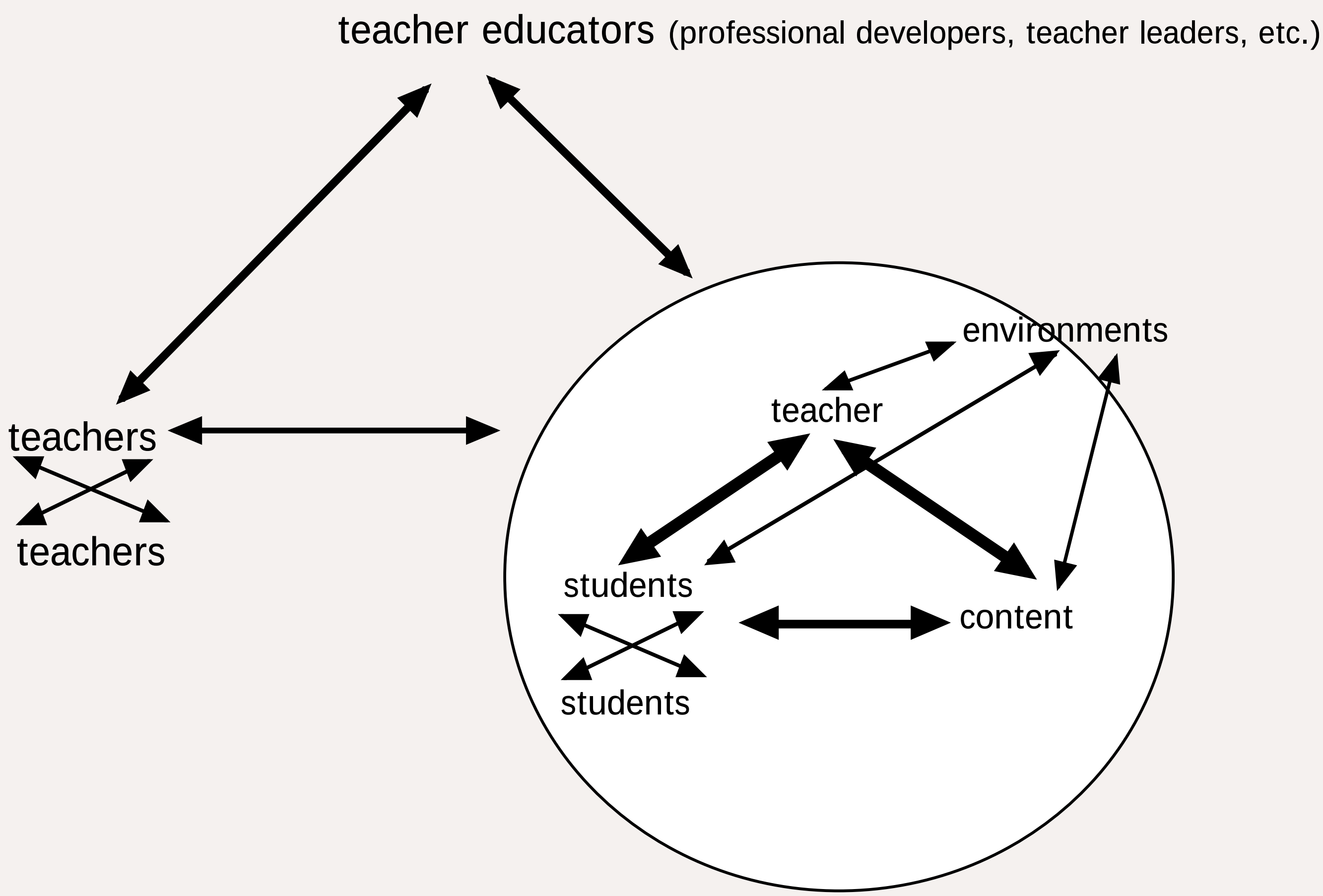
The U.S. lacks a reliable <u>system</u> of professional education

- Basically the same elements as in 1940
- Inappropriate subject matter preparation
- Inadequate preparation for diversity
- Persistent divides of theory and practice; tensions over "ivory tower" ideas and realities of practice
- Weak coordination of training, licensure, assessment of performance; certification, continuing education
- An underspecified theory of experience in learning to teach

Teaching practice as the content of professional education



Redesigning curriculum and pedagogy for teaching practice

- *What* is the "content"?
  - How to articulate the work of practice (Grossman and her colleagues)
  - How to see it when so much is invisible (Lewis, 2007)
  - Figuring out what is generic and what is subject-specific or context-specific
- *How* can practice be practiced?
  - Rehearsal
  - Approximations to real practice
  - In real time
- *Where* can practice be practiced?
  - Virtual, designed, real settings (Lampert)
- How can practice be *taught and assessed* in ways that do not convert it to propositional knowledge and analysis?

Identifying what to teach

- De-compose the work of teaching into smaller practices that:
  - can be articulated, unpacked, studied, and rehearsed
  - can be reintegrated into real-time teaching
- Choose practices that are “high-leverage” for beginners
  - Occur frequently in teaching
  - Core to different approaches to teaching
  - Crucial to improve the learning and achievement of all students
  - Can be articulated and taught

Grossman & Shahan (2005); Ball, Sleep, Boerst, and Bass (2009)

A few examples . . .

- Designing high quality instructional tasks
- Addressing basic skills within more complex work
- Building democratic classroom environments
- Interacting with pupils' families and caregivers
- Posing strategic questions
- Using the public space (whiteboard, posters)
- Assessing and diagnosing pupils' skills and knowledge
- Learning about pupils' out-of-school lives and contexts and considering in designing instruction

How might professional assessments be designed to --

- Validate the specialized skill and capacity it takes to teach well
- Leverage change in centering professional education in practice
- Ensure quality of instruction to which all pupils are entitled

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4. Centering professional education in practice: What are the challenges — and the resources?

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Challenges of centering professional education in practice

1. Lack of an adequate knowledge base about teaching practice
   - Inadequate language (in English)
   - Difficulty parsing the work into basic elements
2. Problem of expertise and tacit knowledge
3. Widely held view of teaching as uncertain, artistic, and unable to be specified
   - Resistance to seeing teaching as high-precision work, requiring high levels of skill
   - View of detail as "prescriptive" and as de-skilling professional work

Resources for centering professional education in practice

- Past history of microteaching and competency-based teacher education
  - Analyze similarities and differences
  - Integrate subject matter knowledge for teaching, skills, discretionary adaptation and judgment
- Progress made on content knowledge for teaching
- Other professions
  - Developing an agreed-upon curriculum of practice
  - Broadening idea of “clinical” and ways to structure and support it
  - Attention to relational work

An alternative (stealth) title:

In Praise of ‘Prescriptiveness’ and Training in Professional Education

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In praise of prescriptiveness and training in professional education

- Teaching mathematics is intricate work, and not natural, and needs to be learned and, hence, taught.
- Seeing teaching as skilled, high-precision work, that is not a matter of personal style and preference, is to <u>acknowledge</u> its professional nature, not to repudiate its 'creativity.'
- We need a reliable system of preparing many people for skillful practice that can
  - help diverse pupils attain complex outcomes, and
  - ensure that it is not a zero-sum game

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Slides will be available at Deborah Ball's website

(Google "Deborah Ball")

Thank you!

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