Critical Issues in Mathematics Education Workshop

Teaching and Learning of Algebra

14 - 16 May 2008

Mathematical Sciences Research Institute • Berkeley, CA

Workshop sponsored by

Mathematical Sciences Research Institute, National Science Foundation,

Math for America, Noyce Foundation

School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu

Core Questions for this Workshop

1. What are some organizing principles around which one can create a coherent pre-college algebra curriculum?
2. What is known about effective ways for students to make the transition from arithmetic to algebra?
3. What algebraic understandings are essential for success in beginning collegiate mathematics?

School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu

Critical Issues in Mathematics Education Workshop

(Teaching and) Learning of Algebra

14 - 16 May 2008

Mathematical Sciences Research Institute • Berkeley, CA

Workshop sponsored by

Mathematical Sciences Research Institute, National Science Foundation,

Math for America, Noyce Foundation

School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu

In search of teaching . . .

“Don’t forget the middle ‘man’” (Ed Silver)

Problem:
Teaching often remains invisible

- Limited and misleading language (classroom processes, development, facilitation, no word for the fundamental transaction)
- Some of the work is “inside” the teacher’s head
- Lack of frameworks for seeing teaching
- Grain size issues: at what scale to look
- Disciplinary traditions and perspectives that focus closely only on part of the story
- Teaching is undervalued and misunderstood

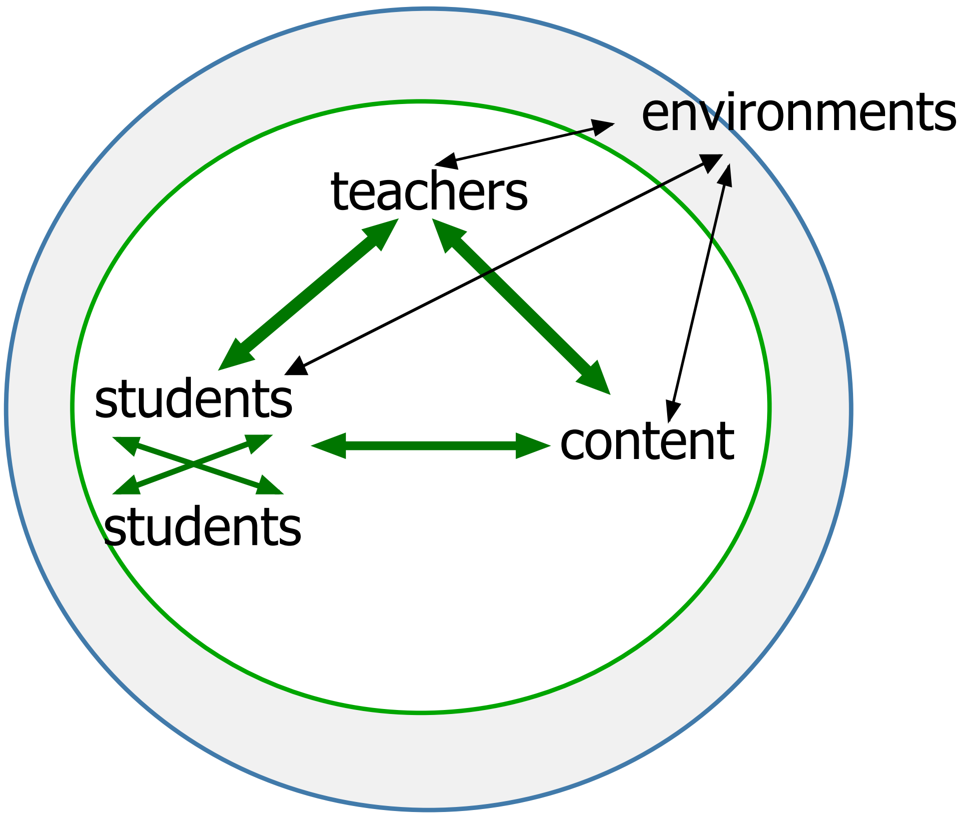
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu

A closer view: Teaching as mathematical work

- Using and analyzing representations, and mapping across different kinds of representations
- Defining terms and attending closely to language
- Using and inventing notation
- Producing and analyzing explanations
- Generating simpler and more complex versions of a problem
- Asking mathematical questions
  - Why does this work? Does this work in all cases? Do we have all the solutions? How are these two representations related?
- Thinking of special cases
  - Boundary cases, or examples that might push an initial idea

School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu

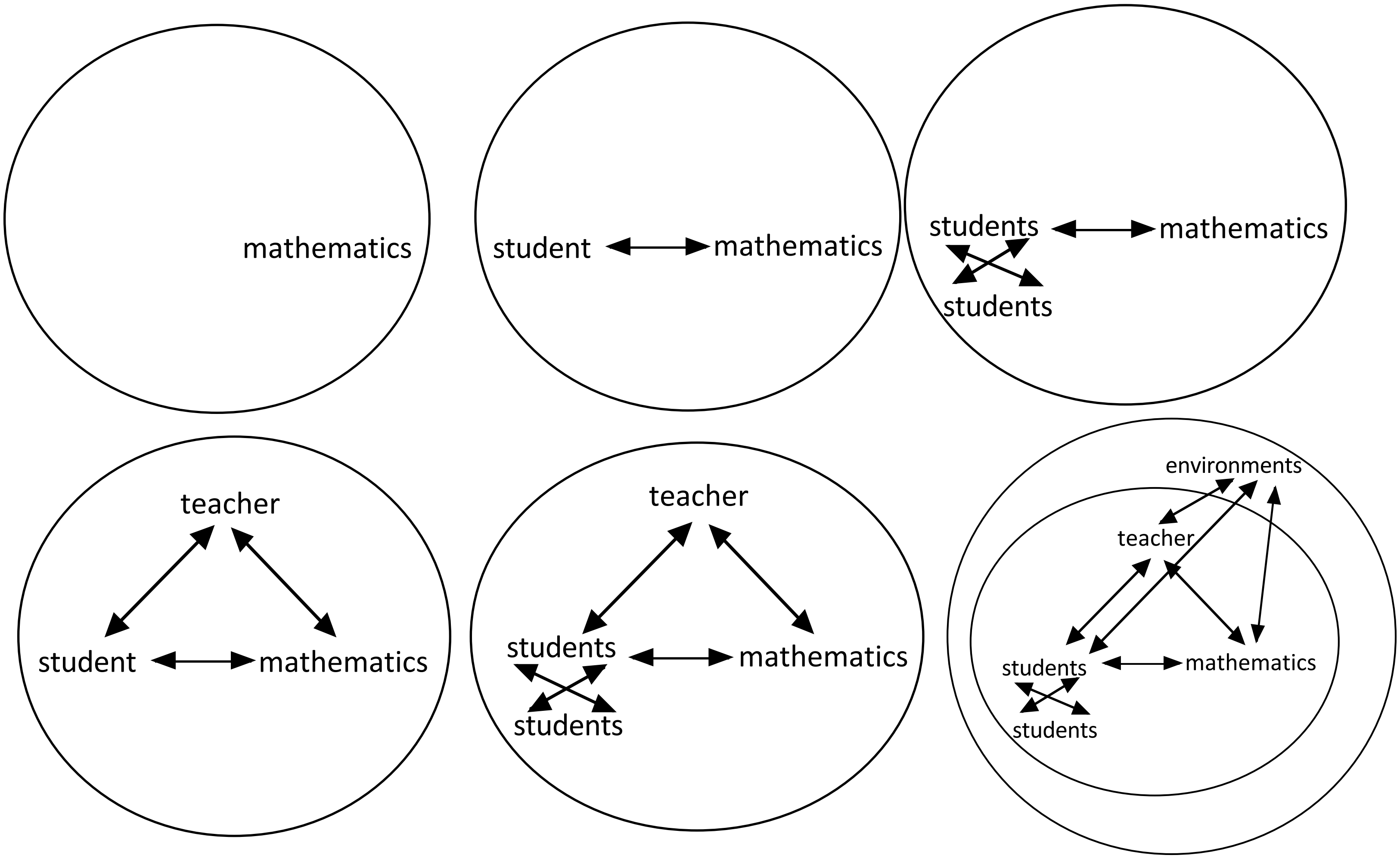
Teaching and learning of algebra



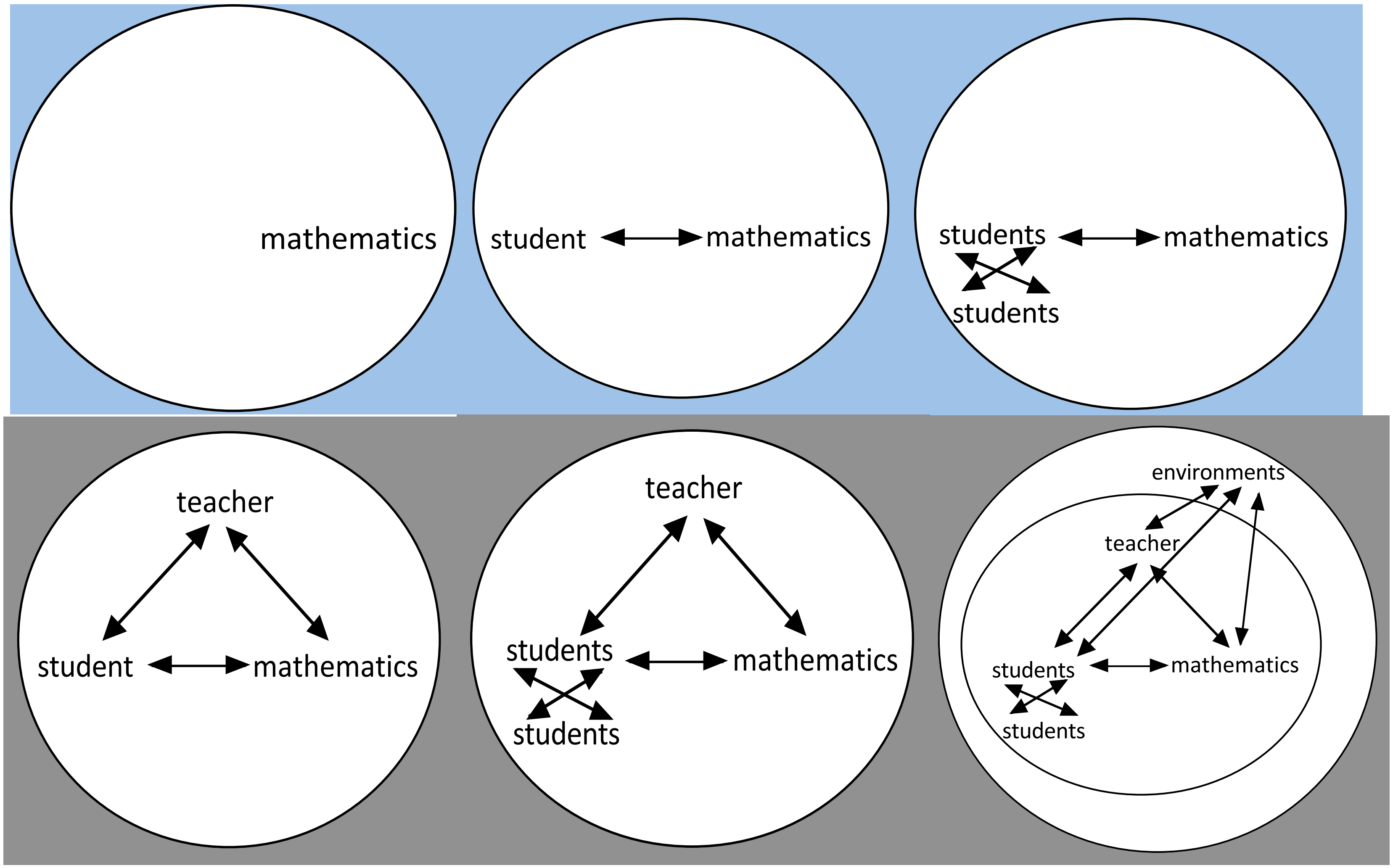
- Instruction as the "black box" of interactions among students, teachers, content
- (Cohen) Teaching is a thoughtful human construction designed to improve learning
- Teaching practice is deliberate and attentive.
- Practicing teachers seek to connect their teaching to students' learning.

Cohen, Raudenbush, & Ball (2003). Resources, instruction, and research. EEPA.

Alternative optics on the learning of algebra



Where we have focused



Knowledge and practice

- Dewey (1902): the logical and psychological structures of a subject
- Bruner (1960): intellectually honesty
- Schwab (1971): substantive and syntactical disciplinary structures (nature of knowing)
- Shulman (1986): pedagogical content knowledge

School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu

Why this matters:
Teaching as
unnatural and intricate work

School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu

Teaching as mathematically "natural" work, and the limits of this perspective

- Some aspects of teaching depend on mathematical instincts, habits of mind, practices
- So an additive view of learning to teach may make sense — add other knowledge to mathematical knowledge and habits

but —

- Teaching mathematics also involves doing things that are mathematically *unnatural*

School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu

Examples of mathematics teaching as mathematically unnatural work

1. Unpacking mathematical ideas
2. Listening to mathematically imprecise language
3. Not automatically affirming correct statements
4. Hearing what others say, not what you think
5. Surfacing “error”

School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu

One core problem of teaching mathematics

- Managing the journey toward compression
- Keeping one eye on the mathematical horizon while avoiding “compression impatience”
- Keeping an eye on the students and on the mathematics and recognizing opportunity moments for compression
- Teaching and learning mathematics as language work whose tools are definitions, representations, symbols, language

- (Hy Bass)

Teaching as intricate work: One look

1. "Off camera": Before this episode
2. During these 2 minutes

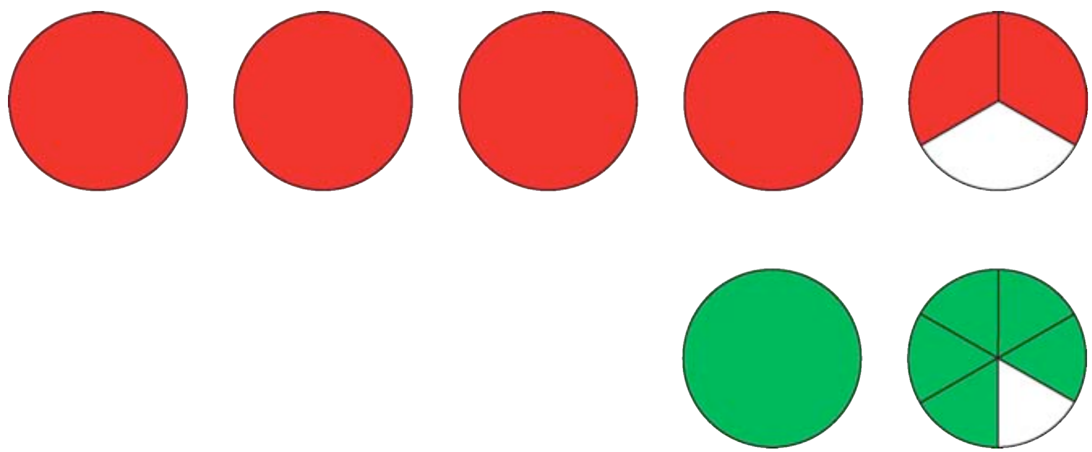
What are some specific tasks required in order to teach this segment?

School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu

Modeling addition of negative and positive fractions

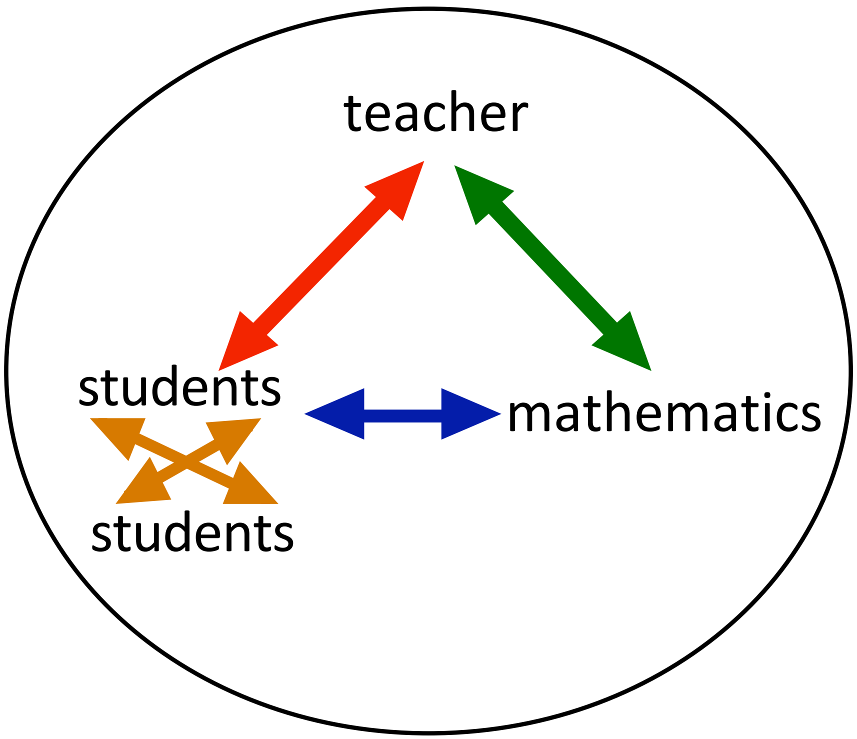
$$-4\frac{2}{3}+1\frac{5}{6}$$

- Red "pies" to represent negative numbers
- Green "pies" to represent positive numbers



School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu

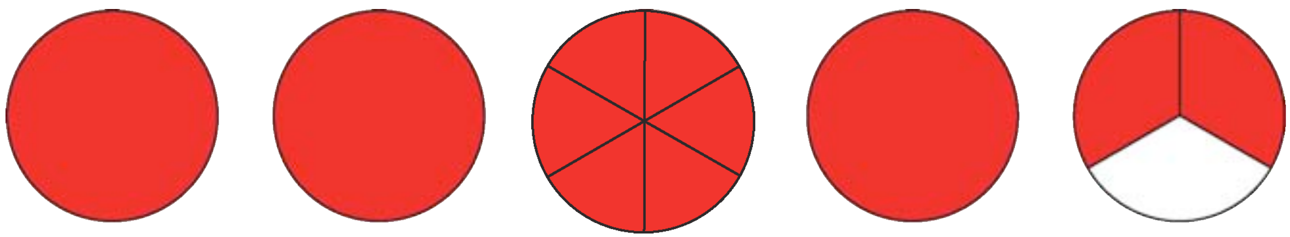
The work of teaching mathematics: What is there to "see"?



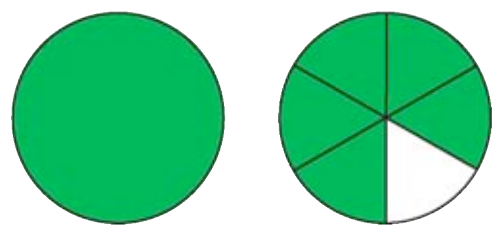
School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu

Modeling $-4\frac{2}{3}+1\frac{5}{6}$

- Red "pies" to represent negative numbers
- Green "pies" to represent positive numbers



Answer: $-2\frac{5}{6}$



School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu

Three terrains of teaching practice

1. Knowledge extended to learners
   a) How knowledge is extended
   b) The nature of knowledge
2. Instructional discourse
3. Teachers' acquaintance with students' knowledge and resources

D. Cohen (2008)

Toward bringing teaching into our conversations . . .

School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • (734) 615-7235 • mtlt@umich.edu