Knowing Mathematics Well Enough to Teach It: What More Does It Take?

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Knowing radical expressions

Simplify: $\sqrt{150}$

Knowing radical expressions for teaching

Which of the following is best for setting up a discussion about different solution paths for simplifying radical expressions?

(a) $\sqrt{54}$

(b) $\sqrt{156}$

(c) $\sqrt{128}$

(d) These examples all work equally well.

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Overview

- Defining "mathematical knowledge for teaching"
- How well do YOU know mathematics **for teaching**?
- Measuring mathematical knowledge for teaching: Results
- The need

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The ambitious goal of a more mathematically skilled population

- Goals for mathematical proficiency have increased and broadened over the last 50 years
  - Who needs to be successful with math
  - What mathematical proficiency is
  - What is important to learn
- This is not “reform,” but radical renovation

The role of teachers, and the problem

- Teachers are key to this ambition: any other tool (e.g., technology, curriculum, class size, time) depends on teachers' capacity to manage and use the tool and to help students do so

But --

- We have a shortage of qualified and skilled teachers, and
- We do not reliably prepare or support people to teach mathematics in ways that meet these aspirations

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What is the “problem”?

The quality of mathematics teaching and learning

Teachers’ knowledge of mathematics and their ability to use it in practice

Some approaches to solving the problem, and the unresolved question

- Require more mathematics for certification
  - More mathematics courses
  - A major (or minor) in mathematics
  - Mathematics test
- Recruit mathematically trained people into teaching
  - Engineers, accountants, mathematicians, …
- Fund mathematically focused professional development

But what kind of mathematical knowledge, skill,and reasoning is needed in teaching, and how can it be developed?

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Teachers need to know math: The common sense argument

Consider some obvious mathematical demands of math teaching. What do teachers do?

- Use textbooks
- Present content (either from the textbook or by one's own design)
- Show students how to solve problems
- Answer students' questions
- Assess students' work (responses in class, homework, tests)

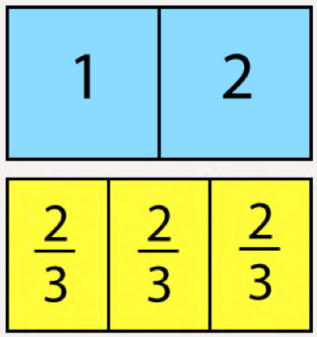
These all require knowing mathematics.

A closer view: Teaching as mathematical work

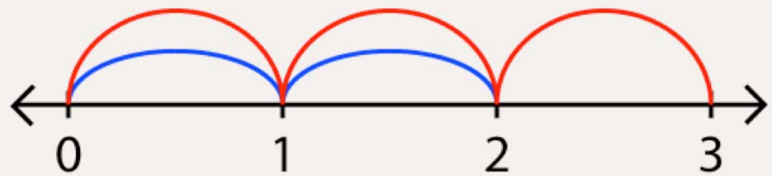
- Using and analyzing representations, and mapping across different kinds of representations
- Defining terms and attending closely to language
- Using and inventing notation
- Producing and analyzing explanations
- Generating simpler and more complex versions of a problem
- Asking mathematical questions
  - Why does this work? Does this work in all cases? Do we have all the solutions? How are these two representations related?
- Thinking of special cases
  - Boundary cases, or examples that might push an initial idea

Analyzing representations: Which of the following can be used to represent 2 ÷ 2/3?

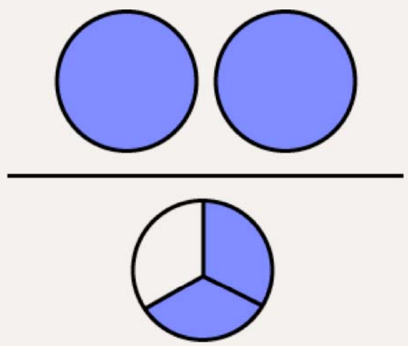
(a)



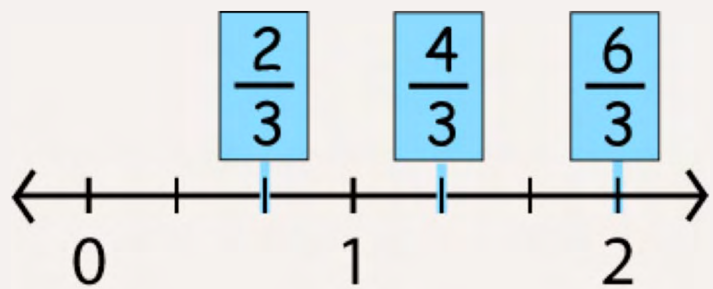
(b)



(c)



(d)



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Toward a practice-based theory of mathematical knowledge for teaching

1. Study instruction, and identify the mathematical work of teaching
2. Analyze what mathematical knowledge is needed to do that work effectively, and how it must be understood to be useful for the work
3. Develop, test, and refine measures of MKT using multiple methods as a means to evaluate professional education, investigate effects on students' learning, and improve theory
4. Develop and evaluate approaches to helping teachers learn mathematical knowledge for teaching

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Mathematical knowledge for teaching (MKT)

- What do we mean when we use this term, "mathematical knowledge for teaching"?
  - Mathematical knowledge, skill, habits of mind that are entailed by the work of teaching
- What do we mean by the "work of teaching"?
  - The tasks in which teachers engage, and the responsibilities they have, to teach mathematics, both inside and outside of the classroom

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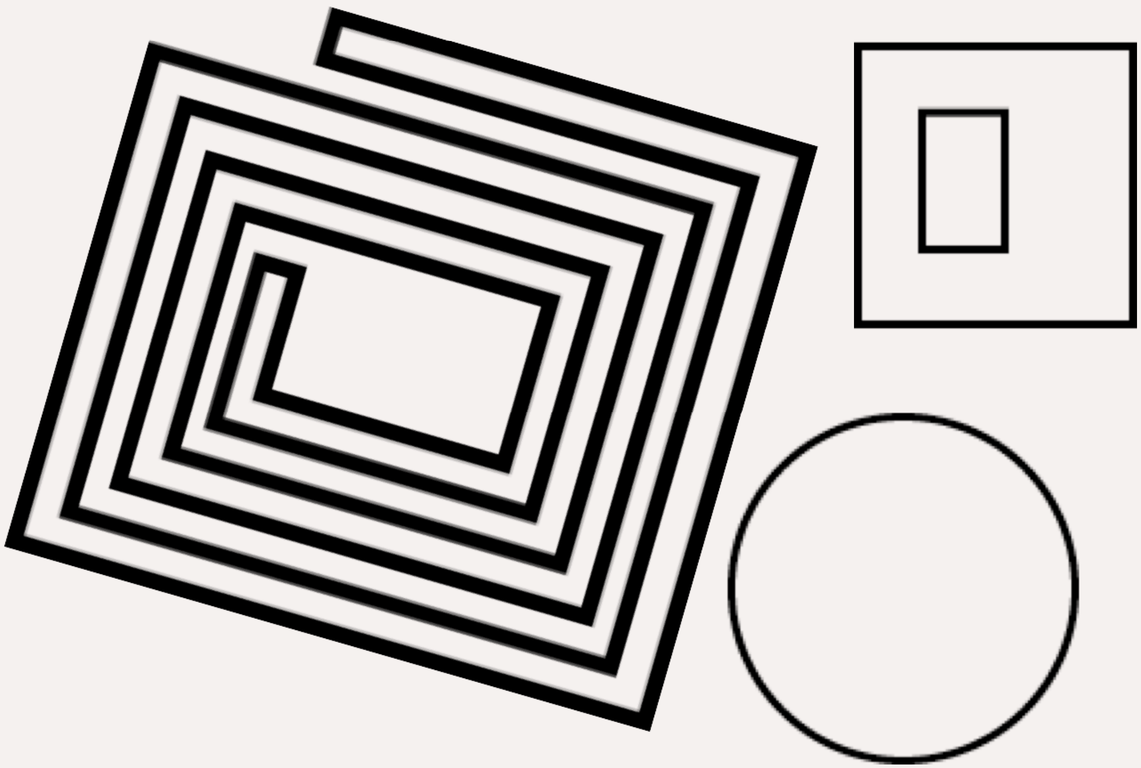
Knowing what a polygon is



Developing a useful definition of "polygon"

A polygon is a simple closed plane curve composed of finitely many straight line segments.

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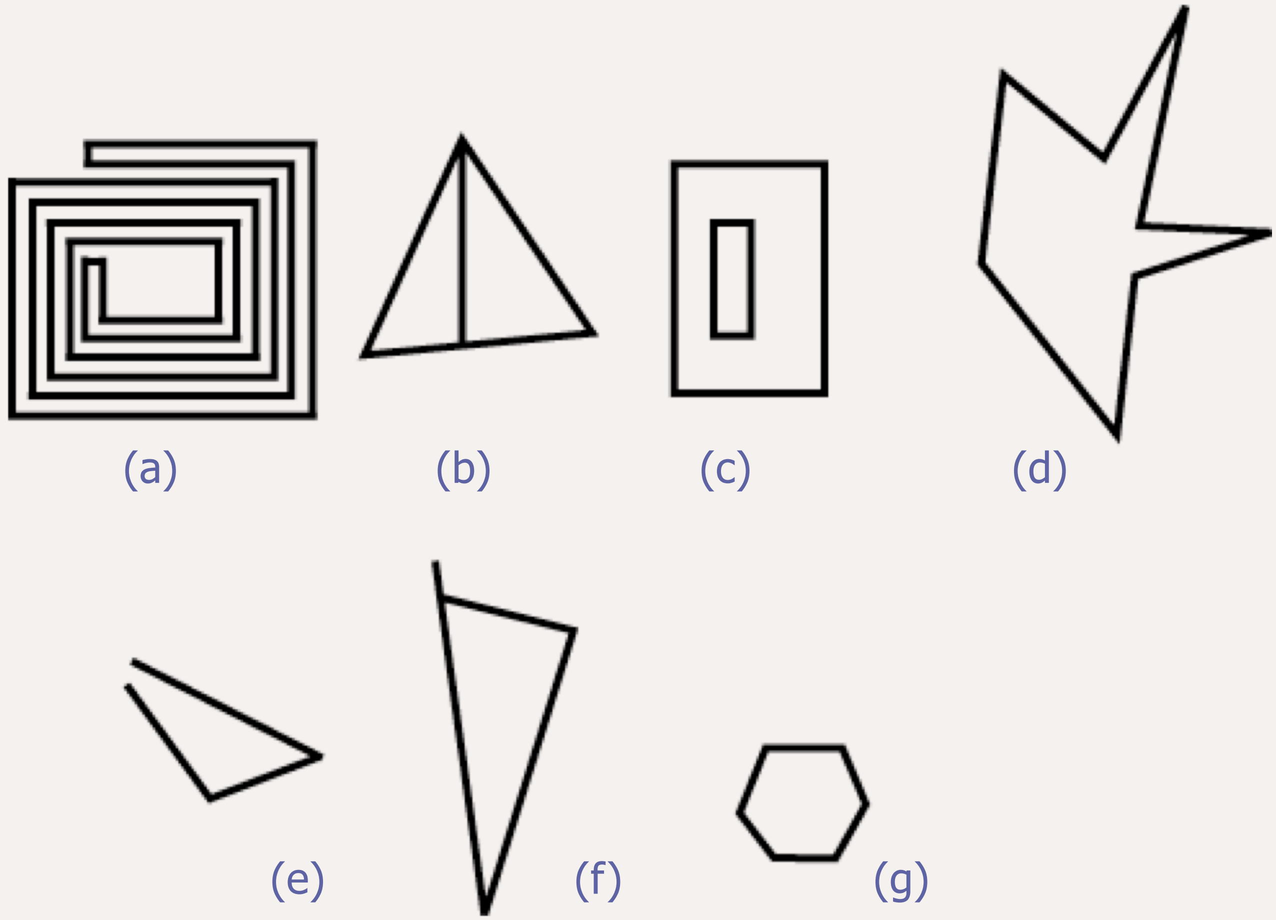
Possible examples: What is the reason for each?



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Other work of teaching

- Examining the mathematical quality of a textbook treatment of a topic
- Using and defining terms
- Supporting the linguistic demands for English language learners (ELLs)
- Producing and evaluating mathematical explanations
- Using notation
- Interpreting and evaluating alternative solutions and thinking
- Choosing contexts with care for mathematical integrity, diversity, and transparency for learning
- Explaining goals and mathematical purposes to others
- Designing homework and quizzes; Selecting and modifying mathematics problems
- Choosing and using representations
- Building correspondences between a model and a concept or procedure
- Evaluating students' work and statements (often quickly)
- Posing questions
- Choosing examples

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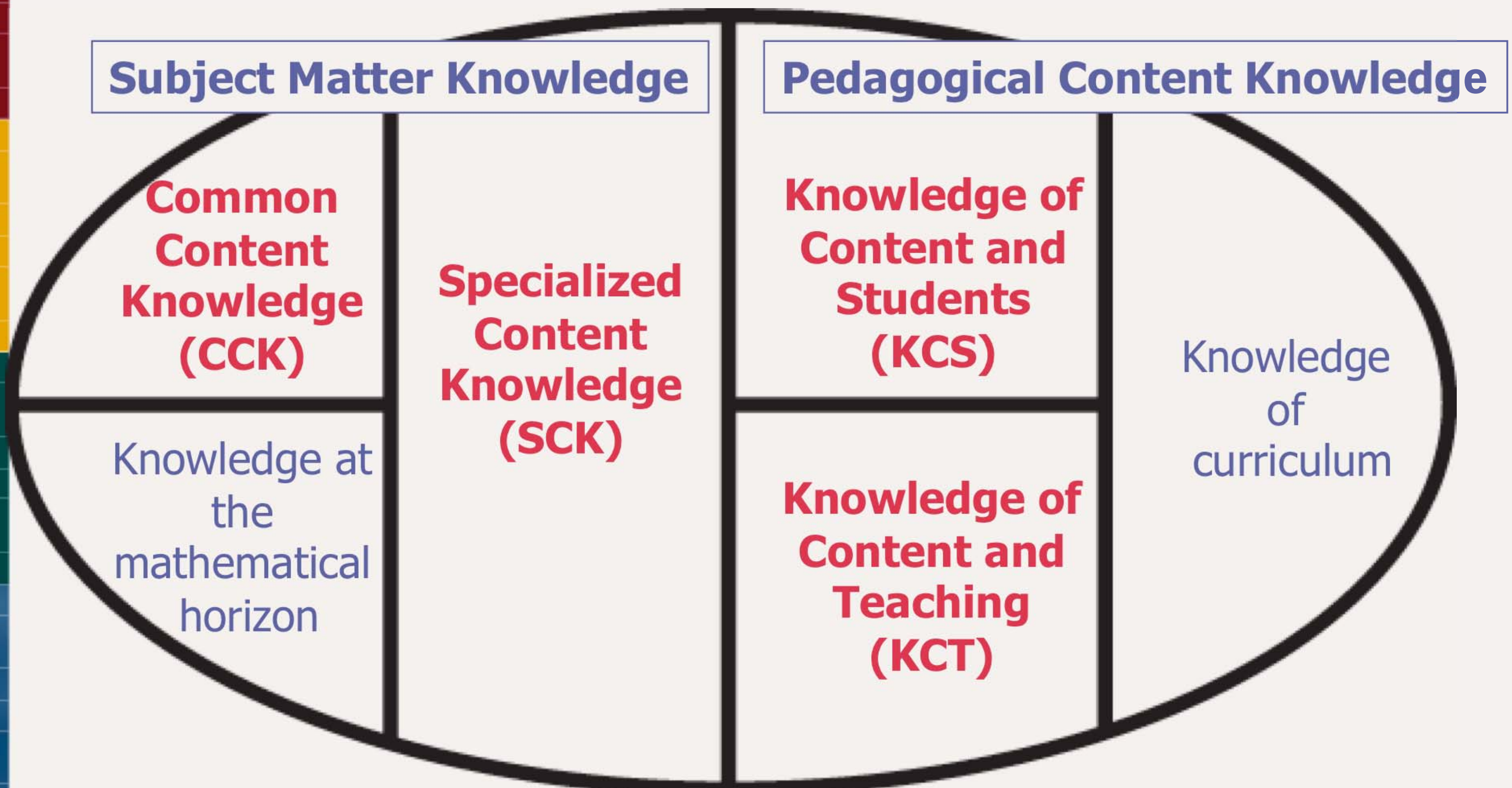
Mathematical knowledge for teaching



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What makes a "good" definition?

- Mathematically precise — correctly identifies the kind of object, process, property
- Usable by user community — based on already-defined and understood term
- Good for growth: Will expand easily as students advance

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Proposed definitions of "even number"

1. A number that can be divided in two equal parts with nothing left over is even.
2. A whole number is even if it can be divided into groups of 2 with nothing left over.
3. A number with 0, 2, 4, 6, or 8 in the ones place is even.

Is 7 even?

Is 32.7 even?

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What do we mean by “mathematical language”?

- Focus on three aspects of mathematical language:
  - Mathematical vocabulary
  - Mathematical notation
  - Grammar and syntax
- Selected because of prevalence in the literature and their impact on mathematics learning and teaching

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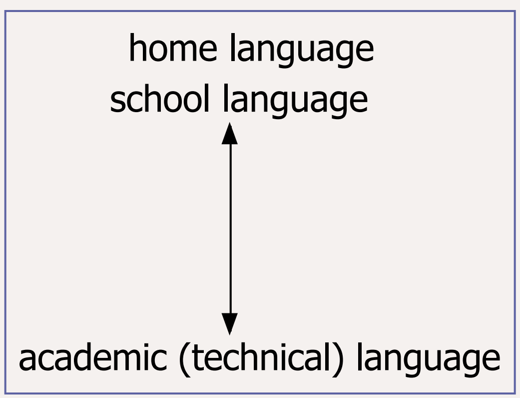
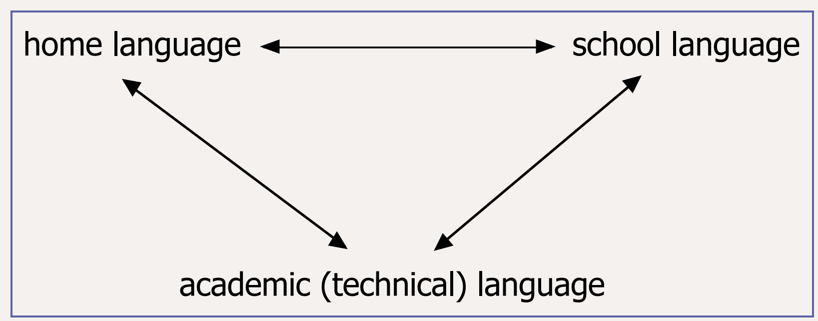
Issues with mathematical language: Mathematical vs. "everyday" language

- Mathematics often uses and specializes everyday language rather than coining a separate technical vocabulary -- both enabling and complicating entry to its register.
- In learning mathematics, students talk informally and imprecisely
- Equity issue: Additional dimension for students who must navigate between home, school, and mathematical languages





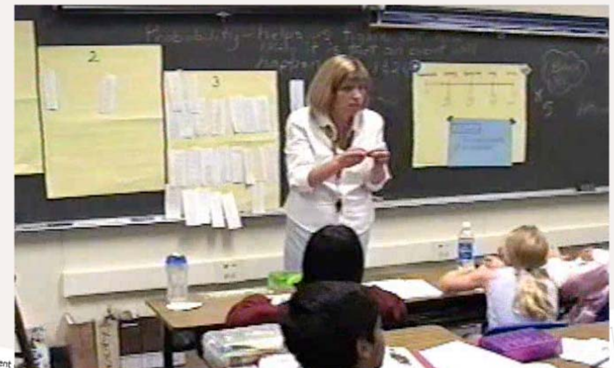
Developing measures of mathematical knowledge for teaching

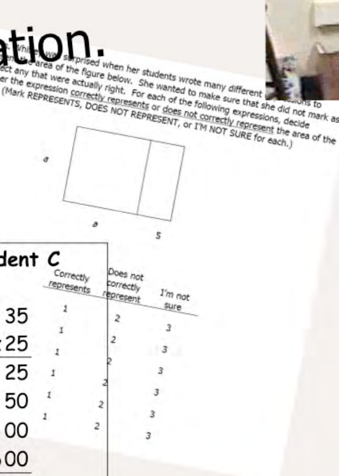
Learning Mathematics for Teaching (LMT) Project

Write a story or a situation or make a diagram for which

$$1\frac{3}{4} \div \frac{1}{2}$$

is the mathematical formulation.





Student A	Student B	Student C
35 ×25 125 +75 875	35 ×25 175 +700 875	35 ×25 25 150 100 +600 875

Which of the students is using a method that would work to multiply any two whole numbers?

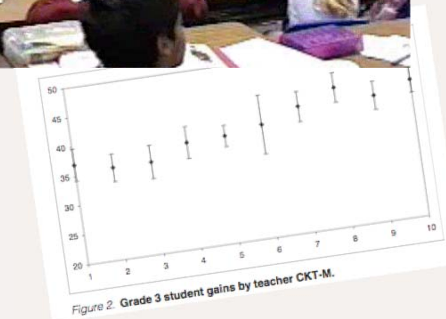
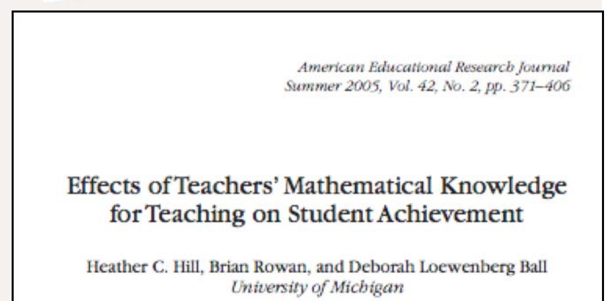


Figure 2. Grade 3 student gains by teacher CKT-M.



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Effects of Teachers' Mathematical Knowledge for Teaching on Student Achievement

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Sample item:
Which student is using a method that would work for any two whole numbers?

Student A	Student B	Student C
35 x25 125 +75 875	35 x25 175 +700 875	35 x25 25 150 100 +600 875

Validating our measures

How do we interpret teachers' performance on our questions?

1. Their score reflects their mathematical thinking
   – Cognitive interviews
2. Higher scores mean higher-quality mathematics instruction
   – Videotape validation study
3. Scores reflect common and specialized knowledge of content
   – Mathematician and non-teacher interviews
4. Higher scores related to improved student learning
   – Student gains analysis

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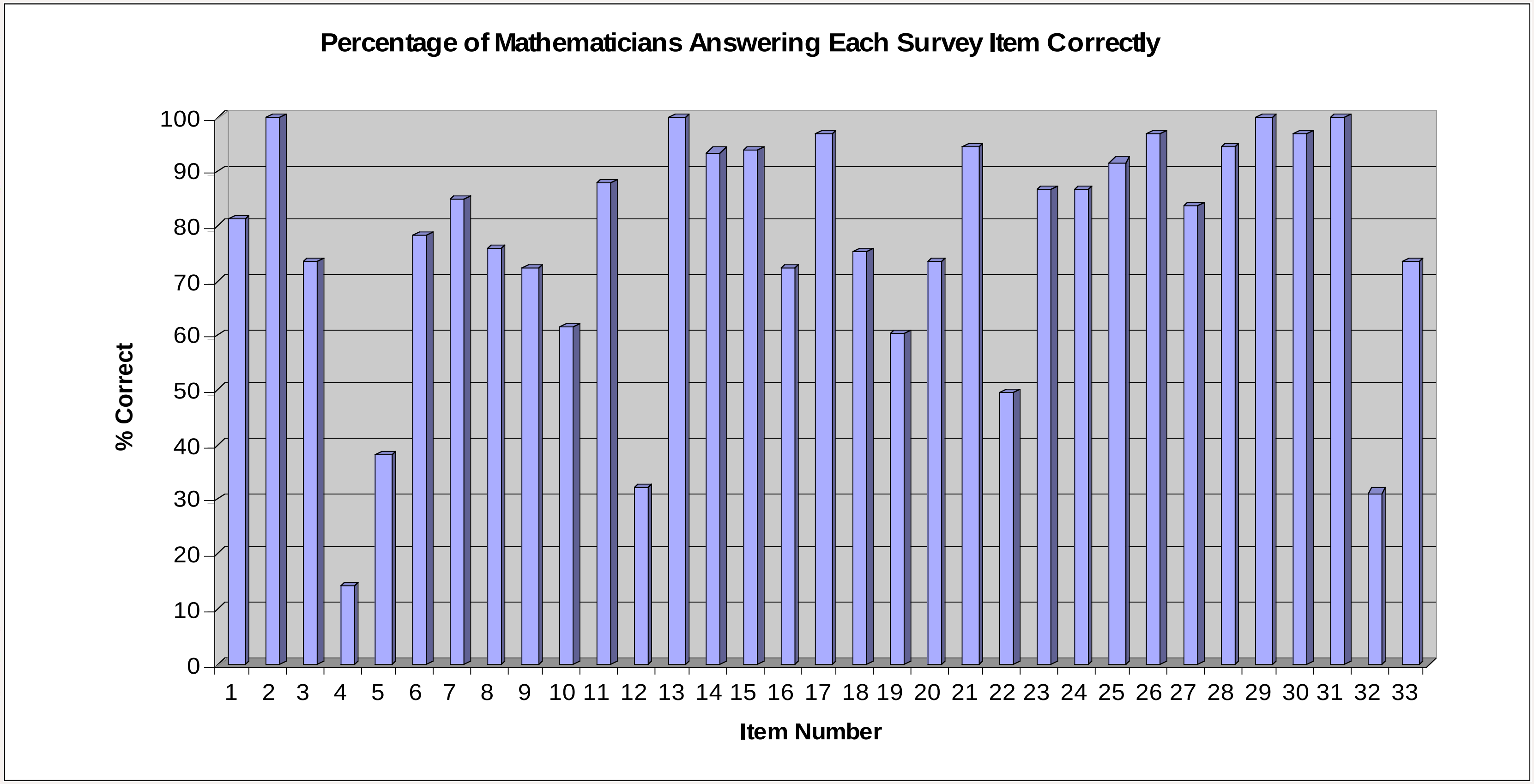
Percentage of mathematicians answering each survey item correctly



Sleep, L., Delaney, S., Ball, D. L., & Hill, H. C. (2005). Validating "mathematics knowledge for teaching": Evidence from mathematicians' performance on teacher knowledge items. Paper presented at the annual meeting of the American Educational Reseach Association, Montreal, Canada. April 14, 2005

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Why did mathematicians get items wrong?

- Items had mathematical flaws.
- Items required knowledge of learners.
- Items demanded mathematical knowledge unique to the work of teaching:
  - Making sense of non-standard solutions or ideas
  - Choosing numerical examples
  - Choosing representations

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Linking teacher knowledge and student achievement

- Questionnaire consisting of 30 items (scale reliability .88)
- Model: Student Terra Nova gains predicted by:
  - Student descriptors (family SES, absence rate)
  - Teacher characteristics (math methods/content, content knowledge)
- Teacher content knowledge significant
  - Small effect (< 1/10 standard deviation): 2 - 3 weeks of instruction
  - But student SES is also about the same size effect on achievement

(Hill,Rowan, and Ball, AERJ, 2005)

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An important problem: Insufficient opportunities for teachers to develop MKT

- Many of teachers' opportunities to learn mathematics are not aimed at developing the capacity to know and use mathematics in teaching
- Some teachers learn MKT from experience, but many do not
- Lack of materials for teaching mathematics to teachers in ways that enable them to know and use mathematics in practice (despite many current projects)

Developing usable knowledge of MKT

- Find/develop tasks that create opportunities for learning mathematical knowledge for teaching
- Situate teachers' opportunities to learn in the contexts of use
- Provide teachers with opportunities to practice the kinds of mathematical thinking, reasoning, and communicating used in teaching
- Enact tasks in ways that maintain the focus on developing MKT and the ability to use it in teaching

Conclusions

1. The mathematical knowledge for teaching needed is specialized and is more than the content knowledge shared in common with other adults.
2. This is a professionally-shaped way of knowing the subject. Who knows this content, and who can teach it to teachers?
3. Identifying the expertise involved in mathematics education is not easy: a problem shared by experts in other domains. But when we cannot identify this expertise, we contribute to the sense that teaching (and education more broadly) depends only on common sense.