

Adapting and Using U.S. Measures Of Mathematical Knowledge for Teaching in Other Countries: Lessons & Challenges

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Adapting and Using U.S. Measures Of Mathematical Knowledge for Teaching in Other Countries



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Overview

- Overview of MKT
- International interest in MKT measures
- Why as currently conceptualised MKT may be specific to the United States
- Overview of presentation
- What is meant by “items”

Challenges of Translating and Adapting the MKT Measures for Norway



Reidar Mosvold & Janne Fauskanger
University of Stavanger



The Stavanger project

- Who we are
- Where we have been
- Where we are now
- Where we are going

Our translation process!

Changes related to:

1. General cultural context
2. School cultural context
3. Mathematical substance
4. Other

Additional changes:

1. Translation into Norwegian
2. Political directives

Example item

10. Students in Mr. Hayes' class have been working on putting decimals in order. Three students – Andy, Clara and Keisha – presented 1.1, 12, 48, 102, 31.3, .676 as decimals ordered from least to greatest. What error are these students making? (Mark ONE answer.)

- a) They are ignoring place value.
- b) They are ignoring the decimal point.
- c) They are guessing.
- d) They have forgotten their numbers between 0 and 1.
- e) They are making all of the above errors.

Our translation

10. Elevene til Hans har arbeidet med å sortere desimaltall i stigende rekkefølge. Tre av elevene, Anders, Klara og Kristin, sorterte desimaltall slik: 1,1 12 48 102 31,3 0,676.

Hvilken feil er det disse elevene gjør? (Marker ETT svar.)

- a) De ignorerer plassverdi/posisjonsverdi.
- b) De ignorerer desimalkomma.
- c) De gjetter.
- d) De har glemt at det fins tall mellom 0 og 1.
- e) De gjør alle feilene ovenfor.

Changes made

10. **Elevene til Hans** har arbeidet med å **sortere desimaltall i stigende rekkefølge**. Tre **av elevene**, **Anders, Klara** og **Kristin**, **sorterte desimaltall slik: 1,1 12 48 102 31,3 0,676**.

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General cultural context

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School cultural context

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Mathematical substance

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From English into Norwegian

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Political directives

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Other publications:

Fauskanger, J., & Mosvold, R. (2009). Teachers' beliefs and knowledge about the place value system. In C. Winsløw (Ed.), *NORDIC RESEARCH IN MATHEMATICS EDUCATION Proceedings from NORMA08 in Copenhagen, April 21-April 25, 2008*, 159-166. Rotterdam, The Netherlands: Sense Publishers.

Mosvold, R., Fauskanger, J., Jakobsen, A. & Melhus, K. (2009). Translating test items into Norwegian - without getting lost in translation? Manuscript submitted for publication.

For more information, check out:

<http://mosvold.info> and
<http://mathedresearch.blogspot.com>

Thank you !

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Validating the Adapted MKT Measures in Korea



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University of Michigan

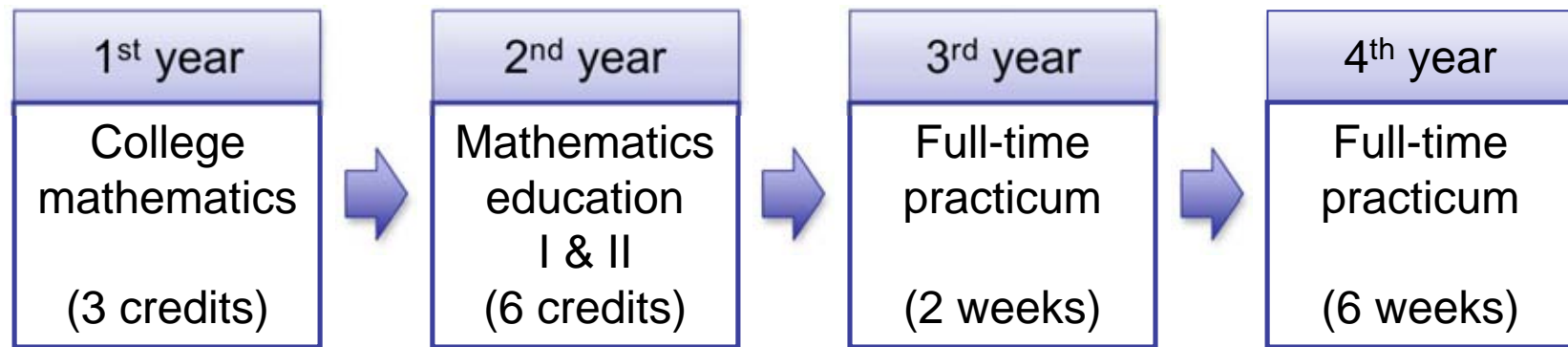
TEACHER EDUCATION IN KOREA

- Highly competitive to get an admission
- Thirteen universities for elementary pre-service teachers
- Four-year undergraduate
- Twelve concentrations (Additional 21 credits)



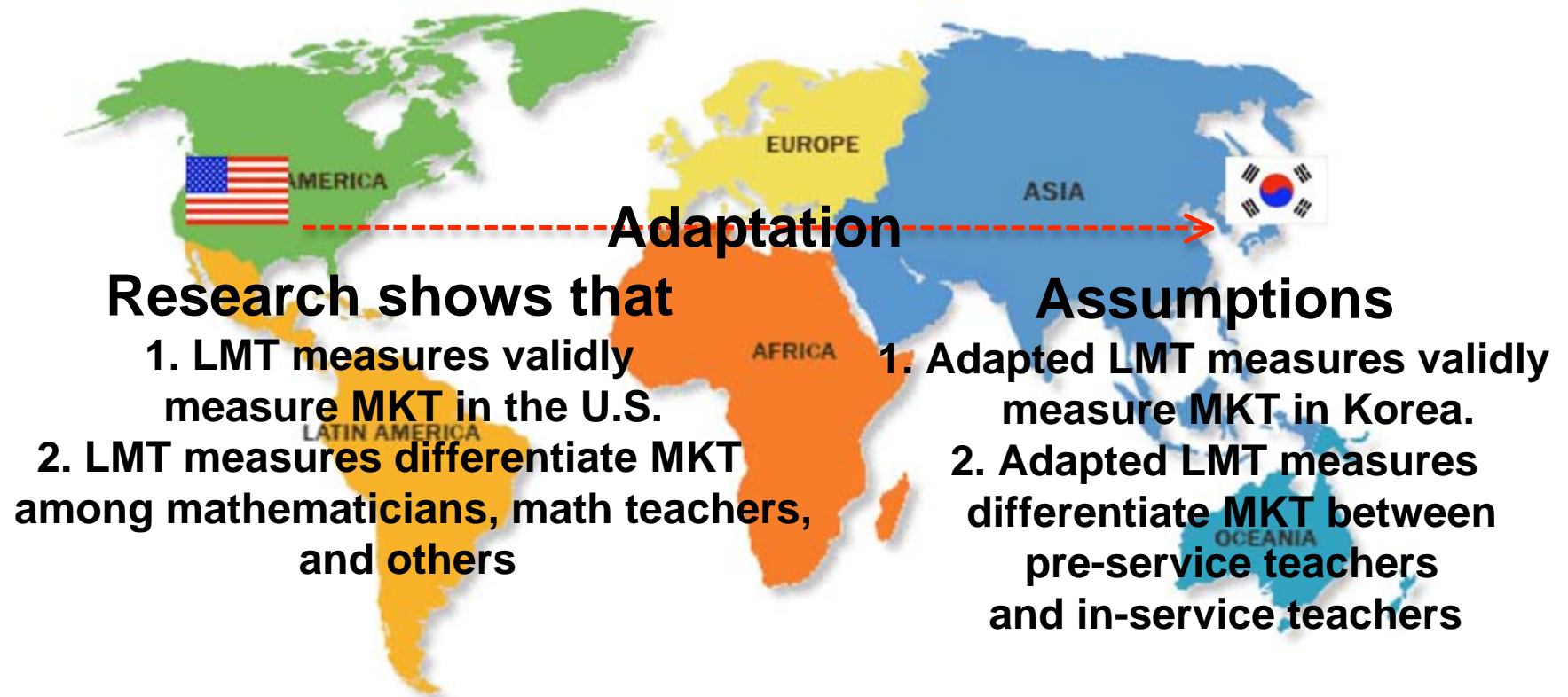
TEACHER EDUCATION IN KOREA

- Mathematics education curriculum



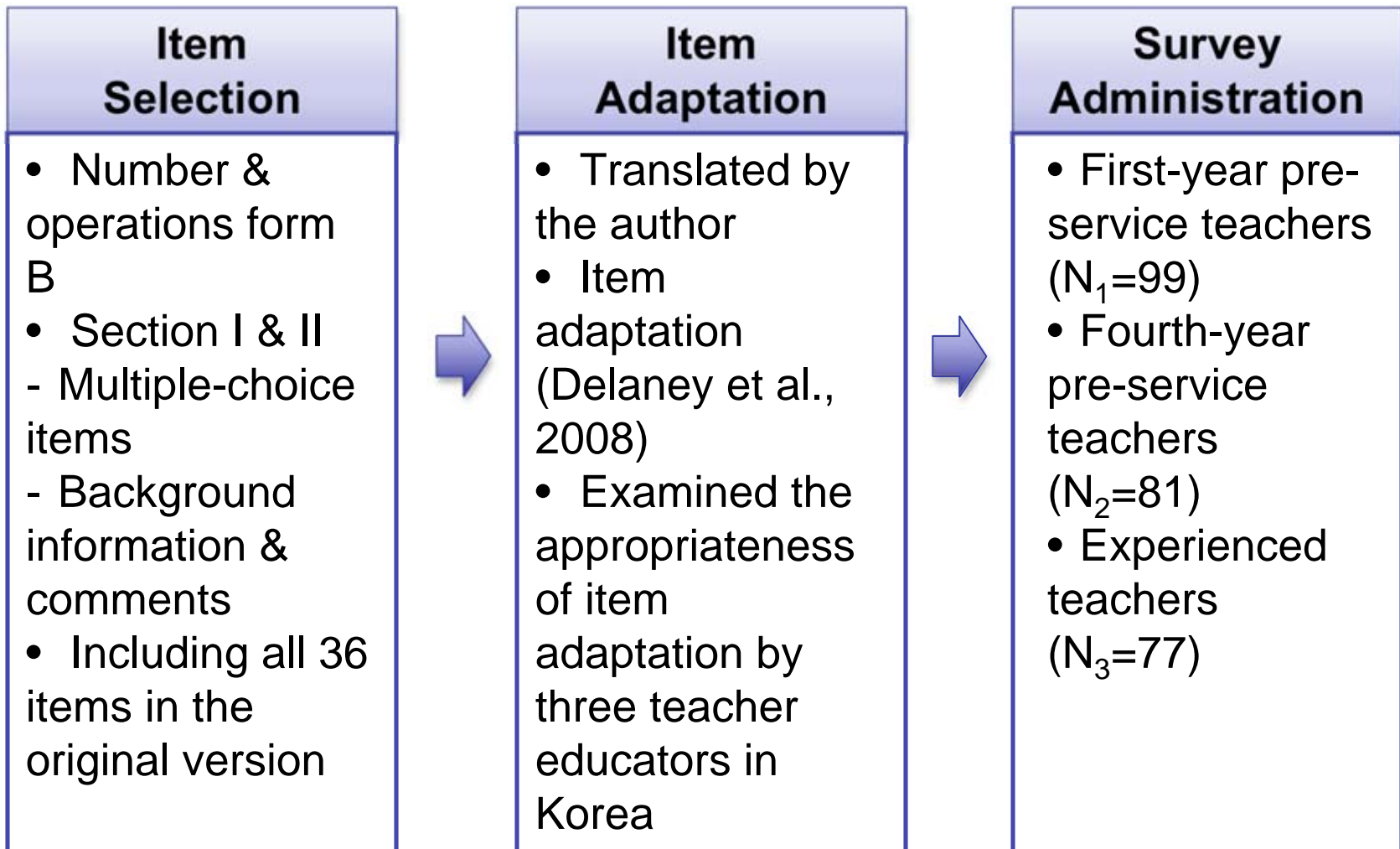
- Math education concentration: At least 30 credits
- Other concentrations: At least 9 credits

RESEARCH QUESTION



Are LMT measures in U.S. validly adapted to measure MKT in Korea?

METHODS



THE PROCESS OF ITEM ADAPTATION

- Item adaptation (Delaney et al., 2008)

1. General cultural context

Item number	Original U.S. form	Adapted Korean form (Re-translated into English)
All	e.g., Mrs. Kwon She	e.g. Teacher Jinyoung Kwon Teacher Kwon

2. School cultural context

Item number	Original U.S. form	Adapted Korean form (Re-translated into English)
#1	Mathematic text	Mathematics textbook
#5	District-sponsored	Local office of education
#19	Students' paper	Students' notebook
#20, 24	Quiz	Performance assessment
#24, 28	State	Metropolitan office of education
#24, 28	Mini-lessons	Make-up classes



THE PROCESS OF ITEM ADAPTATION

3. Mathematical substance

Item number	Original U.S. form	Adapted Korean form (Re-translated into English)
#2	Tenth	The first-place number below the decimal point
#2	Hundredth	The second-place number below the decimal point
#6	Half-way	Middle-point
#7	Five dollars	Five thousand won
#8	Divisible	Divide without remainders
#9	A dollar	One thousand won
#13	100's chart	Number chart from 1 to 100
#15	Identical rectangular cakes	Rectangular cakes with same size and same shape
#17	$1\frac{1}{2} \times \frac{2}{3}$	$\frac{2}{3} \times 1\frac{1}{2}$
#21c	Conventional algorithm	Traditional algorithm
#24	Adding columns of numbers	Adding
#25	Feet	m
#25	Square feet	m ²
#25	Twelfths	$\frac{1}{12}$
#25a	Conventional one	Traditional method
#31	Input-output machines	Function machines
#33	Sequence	Arrangement



THE PROCESS OF ITEM ADAPTATION

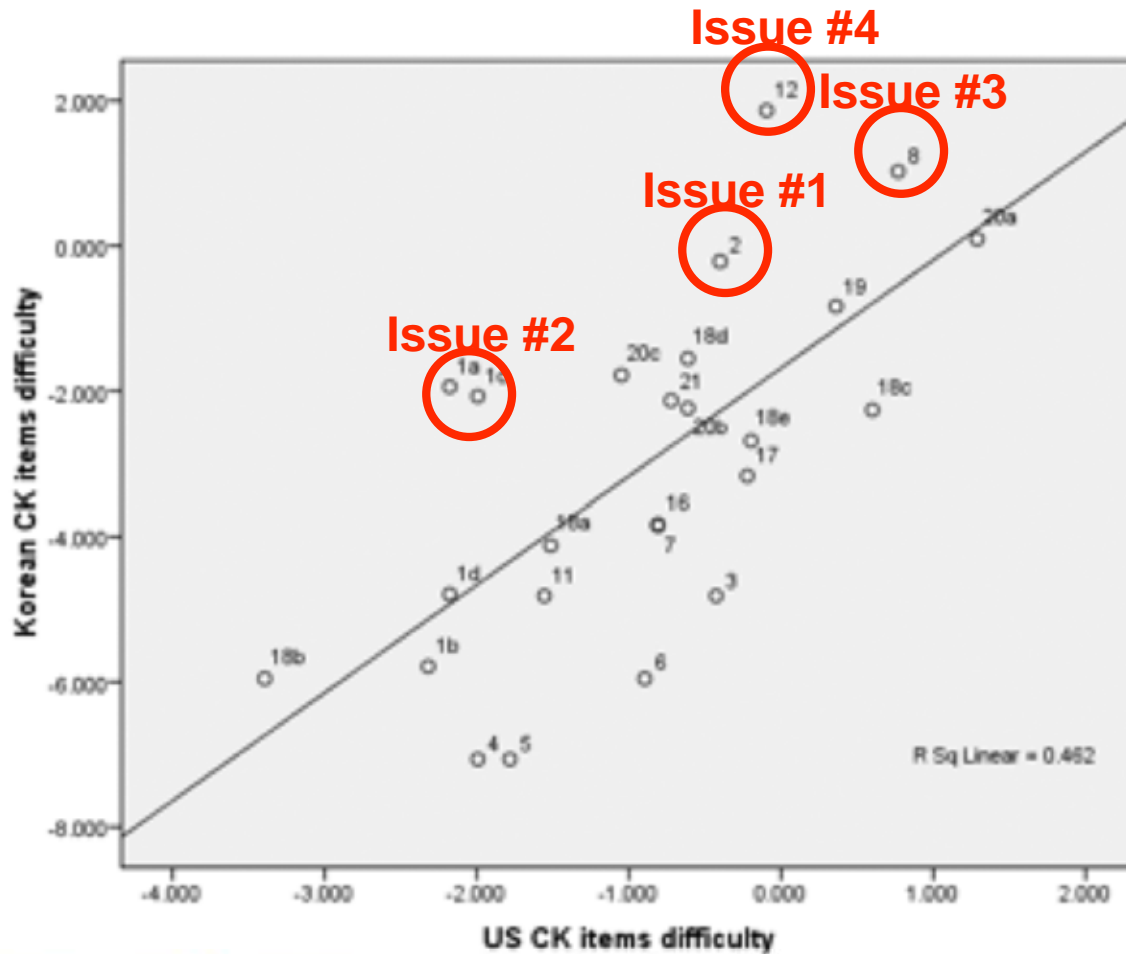
4. General language

Item number	Original U.S. form	Adapted Korean form (Re-translated into English)
All	I'm not sure	Not sure
#1, 11d, 16	Correctly	Exactly
All #9	Most likely Knowing	Best Knowledge
#10	Had trouble	Had difficulties
#26	Containers	Boxes
#27	Feel comfortable accepting	Accept satisfactory
#29	Hunch	Intuition



RESULTS

- Relative item difficulty: Positive strong correlation



RESULTS

ISSUE #1. Mathematical substance (Item #2)

- Teaching decimals using base-ten blocks

[Removed Image from Korean National Curriculum Materials]

RESULTS

ISSUE #1. Mathematical substance (Item #2)

- Teaching decimals using base-ten blocks

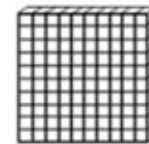
Original version

Hundreds “flats”
become whole; tens
“rods” become
tenths; ones “cubes”
become hundredths.

Adapted version

Hundreds model
“flat” becomes
whole, tens model
“rods” becomes the
first-place number
below decimal point,
and ones model
“cubes” becomes the
second-place
number below
decimal point.

Revised version



become 1,



become 0.1,

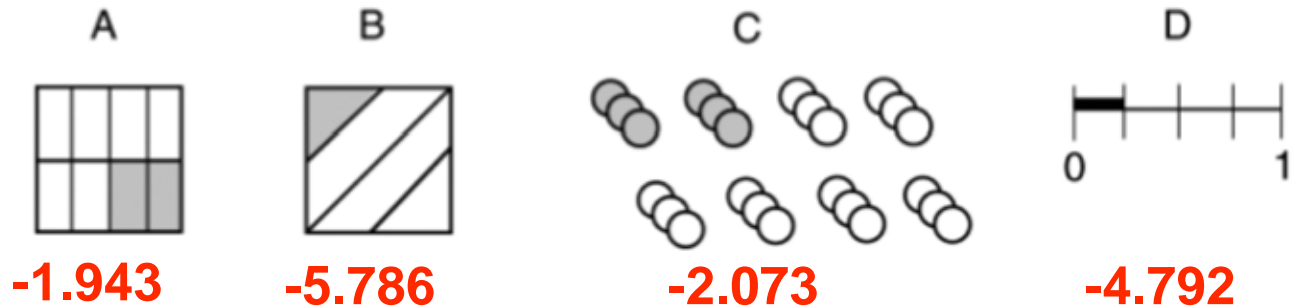


become 0.01.

RESULTS

ISSUE #2. General language (Items 1a & 1c)

- Representation correctly show $\frac{1}{4}$



Original version

correctly show $\frac{1}{4}$

Adapted version

exactly show $\frac{1}{4}$

Revised version

correctly show the
equivalent to $\frac{1}{4}$

RESULTS

ISSUE #3. Curriculum (Item 8 vs. Item 11)

- Prime number vs. Prime factorization

ISSUE #4. Acceptable explanations (Item 12)

- Reducing a fraction produces an equivalent fraction



DISCUSSION

- Strong correlation of item difficulties between LMT measures in U.S. and adapted LMT measures in Korea
- Challenges of adaptation
 - Mathematical substance
 - General language
- Further study
 - Follow-up study using revised items
 - Analyzing the mathematical work of teaching in their teaching practice



Thank you !

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Translating and Adapting the MKT Geometry Measures for Indonesia



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INDONESIAN CONTEXT

- Developing country
- Low performance in mathematics nationally and in international comparisons
- Efforts in improving education:
 1. Raising minimum education for elementary teachers
 2. Applying system of certification
 3. Developing exemplary curriculum materials

RESEARCH QUESTIONS

- What were the challenges encountered when translating and adapting the MKT geometry measures for use in Indonesia?
- How did the adapted MKT geometry measures perform in Indonesia?

METHODS

- Qualitative Method - Item Adaptation
- Psychometric Analyses (N = 210)

ITEM ADAPTATION

1. Changes in general cultural context

Tetris game

2. Changes in school context

“Stump” → “contribute”

ITEM ADAPTATION

3. Changes in mathematical substance:

- Changes in mathematical vocabulary
 - polygon
 - tessellation
 - tetrahedron
 - face and edge
- Changes due to differences in instructional practice - use of manipulatives, e.g. geoboard
- Changes in mathematical representations - decimal point

Results of Psychometric Analyses

- One item with negative point-biserial correlations:
“Is it possible for a parallelogram to have congruent diagonals?”
- Items related to classes of shapes were relatively more difficult for Indonesian teachers.
- Items related to geoboards had similar relative item difficulties.
- Moderate reliabilities (Cronbach’s alphas)

Differences in Treatment of Mathematical Topics

- U.S. expectation:
“identify, compare, and analyze attributes of two-dimensional shapes ... according to their properties and develop definitions of classes of shapes ...”
(NCTM, 2000, p. 164)
- Indonesian expectation:
“identify two- and three-dimensional shapes based on their properties, characteristics, or similarities”
(Depdiknas, 2003)

Conclusions

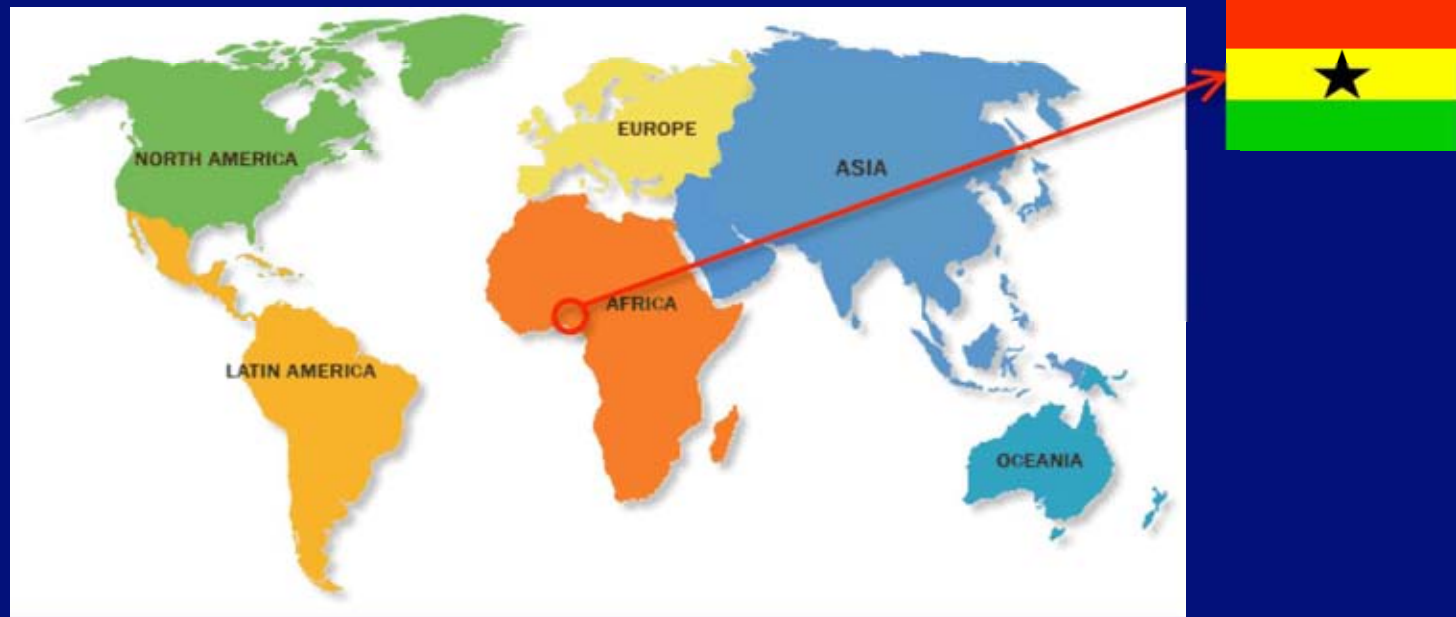
- Use of mathematical language
- Instructional practice
- Use of representations
- Emphasis of mathematical topics

Thank you !

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Relationship between MKT scores and MQI in Ghana

- The case of 3 teachers -



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Overview

- Context
- MQI
 - What is it?
 - How is it measured
 - Results
- Next steps

Context

- More than 40 mutually unintelligible languages
- Instruction in
 - Local language up to class 3 (public)
 - English (private and class 4 and up)
- National syllabus
- 22% private schools
- private school tuition/term between \$1 and \$700
- Teaching and learning
 - Shift from rote to participatory: explanations, evaluation of claims, justifications, etc



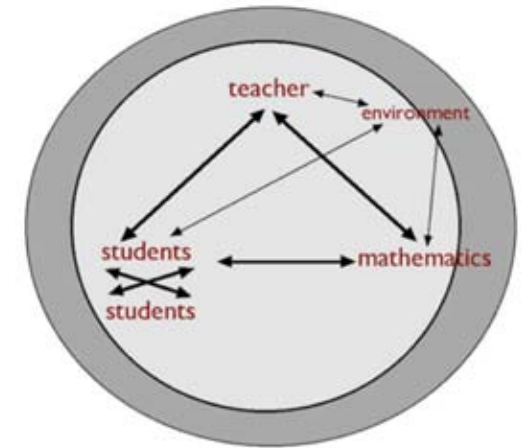
Research Questions

- What can be learned from using the U.S.-developed MKT measures in Ghana by studying their use with three Ghanaian teachers?
- How is the mathematical quality of instruction of three Ghanaian teachers related to their MKT scores?

Data

- 60 teachers surveyed
- 10 teachers
 - Interviewed about survey
 - 2 lessons each videotaped
- 3 teachers selected for case study

What is MQI?



- MQI: specifically mathematical transactions that take place in instruction
 - Language use
 - Explanations
 - Treatment of student errors
 - Instructional time on mathematics, etc

(Hill, Blunk, Charalambous, Lewis, Phelps, Sleep, & Ball, 2008)

How is MQI measured?

- Watch the entire video
- 5-minute clips of video
- Coded with LMT coding instrument

Event	Present	Not present
Appropriate	P-A	NP-A
Inappropriate	P-I	NP-I

- Reconcile with trained coder
- Reliability 75%, 85% for two, I coded one

Assumption for validity of MKT scores

- High MKT score → High MQI
 - Knowledge of mathematical terrain of the lesson
 - Explanations
 - definitions
 - Use of mathematics with students
- Low MKT score → Low MQI
 - Errors and oversights



Findings

Ewusi

Frank

Kofi

MKT

•93rd percentile rank

0th percentile rank

16th percentile rank

MQI

- Use of explanations
- Technical and general language use
- Interpretation of student productions

- Mathematical error
- No explanations
- Interpretation of student production

- Mathematical explanations
- Multiple representations

BUT

- No errors

- General and technical language issues
- Student errors



Fractions representations

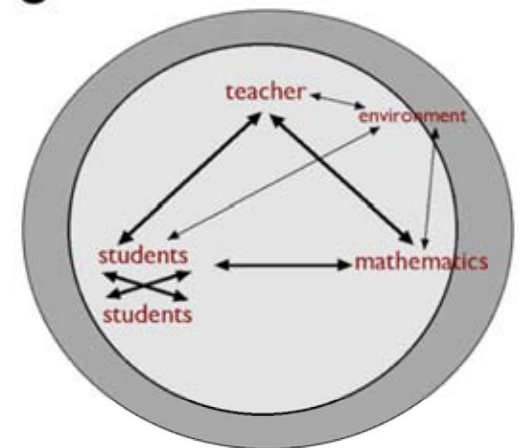


Teacher Explanation



Findings

- MKT scores might be used to identify teachers with high MQI
- Other factors
 - Language proficiency could influence teachers' MQI
 - Ghanaian English vs. Standard English
 - Cultural norms



Finally

Limitations

- No student scores
- No post-lesson interviews
- No connection with curriculum
- Different mathematics content taught

Next steps

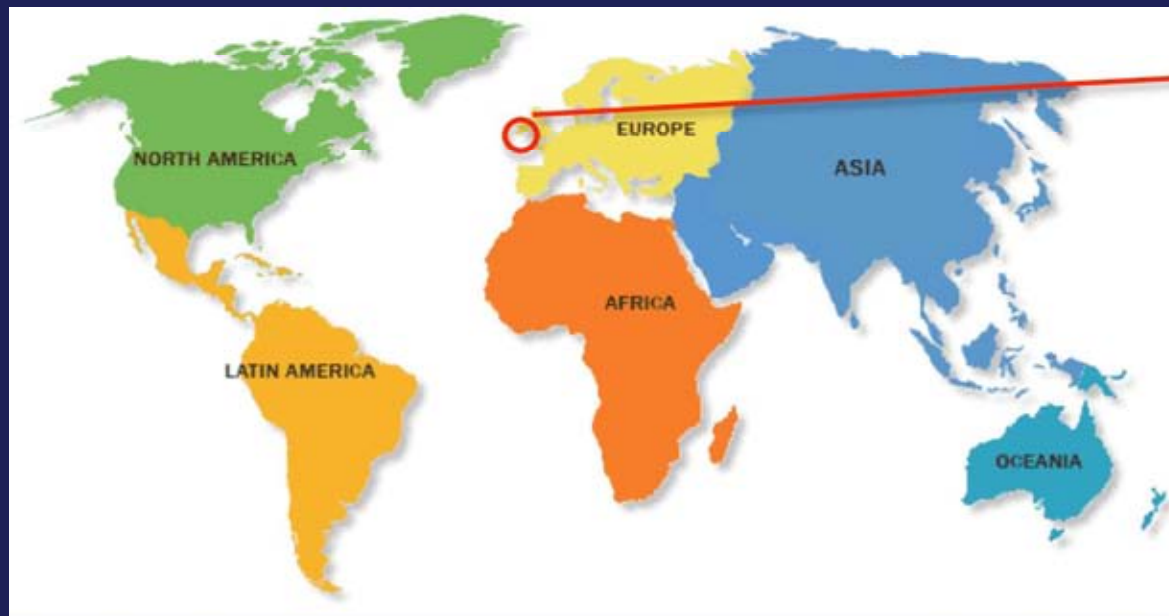
- Investigate validity with a larger sample of teachers
- Curriculum expectations at elementary level
- Explore aspects of teaching in mathematics that MQI instrument did not capture



Thank you !

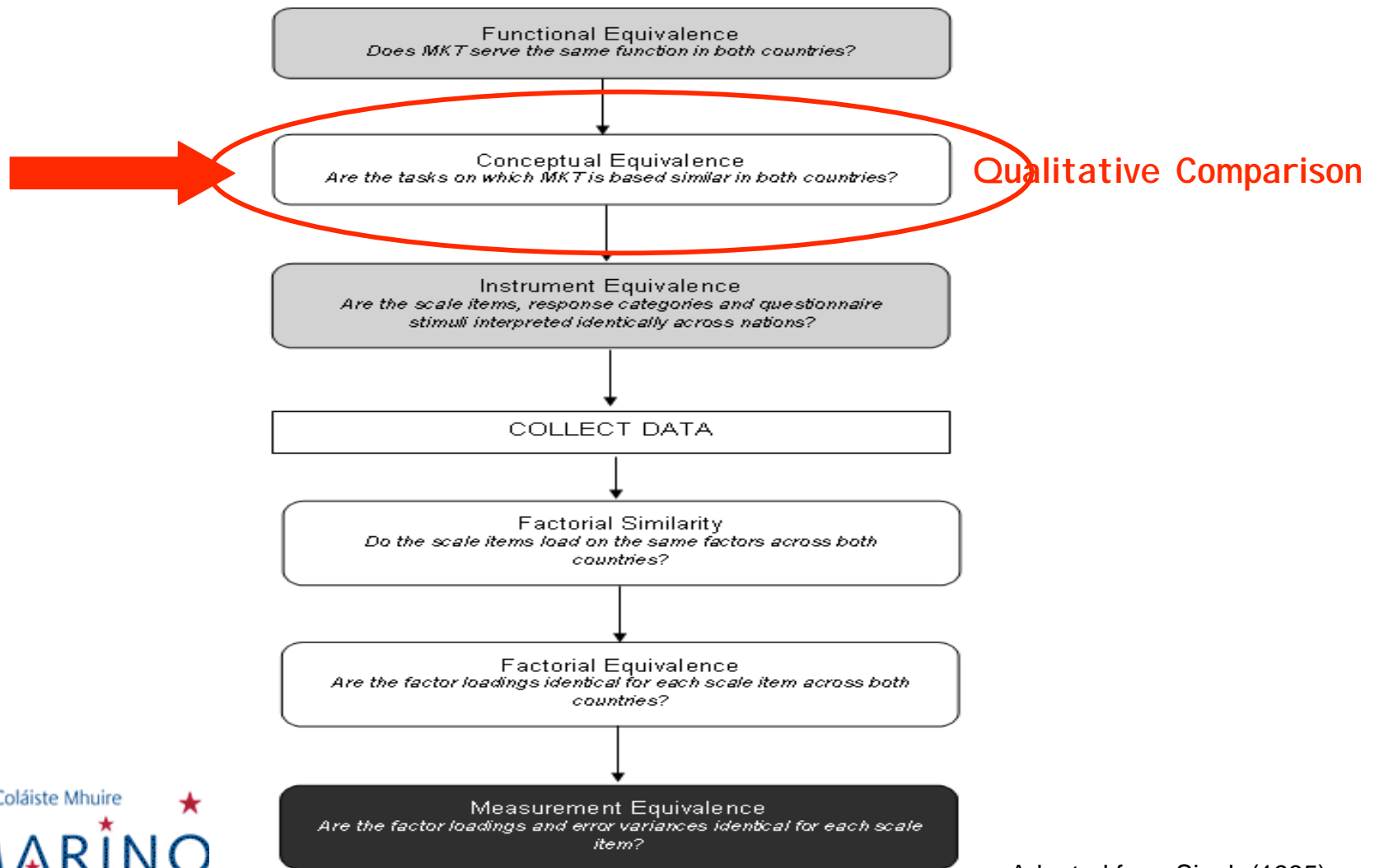
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Using Qualitative and Quantitative Methods to Study Equivalence of a Teacher Knowledge Construct



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Marino Institute of Education

Construct Equivalence



Source of MKT

- Reference to existing literature on teachers' mathematical knowledge
- Study of the mathematical work of teaching
We analyze particular segments of teaching ... to examine how and where mathematical issues arise in teaching ... and ... to understand ... what elements of mathematical content and practice are used – or might be used – and in what ways in teaching.

(Ball & Bass, 2003b, p. 6)

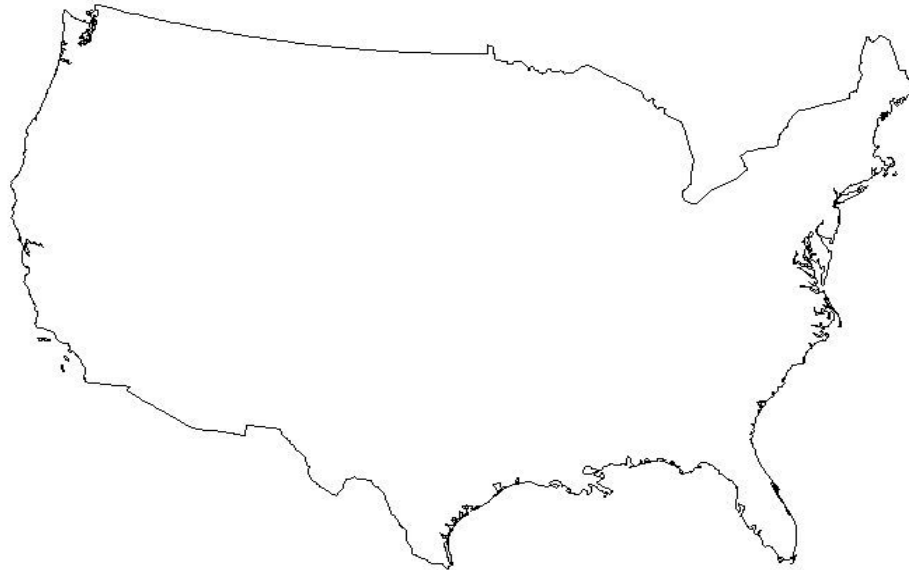
Mathematical Work of Teaching

Tasks that require “mathematical reasoning, insight, understanding, and skill” (Ball & Bass, 2003b, p. 5) or “mathematical sensibilities or sensitivities [or] mathematical appreciation” (Ball, 1999, p. 28).



Videos of math lessons

Personal knowledge



MKT literature

MKT items

Mathematical Work of Teaching in Ireland

- Rarely, if ever, documented
- 10 lessons taught by 10 teachers
- Identified moments where “mathematical and pedagogical issues meet.” (Ball, 1999, p. 28)
- Grain-size of task
- Tasks nested in one another
- Some work not visible in videos of lessons

The U.S. Construct of MKT: Literature & Items

Toward a Practice-Based Theory of Mathematical Knowledge for Teaching¹
 Deborah Loewenberg Ball and Hyman Bass²
 University of Michigan

Mathematics professor: The situation is terrible: Only one of the students in my mathematics content course for teachers can correctly divide .0045789 by 3.45.
 Fifth grader: Ms. Ball, I can't remember how to divide decimals. There's something my stepfather showed me about getting rid of the decimal point, but I can't remember what he said and, besides, I don't think that would work.

With all the talk of teachers' weak mathematical knowledge, we begin with a reminder that the problem on the table is the quality of mathematics teaching and learning, not—in itself—the quality of teachers' knowledge. We seek in the end to improve students' learning of mathematics, not just produce teachers who know more mathematics.
 Why, then, talk about teacher knowledge here? We focus on teacher knowledge based

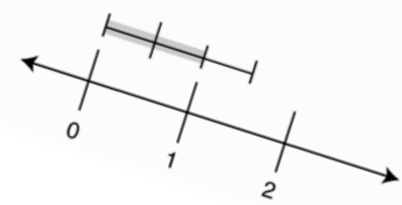
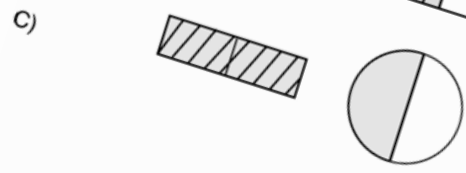
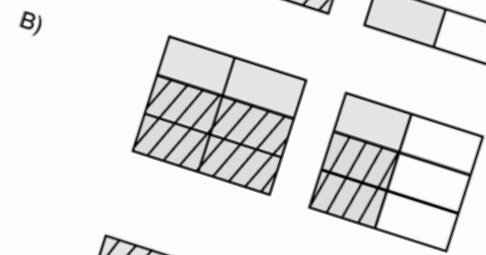
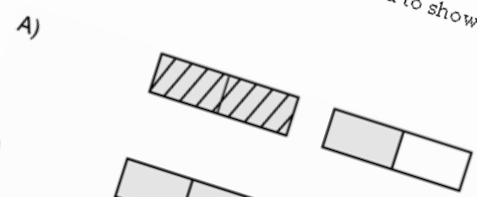
**BRIDGING PRACTICES
 INTERTWINING CONTENT AND PEDAGOGY IN
 TEACHING AND LEARNING TO TEACH**

Deborah Loewenberg Ball
 University of Michigan

Subject matter and pedagogy have been peculiarly and persistently divided in the conceptualization and curriculum of teacher education and learning to teach. This fragmentation of practice leaves teachers on their own with the challenge of integrating subject matter knowledge and pedagogy in the contexts of their work. Yet, being able to do this is fundamental to engaging in the core tasks of teaching, and it is critical to being able to teach all students well. This article proposes three problems that would have to be solved to bridge this gap and to prepare teachers who not only know content but can make use of it to help all students learn. The first problem concerns identifying the content knowledge that matters for teaching, the second regards understanding how such knowledge needs to be held, and the third centers on what it takes to learn to use such knowledge in practice.



17. At a professional development workshop, teachers were learning about different ways to represent multiplication of fractions problems. The leader also helped them to become aware of examples that do not represent multiplication of fractions appropriately.
 Which model below cannot be used to show that $1\frac{1}{2} \times \frac{2}{3} = 1$? (Mark ONE answer.)



Example of Comparing Mathematical Work of Teaching (I)

- Clip from Irish mathematics lesson
- “Draw a picture of 1 divided by a quarter”

Mathematical Knowledge

“It’s just the answer is all of them, not just one. It’s usually one. Because if you’re quartering it, the answer is one of them, but if you’re ahh, dividing by a quarter it’s all of them”

Example of Comparing Mathematical Work of Teaching (II)

- Did a similar task inform the U.S. construct of MKT?
- A teacher needs to represent “ideas carefully, mapping between a physical or graphical model, the symbolic notation, and the operation or process” (Ball & Bass, 2003, p. 11)

Comparing the Mathematical Work of Teaching

Mathematical Task of Teaching	Example in Ireland	Example in the United States
Following students' descriptions of their mathematical work	Teacher listens and responds to students who describe how they would handle a remainder in a division problem	Teacher listens to students' descriptions of how they solved a problem (MKT Item B_01, 26)
Comparing different solution strategies	Teacher elicits 3 different methods to calculate the answer to a problem	"Making sense of methods and solutions different from one's own" (Ball & Bass, 2003b, p. 13)
Responding to students' questions and observations	Teacher responds to a student question about answers found when dividing whole numbers by unit fractions	Responding "productively to students' mathematical questions and curiosities" (Ball & Bass, 2003b, p. 11)
Connecting number patterns and procedures	Teacher asks pupils to develop the division of fractions algorithm by looking for a pattern in problems worked out by folding paper	Teacher asks students to look for patterns on a 100 square (MKT Item B_01,13)
Selecting useful examples	Teacher chooses to demonstrate solving a textbook problem that is too difficult for most students based on previous learning	Choosing useful examples (Ball et al., 2005)

Summary of Qualitative Findings

- 60 tasks identified in Ireland matched to tasks that informed the U.S. construct of MKT
- 8 tasks identified in Ireland not matched directly to tasks that informed MKT
- 4 tasks in U.S. literature not matched to tasks identified in Ireland
- Substantial overlap exists

Conclusions

- Need to be careful about claiming that teachers in one country know more/less mathematics than teachers in another country
- Ask first if the work is similar because if not, different mathematical knowledge may be required in each setting
- This study proposes a way to study the mathematical work of teaching in two countries
- MKT construct as developed in the United States seems suitable for studying Irish teachers' mathematical knowledge
- One challenge: we lack a model for documenting the mathematical work of teaching, which makes it difficult to compare the mathematical work of teaching across settings

Thank you !

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