Learning to Do Mathematics as a Teacher

Mathematics Teaching and Learning to Teach (MTLT) Project

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A problem in teaching

You ask your fourth graders to explain what a rectangle is. One child offers a definition:

A rectangle is a flat shape. It has four square corners, and it is closed all the way around.

What is the work for the teacher?

A rectangle is a flat shape. It has four square corners, and it is closed all the way around.

1. To see that something is missing
2. Decide what to do or say
3. Offer a non-rectangle that meets the proposed definition

What is a shape that satisfies this definition and yet is not a rectangle?

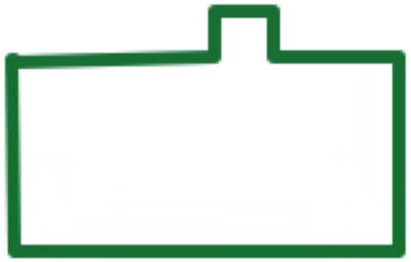
What is the work for the teacher?

A rectangle is a flat shape. It has four square corners, and it is closed all the way around.

1. To see that something is missing
2. Decide what to do or say
3. Offer a non-rectangle that meets the proposed definition

"Is this a rectangle?"

(straight sides)



(exactly four square corners)

A rectangle is a flat shape with **straight sides that are connected at exactly** four square corners. It is closed all the way around.

Overview of session

1. Teaching involves specialized mathematical work
2. Mathematical resources for engaging in this work
3. Learning to do this specialized work

Teaching involves specialized mathematical work

Three key aspects of this work:

1. Solving special kinds of mathematical problems
2. Engaging in specialized mathematical reasoning
3. Using mathematical language precisely but accessibly

Choosing pedagogically strategic examples

Which of the following lists would be best for assessing whether your students understand decimal ordering? Justify your choice.

A.	**.5**	**7**	**.01**	**11.4**
B.	**.60**	**2.53**	**3.12**	**.45**
C.	**.6**	**4.25**	**.565**	**2.5**

Analyzing errors

What mathematical steps could have produced this answer?

(a)
$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline 405 \\ 108 \\ \hline 1485 \end{array}$$

(b)
$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline 225 \\ 100 \\ \hline 325 \end{array}$$

(c)
$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline 1250 \\ 25 \\ \hline 1275 \end{array}$$

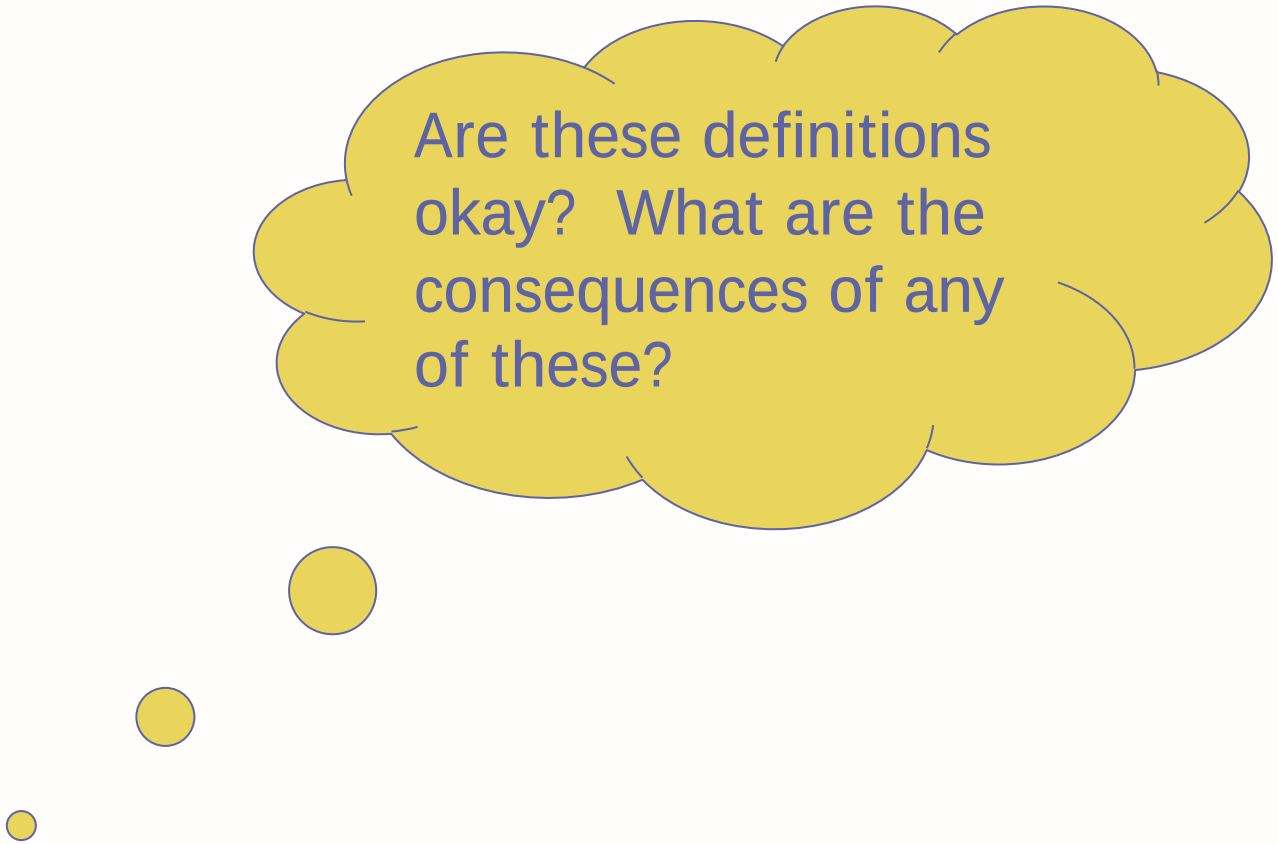
Analyzing non-standard (but correct) responses

Which student is using a method that could be used to multiply any two whole numbers?

Student A	Student B	Student C
35 x25 125 +75 875	35 x25 175 +700 875	35 x25 25 150 100 +600 875

Choosing definitions

a) An even number is a number that can be divided into two equal parts.
b) An even number is any multiple of 2.
c) An even number is any integer multiple of 2.
d) An even number is any number whose unit digit is 0, 2, 4, 6, or 8.
e) A whole number is even if it is the sum of a whole number with itself.



a) An even number is a number that can be divided into two equal parts.

b) An even number is any multiple of 2.

All numbers, for example 7, 3/5, $\sqrt{2}$, π, are even!

c) An even number is any integer multiple of 2.

This is a correct definition of even number.

d) An even number is any number whose unit digit is 0, 2, 4, 6, or 8.

In this case, 36.7 is an even number!

e) A whole number is even if it is the sum of a whole number with itself.

This is a correct definition of evenness for whole numbers, and is consistent with the general definition for integers that will arrive later.

Posing mathematics problems carefully

POSSIBLE PROBLEM:

I have pennies, nickels, and dimes in my pocket. If I pull out two coins, what amounts of money might I have?

Reasoning about different wording

How does the exact wording of the problem affect the mathematical work for my students? Which best fits my goals for them?

Reasoning about different wording

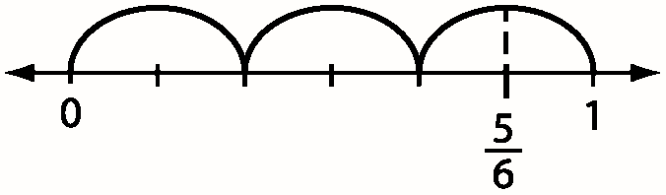
1. I have pennies, nickels, and dimes in my pocket. If I pull out two coins, what amount of money might I have?

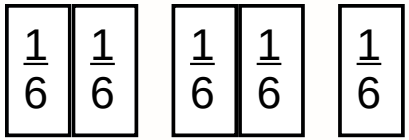
2. I have pennies, nickels, and dimes in my pocket. If I pull out two coins, how many combinations are possible?

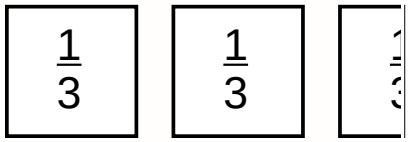
3. I have pennies, nickels, and dimes in my pocket. If I pull out two coins, how many different amounts of money are possible? Prove that you have found all the amounts that are possible.

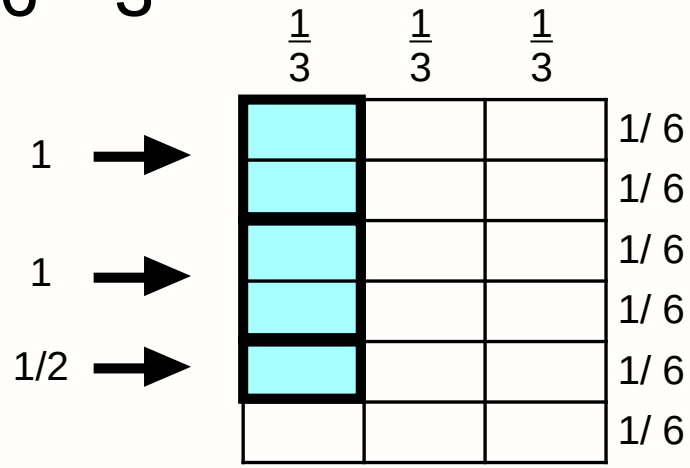
Analyzing — and "talking" — representations

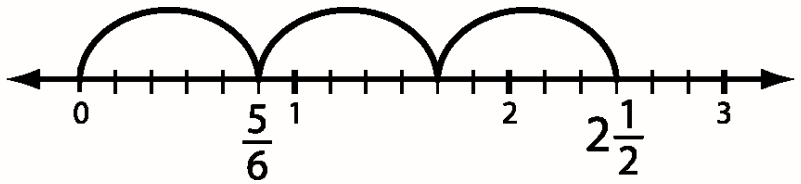
Which of these can be used to represent $\frac{5}{6} \div \frac{1}{3}$? Explain with reference to all parts of the expression.



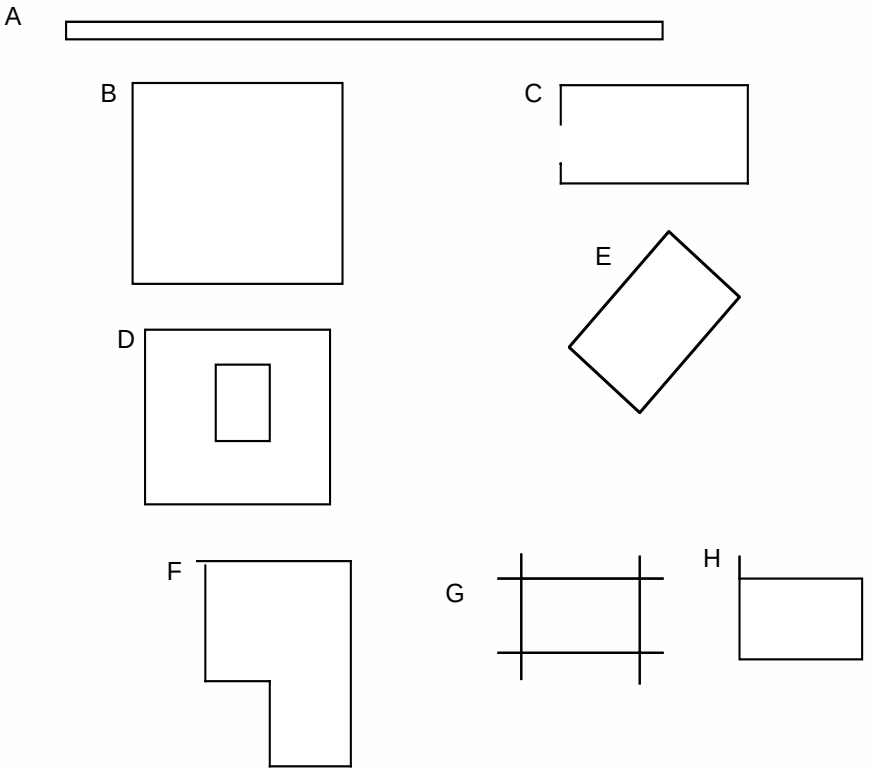








Deciding about starting points and sequencing



- In a whole-class discussion aimed at developing the concept and definition for "rectangle," which figure would be good to discuss first? Why?
- How would you sequence these figures to develop the concept and definition of rectangle? Why?

Teaching involves specialized mathematical work

Three key aspects of this work:

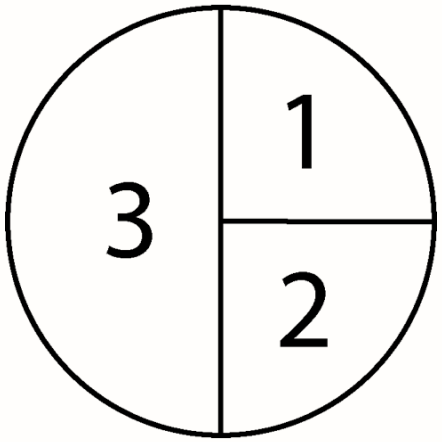
1. Solving special kinds of mathematical problems
2. Engaging in specialized mathematical reasoning
3. Using mathematical language precisely but accessibly

Teaching involves specialized mathematical work

Solving special kinds of mathematical problems	Engaging in specialized mathematical reasoning	Using mathematical language precisely but accessibly
• Selecting or constructing a strategic example, representation, or task • Analyzing representations and definitions • Analyzing non-standard correct responses and incorrect responses	• Comparing mathematical alternatives • Justifying choices of representations, language, and tasks • Reasoning about starting points and sequences	• Coordinating between mathematical details and precision and terms children know or can learn to use • Coordinating between simple, invented, or everyday language and mathematical details • Judging what can be left more casual and what not

Context of video

- 4th grade probability lesson
- Introduction to a spinner experiment

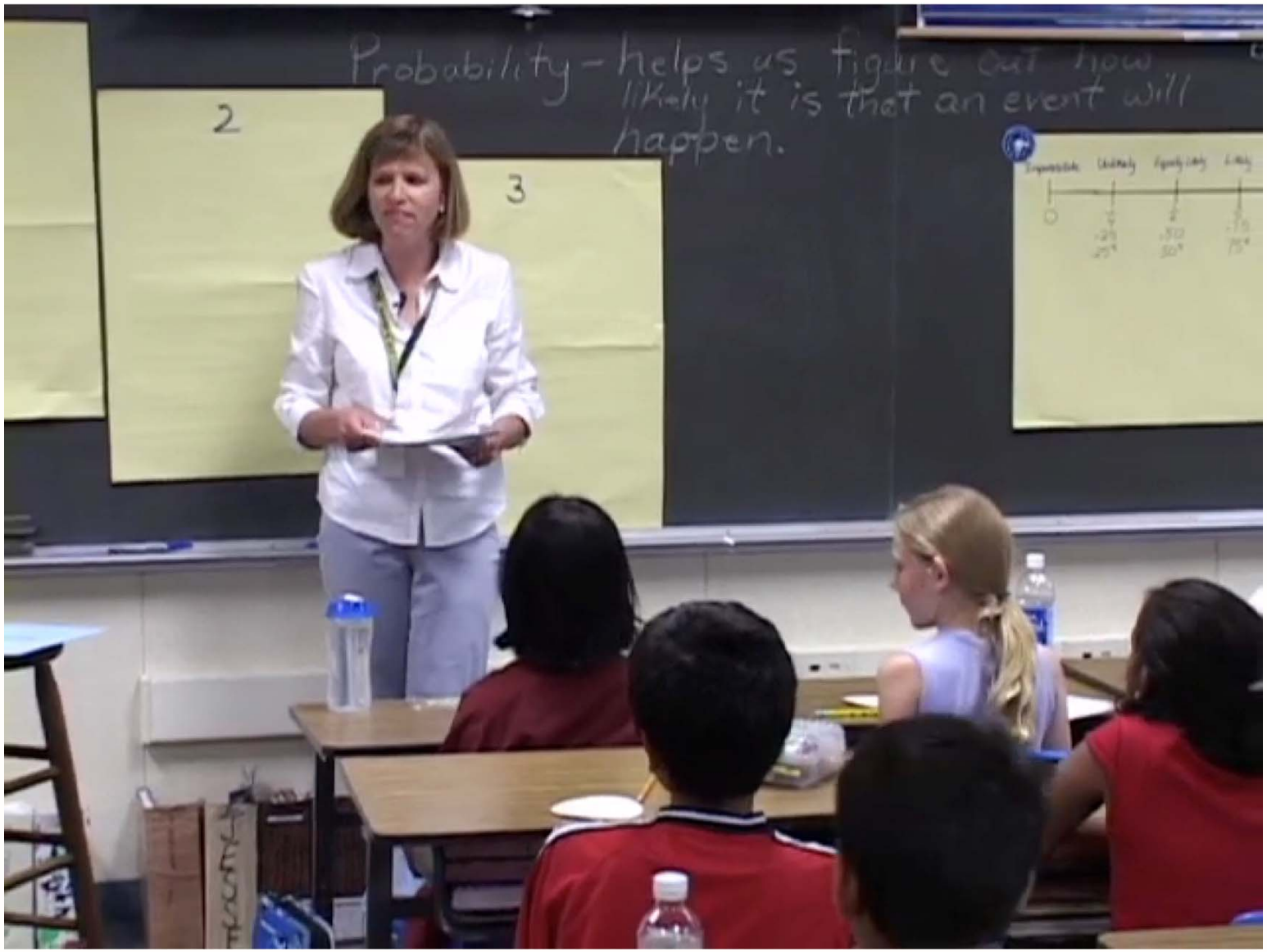


Question #1: When you spin the spinner, is any number more likely to come up than any other number? Why do you think so?

Question #2: What do you think the entire graph will look like when one number reaches the top of the paper?

Viewing focus

1. What examples of mathematical problems of teaching are visible in the video?
2. What specialized mathematical reasoning might be required?
3. Are there examples where there is a need to attend to mathematical language precisely but accessibly?



Viewing focus

1. What examples of mathematical problems of teaching are visible in the video?
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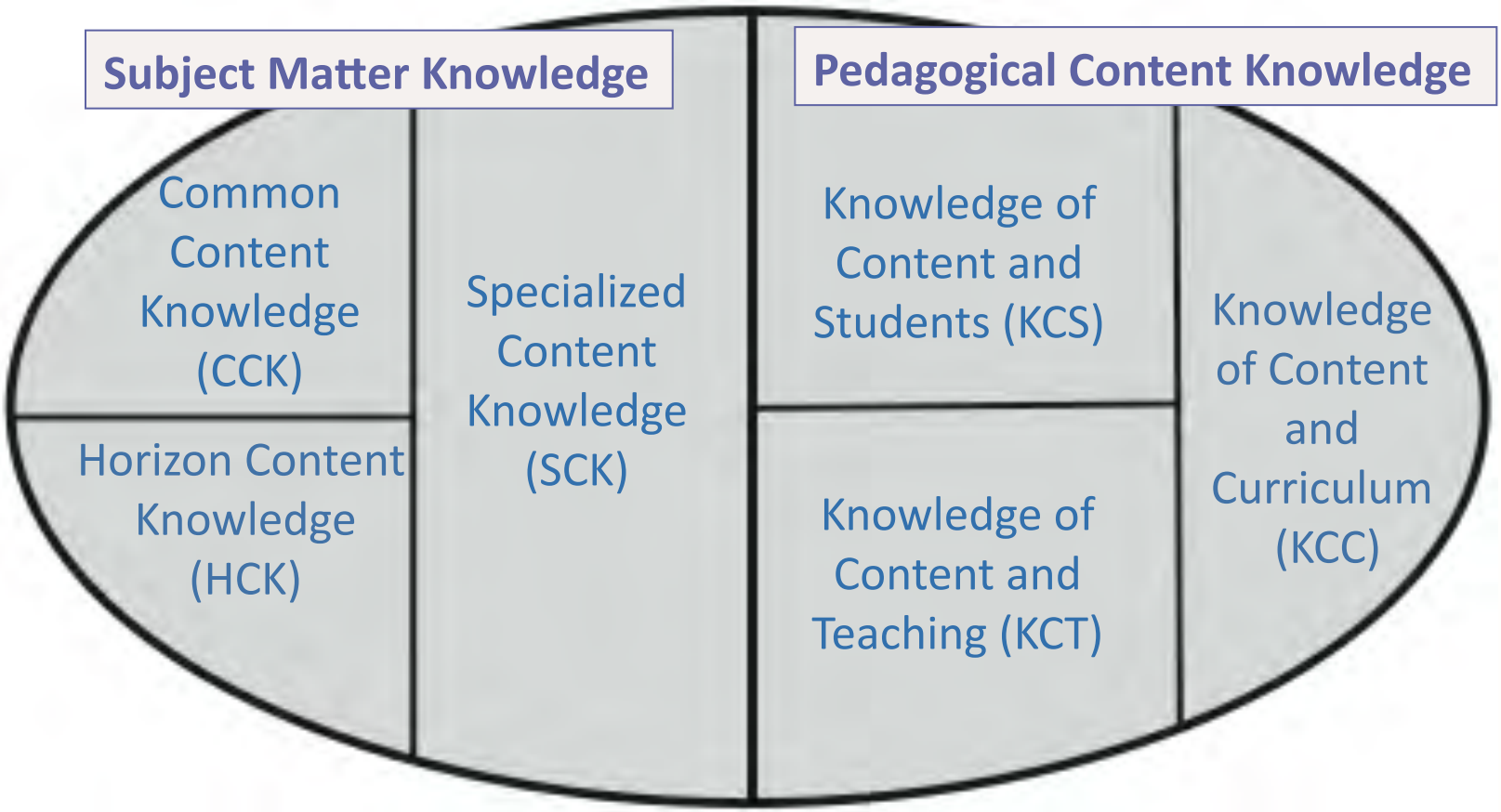
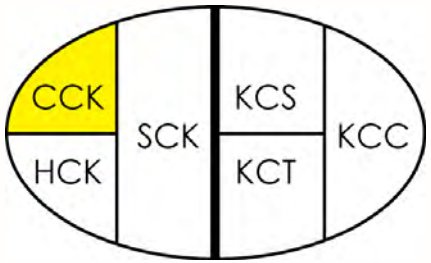
What makes this specialized mathematical work?

1. It is a doing of math conditioned by its ultimate uses.
2. It is not pedagogy—there is much more to teaching than this.
3. It is math, but performed in the service of helping others learn mathematics.
4. Its warrants are tied both to pedagogical purpose and mathematical integrity (Thames, 2009).

What mathematical resources are needed to engage in this specialized mathematical work?

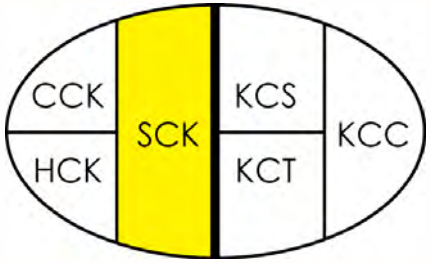
Mathematical Knowledge for Teaching (MKT)





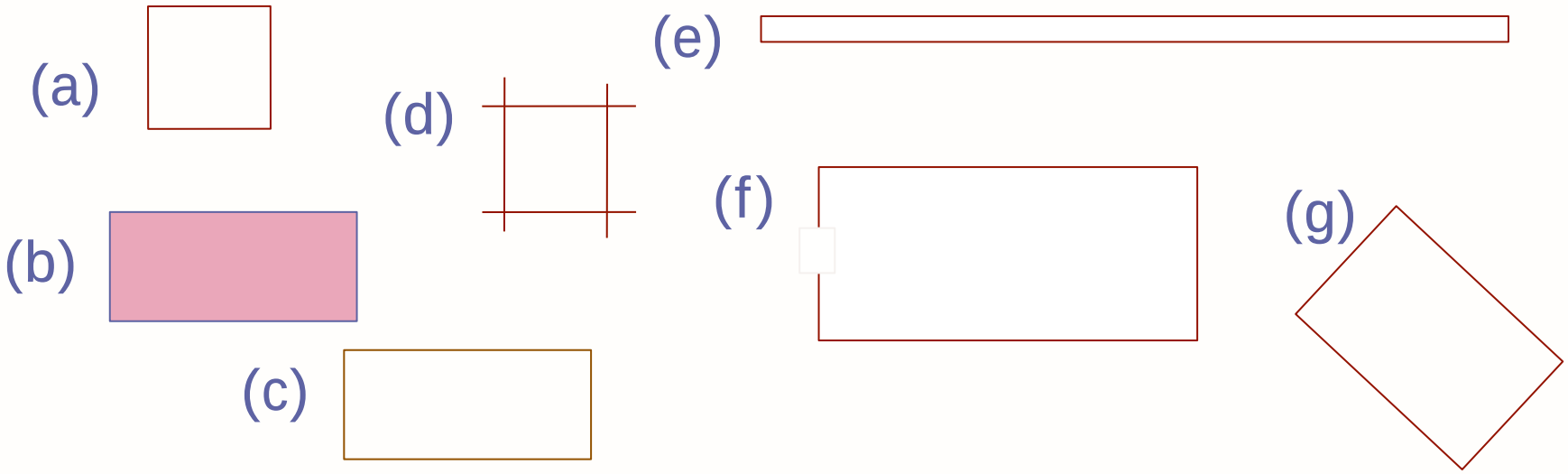
Common content knowledge (CCK)

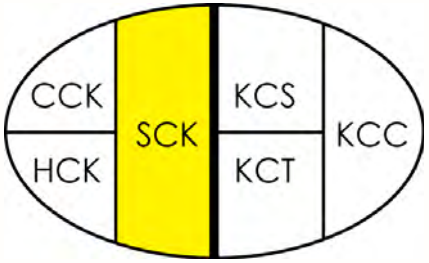
Draw a **rectangle**.



Specialized content knowledge (SCK)

Which of these figures would be good to present to assess whether students understand what a rectangle is, and why?





Specialized content knowledge (SCK)

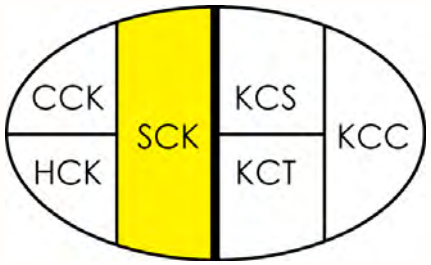
Which of these is a mathematically accurate definition of "rectangle"?

① A rectangle is a figure with four straight sides, two long and two shorter.

② A rectangle is a shape with exactly four connected straight line segments meeting at right angles.

③ A rectangle is flat, and has four straight line segments, four square corners, and it is closed all the way around.

For any that are not mathematically accurate, give an example that shows what is wrong.



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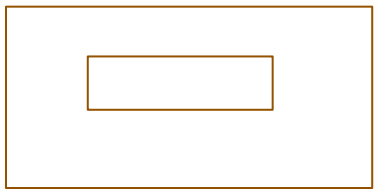
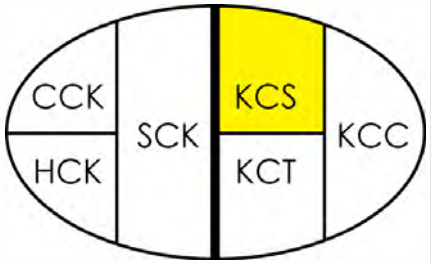
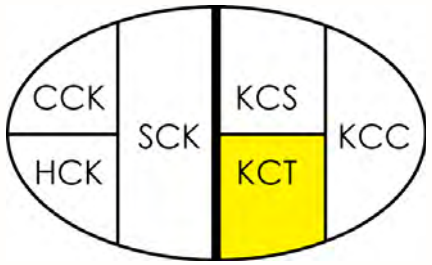


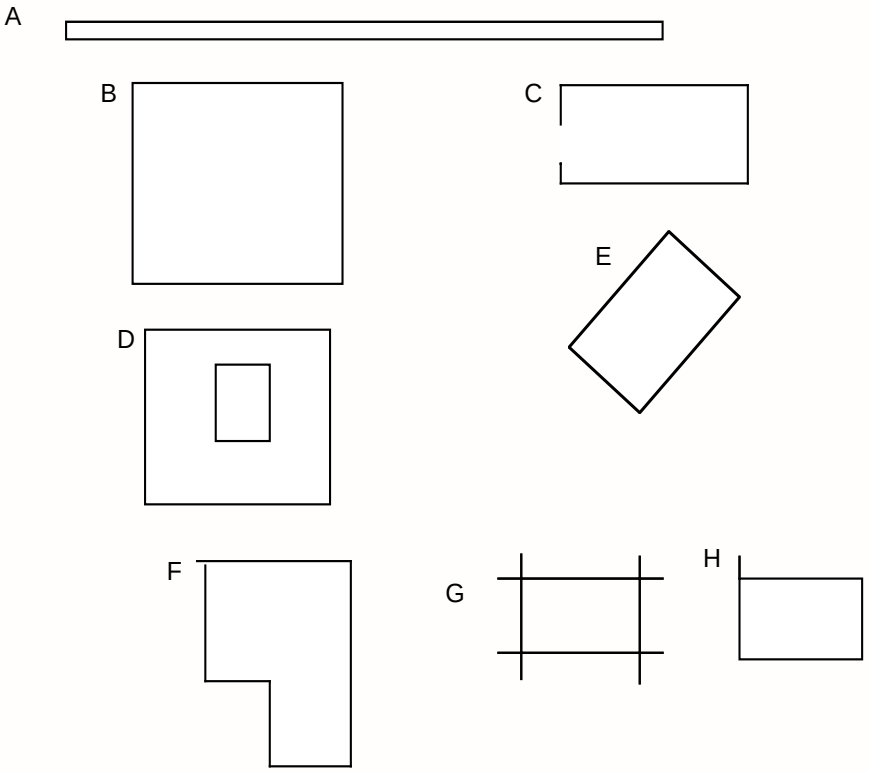


Knowledge of content and students (KCS)

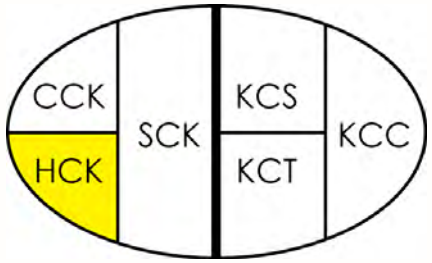
- Write a mathematically accurate definition of "rectangle" that is usable by second graders.
- How can the notion of "simple closed curve" be expressed in a way that is both mathematically accurate and usable? Which part of this phrase is most challenging for children?
- What are students likely to know about what a rectangle is?
- What do students typically have difficulty with in learning about rectangles, and why?



Knowledge of content and teaching (KCT)



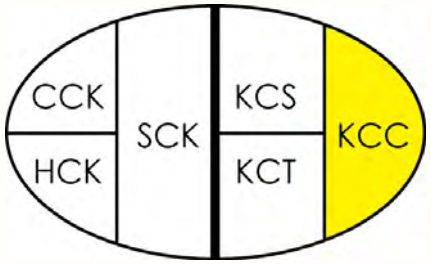
- How would you sequence these figures to discuss the concept of a rectangle?
- What task would you create using these figures (or others) to set up a productive discussion aimed at developing a definition?
- In a whole-class discussion, which one would be good to discuss first?



Horizon content knowledge (HCK)

- Is it okay to shade in the figures shown to students – e.g., 

- What are the issues involved with the fact that children learn about rectangles before polygons?



Knowledge of content and curriculum (KCC)

- At what grade level are students first introduced to rectangles?
- At what grade level are students first introduced to polygons?

What is MKT not?

1. Just knowing the topics of the school curriculum.
2. Knowing school content from an "advanced perspective."
3. Just another term for "pedagogical content knowledge."
4. Being able to teach the content.

What is MKT?

1. Knowing mathematics from the perspective of helping others learn it.
2. Being "mathematically ready" to plan and teach an idea, method, or other aspect of math.
3. Asking questions that inhere "naturally" in the purpose (i.e., helping someone else learn the content

A challenge for teachers — and teacher educators

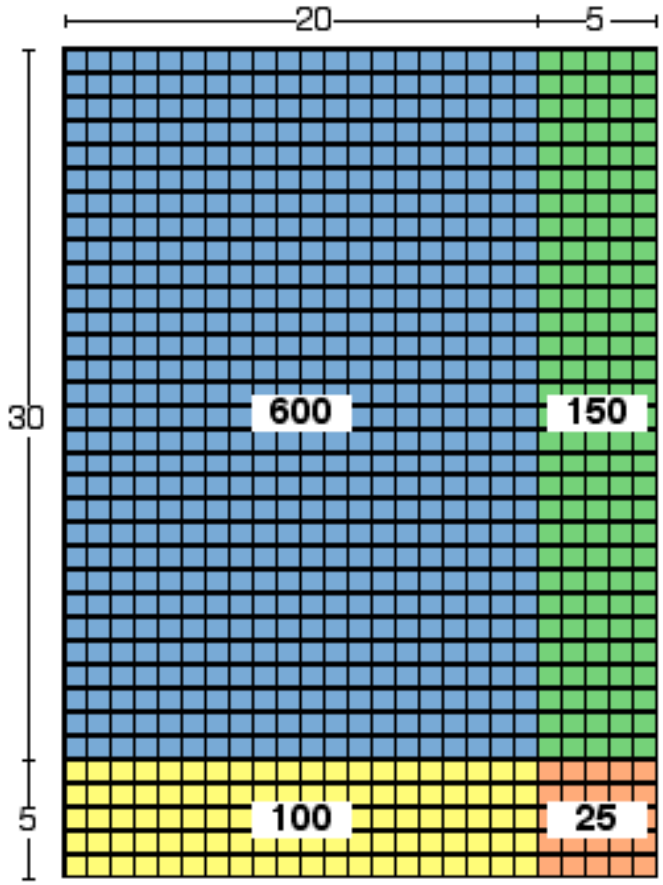
Teaching is a practice.

It is the *use of mathematics in practice* that matters.

How can teachers learn to do the specialized mathematical work of teaching?

1. Unpacking mathematics ideas and practices
2. Developing horizon knowledge
3. Practicing mathematical work of teaching

Unpacking mathematical ideas



A

35
x 25
125
+75
875

B

35
x 25
175
+700
875

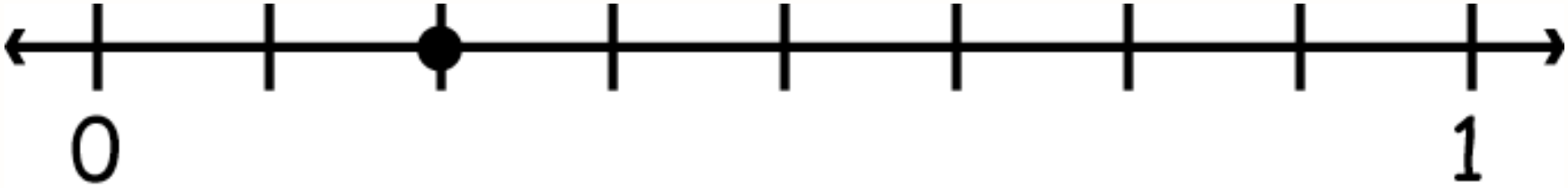
C

35
x 25
25
100
150
+ 600
875

Developing horizon knowledge

Exploring ideas of density of the rational numbers

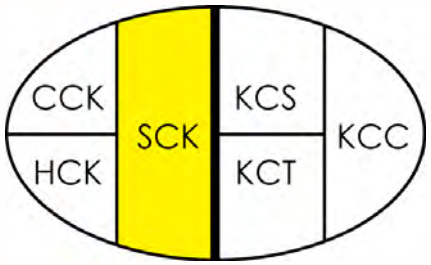
Developing awareness of the two kinds of infinity



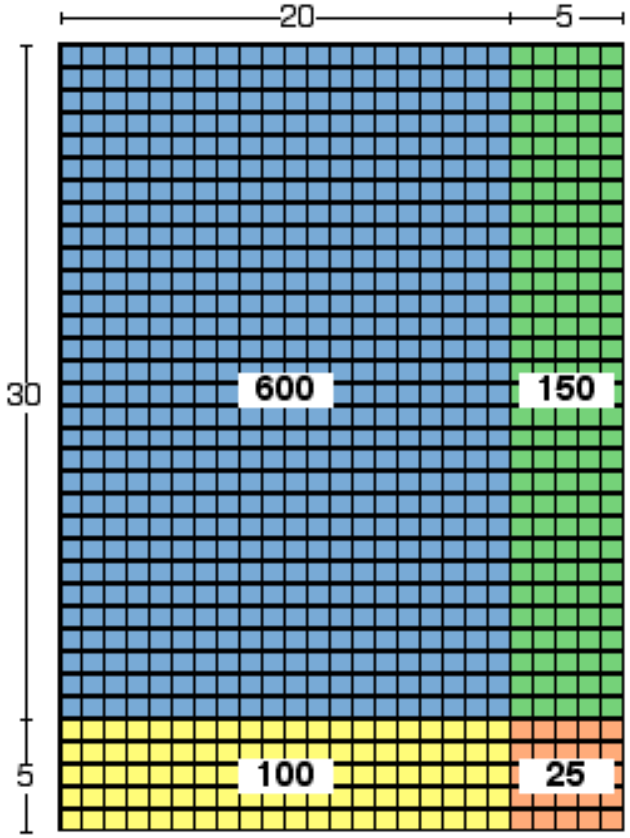
All the numbers of K-8 mathematics "live" on the number line

Practicing the mathematical work of teaching

- Working on examples like we have done, including practicing providing reasons
- Developing fluency and speed
- Developing sensibilities about mathematical language



Practice #1: Representing and mapping across representations



A

35
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35
x 25
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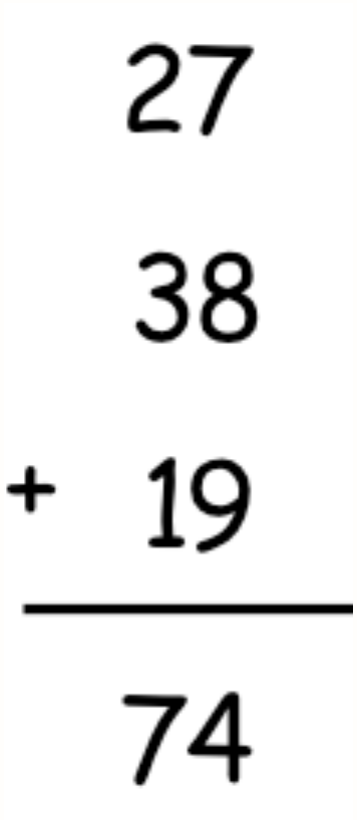
100
150
+ 600
875

Practice #2:
Becoming mathematically agile



What problem would you pose next to this pupil, and why?

Practice #3: Developing sensibilities about mathematical language

- Use of quantifying terms: e.g., exactly, no more than, no less than, at least, at most
- Attention to the wording of mathematical tasks
- Care with definitions, their role, their requirements, and judgments about when, where, and why they are essential
- Awareness of the overlaps and conflicts between mathematical language and everyday language, other school language, and students' home languages

Conclusion

1. Teaching involves doing mathematics in some special ways that are connected to the purposes of teaching practice — i.e., helping others learn mathematics.
2. This special kind of work involves special kinds of problems, engaging in special kinds of mathematical reasoning, and using mathematical language in specially careful ways.
3. Teachers need opportunities to develop skills with this special kind of mathematical work.

THANK YOU!

Slides will be available
at Deborah Ball's website

(Google "Deborah Ball")

Credits

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