The Laboratory Class:
A Multidisciplinary Approach to Studying the Teaching and Learning of Mathematics

Deborah Loewenberg Ball, Hyman Bass, Kara Suzuka, Laurie Sleep,
Imani Masters Goffney, Seán Delaney
University of Michigan

Geoffrey Saxe, Meghan Shaughnessy, Darrell Earnest
University of California Berkeley

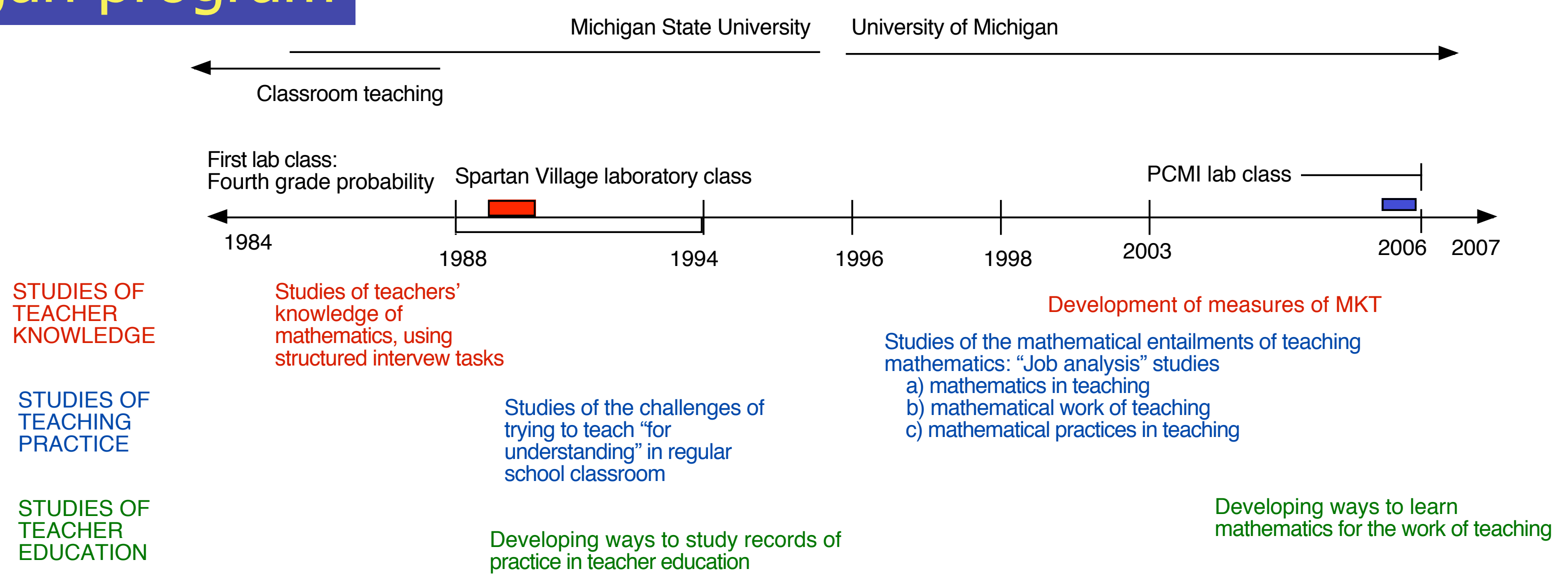
Courtney Cazden (Discussant)
Harvard University

American Educational Research Association • Annual Meeting 2007 • Chicago, IL
April 10, 2007

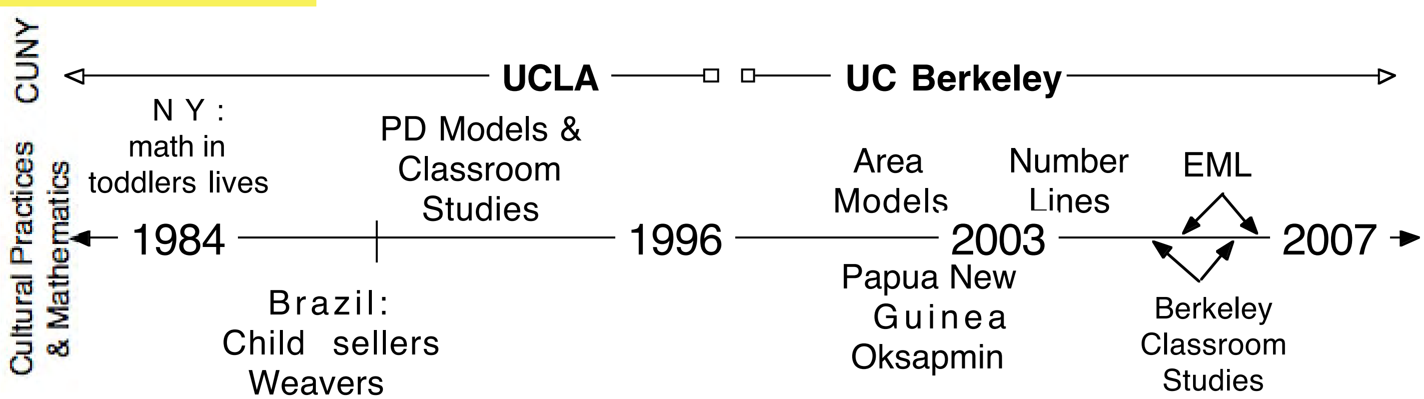
Berkeley University of California   UNIVERSITY OF MICHIGAN

Two research programs intersect in the lab class

Michigan program



Berkeley program



Overview of Session

1. What is a “laboratory class”?
   - How does it function as a site for research by different groups?
2. A video clip: Fifth graders in the early stages of work on a complex math problem
3. UM group
4. Berkeley group
5. Discussion

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What is a “laboratory class”?

- **Laboratory**
  A planned setting developed for real-time studies of the interplay of instructional design, teaching, and learning
- **Class**
  A “live” classroom that can be documented and evaluated as the designs unfold in enactment

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Features of a laboratory class

- Practice that is public and perceptible

- Careful documentation
- Teaching that is studyable

- Experimentation and interdisciplinary study

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A glimpse of the 2006 laboratory class

July 2006

Entering fifth graders

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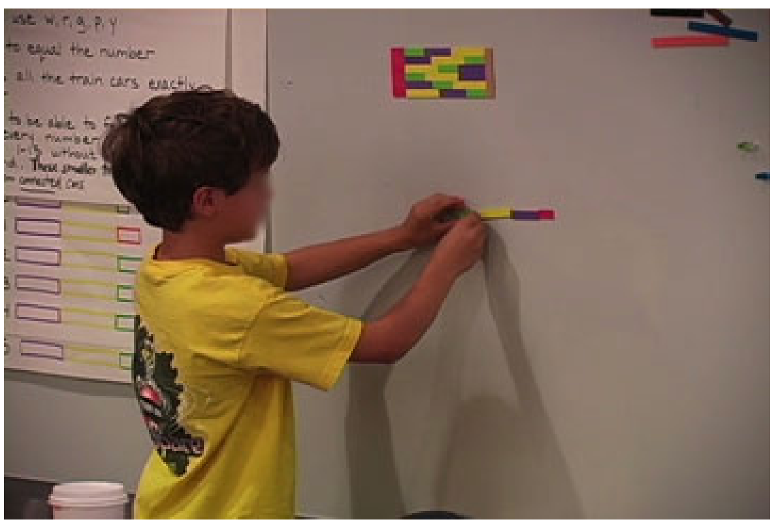
Elementary Mathematics Laboratory Summer 2006



Inner circle:

- An elementary summer school class — 21 students, entering fifth grade, wide range of mathematics achievement and dispositions
- 2.25 hours of instruction per day for 6 days
- Mathematical content: Fractions, permutations, number line as a mathematical context
- Mathematical skills: explaining, representing, proving

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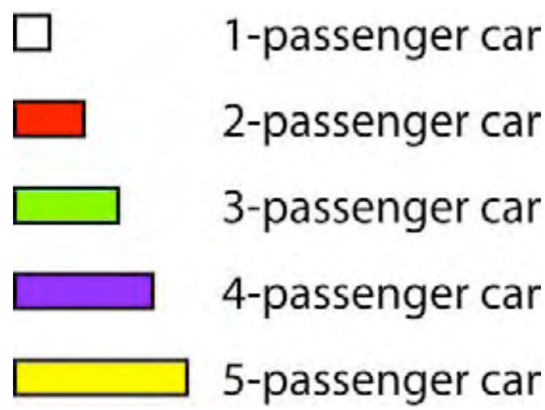
Elementary Mathematics Laboratory Summer 2006

Outer circle:

- A research group comprising mathematicians, K-12 teachers, mathematics educators, educational researchers, & developmental psychologists
- A laboratory for developing and studying mathematics instruction and the mathematical demands for teachers

Video clip: Fifth graders in the 2006 laboratory class

- Day 1, end of first hour
- Students explored Cuisenaire Rods



- Rods as cars of trains

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Setting up the problem

- Warm-up activity, building trains, sizes: 1, 15

- 4 + 5 + 3 = 12
- Conditions
  - Only use cars 1 to 5
  - Use each car at most once
  - Hold stated number of passengers
- Activity presented, discussed, written copy distributed
- Individual/pair work for 15 minutes
- Whole group reconvened, shared results, checked conditions, trains of 1 to 12 passengers
- Clip begins with a pupil's suggestion for a train that holds 13 passengers . . .

video

Overview of UM part

1. Summer 2006 study research questions
2. Studying mathematical reasoning and proving in fifth grade
   a) The mathematics problem and its demands
   b) Students' mathematical work
   c) The mathematical work of teaching reasoning and proving

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Research questions in Summer 2006 lab class work

1. What is involved in fifth graders learning to reason about and construct a proof that no solution exists to a complex problem?
2. Can “fraction” be usefully and correctly defined, and operationalized on the real line, for and with fifth graders?
3. What are the tasks for the teacher in work on (1) and (2)?

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Reasoning and proving in elementary mathematics classrooms

1. The mathematical entailments of the Train Problem

2. The students' work

3. The work of the teacher

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Reasoning and proving in elementary mathematics

1. The **mathematical territory and demands** of the Train Problem

The Train Problem's Mathematical Core

Consider the numbers 1, 2, 3, 4, 5.

What sums can be made using only these numbers, using each one at most once?

For example,

$$1 + 2 + 3 + 4 + 5 = 15$$

is the maximum possible.

It is simple to get 1, 2, 3, 4, 5, as well as

$$15 - 1 = 14, \quad 15 - 2 = 13, \ldots, \quad 15 - 5 = 10.$$

And we can get

$$5 + 1 = 6, \ldots, 5 + 4 = 9.$$

So all numbers from 1 to 15 are possible.

The “Train Problem”: Numerical Form

Fix the numbers 1, 2, 3, 4, 5 in some order, for example,

1	5	3	4	2

Can we choose such an ordering of the numbers so that each number from 1 to 15 is a sum of <u>consecutive</u> terms in our list?

For any ordering we can clearly get the sums 1, 2, 3, 4, 5, and 15.

For the example above we can also get

$6 = 1 + 5, \quad 7 = 3 + 4, \quad 8 = 5 + 3, \quad 9 = 3 + 4 + 2$

What else?

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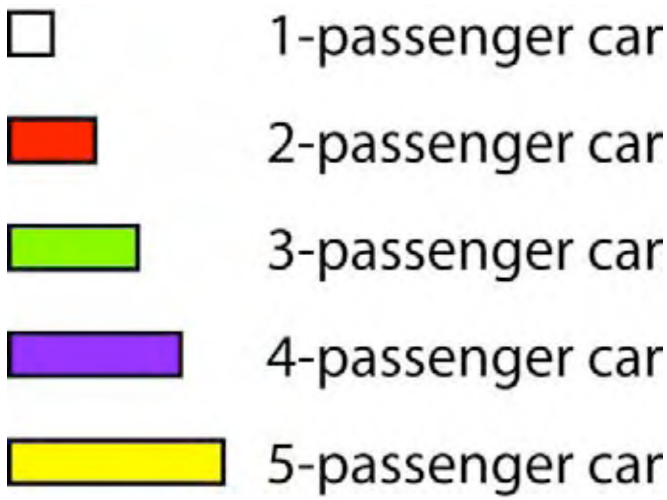
The different orderings of 1, 2, 3, 4, 5

- How many do you think there are?
- 5! = 1 x 2 x 3 x 4 x 5 = 120. A lot!
- A BIG search for a solution
- But— <u>there is no solution</u>
- But, if you don't know that, you could spend a lot of frustrated time before finally finding out
- That is the situation the children faced. They were destined to "fail"
- Rescue: It was impractical to exhaust all cases, so they were supported to find a mathematical proof that no solution can exist; that no one could find a solution!
- Proving the non-existence of a solution as a mathematical problem

The PCMI Train Problem



Mr. Howe wants to order a special five-car train that uses one of each of the different-sized cars. He wants to be able to break apart his 5-car train to form smaller trains that hold exactly 1 to 15 people. In addition, he wants to be able to form these smaller trains using cars that are next to each other in the larger train.

Can the PCMI Train Company fill Mr. Howe's order? Explain how you know.

What mathematical knowledge and skills are involved in the Train Problem?

- Arithmetic skills (adding different numbers of terms quickly)
- Empirical exploration; finding patterns, making conjectures
- Structured record keeping
- Reasoning "forwards" and "backwards", using what is known to reason about next steps
- Representation (physical, written, oral) of mathematical ideas
- Articulation and justification of mathematical claims
- Critical analysis and evaluation of the arguments of others
- Confronting the need for a complex mathematical proof of impossibility (of something that seems reasonable); mathematically akin to a "proof by contradiction"

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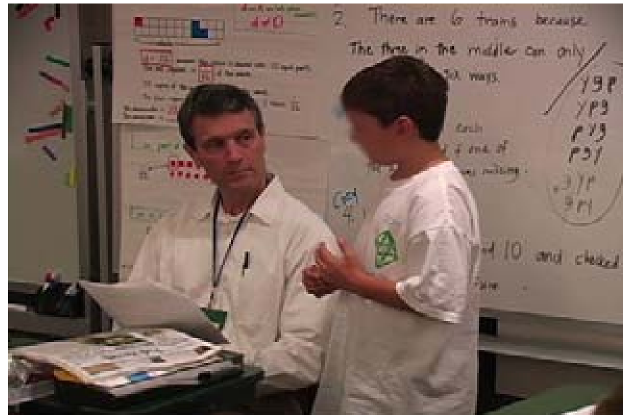
PCMI Train Company

Dear Mr. Howe,

We Are Sorry Your Order Was Not Able To Be Completed. We Will Refund Your Money As Soon As Possible.

Please Turn The Page To See The Work We've Done To Prove That Your Order Cannot Be Filled.



The Companies crew:

Maddie ★ Cozy Hailey Luke
Art SEAN Trevor Ben Sarah
Rebecca Vinnie David Paige
Tori Brianna Ally
Jessica

Sign Here x Mr. Howe

Reasoning and proving in elementary mathematics

2. The work for **students** in trying to resolve the Train Problem

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Three mathematical aspects of student work on the Train Problem

1. Empirically exploring and reasoning
2. Mathematically justifying claims, and articulating them
3. Achieving mathematical authority and conviction

Persistent and systematic work over six days.

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1. Empirical exploration and reasoning

- Students experimented with random trains
- Each arrangement they tried failed to contain trains for certain numbers
- Teacher question: “Why are you beginning to think that Mr. Howe’s order can’t be built?”

> Tori: Because there are a lot of people in here and there are probably a lot of combinations that we can do too, and if you think about the amount of time we had and the amount of people and how much they got, and I’m assuming a lot of people got at least like six, one of us if we didn’t all do the exact same thing would have probably found it if there was a way.

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2. Producing and articulating mathematical justifications

- Mr. Howe's Consultant: To make Mr. Howe's train you would have to put the white car (1) and red car (2) on the ends (no explanation given)
- Mr. Howe: Why?
- The students, with scaffolding, developed an explanation; the teacher asked if someone would remind the class

> Rebecca: Because like you need to make … to be able to make 14 you just take off the one, but if it's in the middle somewhere … it leaves a hole and so you can't make it. But then to make a 13 you have to … you have to take away the two car train and if that's in the middle somewhere there's another hole and so you have to have 'em on the end, so … there aren't any holes when you …

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3. Striving for mathematical authority and conviction

- Failure to build Mr. Howe's train: Was it their failure, or was it just impossible?
- And, if impossible, how could they be sure?
- Too many cases to test all of them
- The teacher pressed, and guided them toward mathematical ways (proof) to be sure (without testing all cases)
- By day 6 the students had assembled a complex multi-step proof of impossibility, a new mathematical experience for them
- As the students prepared to report their findings to Mr. Howe, the teacher questioned whether they felt confident in their conclusion and proof

"Show him that the train can't be built by <u>anybody</u>."

Teacher: We don't want him to come in here and then think, oh, he asked the wrong people and he should go find some smarter people. We need to prove to him that … we know exactly that we're right, so we need a proof that he'll be completely convinced by, and I think that we have all the pieces now.

Cosy: Well, I'm not positive on that, but they (other mathematicians) could try, but they won't won't get it for a while. They would have to work really hard on it.

Rebecca: If you (Mr. Howe) go to another mathematician you'll be wasting your time.

Tori: They'll give him the same answer we did and so it'll be a waste.

Holly: Not another mathematician (could do it), except maybe Albert Einstein or Thomas Edison.

Reasoning and proving in elementary mathematics

3. The work of **teaching** with the Train Problem

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The work of teaching:

Scaffolding the learning of complex practices through task design and discourse structuring

1. Unpacking and articulating the conditions of a problem
2. Unfolding the stages of work on a problem
3. Supporting mathematical explanation

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1. Helping students unpack and articulate the conditions of the problem

- Only use w, r, g, p, y
- Must use each rod exactly once
- Can't use the same car twice
- Has to be able to form trains for every number of passengers from 1-15 without moving cars around. These smaller trains must be built from connected cars

2. The work of teaching: Unfolding the stages of work on a problem

Phase I (Class #1)	Trains for 1 - 15 passengers	Practice with the basic elements of the problem, establish common knowledge of the materials and developing norms of explaining solutions
Phase II (Class #1)	The Train Problem	Setting a context for the problem that supports the core mathematical work, captures students' attention, and does not distort or distract
Phase III (Class #2)	White-red ends	Focusing the students' work from empirical experimentation to more systematic inquiry
Phase IV (Class #3)	The only way to get 13 and 14	Provoking the need to prove and further focusing the students' work
Phase V (Class #4)	Six white-red end trains	Building a key sub-part of the proof
Phase VI (Class #5)	Consolidating and assembling the proofs	Organizing the argument
Phase VII (Class #6)	Final report to Mr. Howe	Experiencing the sense of conviction

2. The work of teaching: Unfolding the stages of work on a problem

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Designing the problem with a social framing to provoke the imperative for proof

Mr. Howe became discouraged with our lack of progress, so he hired a consultant to get some help with his order. The Consultant told him:

"To make a train like the one you ordered, the PCMI Train Company <u>has to</u> put the one-passenger car on one end and the two-passenger car on the other end."

Unfortunately, the Consultant left right away and she didn't explain her reason. Is she right, and if so, why? If she is not right, can you show why she is not?

Mr. Howe wants us to help him determine if the Consultant's claim is true and prepare a report to show him what we decide and explain it to him.

"Show him that the train can't be built by <u>anybody</u>."

Teacher: We don't want him to come in here and then think, oh, he asked the wrong people and he should go find some smarter people. We need to prove to him that … we know exactly that we're right, so we need a proof that he'll be completely convinced by, and I think that we have all the pieces now.

- Preparing to convince Mr. Howe, who had placed the order for a train with them
- Fanciful context (not "real world") but with strong social imperative to prove

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Using the social context to provide experience as well as skills of proof

Teacher: One of the things we're trying to work on here is to see that there is a way to actually prove things in math, that might not take you forever …. And a really important idea in math is that sometimes you can find ways to find something is true without checking every last case. In this problem we're probably going to be able to check every case. But I want you to start thinking that there are actually ways that you can be sure, and you don't have to worry that somebody's going to come along later and say that , oh, you didn't think of this or that …

So … be really careful in your explanations so that you can say to Mr. Howe either, "Here's your train." Or "No way, Mr. Howe, you can't have your train!"

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3. Supporting mathematical explanation

- Using the conditions of the problem
- Using language, notation, and representations carefully
- Using audience as central to mathematical explanation
- Interrogating explanations
- Developing concept of conviction, and what mathematical conviction requires

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Teaching explanation is more than requesting explanations

Teacher: What about a train that holds 8 people? Is there one?

Paige: Yellow and light green (*the teacher makes the train on the whiteboard)*

Teacher: Okay, can someone see if Paige's solution fits all three of the things that have to be? Can someone check it? … Who can check it against these three things (*pointing to the conditions*)? Who can check it? (*Waits*) Can someone do this? I only see a few hands up. (*Waits*) I'm asking, can someone take this solution (*points to Paige's yellow-green train*)… take this solution and check to see if it fits all three of our points (*points to conditions*) that it has to be to be a good solution? Okay, Tori.

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Teaching explanation is more than requesting explanations

Tori: Well it doesn't use any of the same cars twice.

Teacher: Okay, it doesn't use the same car twice. Good.

Tori: And it uses yellow and green and those are the cars that the PCMI Train Company uses. And it is equal to the number!

Teacher: Okay, how do we know?

Tori: Because three plus five is eight.

Teacher: Okay. You checked that solution really well, Tori.

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More to analyze . . .

1. The role of the problem context and designed unfolding
2. Learning to believe in mathematical proof
3. Using knowledge of the mathematical territory of the problem to scaffold the work without thinning the problem's challenge
4. Using the social setting of the classroom and student diversity to achieve mathematical goals

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Back out to the laboratory class as a setting for overlapping research agendas

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Overview of UC Berkeley Part

- A research approach to lesson design: Fractions and Number Lines
- Studying the *travel of ideas* in designed lessons
- *Travel* & trains

UC Berkeley: Part I

A Research Approach to the Design and Support of Lesson Sequences

Geoffrey Saxe, Meghan M. Shaughnessy, Darrell Earnest

UC Berkeley

Additional Research Group Members at UC Berkeley:

Sarah Cremer, Linda Platas,

Yasmin Sitabkhan, and Adena Young

Lesson Structure

Problem of the Day

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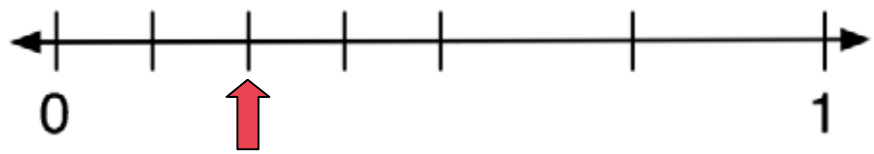
Lesson Structure

Problem of the Day

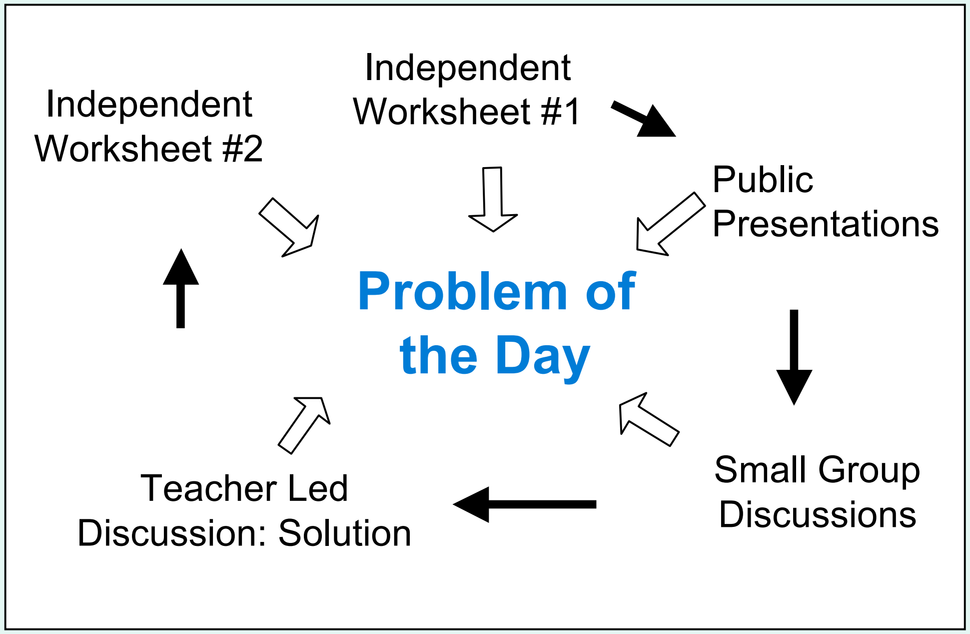
Figure out what this point is called…



Circle the correct answer:

2/6 2/7 1/4 2/4 2

Explain why



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Design of Lesson Sequences

- Design Principles
- Empirical Research

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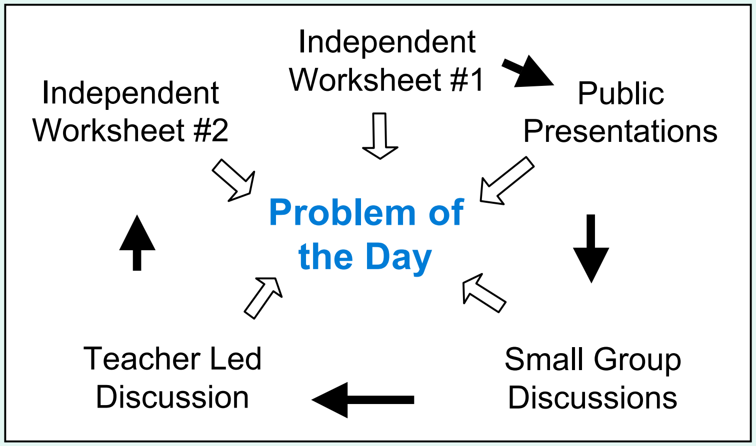
Design Principles

- Principles
  - Target core math ideas
  - Build on student thinking
  - Problematize students' math
  - Coherent lesson series
  - Use sustainable structures



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Design Principle: Target Core Mathematical Ideas

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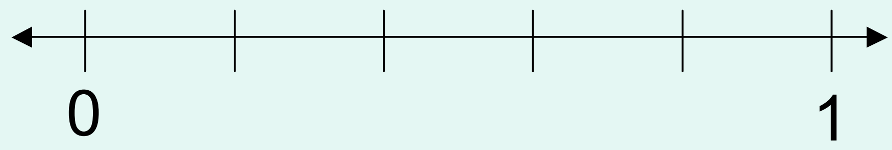
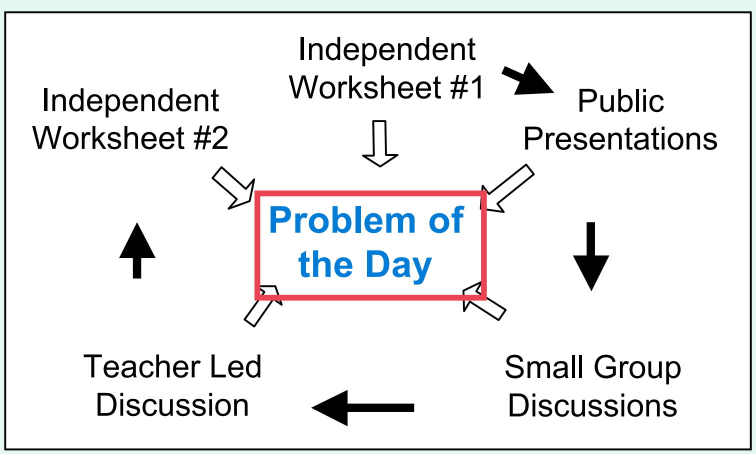


- Number as measure
- Necessity of equal intervals
- Many equivalent names
- Every number has a place
- Density of the line



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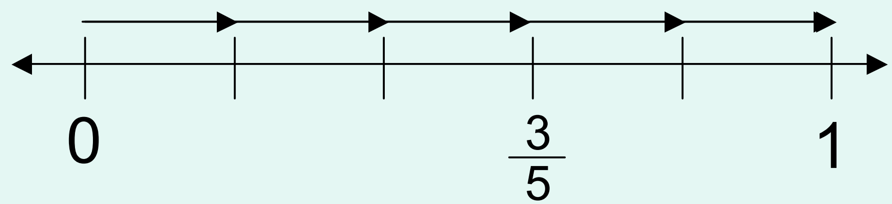

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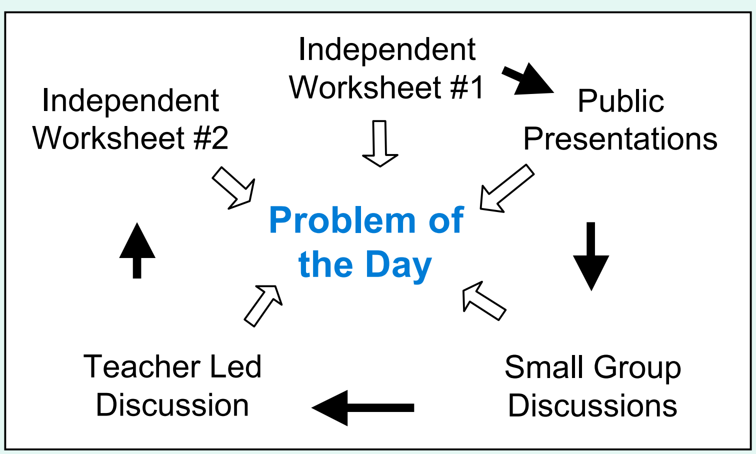

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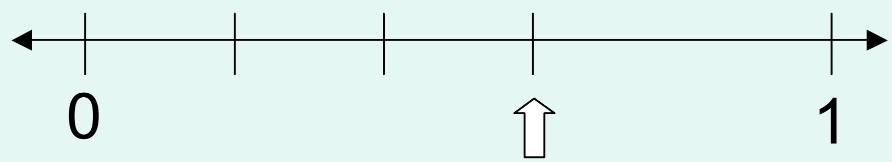

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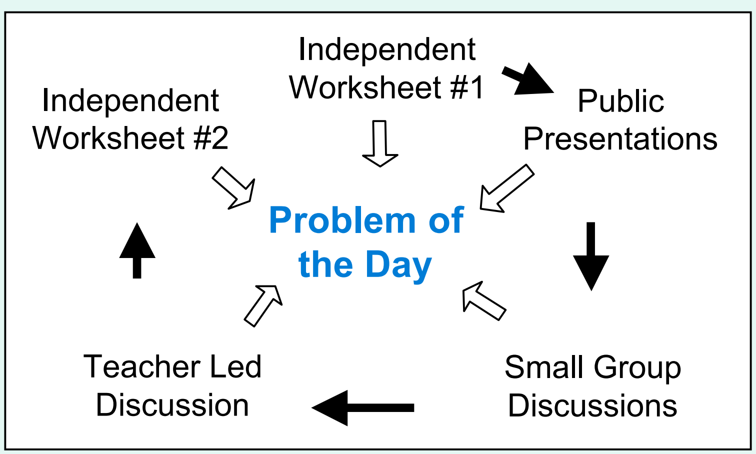

Design Principle: Build on Student Thinking

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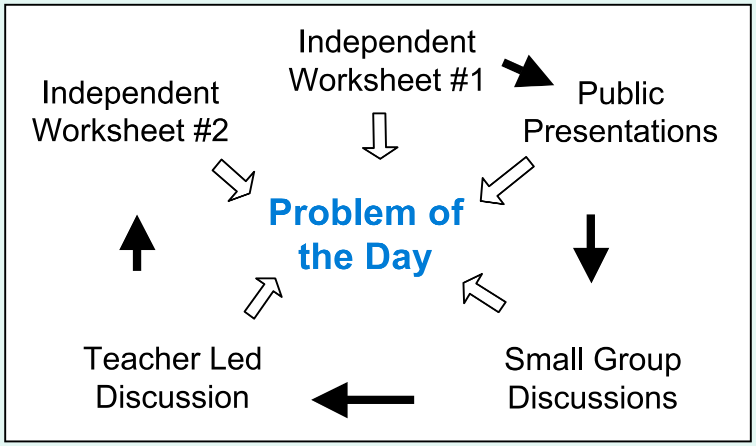

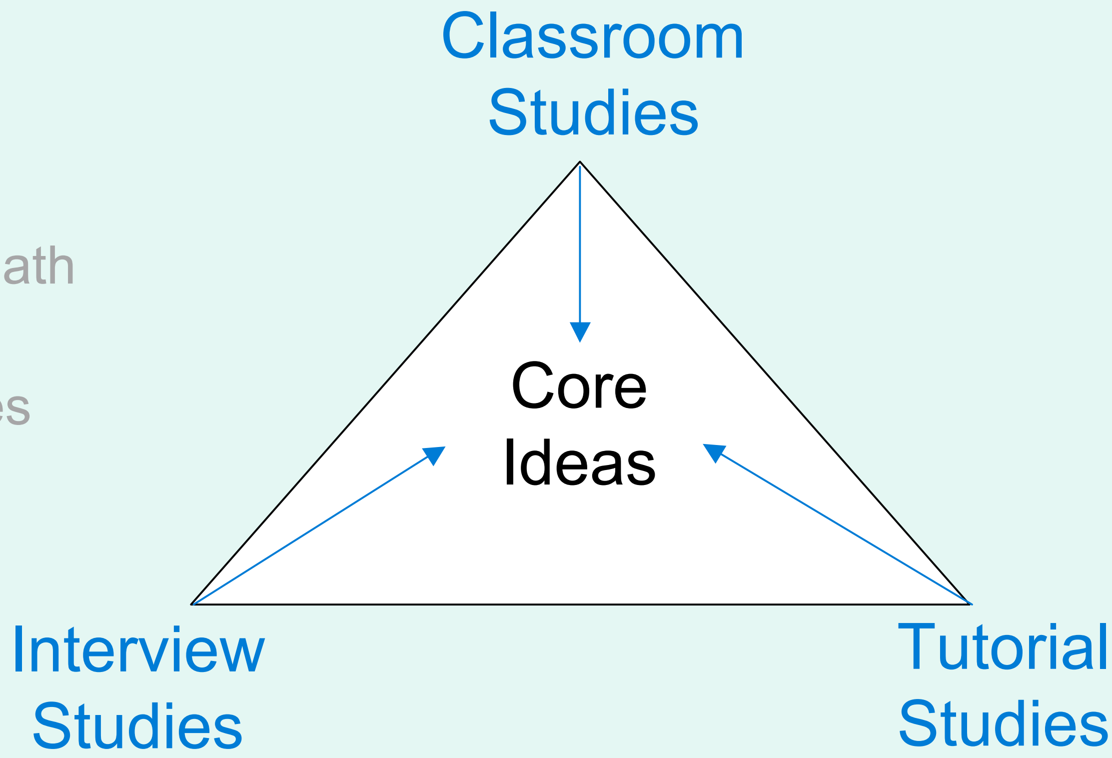

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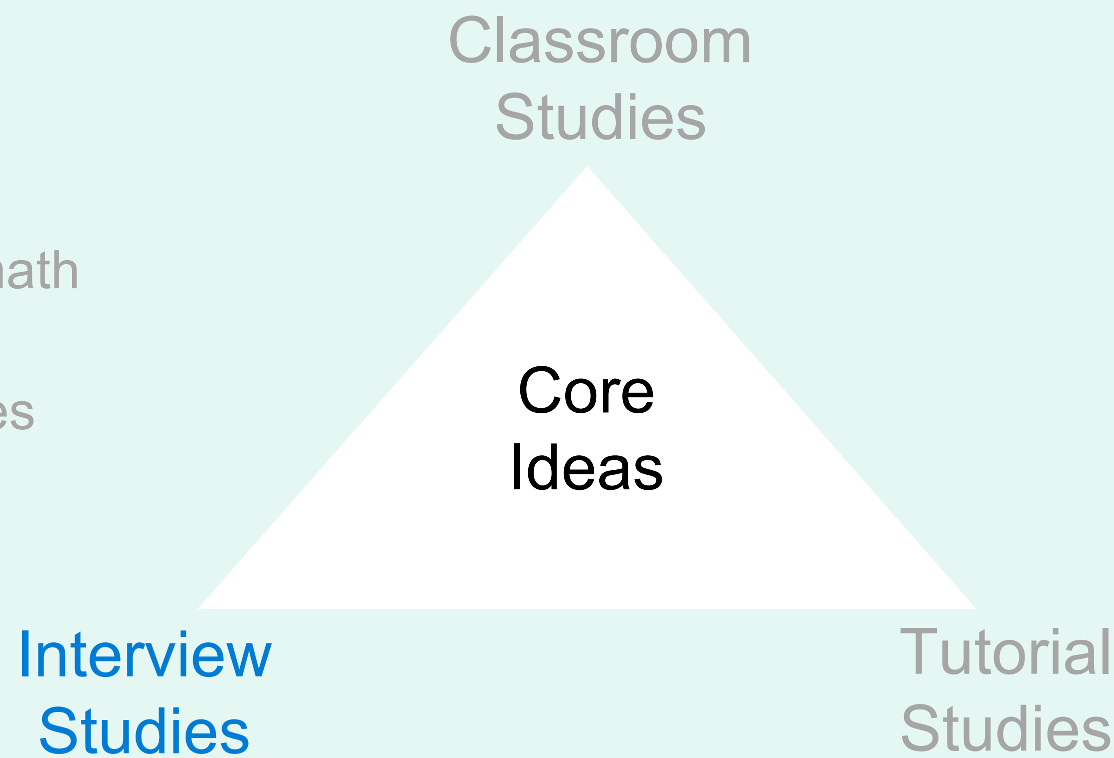
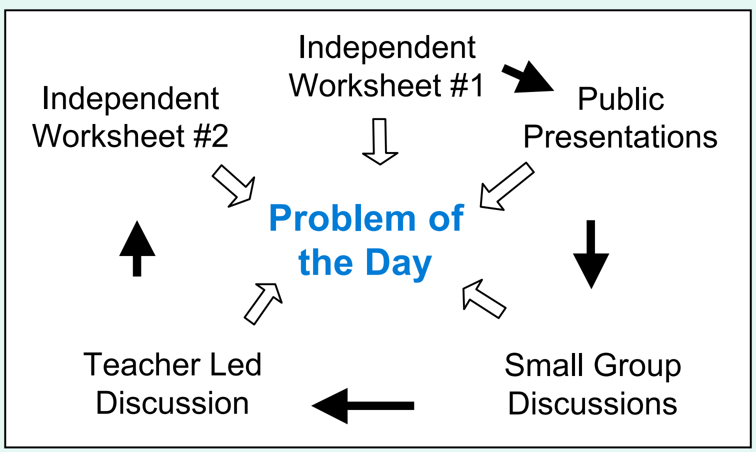
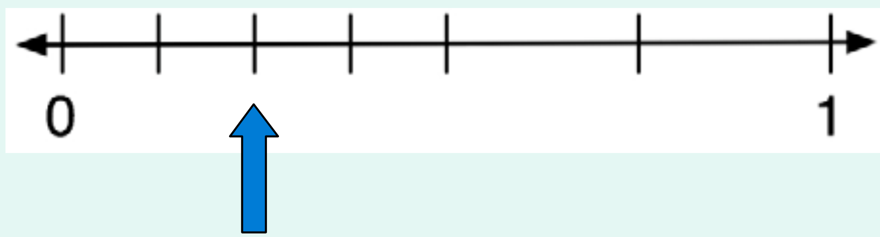


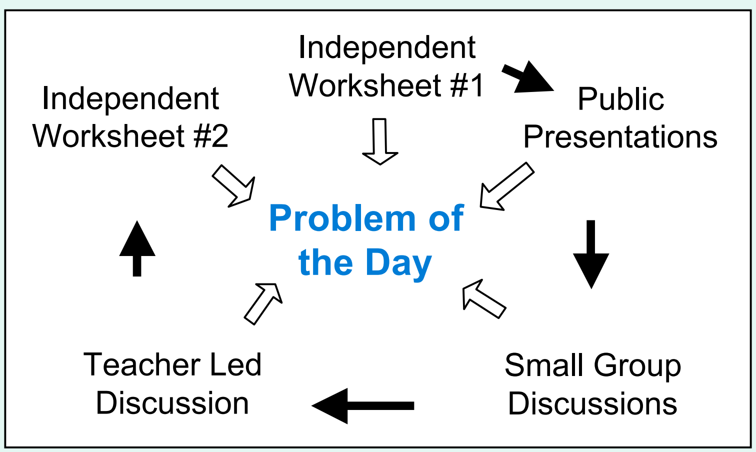


Interview

Figure out what this point is called on the number line



2/6 2/7 1/4 2/4 2



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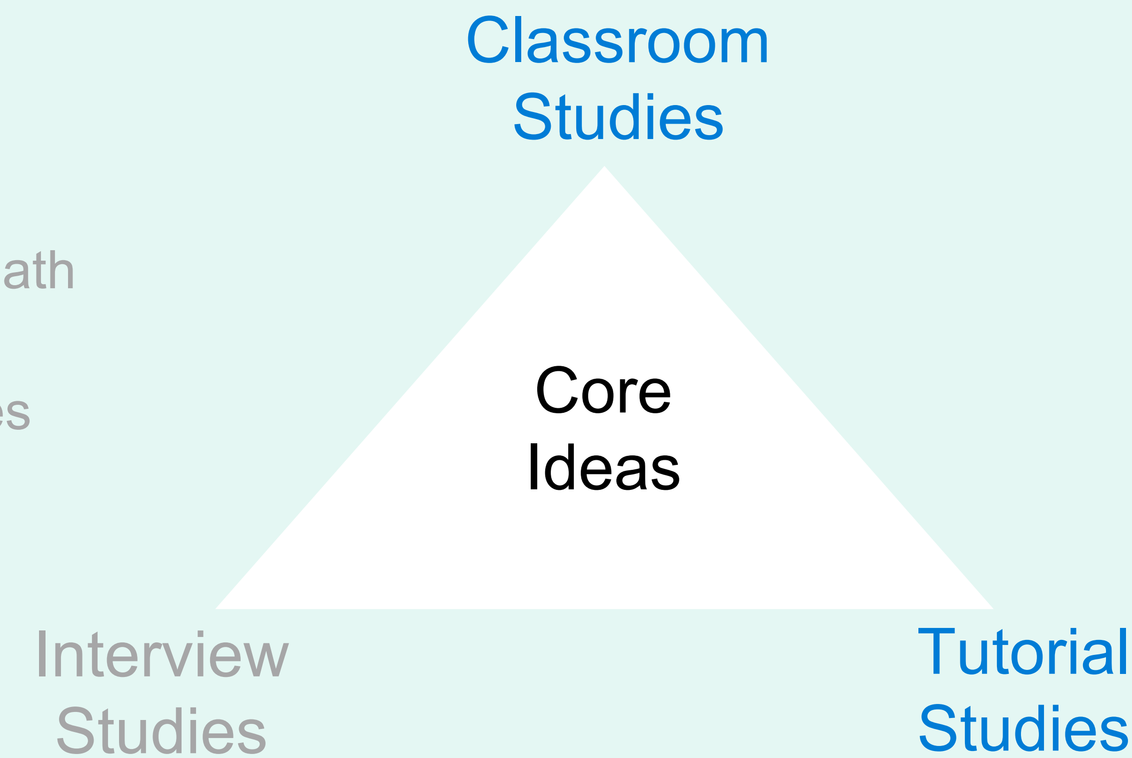
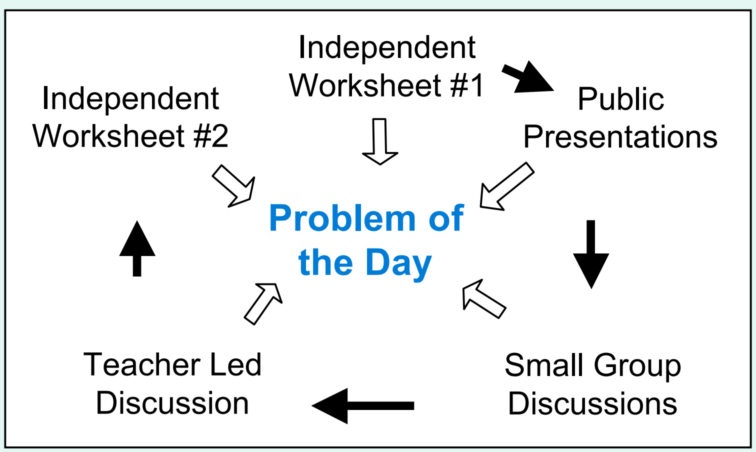
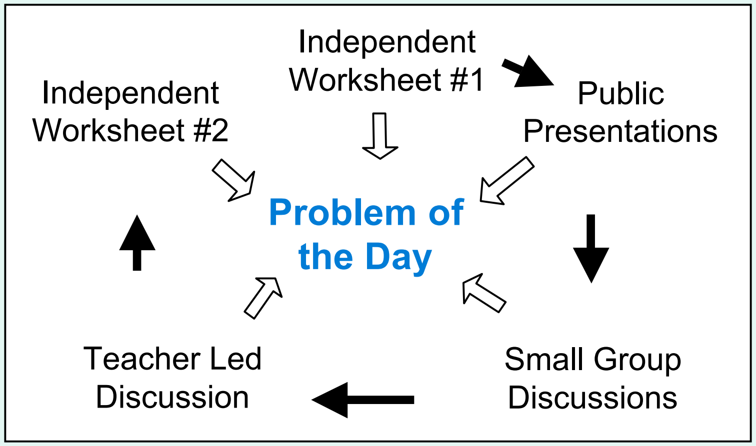




Design Principle: Problematize Students' Math

- Principles
  - Target core math ideas
  - Build on student thinking
  - Problematize students' math
  - Coherent lesson series
  - Use sustainable structures

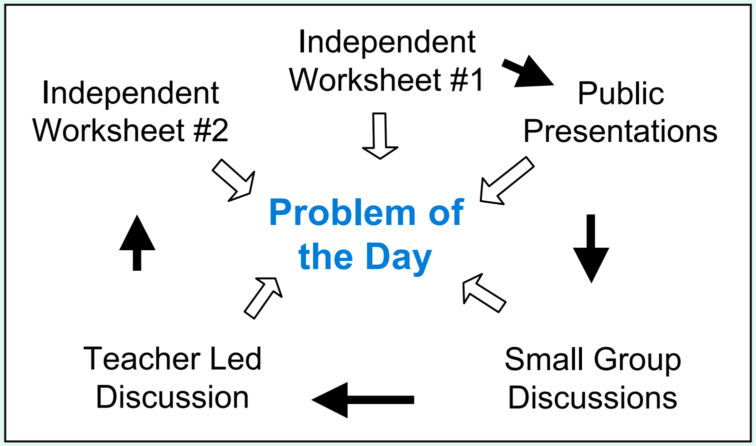
- Students generate conceptual challenges as they
  - put their own thinking in relation to others
  - as they make use of learning from one lesson to solve problems in subsequent lessons



Design Principle: Coherent Lesson Series

- Principles
  - Target core math ideas
  - Build on student thinking
  - Problematize students' math
  - Coherent lesson series
  - Use sustainable structures

- Five lessons in our mini-series
- Series affords coordination of ideas across lessons…



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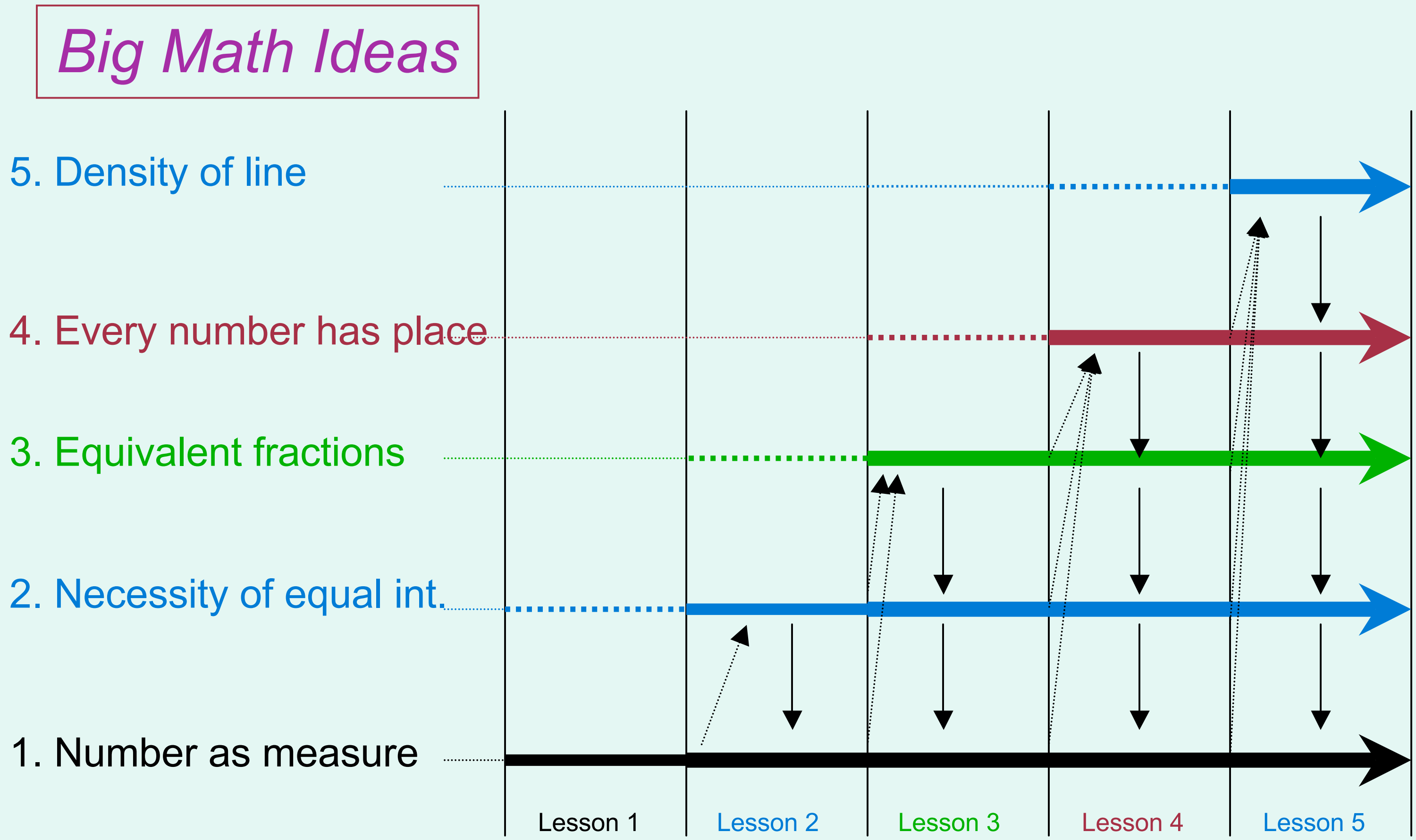
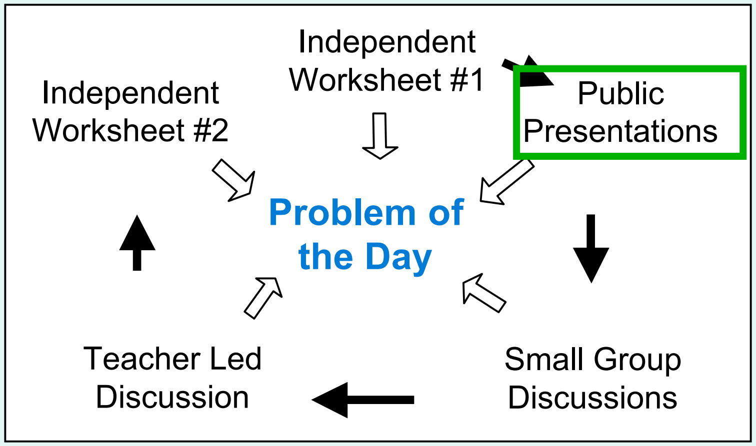
Design Principle: Coherent Lesson Series



Design Principle: Sustainable Lesson Structures

- Design Principles
  - Target core math ideas
  - Build on student thinking
  - Problematize students' math
  - Coherent lesson series
  - Use sustainable structures

- Resources
- Management
- Systematic development of ideas



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Prospects

- Broading scope of lessons
- Using lessons as a laboratory to explore the *travel of ideas*

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UC Berkeley: Part II

Studying Learning in Inquiry-Oriented Classrooms: The Travel of Ideas

Geoffrey Saxe, Darrell Earnest, Meghan M. Shaughnessy
UC Berkeley

Student Thinking as a Resource in Inquiry-Oriented Classrooms…

- Teachers
  - pose problems, solicit contributions, orchestrate discussions
- Students
  - elaborate, transform, refine ideas as they communicate with others and themselves
- In this process…
  - Some ideas are taken up, others altered, others rejected

- ***What travels and how?***

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Method of Inquiry

- Forms & functions
- Developmental (genetic) analysis

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Forms and Functions

- Ideas ---> Forms & Functions

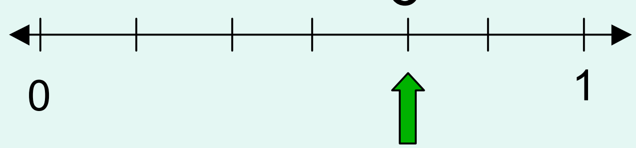
Equivalent fraction names

Geometric Forms

Arithmetic Forms

How many names?

$$\frac{4}{6}$$



$$\frac{2}{3}$$

$$\frac{2}{3} \text{ x } \frac{2}{2} = \frac{4}{6}$$

Developmental Analyses: form-function relations

- Microgenesis
  – Forms becoming means to serve particular functions
- Sociogenesis
  – Forms and functions becoming reproduced, altered, and taken up
- Ontogenesis
  – Forms and functions becoming reproduced and altered as individuals re-construct solutions in their own development

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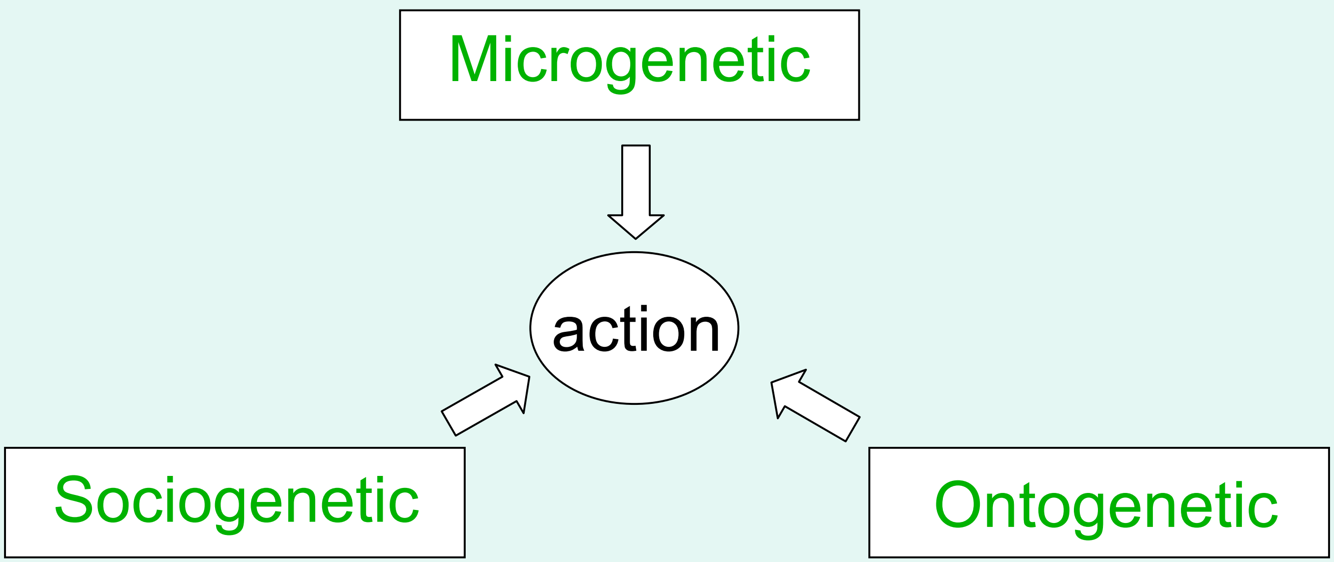
Any conceptual act is a nexus of three developmental strands



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An Illustration -- Method of Inquiry and Empirical Techniques

How many names?

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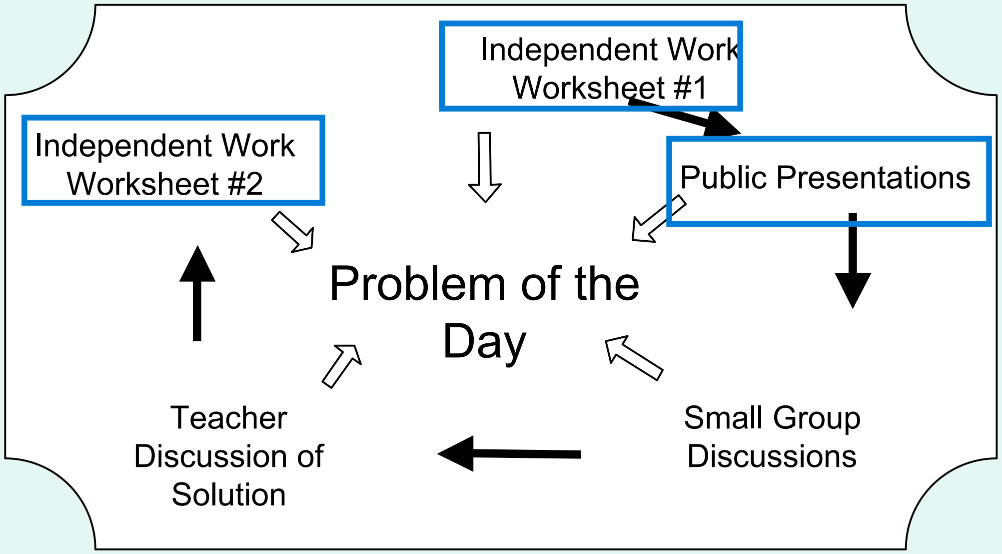
Method, Challenges, & Empirical Techniques

- Microgenesis
- Ontogenesis
- Sociogenesis



Sociogram

Post Lesson Interview

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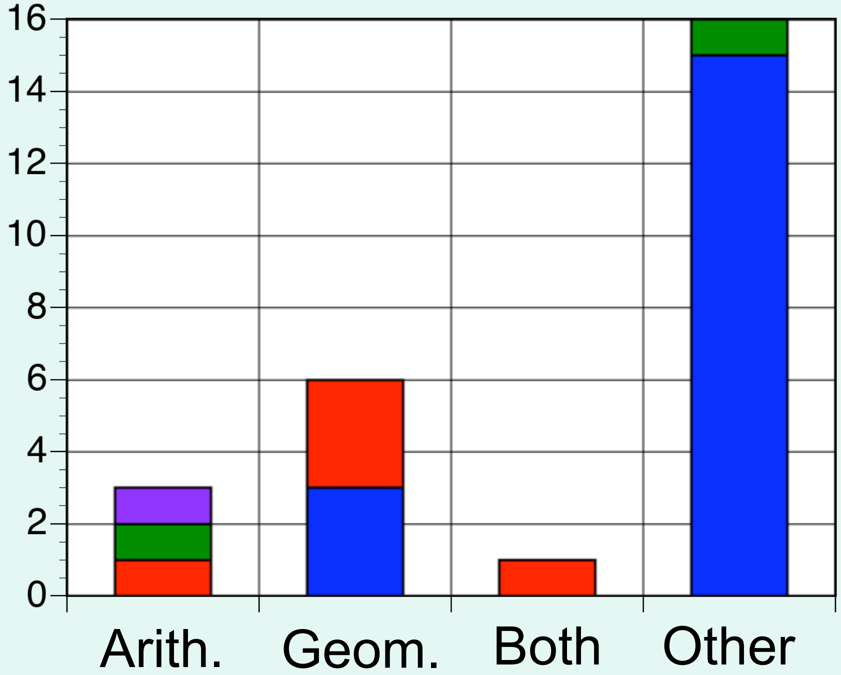
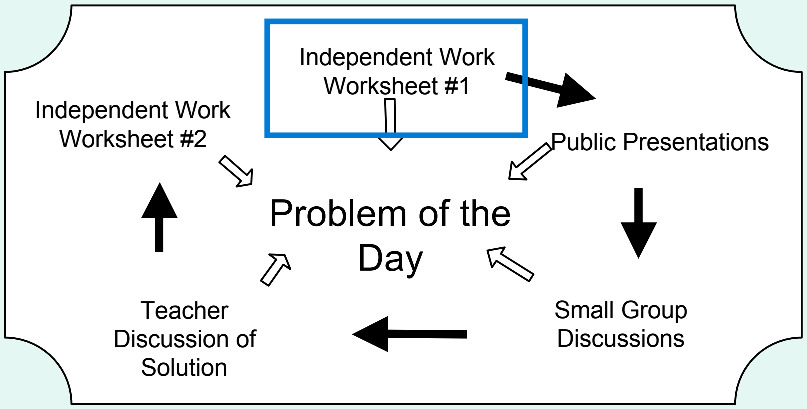
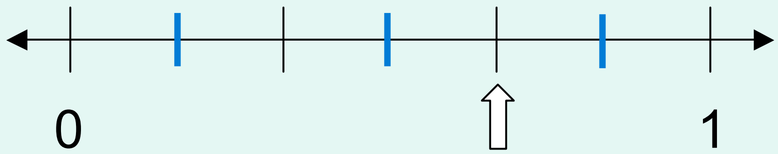
Microgenesis

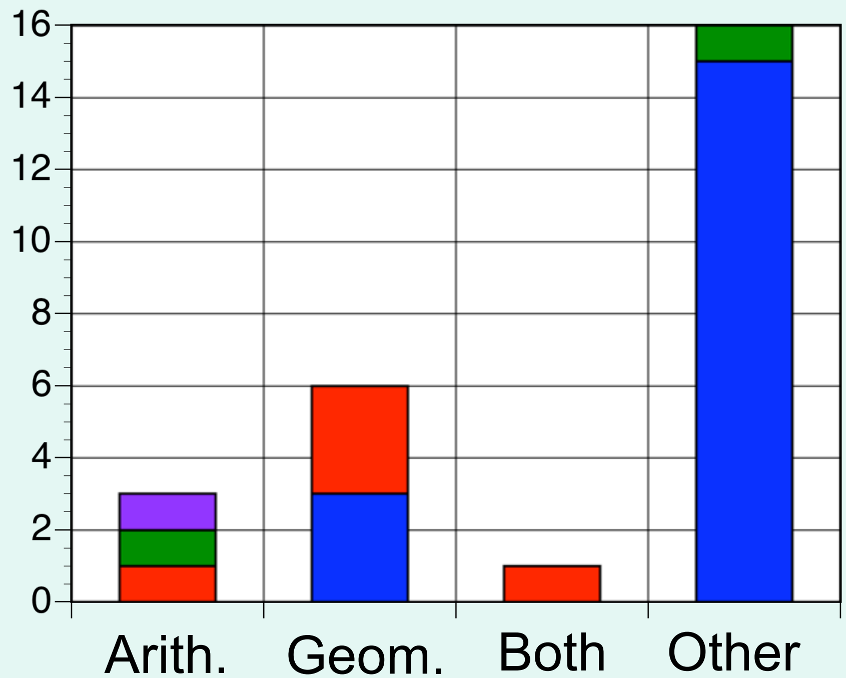




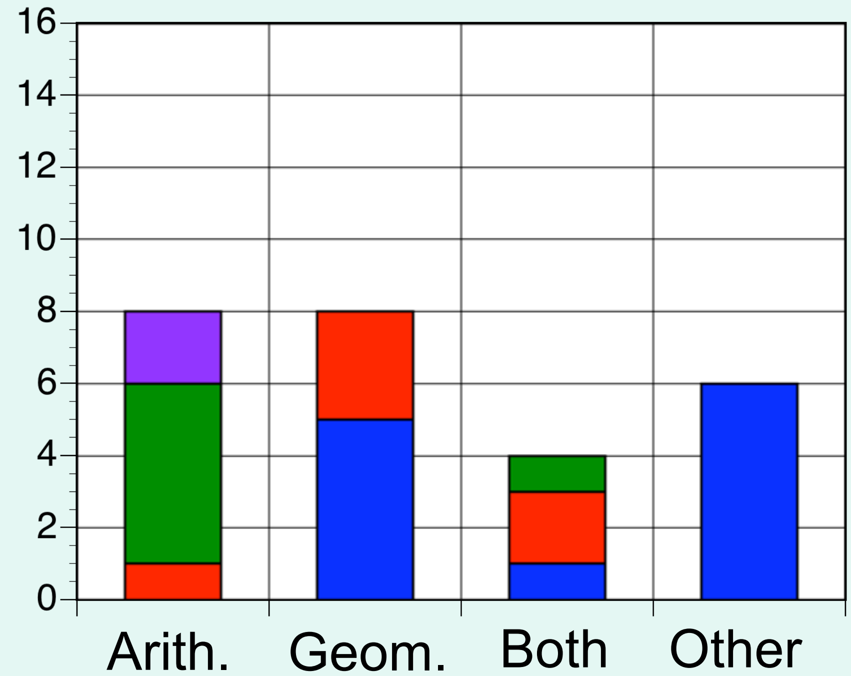


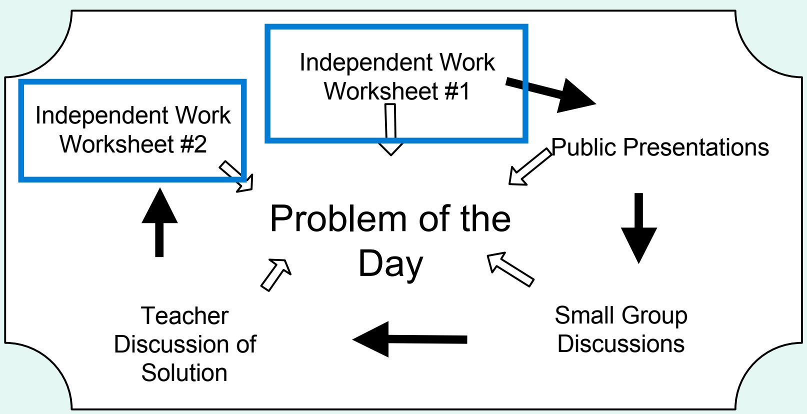
Ontogenesis

First Worksheet



Last Worksheet





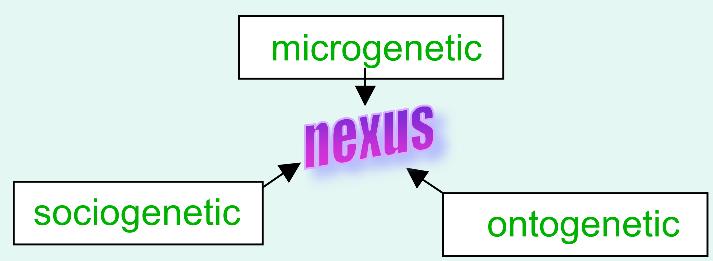
Sociogenesis

Student Presentations: Sienna

"Keep multiplying the fraction by 2. There are infinitely many names."

Who influenced your thinking?

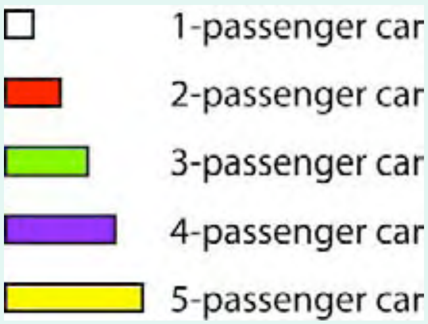
Jose: "Sienna said more than 20 names because you multiply on and on."



UC Berkeley: Part III
The Train and Mr. Howe

- Some parallels

  Forms & functions



PCMI Train Company

Dear Mr. Howe,

We Are Sorry Your Order Was Not Able To Be Completed. We Will Refund Your Money As Soon As Possible.

Please Turn The Page To See The Work We've Done To Prove That Your Order Cannot Be Filled.

Identifying rods
Summing rod values quickly
Exploring and recording permutations…

Parallels…

5. Proof by contradiction

4. Systematic approaches to permutations

3. Explore permutations

2. Summing rods

1. Identifying values

‘Problem of the Week’

Unfolding problem & domain

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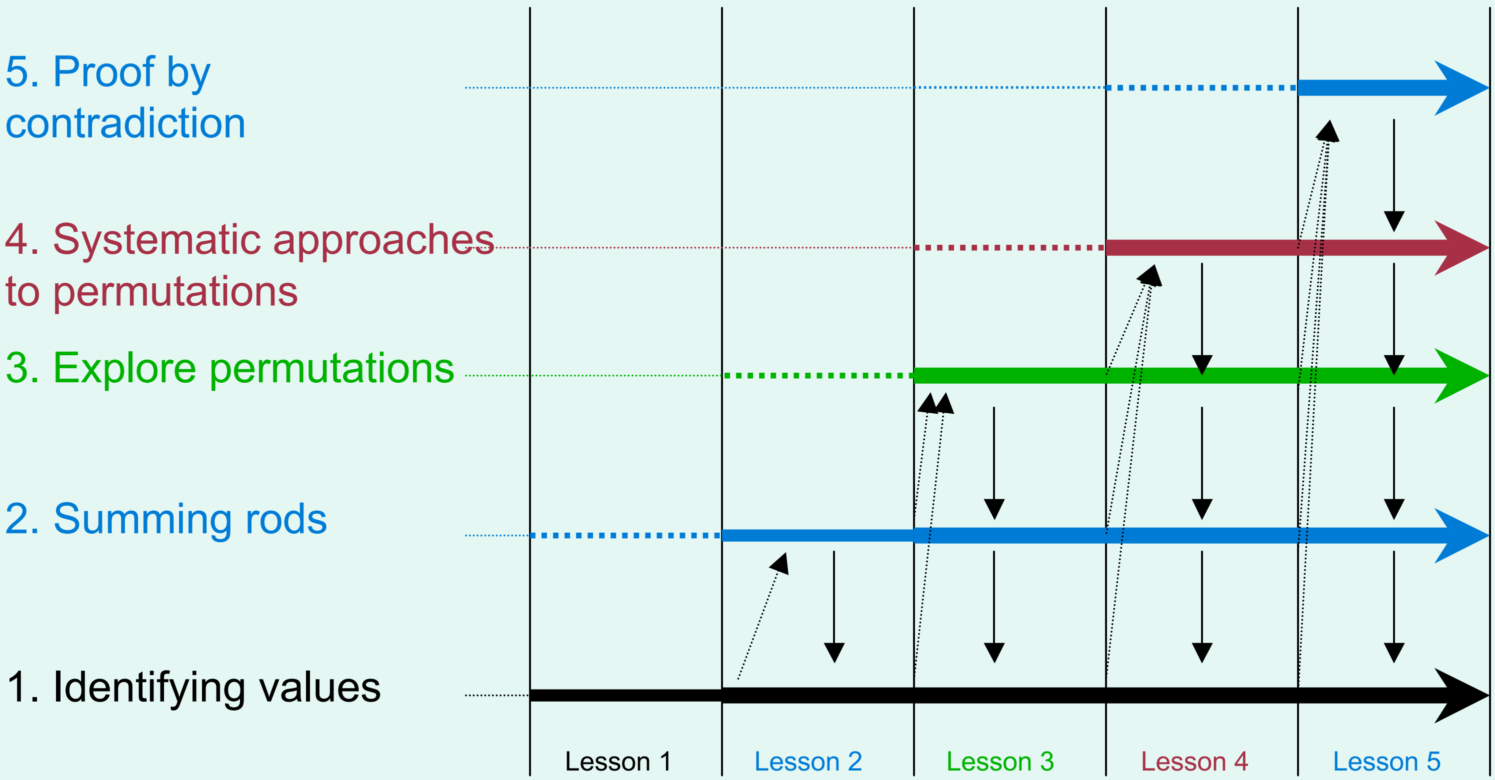
Bird's Eye View of Train Lessons?

Big Math Ideas



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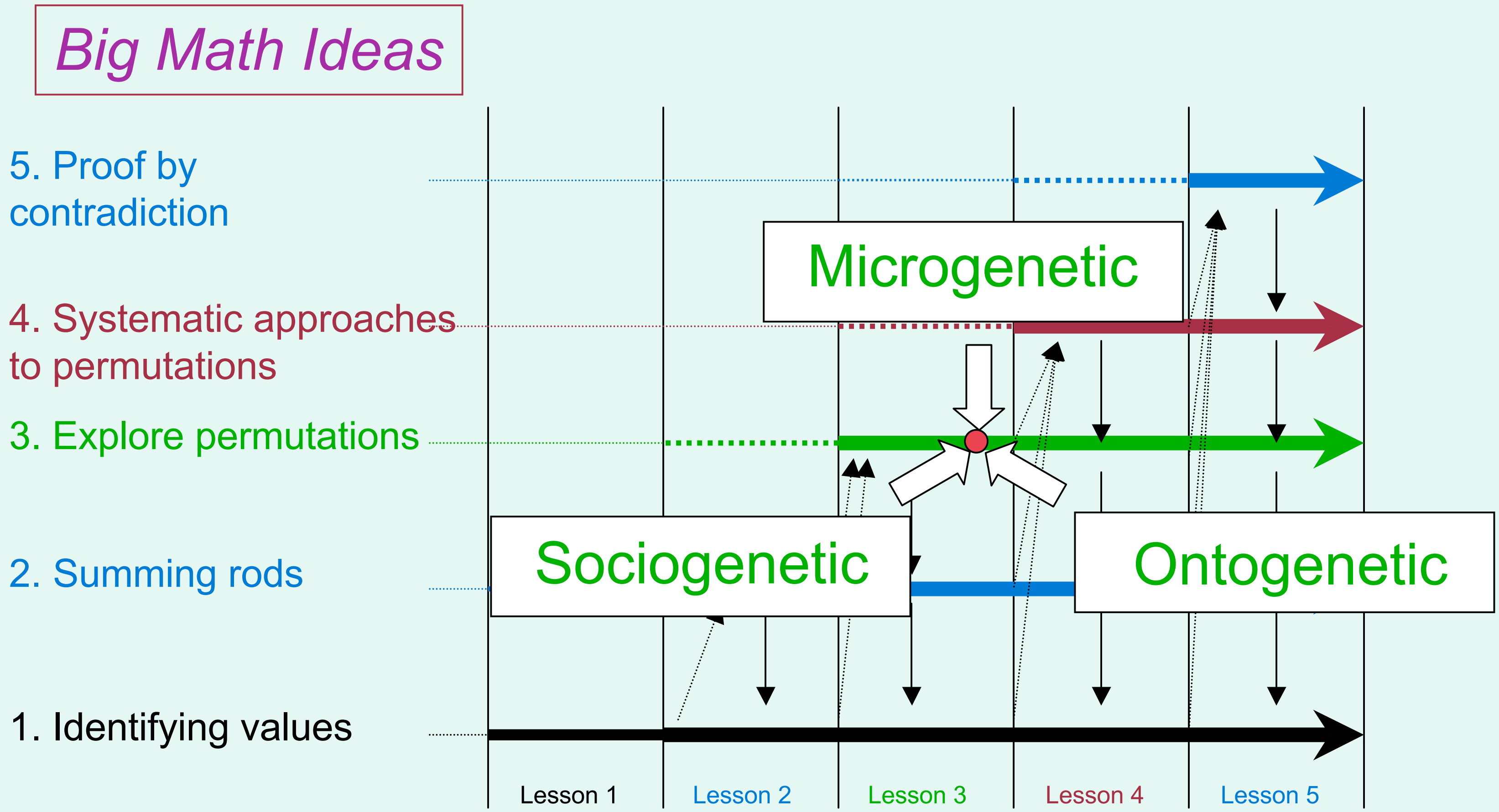
Bird's Eye View of Train Lessons?



Data Sources: From is to ought for trains & travel…

- EML Data Sources
  - Students' independent work in notebooks
  - Spoken displays in whole class discussions
  - Video records of displays and uptake
- Additional sources that would enable corroborative analysis
  - Interviews before & after lessons on thinking that generated displays
  - Social positions
    - Sociograms
    - Self reports of influence
    - Videos of small group interaction

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