

# Content Knowledge for Teaching Reading and Mathematics: Developing Theory and Measures

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# Introduction

**Measures of content knowledge for teaching that we will discuss in this session were developed for:**

- Use in the Study of Instructional Improvement, a large scale, mixed method study being conducted at the University of Michigan;
- Use in studying teacher knowledge and learning in the California Mathematics and Reading Professional Development Institutes.

# Introduction

## **This session is about:**

- How teachers use content knowledge in teaching practice
- What content knowledge for teaching items look like
- What we have learned from our work developing content knowledge for teaching items and scales

# Session Goals

**Demonstrate the theory and purposes for developing content knowledge for teaching items**

**Promote audience discussion of the following issues:**

- Content knowledge and its use in mathematics and reading instruction
- Problems educators and researchers face generating high quality measures of teacher content knowledge

# Session Presenters

- Deborah Loewenberg Ball (mathematics)
- Heather C. Hill (mathematics)
- Geoffrey Phelps (reading)

University of Michigan, Study of Instructional Improvement

- Patrick Callahan (mathematics)

University of California, Office of the President

# Plan for today's session

- **What do we mean by “content knowledge for teaching”?**
- **Why develop measures of content knowledge for teaching?**
- **What do measures of content knowledge for teaching look like?**
- **What have we learned?**

**Theoretical background:**

**What do we mean by  
“content knowledge for teaching”?**

**What is the actual problem  
that we want to solve?**

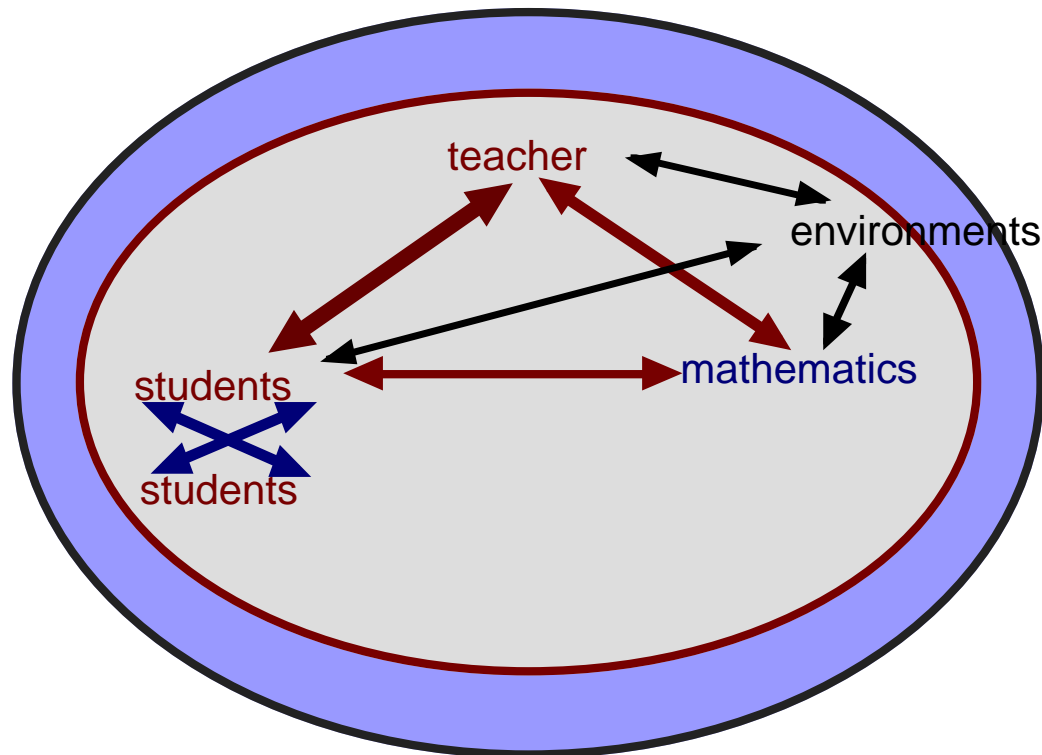
**The quality of  
teaching and learning**

**U.S. teachers' lack of  
content knowledge**

# **Mathematics teaching and mathematics knowledge for teaching**

# What is teaching?

I have pennies, nickels, and dimes in my pocket. If I put three coins in my hand, what amounts of money might I have?



# What is mathematics knowledge for teaching?

- profound understanding of fundamental mathematics
  - unpacked
  - connected
- in the curriculum
- beyond the curriculum
- useful to and usable in the work of teaching

# Knowing mathematics for teaching

- What mathematics knowledge and resources-- skills, understanding, sensibilities -- are entailed by teaching?
- How are such mathematical knowledge and resources used in the course of teaching?
- How might teachers be helped to develop usable mathematical knowledge and resources?

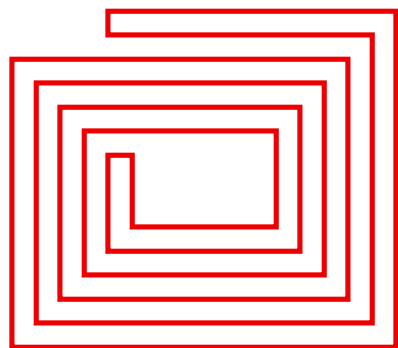
**Where and how do teachers  
use mathematics in teaching?**

## ***Situation #1:***

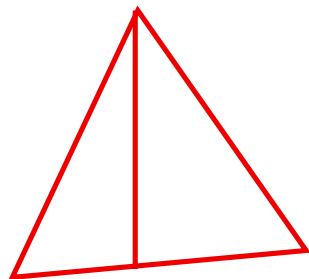
You are teaching the concept of a “polygon” in geometry.

Your students draw some shapes as examples and you realize that you are not sure whether they are polygons or not.

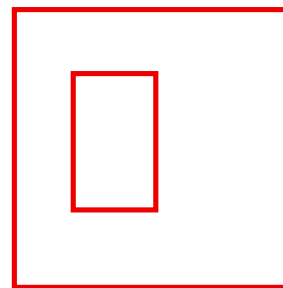
# Are these shapes polygons?



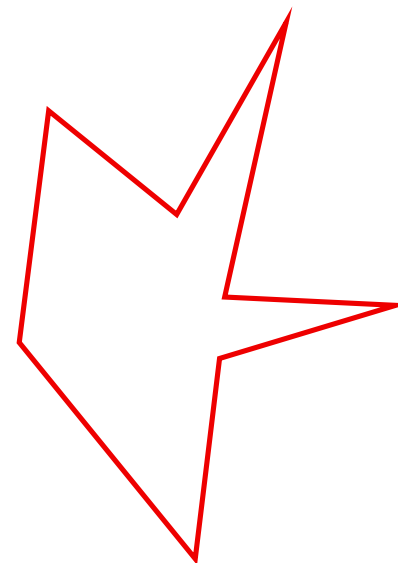
(a)



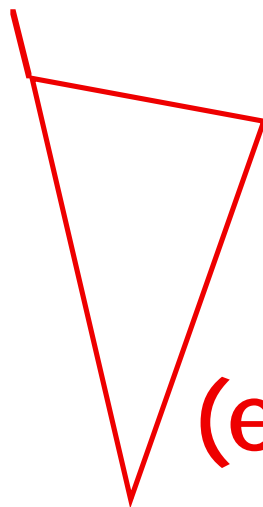
(b)



(c)



(d)



(e)



(f)

**Textbook:** A closed flat two-dimensional shape whose sides are formed by line segments.

# What's an adequate and mathematically acceptable definition of "polygon" for fifth graders?

- a) A simple closed curve with all straight sides.
- b) A closed figure made entirely of line segments.
- c) Plane figures with 3, 4, 5, or more sides such as the following examples (*pictures given*).

## *Situation #2*

You have worked on a problem to get at the meaning of division of fractions. You worked on a very productive problem, and would like to make a similar one as an extension.

# Original problem

Make a story problem for

$$1\frac{3}{4} \div \frac{1}{2}$$

Choose another pair of numbers to make another version of the problem.

Does your new problem involve the same mathematical work or not? On what basis would you claim this?

## Dimensions of the problem

- Does the division leave a remainder?
- Do the fractions afford a range of choices for models?
- Are people likely to consider the problem using a measurement or a partitive interpretation of division?
- Can you end up getting answers that appear right but are produced from a misinterpretation of the unit?

# Other examples of where and how teachers use mathematics in teaching

- Clarifying mathematical goals and approaches
- Communicating with teachers of other grade levels, principals, specialists
- Deciding what to take up and what to leave in a discussion
- Designing lessons
- Evaluating explanations, arguments, proofs
- Examining students' work
- Explaining the curriculum to parents
- Interpreting curriculum materials
- Interpreting, using, and managing state curriculum and assessment policies
- Listening to students
- Making homework assignments
- Posing questions
- Setting up mathematical tasks
- Writing quizzes
- Writing, representing, and recording mathematics

# **Where and how do teachers use knowledge of reading in teaching?**

# Using knowledge of reading to teach reading

While reading a story called *The Marvelous Manatee*, Jamal says, “I’ve heard the word habitat before but I cannot remember what it means.” Ms. Gomez has been helping her students learn to use a variety of approaches to figure out unknown words. She quickly glances over the text of *The Marvelous Manatee* as she decides what to do next.

The peaceful manatee, like its cousin the elephant, is an endangered animal. Manatees have few natural enemies, other than humans. Boat collisions are the single greatest reason for manatee mortalities. Ultimately, however, loss of habitat is the most serious threat facing manatees today. There are approximately 3,000 manatees left in the United States. Without our help the manatee could soon be extinct.

Ms. Gomez notices that a number of students have their hands up. She has been encouraging students in her class to help each other when they are stuck and decides to let the students share some ideas.

Crystal suggests that Jamal should look for a root word in “habitat” and use this to figure out the word.

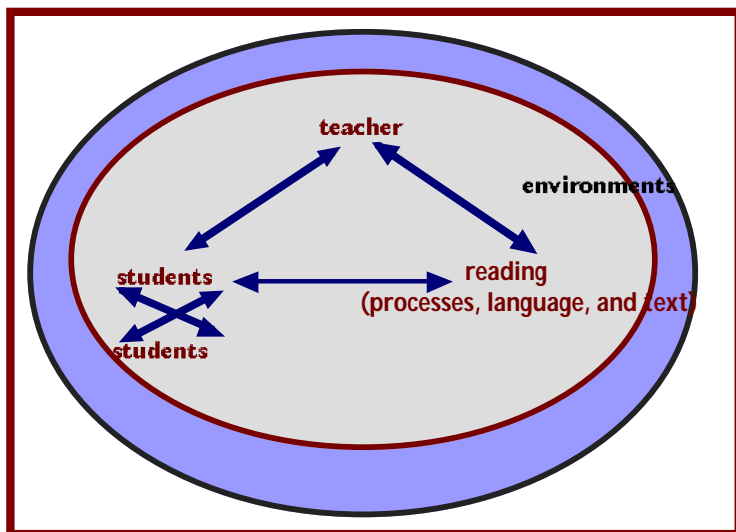
Robert argues that it would be smart to read *The Marvelous Manatee* again, looking for clues in the text.

Ethan says that he would just think back over what he has read in *The Wonderful Manatee* and substitute another word for “habitat.”

The peaceful manatee, like its cousin the elephant, is an endangered animal. Manatees have few natural enemies, other than humans. Boat collisions are the single greatest reason for manatee mortalities. Ultimately, however, loss of habitat is the most serious threat facing manatees today. There are approximately 3,000 manatees left in the United States. Without our help the manatee could soon be extinct.

# Some examples of how teachers use content knowledge in teaching reading

What does teaching reading entail?



*Some examples:*

Interpreting curriculum materials

Designing lessons

Modifying instructional tasks

Clarifying instructional goals

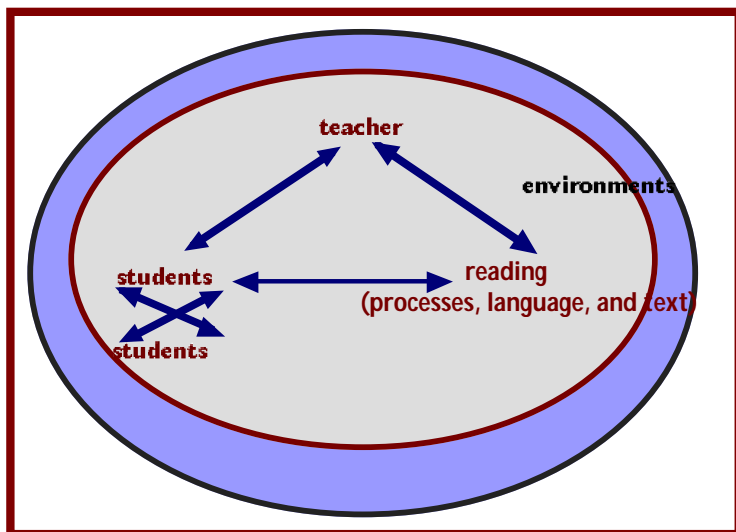
Choosing and using texts

Setting up instructional tasks

Posing questions

# Some examples of how teachers use content knowledge in teaching reading

What does teaching reading entail?



*More examples:*

Examining students' work

Listening to students

Interpreting students' ideas  
and approaches

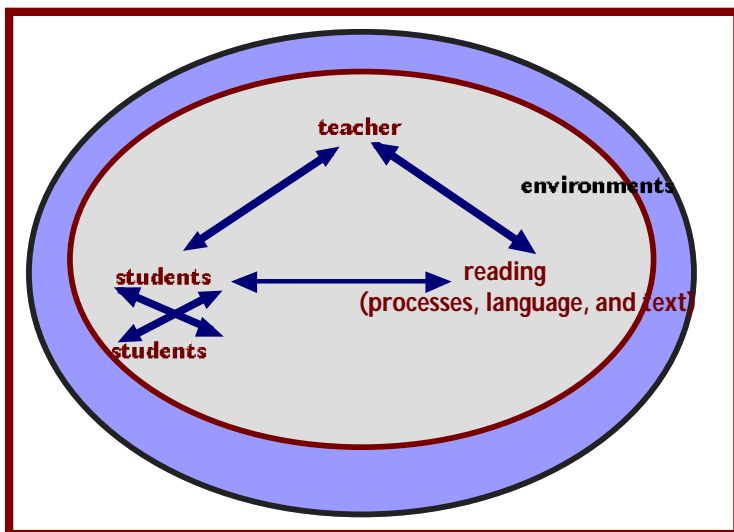
Evaluating students' approach  
to decoding text

Evaluating explanations,  
interpretations

**Motivating students**

# Some examples of how teachers use content knowledge in teaching reading

What does teaching reading entail?



*Still more examples:*

Making homework assignments

Managing discussions

Writing, representing, and recording language

Explaining

Deciding what to take up and what to leave

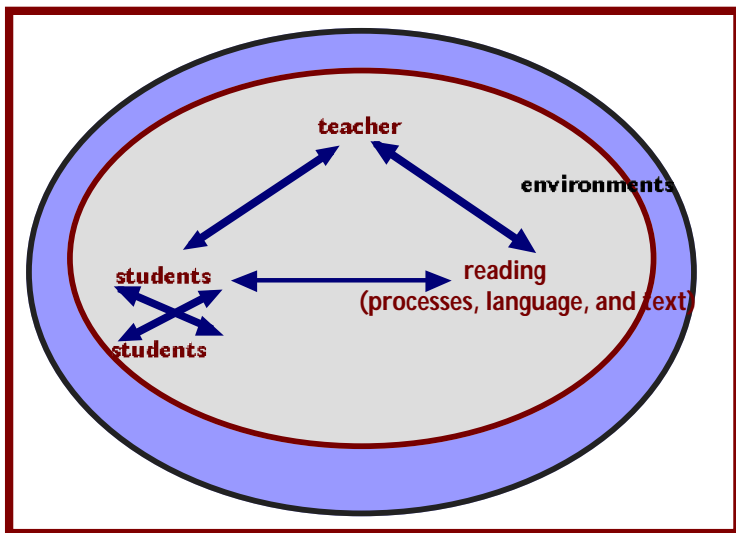
Posing questions

Designing assessments

Interpreting assessments

# Some examples of how teachers use content knowledge in teaching reading

What does teaching reading entail?



*And a few more:*

Explaining curriculum to parents  
Interpreting, using, and  
managing state curriculum and  
assessment policies

Communicating with teachers of  
other grade levels, principals,  
specialists

Professional reading

# Why develop measures of content knowledge for teaching ?

# Aims

- Study teacher learning
- Investigate relations of teachers' usable knowledge of content, their conventional knowledge of content, and their teaching
- Develop theory of content knowledge about teaching useful for professional education
- Study these on a broad scale, not with small, select groups

# **Developing measures of content knowledge for teaching**

# Principles of item development

- Seek to measure use of content knowledge by embedding questions in the work of teaching
- Develop items that have right and wrong answers
- Seek to develop both easy and difficult items; develop scales

# Basic conceptual structure for reading item development

	<b>Content knowledge (CK)</b>	<b>Interpreting student thinking (IST)</b>	<b>Content-specific teaching (CST)</b>
<b>Word analysis</b>			
<b>Comprehension</b>			

# Basic conceptual structure for mathematics item development

	<b>Content knowledge (CK)</b>	<b>Interpreting student thinking (IST)</b>	<b>Content-specific teaching (CST)</b>
<b>Number concepts</b>			
<b>Operations</b>			
<b>Patterns, functions, and algebra</b>			

# What do measures of content knowledge for teaching look like?

# **Mathematics knowledge for teaching: Sample items**

# Working on items

- What mathematical knowledge is used to answer the item?
- In what ways is this mathematical knowledge specialized to the work of mathematics teaching?

**Task of teaching mathematics:  
Responding to students'  
mathematical ideas**

**Alternative methods for  
multi-digit multiplication**

Imagine that you are working with your class on multiplying large numbers. Among your students' papers, you notice that some have displayed their work in the following ways:

**Which of these students is using a method that could be used to multiply any two whole numbers?**

Student A	Student B	Student C
$\begin{array}{r} 35 \\ \times 25 \\ \hline 125 \\ + 75 \\ \hline 875 \end{array}$	$\begin{array}{r} 35 \\ \times 25 \\ \hline 175 \\ + 700 \\ \hline 875 \end{array}$	$\begin{array}{r} 35 \\ \times 25 \\ \hline 25 \\ 150 \\ 100 \\ + 600 \\ \hline 875 \end{array}$

# **Task of teaching mathematics: Evaluating student explanations**

## **Divisibility**

- Ms. Harris was working with her class on divisibility rules. She told her class that a number is divisible by 4 if and only if the last two digits of the number are divisible by 4.
- One of her students asked her why this rule for divisibility by 4 worked. Ms. Harris asked the other students if they could come up with a reason, and several possible reasons were proposed.
- Which of the following statements comes closest to explaining the reason for the divisibility rule for 4?

- a) Four is an even number, and odd numbers are not divisible by even numbers.
- b) The number 100 is divisible by 4 (and also 1000, 10,000, etc.).
- c) Every other even number is divisible by 4, for example, 24 and 28 but not 26.
- d) It only works when the sum of the last two digits is an even number.

# **Knowledge of reading for teaching: Sample items**

# **Task of teaching reading: Inspecting a worksheet**

## Grapheme-phoneme connections

Ms. Peters' students are completing a worksheet requiring them to match two words that contain the same sound made by the letter combination "ough." The worksheet is laid out in two columns and directs the students to draw a line between a word in the first column and the word in the second column that has the same "ough" sound. The worksheet looks like this.

Connect words that sound the same  
with a line.

bought

tough

dough

bough

though

cough

plough

enough

thorough

thought

In the middle of the activity, one of the students in the class raises her hand and complains that some words do not have a match.

Is there an error in the worksheet?

**Connect words that sound the same  
with a line.**

bought

tough

dough

bough

though

cough

plough

enough

thorough

thought

**Task of teaching reading:  
Choosing a productive discussion  
question for a specific text**

**Analysis of nonfiction text**

Mr. Stier is using the following text to determine which of his students can evaluate an author's perspective and the evidence the author uses to support that perspective. Mr. Stier refers to this skill as "critical reading."

The Civil War was fought from 1861 to 1865. The Northern states fought against a collection of 11 states, mainly in the Southern United States. According to most authors, the Civil War was fought to earn freedom for African-American slaves in the Southern United States. But the Civil War was really fought to preserve the sovereignty of states over the national government. While issues such as slavery may have accelerated the conflict, the states' rights issue would have surfaced eventually. Even if slavery had not existed as an issue, the North and South would have eventually gone to war.

The teacher's guide includes a few questions a teacher might ask about this passage. Which of these questions could Mr. Stier choose to assess his students' ability to read critically?

	Yes	No	I'm not sure
a) During what years was the Civil War fought?	1	2	3
b) According to most authors, why was the Civil War fought?	1	2	3
c) Do you agree with the author? Why or why not?	1	2	3
d) What attitude toward the Civil War does the author of this text want you to take?	1	2	3
e) What evidence does the author present to support his claim about the inevitability of the war?	1	2	3

# **Task of teaching: Helping students understand an idea**

## **Prefixes**

Ms. Hill's class is working on identifying prefixes. One of the students in the class has found six words in a story that start with "re" (relative, reach, regal, reported, return, reason). Some of his words merely begin with the letters "r" and "e," but not with the prefix "re."

Which of the following rules could Ms. Hill provide to help the student determine if his words start with the prefix "re?"

	Yes	No	I'm not sure
a) Pronounce the word, listening to each syllable. If “re” is its own syllable in the word, then it is a prefix.	1	2	3
b) The letter combination “re” is a prefix if it is followed directly with a root word like “visit” in <u>revisit</u> .	1	2	3
c) The letter combination “re” is a prefix if the dictionary shows a syllable division directly after the “re” as in “ <u>re/peat</u> .”	1	2	3
d) The prefix “re” means “again” or “back.” Check for those meanings in the word to determine if the letter combination “re” is a prefix.	1	2	3

# What have we learned?

# General findings

- It is possible to generate multiple choice items in reading and mathematics that form reliable scales and discriminate among teachers.
- It is possible to write items that teachers and other educators view as valid measures of teaching knowledge.

# Future directions

- Continue item analysis to explore the dimensionality of the knowledge measures
- Interview teachers to learn the knowledge used in answering items
- Interview non-teachers and content specialists to learn if and how teacher knowledge differs.

# Future directions

- Generate difficult items
- Generate items that better discriminate between high and low performing teachers
- Generate items in a wider range of topical domains

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**Study of Instructional Improvement**

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