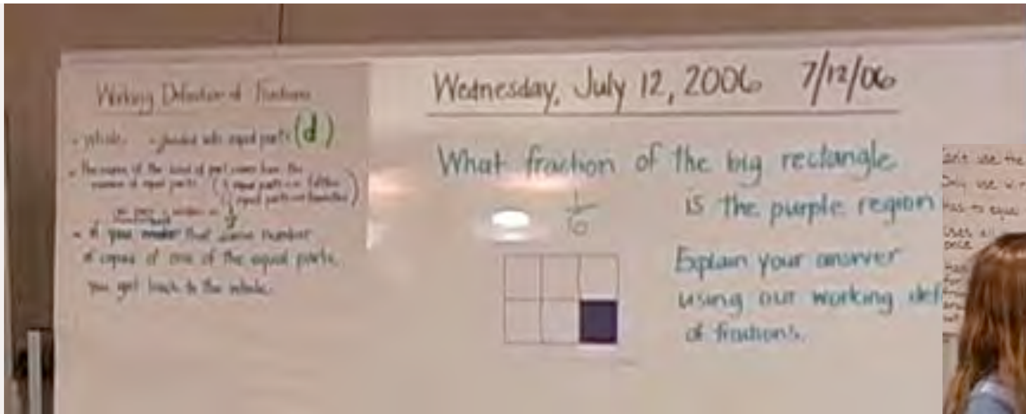

Getting to Practice

Challenging the Conceptual Fallacies that Interfere with Professional Work

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A first stop

- The "Shea video"
  - First shown at NCTM in New Orleans 16 years ago
  - Shown in many venues, to many audiences
  - What do you think are the most common comments people make, even when they are given a focused viewing task?

What do people see and talk about?

- Only a few of the students are engaged.
- The teacher is letting the students do all the work.
- Why doesn't the teacher correct Shea's mistake?
- I wish my (high school or college) students would reason like these students.
- This classroom is a true learning community.
- Have you ever followed up these students?

Two major conceptual fallacies that impede our progress

1. Confounding a specific implementation of an idea with the idea itself

2. Falsely dichotomizing a complex idea

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Why do these fallacies matter?

On one hand,

- These refer to important distinctions.
- They represent core educational orientations.

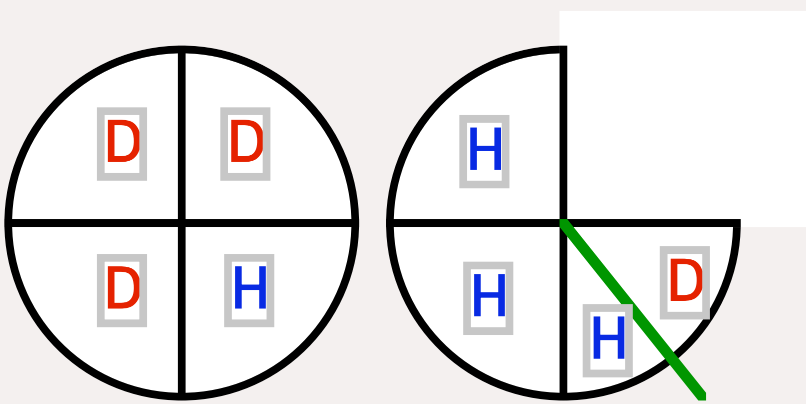
On the other hand,

- Practice is complex, and slogans map sloppily onto reality.
- Ideologies interfere with analytic and disciplined work on practice.

A second stop

$$1\frac{3}{4} \div \frac{1}{2}$$

- What is a common reaction to this phenomenon?



Who ever divides fractions? Does anyone REALLY need to know this?

1. Confounding a specific implementation of an idea with the idea itself: The case of algorithms

One view:

- Algorithms interfere with meaningful mathematical work.
- They lead to mindless symbol manipulation and interfere with understanding.

A second view:

- Algorithms are efficient and reliable ways of solving classes of problems.
- Students' lack of algorithmic proficiency leads to weak mathematical performance.

al·go·rithm *n*

1. a logical step-by-step procedure for solving a mathematical problem in a finite number of steps, often involving repetition of the same basic operation
2. a logical sequence of steps for solving a problem, often written out as a flow chart, that can be translated into a computer program

- Allows compressed and efficient processes
- Enables mind to focus on problem solving

So if algorithms are useful, what is the issue?

- Learning algorithms and being able to use them reliably requires control of them.
- For this, developing competence with algorithms must include some unpacking (many ways to do this) to justify and understand how they work.

Transitional algorithms can help support the development of mastery

```
   56
 x 28
 ----
   48
  400
  120
 1000
 ----
 1568
```

- They can make the procedure more transparent
- They can help show why a procedure works
- The steps can be simpler because the procedure is less compressed

73
- 11
62

1

62
- 11
51

2

51
- 11
40

3

40
- 11
29

4

29
- 11
18

5

18
- 11

6

Division as repeated subtraction

- Divide 73 ÷ 11
  - Subtract 11 repeatedly
  - Result: 6 **11**s can be subtracted from 73, with 7 left over
  - 73 ÷ 11 = 6 R7
  - Measurement interpretation of division

73 ÷ 11 =

Next:

- Become impatient, and work to remove groups of the divisor at once
- Become efficient: remove groups of the divisor bundled in multiples of powers of ten of the divisor

So what is the issue?

Some implementations:

- Focus on rules without reason
- Produce mindless and inadaptive use

Other implementations:

- Do not provoke the needed impatience with transitional algorithms
- Revere complex and inefficient methods, over skilled use of general and efficient ones

Efficiency depends:

Even if they can be, not all problems are best solved with an algorithm

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$$\begin{array}{r} 101 \\ \times\ 99 \\ \hline \end{array}$$

$$\begin{array}{r} 1003 \\ -\ 895 \\ \hline \end{array}$$

$$\frac{15}{32} \div \frac{3}{4}$$

How would each of these be solved most efficiently?

The study of algorithm can provide opportunities for —

- Developing criteria for evaluating algorithms
- Learning to analyze and critique methods
- Mathematical reasoning and proving
- Studying mathematical structure

A closer look at algorithms: Is each below an algorithm? Does it work in general? Why or why not?

(a)

```
 29 1
 302
- 197
 105
```

(b)

```
   56
 x 28
  168
 140
 1568
```

(c)

```
   5  2
 - 1  7
   4 -5
```

(d)

```
    1
   5 2
 - 1 7
    1
   3 5
```


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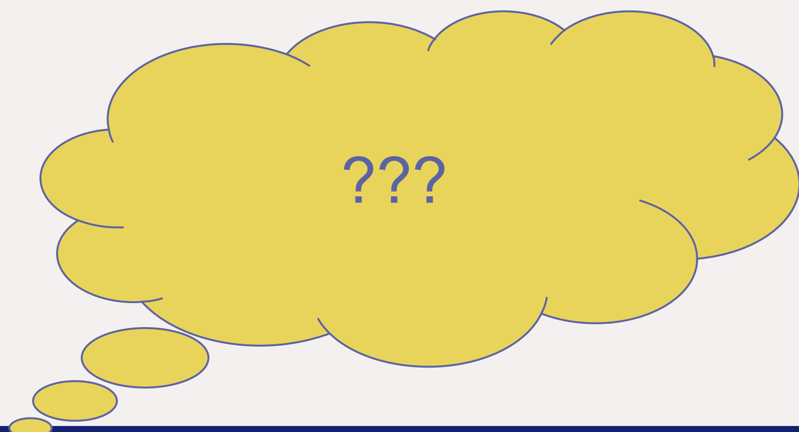
Avoiding the fallacy of confounding an idea with its implementation

- Algorithms can't be "wrong" or "bad"; they are an impressive mathematical accomplishment and resource

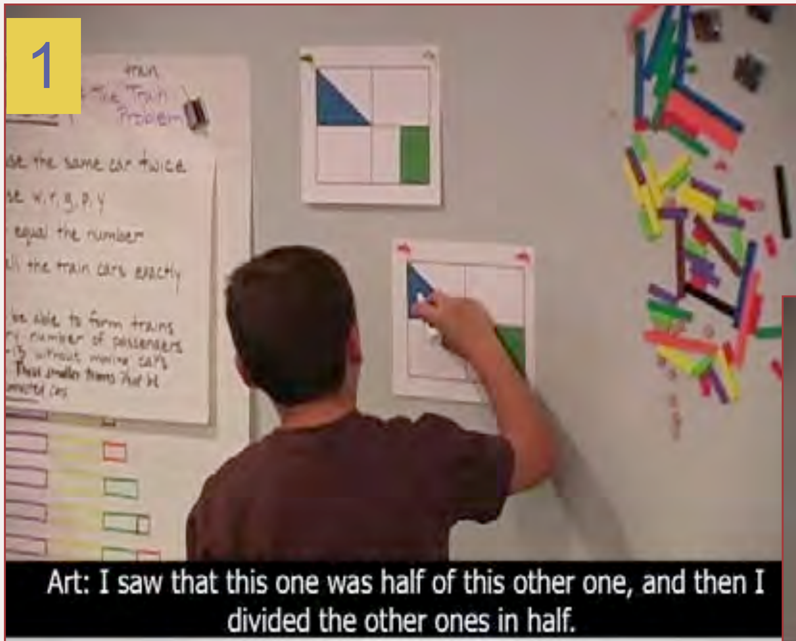
- Algorithms can be (and often are) taught badly

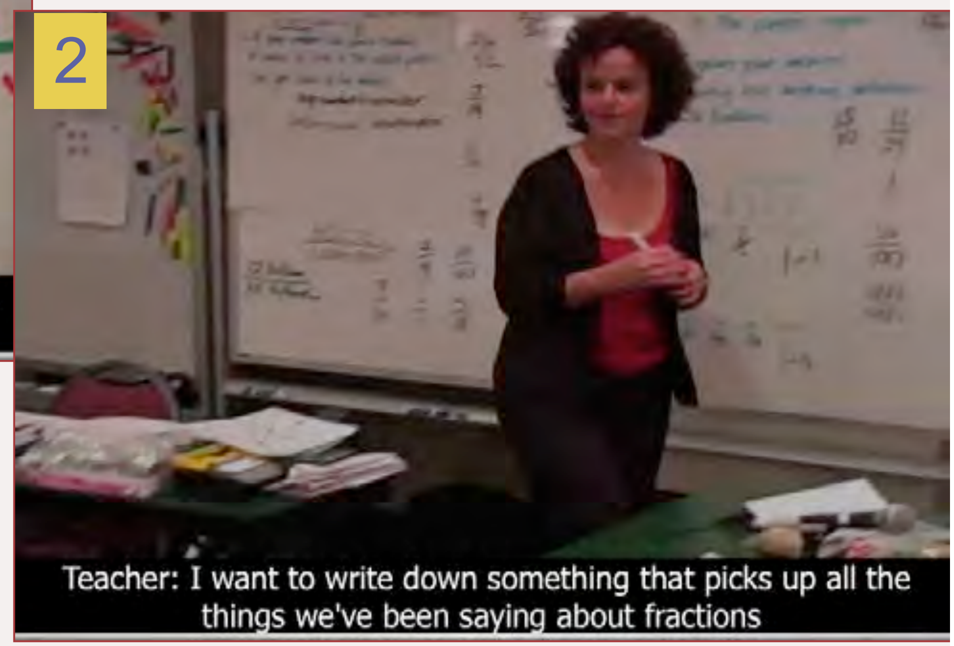
2. Falsely dichotomizing complex ideas: The case of instruction

Teacher-centered	Student-centered
Teacher directed	Inquiry-based
Direct instruction	Constructivist



Video investigation of instruction

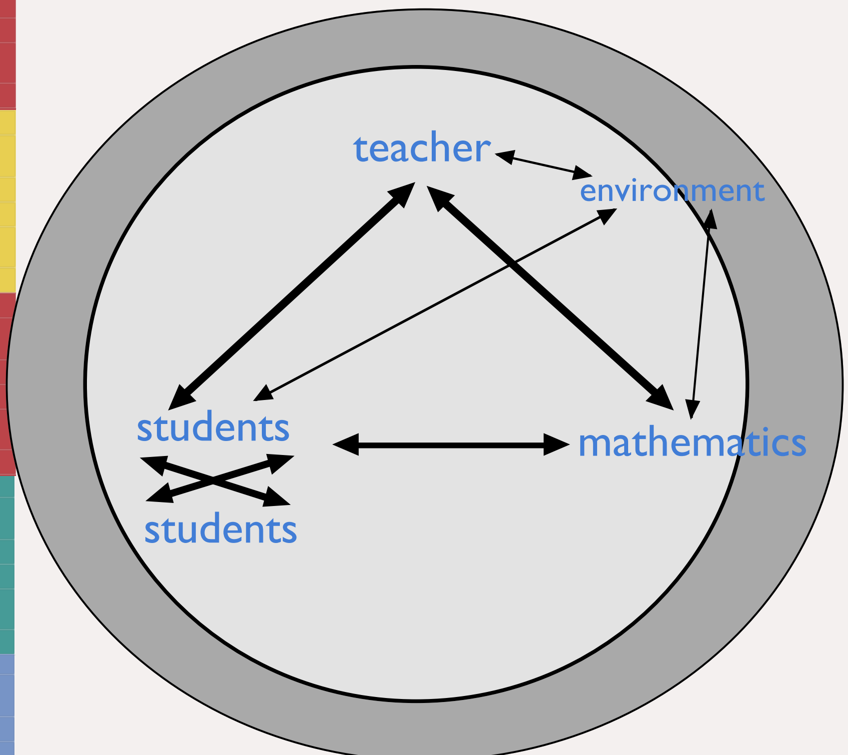




A first consideration

- Is this a teacher-centered or student-centered class?
- Is this direct instruction or inquiry-based instruction?

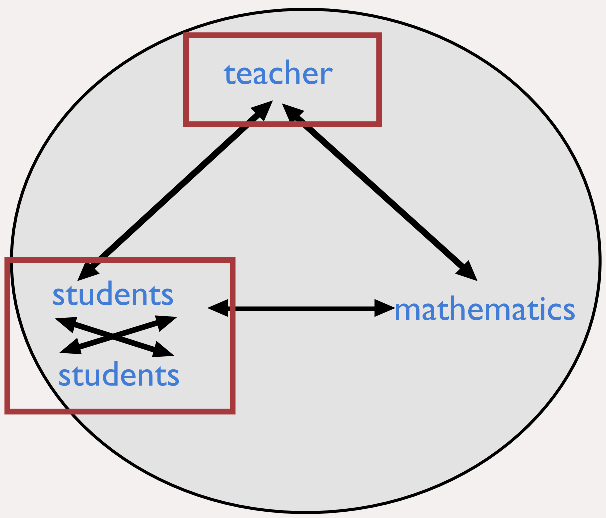
What is instruction?

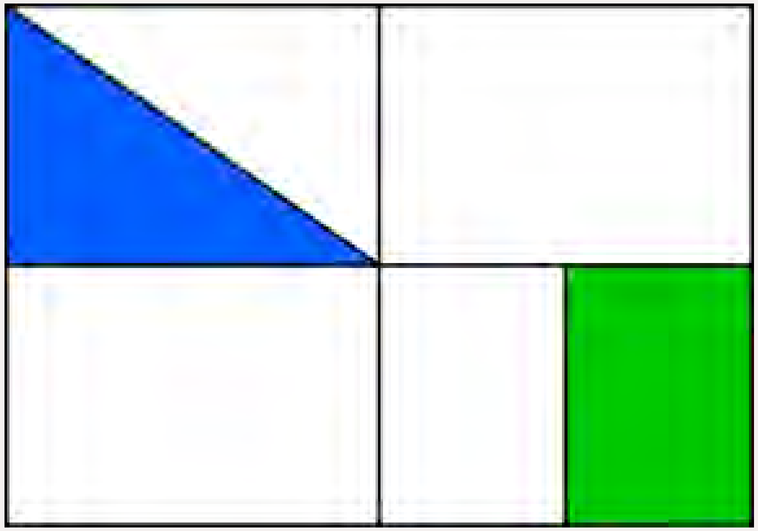


- An interaction between teacher and students about the content, over time
- Influenced by environments (but subjectively) — in school, in communities, parents, policies, school leaders

Viewing focus

- Is Clip#1 student-centered?
- Is Clip#2 teacher-centered?





What fraction of the big rectangle is the blue region?

What fraction of the big rectangle is the green region?

Explain your answers.

A working definition of fraction in fourth grade

If some whole is divided into d equal parts, then we write $\frac{1}{d}$ to name one of those parts.

If a part is $\frac{1}{d}$ of a whole, then it takes d copies of $\frac{1}{d}$ to make the whole. $\frac{n}{d}$ is then defined as n • ($\frac{1}{d}$), or n copies of $\frac{1}{d}$.

What are the key resources that teachers manage to create instruction?

1. Talk (who, how much)
2. Organization and participation structures
3. The mathematical content and their understanding of it
4. Tasks and how the tasks structure the mathematical territory
5. Their students' ideas
6. Time
7. The environment

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An experiment: The lecture

- Teacher does most of the talking
- Students listen and may respond, may take notes
- In some ways, it is the ultimate "constructivist" pedagogy!

1. Talk (who, how much)
2. Organization and participation structures
3. The mathematical content and their understanding of it
4. Tasks and how the tasks structure the mathematical territory
5. Their students' ideas
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7. The environment

Avoiding the dichotomy of teacher-centered vs. student-centered

- All teaching is teacher-directed.
- All learning depends on learners' constructions.
- It's shaped by the decisions the teacher makes about how to distribute talk, time, and the mathematical work, with what sorts of tasks.

So why is this dichotomy about the "center" of instruction a problem?

- It impedes closer analytic study of and talk about teaching. We do much more talking about students and about curriculum than we do about teaching.
- It deskills teachers.
- It inhibits the sort of research we need to identify specific instructional practices and strategies and ways to deploy them under what circumstances.

Learning to challenge conceptual fallacies that prevent us from getting to practice

1. Notice confusions of ideas with their implementation
2. Be skeptical of dichotomies
3. Ask the meaning of slogans
4. Remember that the point of instruction is to help each student develop mathematical proficiency; this is a profession, not a club