SETTING THE STAGE

What considerations are important in thinking about the mathematics that elementary teachers must know for teaching, and about how they can learn — and learn to teach — that mathematics?

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Understanding Teaching as a Form of Mathematical Problem Solving

Deborah Loewenberg Ball

Hyman Bass

Heather Hill

Mark Hoover

University of Michigan

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What is the actual problem that we are trying to address?

The quality of mathematics teaching and learning

U.S. teachers' lack of mathematics knowledge

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Knowing Mathematics For Teaching

- *What* mathematics knowledge and resources-- skills, understanding, sensibilities -- are entailed by teaching?

- How are such mathematical knowledge and resources *used* in the course of teaching?

- How might teachers be helped to *develop* usable mathematical knowledge and resources?

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What mathematical problems do teachers have to solve?

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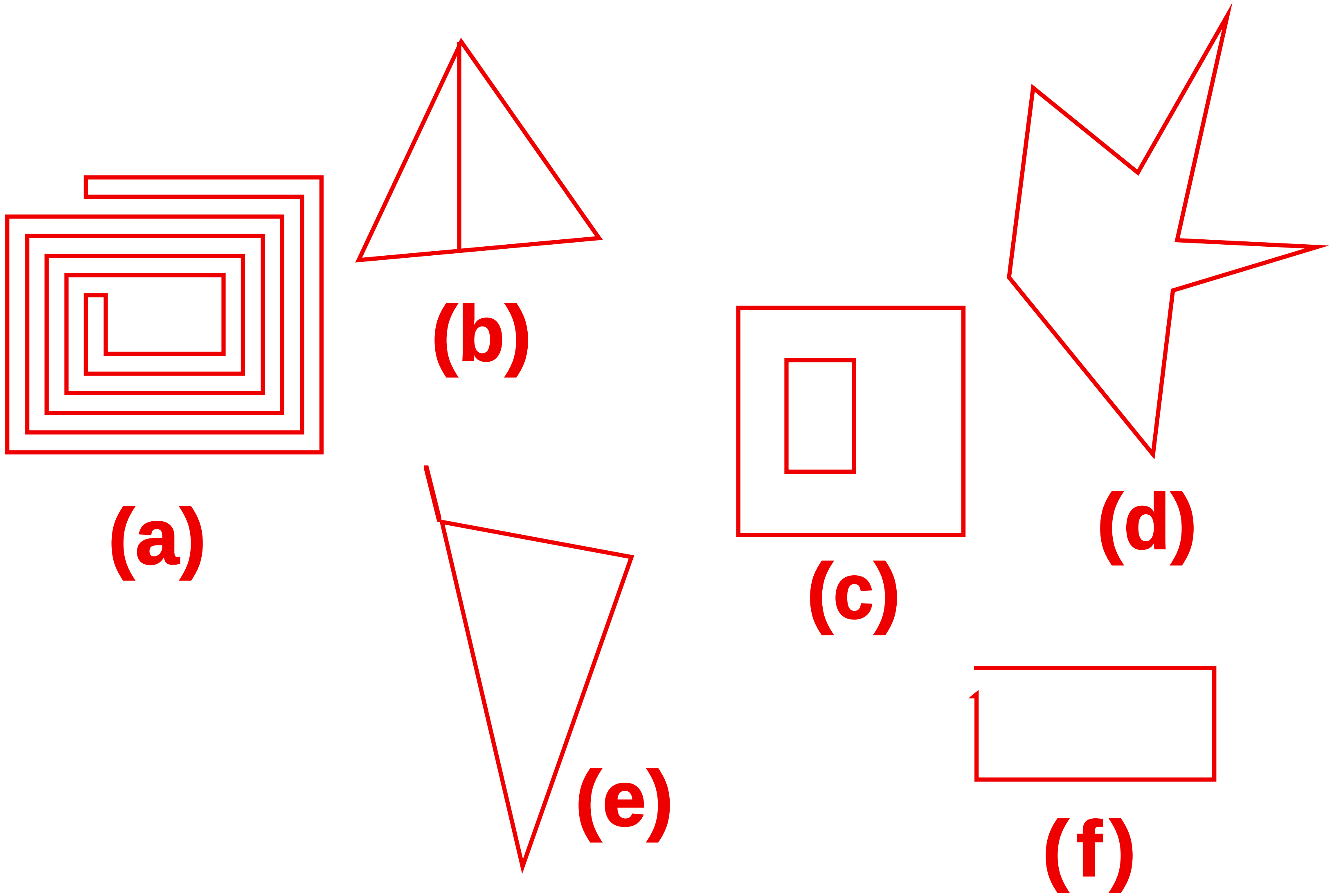
Which of these students is using a method that could be used to multiply any two whole numbers?

Student A	Student B	Student C
35	35	35
x 25	x25	x 25
1 25	175	25
+7 5	+700	1 50
875	875	1 00
		+600
		875

Are these shapes polygons?



Textbook: A closed flat two-dimensional shape whose sides are formed by line segments.

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What’s an adequate and mathematically acceptable definition of “polygon” for fifth graders?

- A simple closed curve with all straight sides.

- A closed figure made entirely of line segments.

3) Plane figures with 3, 4, 5, or more sides such as the examples above (*pictures*).

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Re-framing the problem of knowing and using mathematics for teaching

- While some teachers lack basic knowledge of mathematics, many do know basics
- Many teachers lack mathematical knowledge that is useful and usable for teaching
- Some teachers do develop this kind of mathematical knowledge from teaching
- Inadequate opportunities exist for teachers to develop this kind of mathematical knowledge and learn to use it in their work
- Mathematicians do not necessarily know and are not necessarily able to solve the mathematical problems teachers face`

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One crucial resource for the problem: Mathematical knowledge that is useful and usable for teaching

- Profound understanding of fundamental mathematics
- Unpacked
- Connected
- In and beyond the curriculum
- Useful for the work of teaching

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Examples of the mathematical problems teachers have to solve

- Clarifying mathematical goals and approaches
- Communicating with teachers of other grade levels, principals, specialists
- Deciding what to take up and what to leave in a discussion
- Designing lessons
- Evaluating explanations, arguments, proofs
- Examining students' work
- Explaining the curriculum to parents
- Interpreting curriculum materials
- Interpreting, using, and managing state curriculum and assessment policies
- Listening to students
- Making homework assignments
- Posing questions
- Setting up mathematical tasks
- Writing quizzes
- Writing, representing, and recording mathematics

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What mathematical problems do teachers have to solve?

Five examples

1. Choosing a task to assess student understanding
2. Building correspondence between a model and a mathematical idea
3. Responding to students' ideas
4. Making decisions about what to do with students' mathematical ideas in a discussion
5. Rescaling problems

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Example #1:

CHOOSING A TASK TO ASSESS STUDENT UNDERSTANDING

Ordering Decimals

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Suppose you wanted to find out if your students understand how to put decimal numbers in order. Which of the following lists of numbers would give you the best evidence of students' understanding? Why?

A.	**.5**	**7**	**.01**	**11.4**
B.	**.60**	**2.53**	**3.14**	**.45**
C.	**.6**	**4.25**	**.565**	**2.5**

D. Any of these would work well for this purpose. They all require students to read and interpret decimals.

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Example #2:

BUILDING CORRESPONDENCES BETWEEN A MATHEMATICAL IDEA AND A MODEL

Division

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Write three <u>different</u> stories that can be represented by the expression:

$$38 \div 4$$

and that reveal different interpretations of division or its mapping to specific situations.

Example #3:

RESPONDING TO STUDENTS' MATHEMATICAL IDEAS

Alternative Methods for Dividing Fractions

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A student proposes an alternative way to divide by a fraction:

"I don't see why you have to do all that invert and multiply stuff. To divide fractions, I just divide the numerator of one fraction by the numerator of the second and it works just fine. Look:

I have **6/8 ÷ 1/2**

and then I just divide like this:

$$= \frac{6 \div 1 = 6}{8 \div 2 = 4} \text{ or } 3/2$$

which I can rewrite as 1 1/2, which is the right answer."

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How should the teacher respond?

What mathematical questions must the teacher consider in order to decide what to do?

Make these questions explicit.

Answer them.

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- **Does this way make sense? Or was it just lucky, or worked in this case?**

- **Would this work for other numbers? What would it look like?**

- **Would it always work?**

Example #4:

MAKING DECISIONS ABOUT WHAT TO DO WITH STUDENTS' MATHEMATICAL IDEAS IN A DISCUSSION

Comparing Fractions

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Which is more --

$$\frac{4}{4} \text{ or } \frac{4}{8} \text{ ?}$$

What are some of the mathematical issues here?
What might third graders do, say, think, try?

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PM 12:58:22

Four students' contributions

- Lin: explains why 4/4 is greater than 4/8
- David: discusses how to decide how many lines to draw when making a representation of a particular fraction
- Bernadette: wants to show it using the number line
- Kevin: "First I did something different and I was wrong"

What are the mathematical considerations that might shape a teacher's decisions about how to treat each student's contribution?

Example #5:

PROBLEM RESCALING

?

Original problem

Make a story problem for

$$1\frac{3}{4} \div \frac{1}{2}$$

Choose another pair of numbers to make another version of the problem.

Does your new problem involve the same mathematical work or not? On what basis would you claim this?

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Dimensions of the problem

- Does the division leave a remainder?
- Do the fractions afford a range of choices for models?
- Are people likely to consider the problem using a measurement or a partitive interpretation of division?
- Can you end up getting answers that appear right but are produced from a misinterpretation of the unit?

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Learning Mathematics for Teaching

How can we help teachers learn mathematics in ways that would enable them to solve the mathematical problems of teaching?

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- Learn to unpack and explain mathematical ideas
- Practice examining, analyzing, comparing alternative mathematical solutions, representations, ideas
- Learn to recognize, identify, and engage in mathematical practices, such as reasoning, representation, examining equivalences, careful use of language
- Practice solving and justifying solutions to mathematical problems of teaching

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