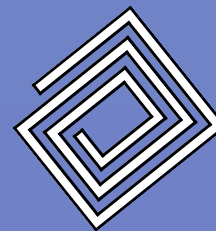
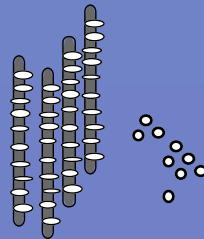
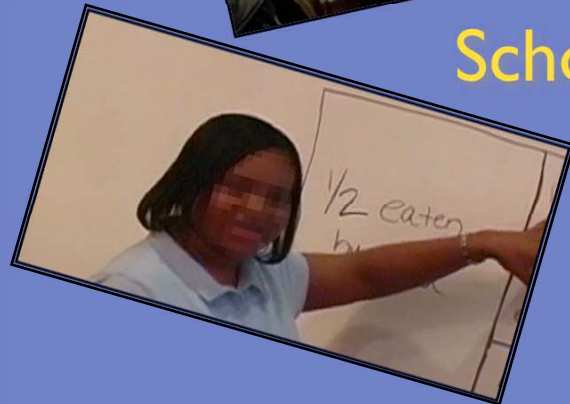


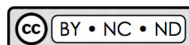
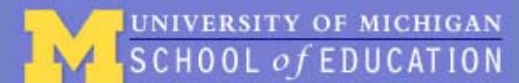


Who Knows Math Well Enough to Teach Third Grade — and How Can We Decide?



Presentation to the
Wolverine Caucus, Lansing
Deborah Loewenberg Ball
School of Education — University of Michigan
March 15, 2006

1



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Overview

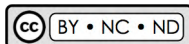
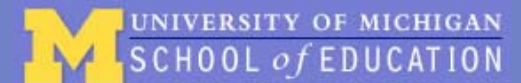
1. The “problem” of teacher knowledge
2. Knowing mathematics for teaching
3. What do we know about the role of mathematical knowledge in teaching?
4. Teacher education for better mathematics teaching



Education in the U.S. in 2006



3



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Clarifying the Problem

*The quality of mathematics
teaching and learning*

Teachers' knowledge of mathematics
and their ability to use it in teaching



But: U.S. Teachers Often Lack Sufficient Mathematical Knowledge

- What's the evidence?
 - Anecdotes
 - Research
- What does it tell us and what is not well understood?
 - Lack of a system
 - This is a societal problem, widespread



Some Approaches to Solving the Problem

- Require more mathematics for certification
 - More mathematics courses
 - A major (or minor) in mathematics
 - Mathematics test
- Recruit mathematically trained people into teaching
 - Engineers, accountants, mathematicians, ...
- Fund mathematically focused professional development

But -- these approaches assume we know what mathematical knowledge and skill is needed for high-quality teaching.



Clarifying the Problem

*The quality of mathematics
teaching and learning*

Teachers' knowledge of mathematics
and their ability to use it in teaching

~~How MUCH mathematics do teachers need to know?~~

But: What mathematical knowledge and skill
does high quality teaching require?



Mathematical Knowledge for Teaching

What do we mean when we use this term,

“mathematical knowledge for teaching”?

- Mathematical knowledge, skill, habits of mind that are entailed by the work of teaching

What do we mean by the **“work of teaching”?**

- The tasks in which teachers engage, and the responsibilities they have, to teach mathematics, both inside and outside of the classroom



Knowing Multiplication

$$\begin{array}{r} 49 \\ \times 25 \\ \hline \end{array}$$



Knowing Multiplication for Teaching: Analyzing Student Errors

(a)

$$\begin{array}{r} 49 \\ \times 25 \\ \hline 405 \\ 108 \\ \hline 1485 \end{array}$$

(b)

$$\begin{array}{r} 49 \\ \times 25 \\ \hline 225 \\ 100 \\ \hline 325 \end{array}$$

(c)

$$\begin{array}{r} 49 \\ \times 25 \\ \hline 1250 \\ 25 \\ \hline 1275 \end{array}$$



Knowing How to Order Decimals

Put these numbers in order:

.7 .2 3.4 .05



Knowing Decimals for Teaching: Designing Quiz Questions

A. .5 7 .01 11.4

B. .60 2.53 3.12 .45

C. .6 4.25 .565 2.5

D. These lists are all equally good for assessing whether students understand how to order decimal numbers.



Knowing Radical Expressions

Simplify:

$$\sqrt{150}$$



Knowing Radical Expressions for Teaching: Choosing Numerical Examples

(a)

$$\sqrt{54}$$

(b)

$$\sqrt{156}$$

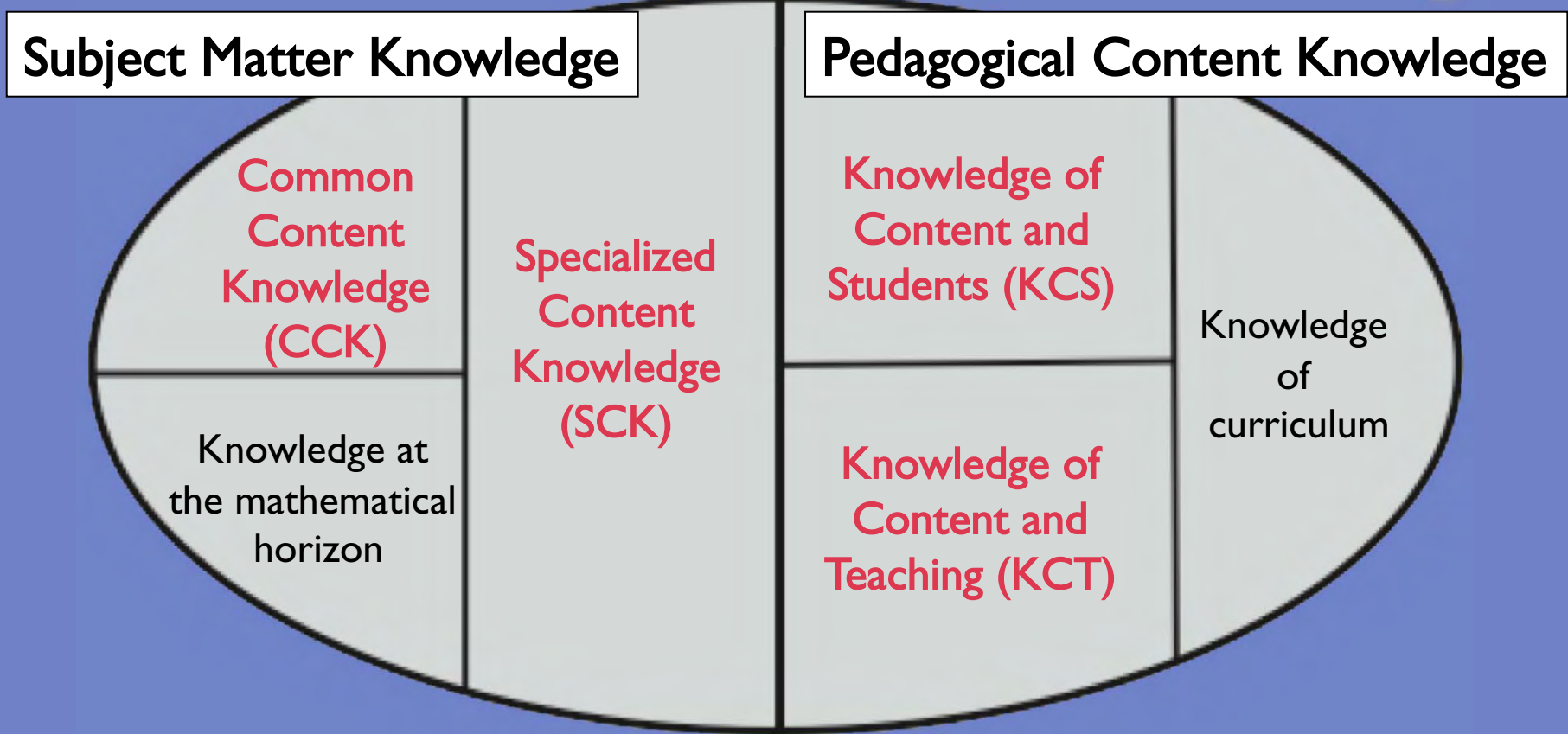
(c)

$$\sqrt{128}$$

(d) These examples all work equally well.



Mathematical Knowledge for Teaching





Common Content Knowledge

The mathematical knowledge and skill
expected of any well-educated adult

Teachers need to be able to:

- Recognize wrong answers
- Spot inaccurate definitions in textbooks
- Use notation correctly
- Do the work assigned to students

Example Items

What is $1\frac{1}{4} \div \frac{1}{2}$?

What number is halfway
between 1.1 and 1.11?



Specialized Content Knowledge

The mathematical knowledge and skill needed by teachers in their work and beyond that expected of any well-educated adult

Teachers need to be able to:

- Analyze errors and evaluate alternative ideas
- Give mathematical explanations and use mathematical representations
- Be explicit about mathematical language and practices



Specialized Content Knowledge

Which of these students is using a method that could be used to multiply any two whole numbers?

Student A	Student B	Student C
$\begin{array}{r} 35 \\ \times 25 \\ \hline 125 \\ + 75 \\ \hline 875 \end{array}$	$\begin{array}{r} 35 \\ \times 25 \\ \hline 175 \\ + 700 \\ \hline 875 \end{array}$	$\begin{array}{r} 35 \\ \times 25 \\ \hline 25 \\ 150 \\ 100 \\ + 600 \\ \hline 875 \end{array}$

M Knowledge of Content and Students Knowledge of Content and Teaching (PCK)

Combines knowledge of mathematics with
knowledge of students or knowledge of teaching

Teachers need to be able to:

- Anticipate student errors and common misconceptions
- Interpret student thinking
- Predict what students are likely to do with specific tasks and what they will find interesting or challenging

Teachers need to be able to:

- Sequence content for instruction
- Recognize instructional pros and cons of different representations
- Size up mathematical issues in responding to students' novel approaches



Knowledge of Content and Students

Ms. Violeta was looking carefully at her students' papers, and she saw the following responses to the problem:

$$8 + 4 = \underline{\quad} + 5$$

- i) 12 ii) 17 iii) Can't do it iv) 1

Which of the following is the most likely explanation of the difficulty the students are having? (Mark ONE answer.)

- a) They do not know their basic addition facts.
- b) They cannot do multi-step problems.
- c) They do not know that addition is commutative.
- d) They do not understand the meaning of the equals sign.



Knowledge of Content and Teaching

To introduce the idea of grouping by tens and ones with young learners, which of the following materials or tools would be most appropriate? (Choose ONE.)

- a. A number line
- b. Plastic counting chips
- c. Pennies and dimes
- d. Straws and rubber bands
- e. Any of these would be equally appropriate for introducing the idea of grouping by tens and ones.



Contrasting Knowledge Common, Specialized, and PCK

Common
Recognize
incorrect
answers

$$\begin{array}{r} 307 \\ - 168 \\ \hline 261 \end{array}$$

Specialized
Analyze
errors

$$\begin{array}{r} 307 \\ - 168 \\ \hline 169 \end{array}$$

Students
Know
common
errors

$$\begin{array}{r} 307 \\ - 168 \\ \hline 261 \end{array}$$

Teaching
Know
what to
do next

$$\begin{array}{r} 307 \\ - 168 \\ \hline 261 \end{array}$$



Linking Teacher Knowledge and Student Achievement

- Questionnaire consisting of 30 items (scale reliability .88)
- Model: Student Terra Nova gains predicted by:
 - Student descriptors (family SES, absence rate)
 - Teacher characteristics (math methods/content, content knowledge)
- Teacher content knowledge significant
 - Small effect ($< 1/10$ standard deviation): 2 - 3 weeks of instruction
 - But student SES is also about the same size effect on achievement
(Hill, Rowan, and Ball, AERJ, 2005)

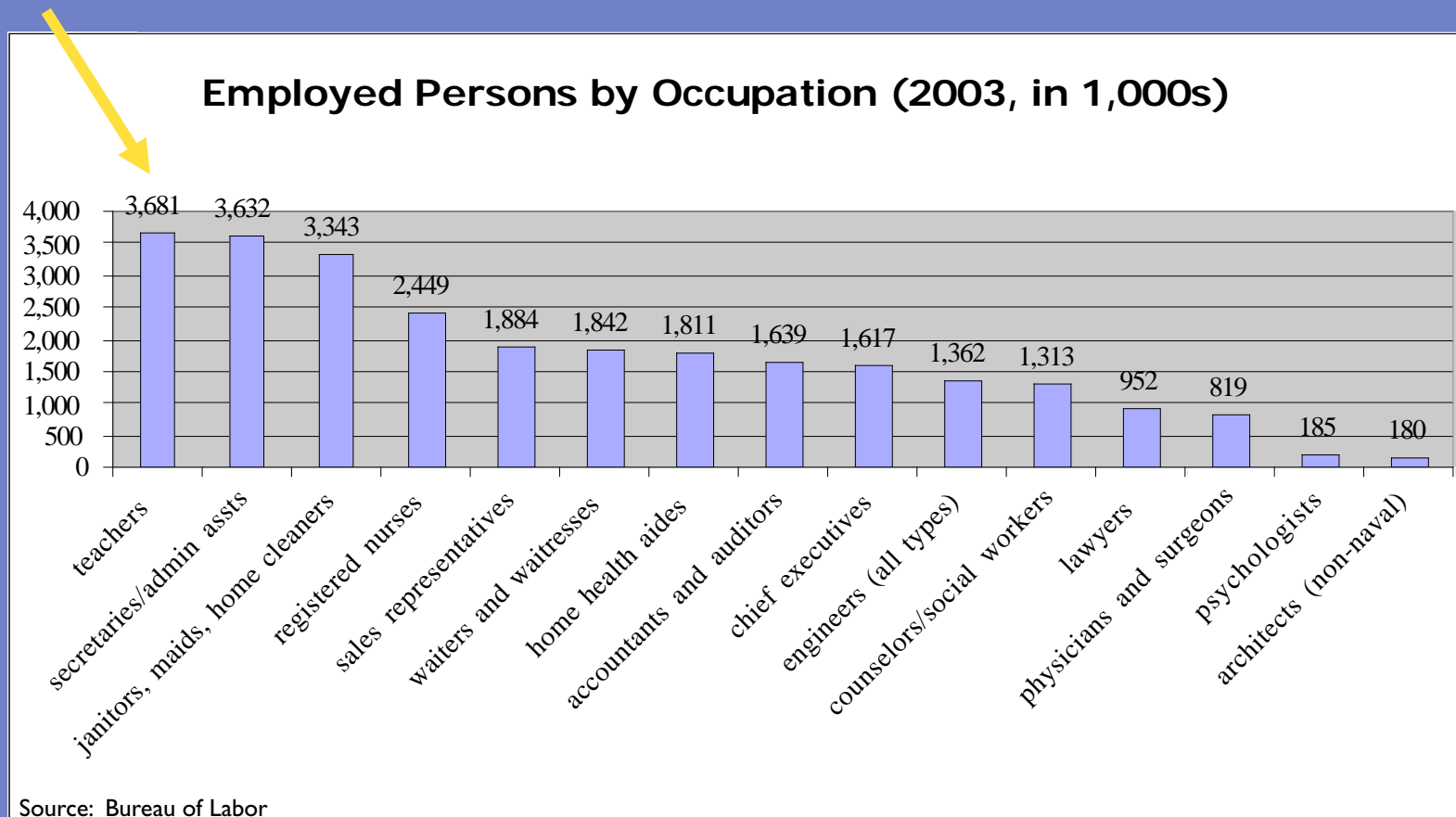


What Do We Know About What Teachers Need to Learn?

- Mathematical knowledge for teaching is the knowledge needed for the work of teaching.
- A kind of content knowledge distinct from that required by other mathematically-intensive professions
- Importance of building this theory IN practice
- Possible to write valid and reliable survey measures of MKT
- Credibility of this approach with teachers
- This kind of testing is professional



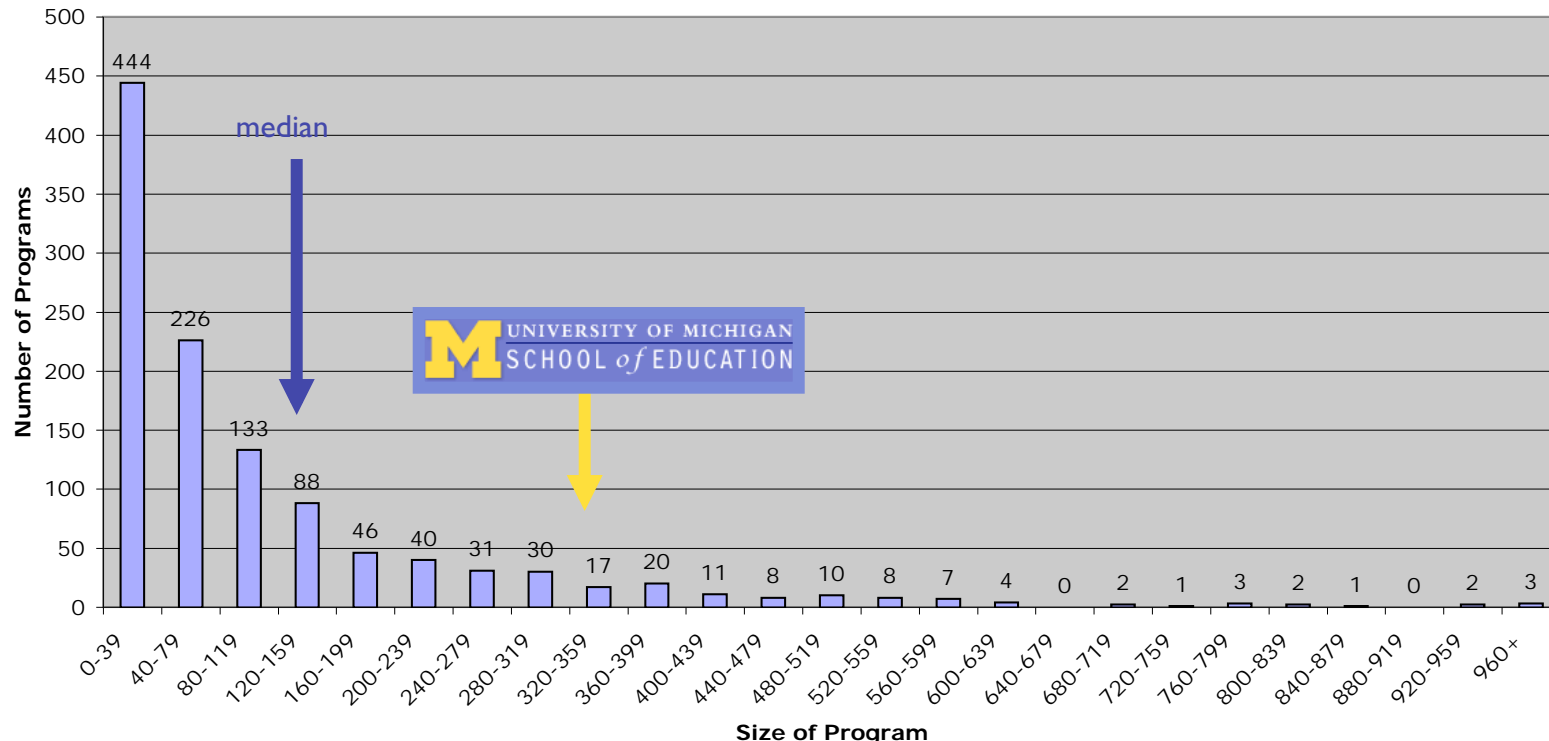
Teachers: The Largest Group in the U.S. Workforce





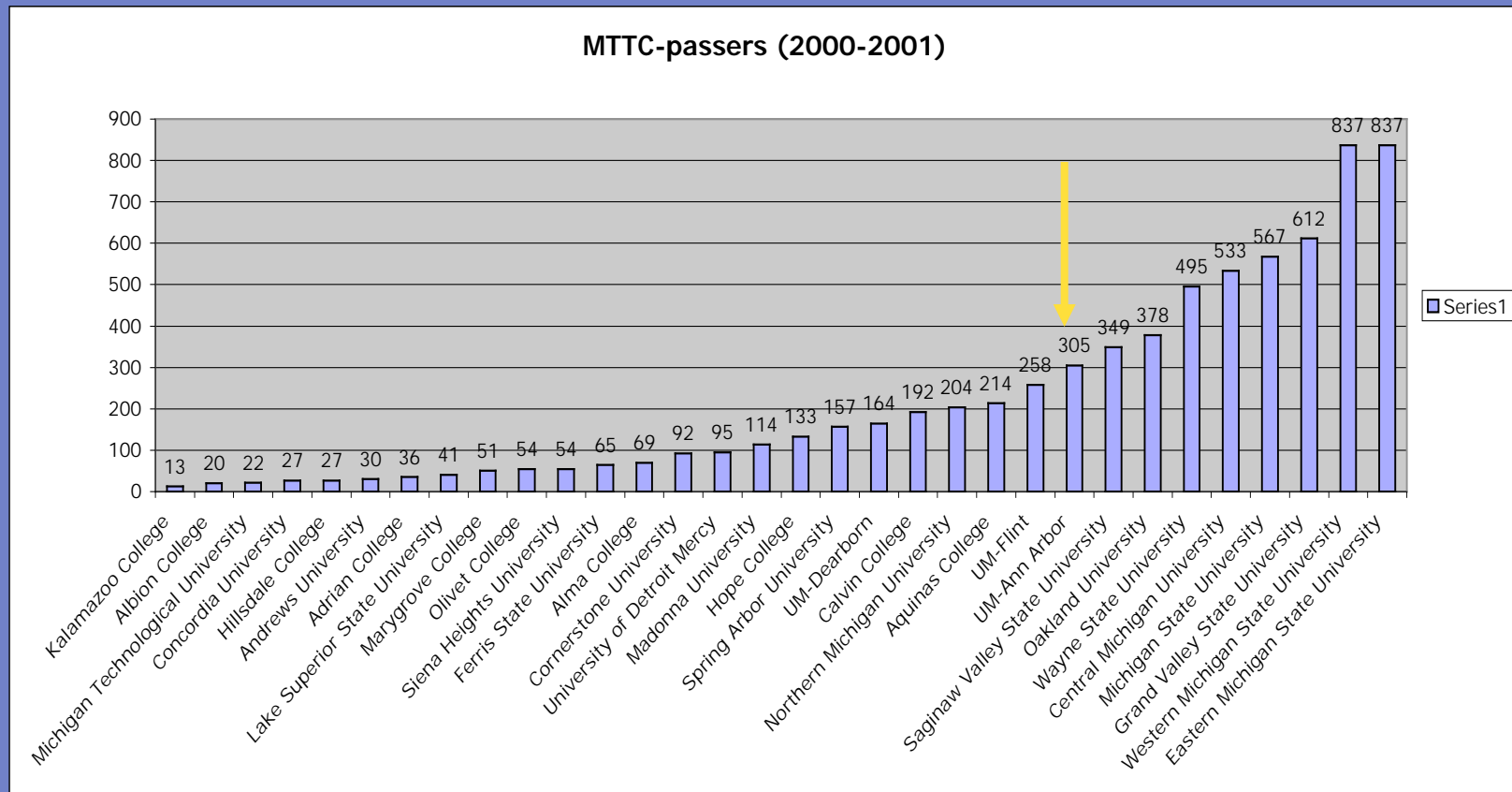
Tiny Size of Many Teacher Education Programs: UM Program in the Top Quartile

Distribution of TE Programs by Size





Teacher Education in Michigan





What is Wrong with Teacher Education in the U.S.?

- Inappropriate subject matter preparation
- Inadequate preparation for diversity of U.S. classrooms
- Lack of focus on practice
- No professional system for training, licensure, certification, ongoing professional education



Building a New Brand of Teacher Education at UM

Key elements

- Practice-based (focused on core practices, study of artifacts and records, use of a variety of school settings as labs)
- Knowledge of academic subjects for teaching
- Skills for working to address inequities
- Interpersonal and relational skills (diversity, families, colleagues)
- Preparation to manage and change school and policy environments
- New strategies for recruitment, admissions requirements
- Assessment of performance throughout the program
- Programs for early career professionals