

Supporting High-Quality Mathematics Instruction

Deborah Loewenberg Ball

Math Intervention Leadership Summit • Austin TX

March 2, 2011

SCHOOL OF EDUCATION UNIVERSITY OF MICHIGAN

Today's session

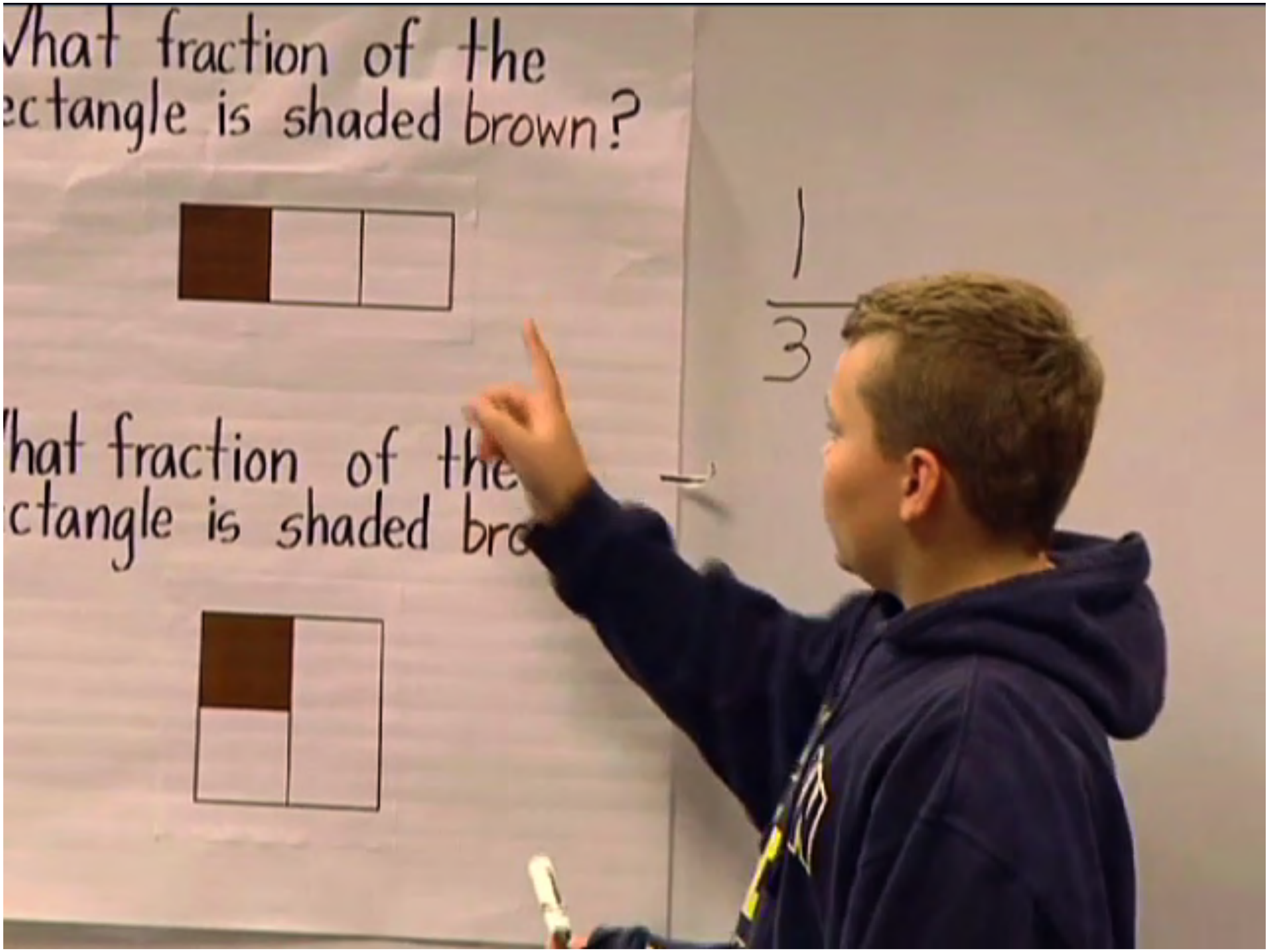
- Designed as instruction itself
- All examples, videos, materials are available for your use
- Everything will be posted on my website *("google" or "bing"* ***Deborah Ball****)*

Warm-up problem

View the video segment:

1. What does Mackenzie seem to know?
2. What should the teacher do next?

Discuss with the person sitting next to you.


hat fraction of the
ctangle is shaded brown?
1
3
hat fraction of the
ctangle is shaded bro

Overview

1. What is the issue and why is it so pressing? Why is there reason to be hopeful?
2. What is “good” teaching, and what are “high-leverage practices”?
3. “High-leverage practices” for teaching mathematics
4. Developing practice: What is high-leverage practice in teacher training?

Write your professional learning goals for this session.

1. What is the issue?

Why is there a reason to be hopeful that we could make a difference in students' learning? But what would it take?

The urgency

1. Enormous gaps in learning opportunities and disparities in achievement (within U.S. and in international comparisons)
2. Rapidly changing school population
3. Higher, more complex academic goals
4. High expectations for <u>all</u> students

Teachers matter—a lot

- Differences in teachers account for 12%-14% of total variability in children's mathematical achievement in each of grades 1, 2, and 3.
- Children assigned to three effective teachers in a row score at the 83rd percentile in math at the end of 5th grade; children assigned to three ineffective teachers in a row score only at the 29th percentile.
- The cumulative effects of being taught by a highly effective teacher can substantially reduce differences in student achievement that are due to family background.

Alternative systems: France, Finland, Singapore

- Common K-12 curricula and criterion-referenced exams
- Prospective teachers study the common curriculum in detail and practice how to teach it
- Inspection system evaluates teachers' instructional capability in reference to the common curriculum

The dawn of a new day in the U.S.?

- Common Core State Standards
- Widespread agreement that teachers matter
- Major shift: teaching can — and *must* be — taught

March 7, 2010
Sunday NYT magazine
•Elizabeth Green

2. What is important about "good" teaching?

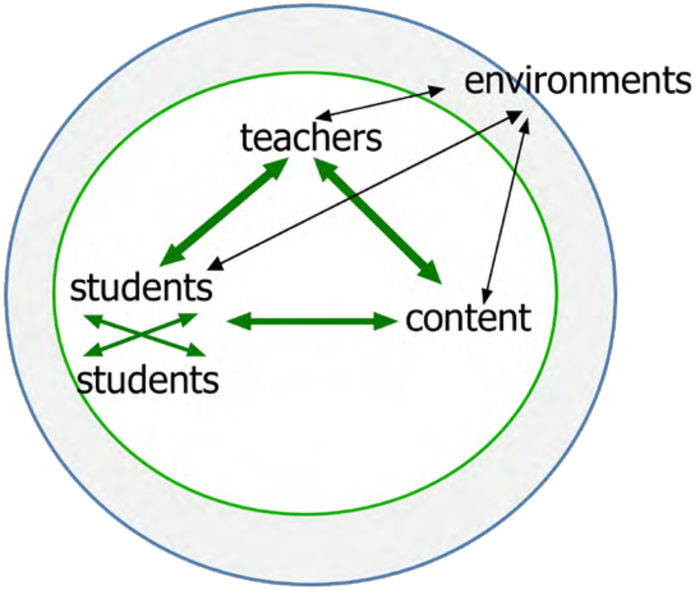
What are "high-leverage practices"?

What <u>is</u> "teaching"?

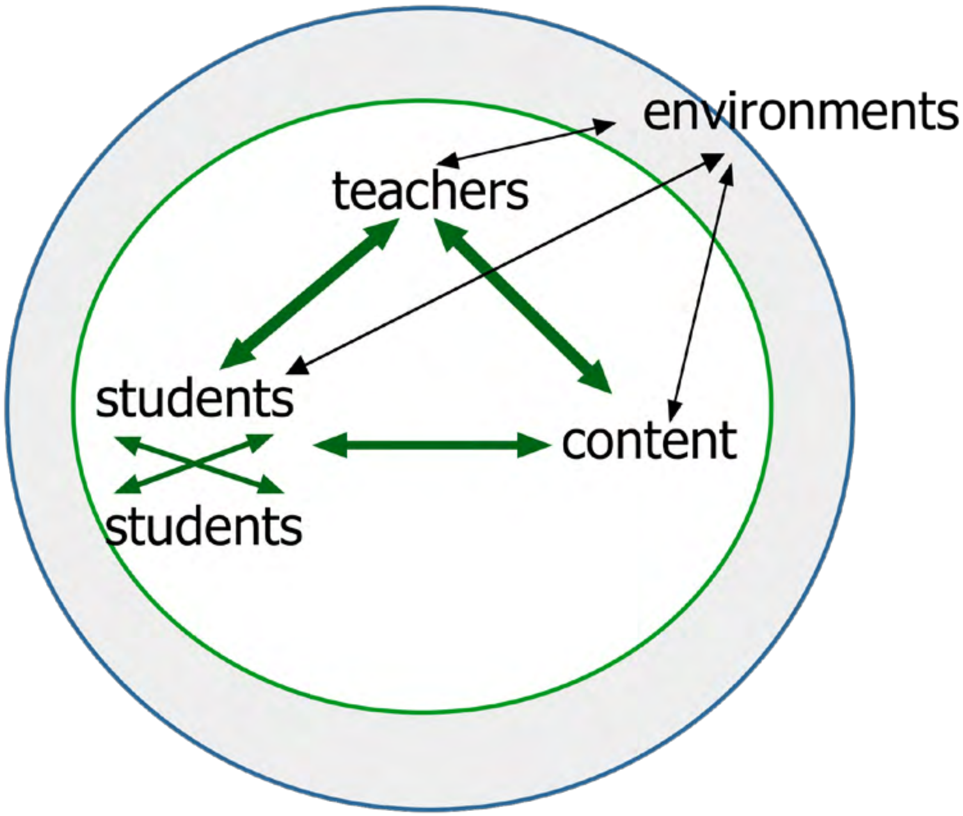
It is <u>not</u>:

- What teachers do
- The cause of students' learning
- "Natural"
- Primarily intuitive and improvisational

Teaching is what is *co-produced* by students and teachers in contexts, around specific content and curriculum.



The power of teaching



Takes responsibility for:

1. deliberately maximizing the quality of the interactions . . .
2. . . . in ways that maximize the probability that students learn
3. . . . worthwhile content and skills

We know that every student does not get responsible teaching every day, in every subject.

Why not?

How could we achieve that goal?

Two schools of thought, or bets

1. Focus on teachers
   - recruitment, selection
   - rewards and sanctions

2. Focus on teaching
   - training
   - assessment
   - curriculum

What are the reasons to bet more on one or the other?

Responsible teaching

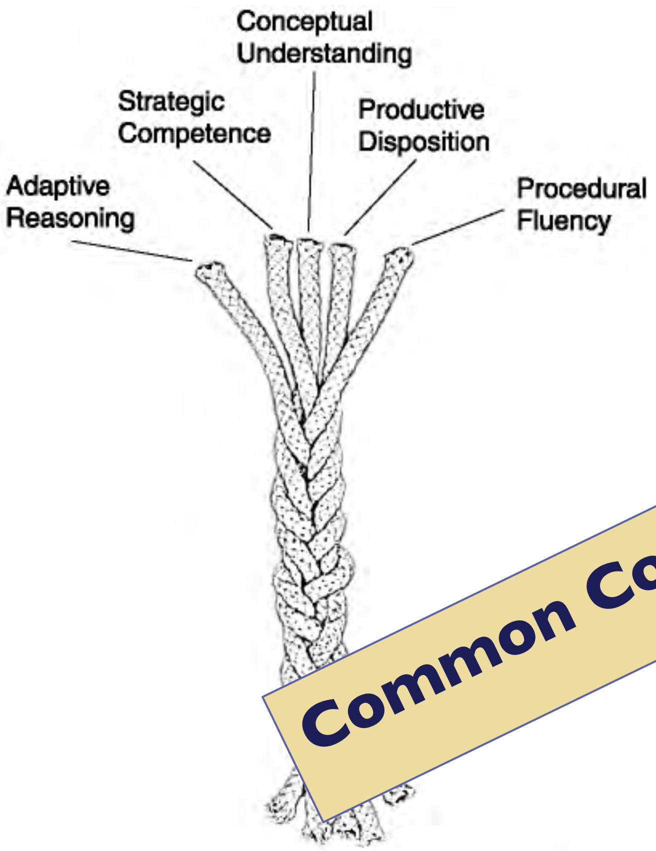
Making mathematical proficiency attainable by all

Strands of mathematical proficiency

- ***Conceptual understanding*** - comprehension of mathematical concepts, operations, and relations
- ***Procedural fluency*** - skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- ***Strategic competence*** - ability to formulate, represent, and solve mathematical problems
- ***Adaptive reasoning*** - capacity for logical thought, reflection, explanation, and justification
- ***Productive disposition*** - habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy

Kilpatrick, J., J. Swafford, and B. Findell. (2001). *Adding It Up: How Children Learn Mathematics*. Washington, DC: National Academy Press.

Strands of mathematical proficiency



- ***Conceptual understanding*** - comprehension of mathematical concepts, operations, and relations
- ***Procedural fluency*** - skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- ***Strategic compete*** ... ormulate, represent, and ... al problems
- ***Ad*** ... ***ing*** - capacity for logical thought, ... planation, and justification
- ***Productive disposition*** - habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy

Common Core State Standards (CCSS)

Kilpatrick, J., J. Swafford, and B. Findell. (2001). *Adding It Up: How Children Learn Mathematics*. Washington, DC: National Academy Press.

Video clip from a classroom

- Third graders
- Unplanned segment, in the midst of a unit on even and odd numbers
- Student says he has “been thinking about the number 6”

Viewing focus

1. Are there important learning opportunities for students, or is this a waste of class time?
2. If our goal is to focus on increasing the power of teaching, what are some of the teaching practices to identify in this segment?


PM 1:06:38

Making reasoned decisions about the use of instructional time

NOT GOOD USE OF TIME

1. Students might get confused about even and odd numbers
2. Teacher should correct Sean's idea about 6 being both even and odd
3. Too much to cover to spend time on a confused discussion of a simple topic
4. Only some students participating

TEACHABLE MOMENT

1. Reconciling three different definitions of even and odd numbers
2. Learning to make mathematical arguments and counterarguments
3. Critiquing mathematical claims and arguments
4. Developing a definition of even and odd number

What are some practices of teaching visible in the segment?

1. Identifying appropriate mathematical learning goals
2. Managing time closely (in relation to mathematical learning goals)
3. Establishing and maintaining a safe and respectful learning environment
4. Interpreting and responding to student "errors"
5. Leading a class discussion of a "discussable" mathematical idea, question, or solution
   - Developing norms for mathematical and respectful talk
   - Identifying and "steering" to the mathematical point
   - Allocating turns
   - Monitoring and ensuring "engagement"
6. Posing strategic questions
7. Explicitly focusing on mathematical language
8. Signalling what it means to be "good at math"
9. Others?

Responsible teaching is unnatural and intricate work

1. Teaching requires teachers to do things that are more than commonsense.
2. Teaching involves some moves or orientations that are unnatural in regular adult life.
3. Teaching practice is intricate, requiring a complex combination of knowledge, skill, timing, and relational work.

“High-leverage” practices

- Have significant power because they:
  - Are central to the daily work of teaching
  - Attend to considerations of equity — effective in using and responding to differences among pupils
  - High probability of making a difference in teaching quality and effectiveness for students’ learning
  - Useful broadly across contexts and content

Examples of high-leverage practices

- Explaining ideas and processes
- Choosing and using representations, examples, and models of core content
- Setting up and managing small-group work
- Recognizing and identifying common patterns of student thinking
- Selecting and using specific methods to assess students' learning on an on-going basis

4. High-leverage mathematics teaching practices

Focusing on teaching practice

Ten high-leverage practices for ambitious mathematical learning for all

1. Choosing and using mathematical tasks that entail complex mathematical work <u>and</u> build basic skills
2. Choosing examples
3. Teaching and using academic language
4. Leading a productive whole-class math discussion
5. Responding productively to students "errors"
6. Using homework equitably
7. Using specific mathematically-focused positive reinforcement
8. Using public recording (posters, whiteboard)
9. Diagnosing common patterns of student thinking (and not-so-common)
10. Assessing students' mathematical proficiency and teaching responsively

Ten high-leverage practices for ambitious mathematical learning for all

① Choosing and using mathematical tasks that entail complex mathematical work <u>and</u> build basic skills

② Choosing examples

③ Teaching and using academic language

④ Leading a productive whole-class math discussion

⑤ Responding productively to students "errors"

⑥ Using specific mathematically-focused positive reinforcement

⑦ Using homework equitably

⑧ Using public recording (posters, whiteboard)

⑨ Diagnosing common patterns of student thinking (and not-so-common)

⑩ Assessing students' mathematical proficiency and teaching responsively

Ten high-leverage practices for ambitious mathematical learning for all

① Choosing and using mathematical tasks that entail complex mathematical work and build basic skills
② Choosing examples
③ Teaching and using academic language
④ Leading a productive whole-class math discussion
⑤ Responding productively to students "errors"
⑥ Using specific mathematically-focused positive reinforcement
⑦ Using homework equitably
⑧ Using public recording (posters, whiteboard)
⑨ Diagnosing common patterns of student thinking (and not-so-common)
⑩ Assessing students' mathematical proficiency and teaching responsively

TODAY: Six high-leverage practices for ambitious mathematical learning for all

① Choosing and using mathematical tasks that entail complex mathematical work and build basic skills

② Teaching and using academic language

③ Responding productively to students "errors"

④ Using specific mathematically-focused positive reinforcement

⑤ Using homework equitably

⑥ Using public recording (posters, whiteboard)

① Choosing and using mathematical tasks that entail complex mathematical work and build basic skills

Analyzing others' solutions

A fourth grade student did the calculations below. For each one, decide whether the answer is correct or incorrect and explain how you know. If the answer is incorrect, try to explain what the error is.

$$\begin{array}{r} 48 \\ +\ \ 27 \\ \hline 615 \end{array}$$

$$29 + 37 + 18 = 84$$

Is the answer correct or incorrect?

How do you know?

If the answer is incorrect, try to explain what the error is.

If the answer is incorrect, please do it correctly here:

Reasoning with tools

Which Cuisenaire rod is three times as long as a red one?

Which rod is half as long as an orange one?

One rod is one-fourth as long as another rod. What colors might they be?

Which rod is one-third as long as the dark green one?

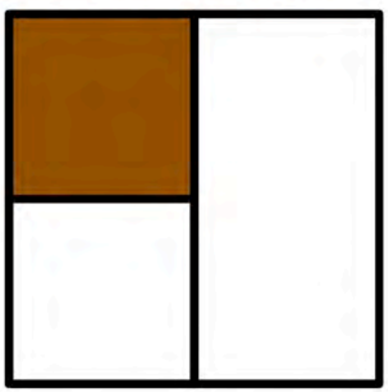
Which rod is five times as long as a red one?

The blue rod is how many times as big as the white Cuisenaire rod?

Reasoning with representations

What fraction of the rectangle below is shaded brown?

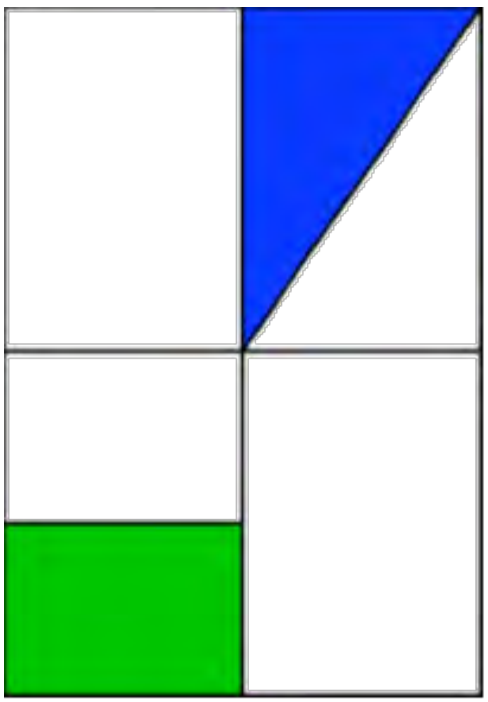
What fraction of the rectangle below is shaded brown?



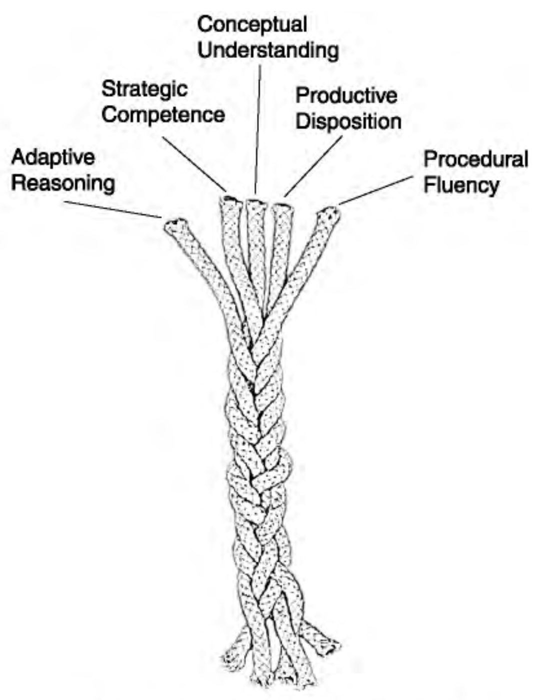
What fraction of the big rectangle is shaded blue?

What fraction of the big rectangle is shaded green?

What fraction of the big rectangle is shaded altogether?



Analyzing tasks



1. How do these tasks make possible learning of complex mathematics?
2. How do these tasks provide opportunities for practice of basic skills?

Attending closely to wording

POSSIBLE PROBLEM:

I have pennies, nickels, and dimes in my pocket. If I pull out two coins, what amounts of money might I have?

Reasoning about different wording

1. I have pennies, nickels, and dimes in my pocket. If I pull out two coins, what amount of money might I have?

2. I have pennies, nickels, and dimes in my pocket. If I pull out two coins, how many combinations are possible?

3. I have pennies, nickels, and dimes in my pocket. If I pull out two coins, how many different amounts of money are possible? Prove that you have found all the amounts that are possible.

What are the differences among these four formulations?

Necessary for using complex mathematical tasks with "high expectations"

- Helping students to believe they can do hard work
- Scaffolding complex work appropriately
- Making wise judgments about what to leave open and what needs to be made explicit
- Commenting on mathematics, and mathematical productions, not features of students
- Supporting error as a fruitful site for mathematical work, and teach students to use error productively

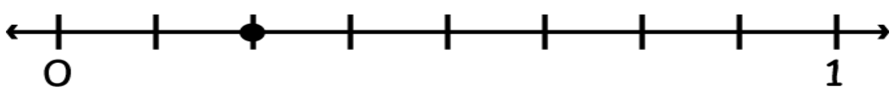
② Responding productively to students' "errors"

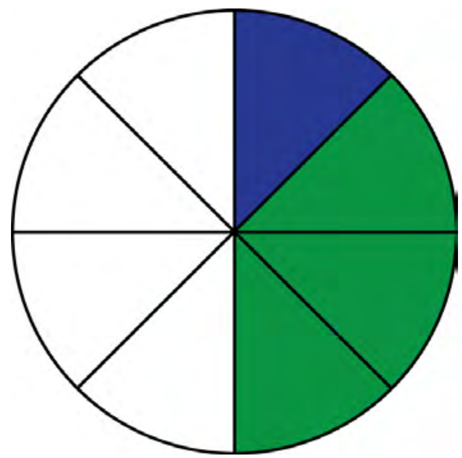
Students' errors as a crucial site for teaching

1. Often occur at the heart of the content; working on them can help make instruction more focused
2. Expose and can provide "inoculation" for probable confusions
3. Develop students' independent skills at analysis (mathematical self-regulation)
4. Can change students' conceptions of what "being good at math" means

“Safe” tasks

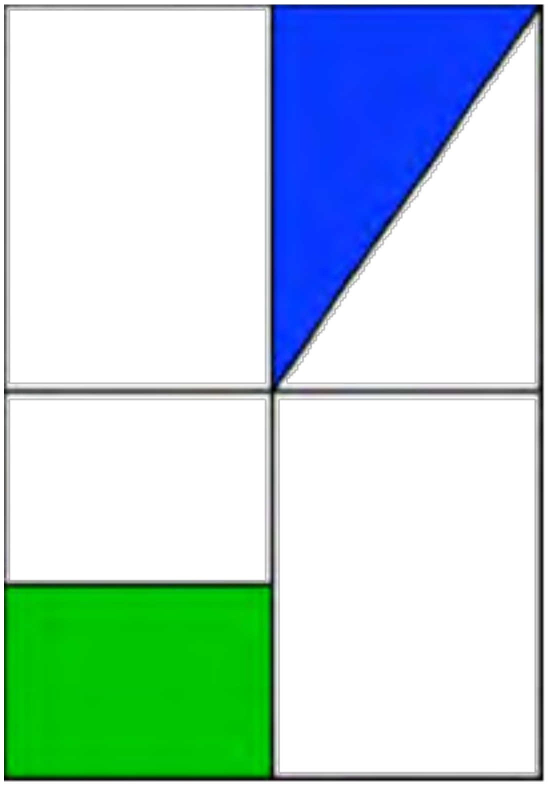
What makes these tasks “safe”?





1. Much of the thinking is done <u>for</u> the students (e.g., equal partitioning; shading; labeling
2. Protect from error — supports getting “right” answer

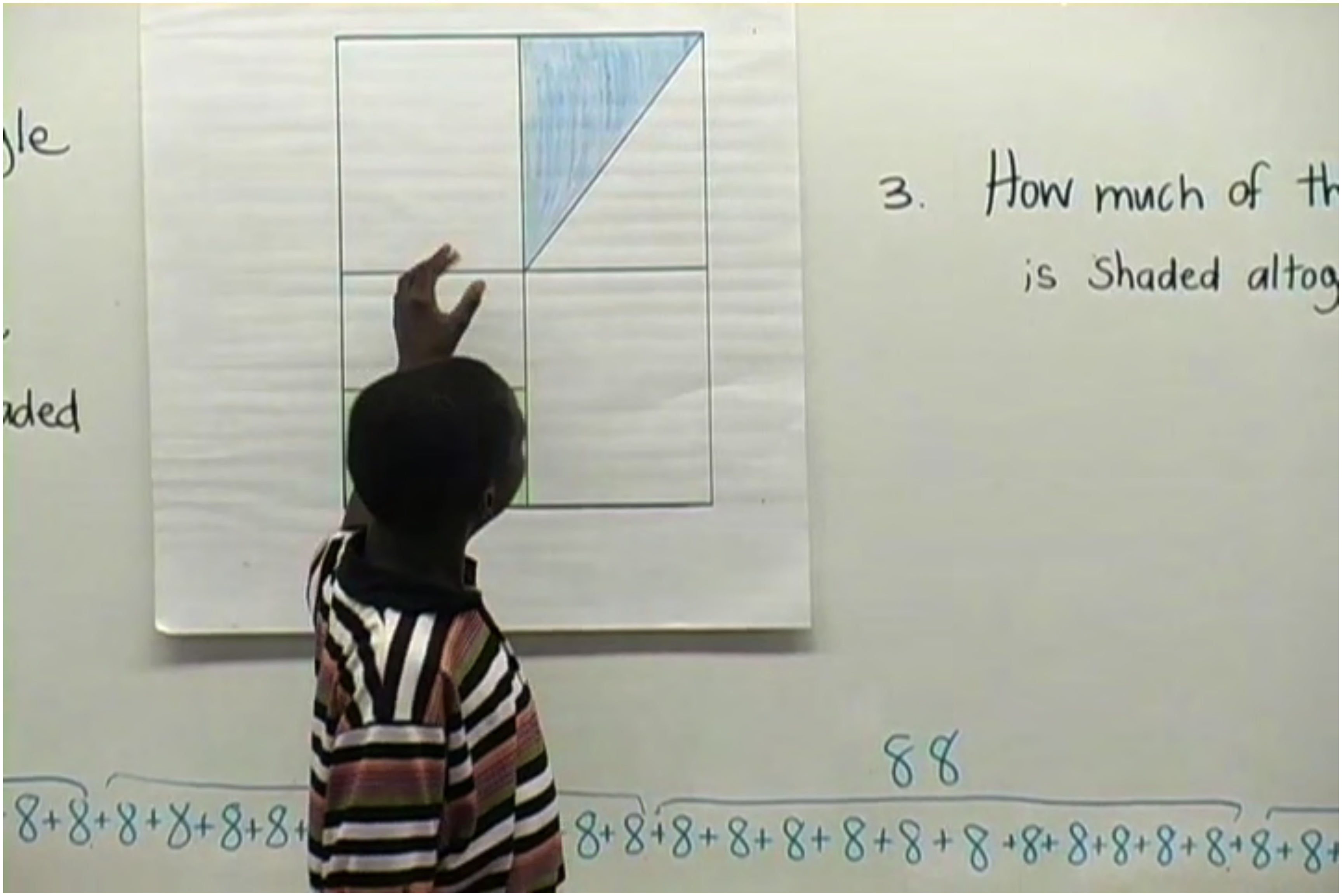
THUS, THESE TASKS ARE ACTUALLY POTENTIALLY <u>UN</u>SAFE.



What fraction of the big rectangle is shaded blue?

What fraction of the big rectangle is shaded green?

What fraction of the big rectangle is shaded altogether?


3. How much of th
is Shaded altog
88
8+8+8+8+8+8+
8+8+8+8+8+8+8+8+8+8+8+8+8+8+8+8

Interpreting and responding to students' "errors"

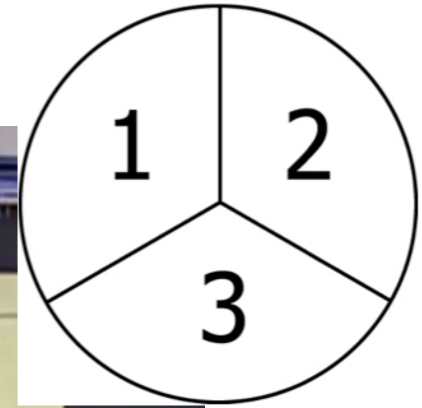
1. Is it really an error or is it in the way the student is expressing or writing it? (e.g., Shea)
2. Is the student actually correctly answering a different question? (e.g., Mamadou)
3. Is the "error" an important conceptual issue that could be exploited productively for everyone's learning? (e.g., Shea, Mackenzie, Mamadou)

③ Teaching and using academic language

Viewing focus

What are the components of the teacher's approach to introducing a new mathematical term?


1
2
3
Probability – helps us figure out how likely it is that an event will happen.


1
2
3

Examples of high-leverage practices visible in this clip

- Making transitions
- Getting and holding the floor
- Introducing a mathematical term
- Creating a safe classroom learning environment
- Designing and sequencing lessons for specific mathematical goals
- Engaging students in experimentation to develop probability concepts
- Posing questions
- Assessing students' prior knowledge and their learning
- Launching a task
- Providing positive reinforcement

Introducing a mathematical term

1. "What numbers can you get?" (concept)
2. "We call those outcomes." (new term)
3. "Possible results of an experiment." (definition)
4. Puts up poster. (signals importance, supports remembering and using term)
5. "How many outcomes are there?" (practice)

Why is language so important?

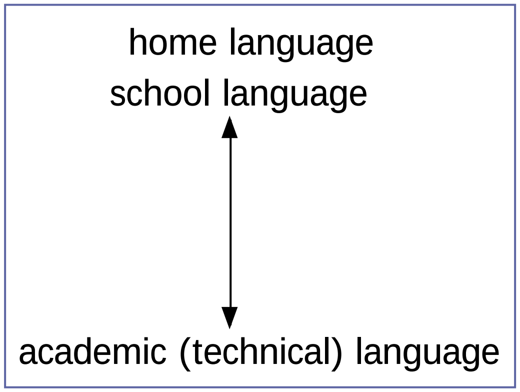
- Mathematical knowledge and reasoning depends on and is supported by mathematical language.
- Teaching and learning mathematics depends on and is supported by language.
- Mathematical language is both mathematical content to be learned and medium for learning mathematical content.

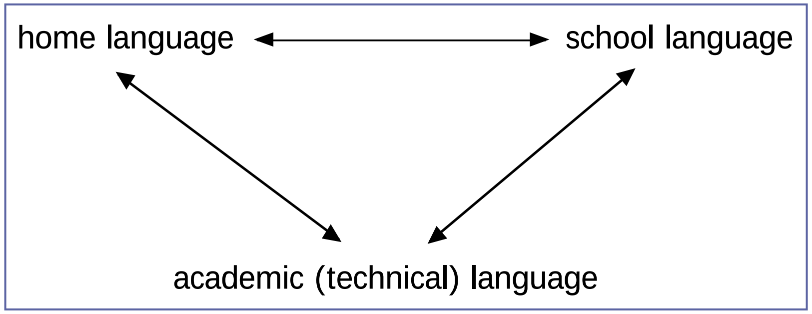
Academic and intellectual vocabulary

1. Specific mathematical concepts (e.g., outcome, even number, rectangle, equal)
2. Terms for knowledge or knowing (e.g., prove, conjecture)
3. Terms for intellectual work (e.g., confer, propose, argue)

Issues with mathematical language: Mathematical vs. everyday language

- Mathematics often uses and specializes everyday language rather than coining a separate technical vocabulary -- both enabling and complicating entry to its register.
- Equity issue: Additional dimension for students who must navigate between home, school, and mathematical languages





Challenges of using language

You ask your learners to explain what a rectangle is. One child offers a definition:

> A rectangle is a flat shape. It has four square corners, and it is closed all the way around.

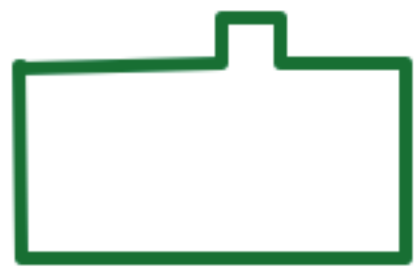
What is the work for the teacher?

A rectangle is a flat shape. It has four square corners, and it is closed all the way around.

1. To see that something is missing
2. Decide what to do or say
3. Offer a counterexample

"Is this a rectangle?"

(straight sides)



(exactly four square corners)

A rectangle is a flat shape with straight sides that are connected at exactly four square corners. It is closed all the way around.

Dilemmas of using language

1. Learners necessarily use imprecise language.
2. We want to encourage students to express themselves.
3. Informal language can serve as a bridge or a support.
4. Attending to the needs and resources of English language learners.

What makes a "good" definition?

- Mathematically precise — correctly identifies the kind of object, process, property
- Usable by user community — based on already-defined and understood term

What is a mathematically **precise and usable definition** of “even number” for third graders?

Using a definition of "even number"

a) An even number is a number that can be divided into two equal parts.
b) An even number is any multiple of 2.
c) An even number is any integer multiple of 2.
d) An even number is any number whose unit digit is 0, 2, 4, 6, or 8.
e) A whole number is even if it is the sum of a whole number with itself.

a) An even number is a number that can be divided into two equal parts.

b) An even number is any multiple of 2.

All numbers, for example 7, 3/5, $\sqrt{2}$, π, are even!

c) An even number is any integer multiple of 2.

This is a correct definition of even number.

d) An even number is any number whose unit digit is 0, 2, 4, 6, or 8.

In this case, 36.7 is an even number!

e) A whole number is even if it is the sum of a whole number with itself.

This is a correct definition of evenness for whole numbers, and is consistent with the general **definition for integers that will arrive later.**

④ Using homework equitably

Principles about homework

1. Homework, understood as work "between class days" is an important part of students' learning opportunities.
2. In order to manage potential problems of inequity and mathematical distortion or misunderstanding, homework can be designed and used in special ways.
3. Explicit expectations, routines, and design, are important.
4. Other strategies include three different types of tasks and clear differentiation of the mathematical demand of in-school and homework tasks.
5. Next steps: Ways to build or adapt new forms of homework; better supports for English language learners; experimenting with other types of tasks.

Support responsible and possible individual work

1. Formal contracts that specify expectations
2. Homework boxes for supplies at home (ruler, colored pencils, glue, scissors)
3. Norms of not working with others (can include one problem to "show" to someone at home) and tasks appropriate for completely independent work
4. Problems that "stretch" (teach students to try even when not sure, and to distinguish from ones where they should know how to do them)

Three types of homework tasks

1. Independent practice: Skill development and reinforcement, developing students' ability to reflect on their own learning
2. Preparing for new work: Anticipating, growing tolerance for difficult work (productive disposition)
3. Share with others: Home-school communication, development of students' ability to narrate their work

Differentiating in-class and homework tasks

HOMEWORK

- Can be done independently
- As unambiguous as possible
- Structures the workspace more explicitly

IN-CLASS

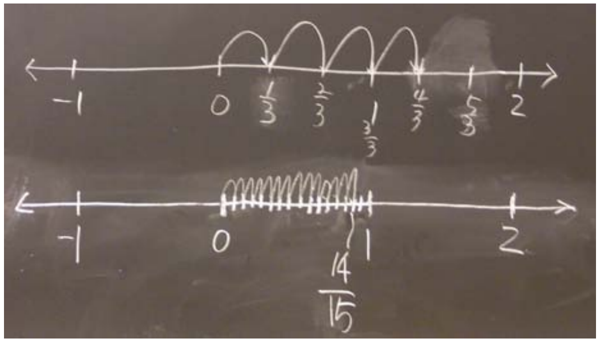
- Profits from exchange with others
- Requires interpretation
- Requires more decisions about representation and recording
- Likely to yield multiple solution methods or solutions

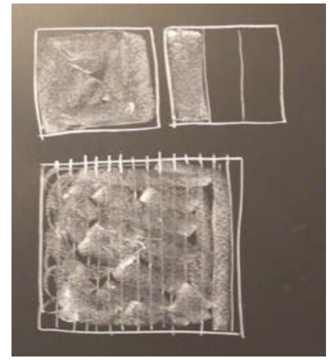
⑤ **Using specific** mathematically-focused positive reinforcement

Using mathematically-focused reinforcement

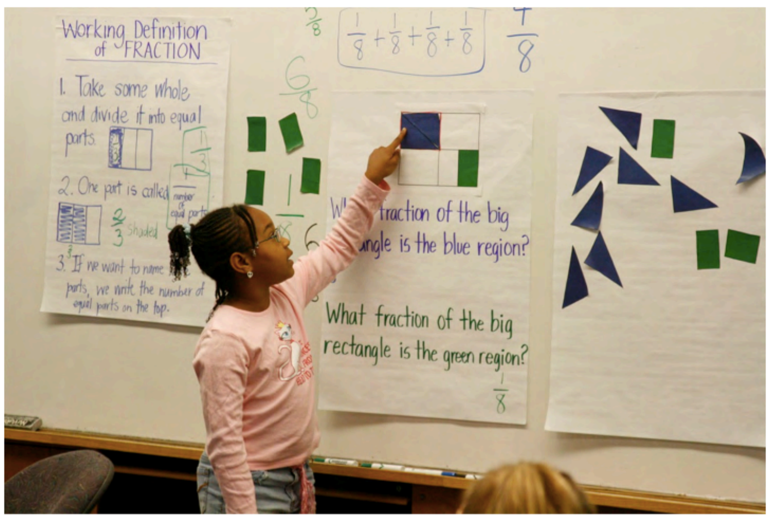
1. Distribution of intellectual turns (e.g., who gets to show their work on the board or call on others' for comments)
2. Focus on a mathematical accomplishment (e.g., "Your drawing is very clear"; "Julia's explanation was very complete")
3. Focus on mathematical productivity and analysis: (e.g., Call on a student to show something he or she was "trying" or "stuck" and comment on why this is important)
4. Accord authorship of mathematical ideas (e.g., "Lily's conjecture" or "James's proof of odd + odd = even"; "Chloe's question")

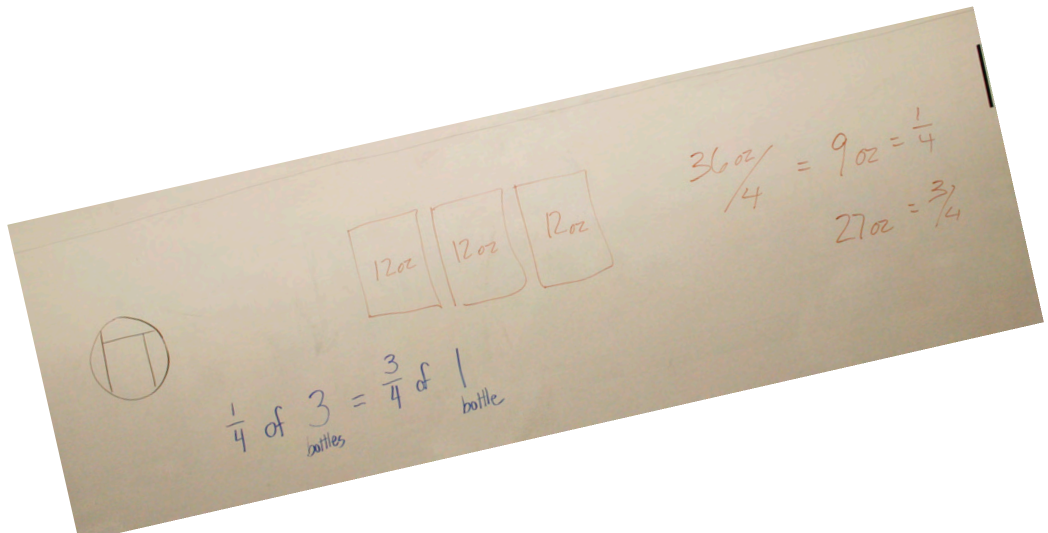
*In common: Communicating (and broadening) what "being good at math" means.



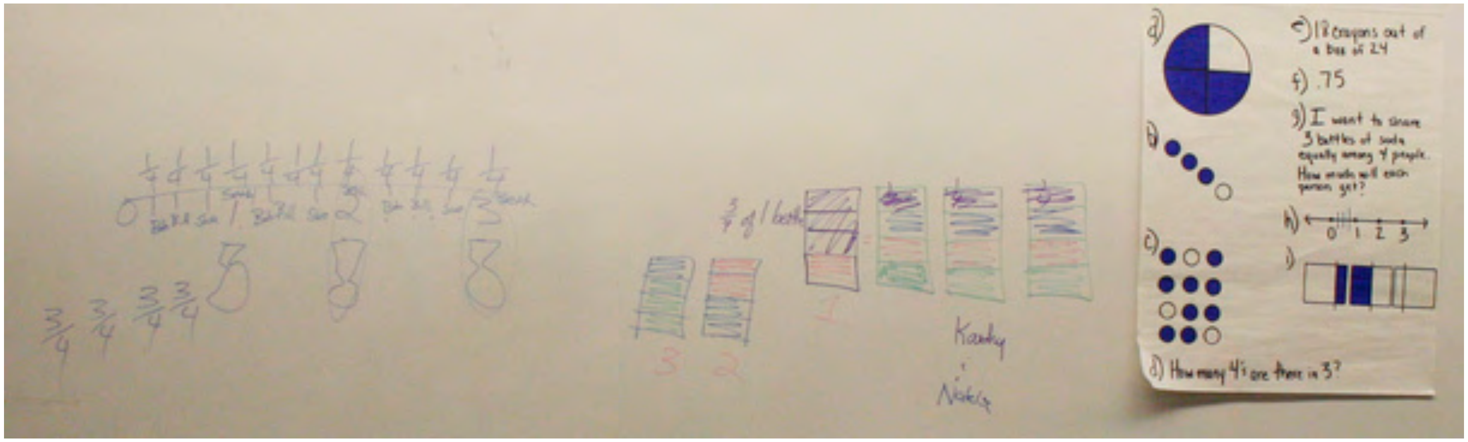


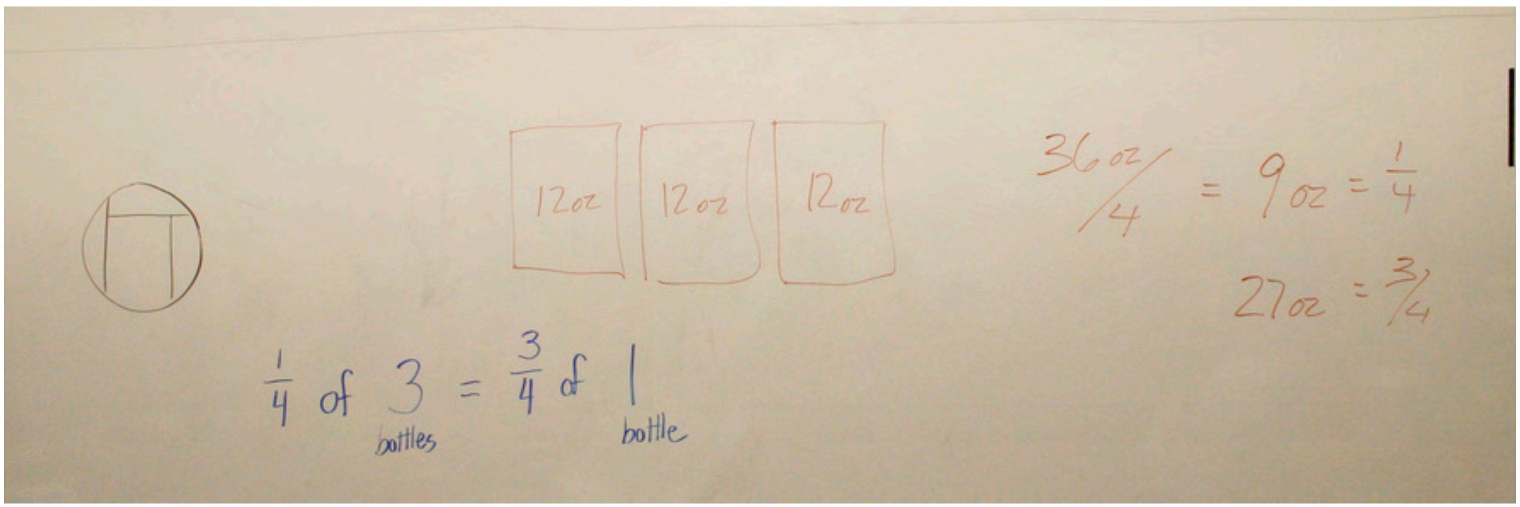
⑥ Using public recording





Images from the whiteboard





Using public recording space

- An idea imported from the practice of Japanese teachers
- "bansho": the study of blackboard use
- "Public recording space": whiteboard, chalkboard, pre-made posters, chart paper during class
- Principles for use of the board and connecting this to students' opportunities to learn mathematics

Why attend to public recording space in mathematics teaching?

Teachers can use public recording space in classrooms to:

- Make the use of representations more effective
- Capture ideas across a lesson to make it possible for students to remember and revisit what has been discussed
- Model mathematical practices and the use of language
- Connect ideas and summarize lesson
- Make records from a lesson for subsequent use in later work

(ideas adapted from work of Makoto Yoshida)

4. Developing practice

What is high-leverage practice in teacher training?

What are ways to support teachers' learning of practice?

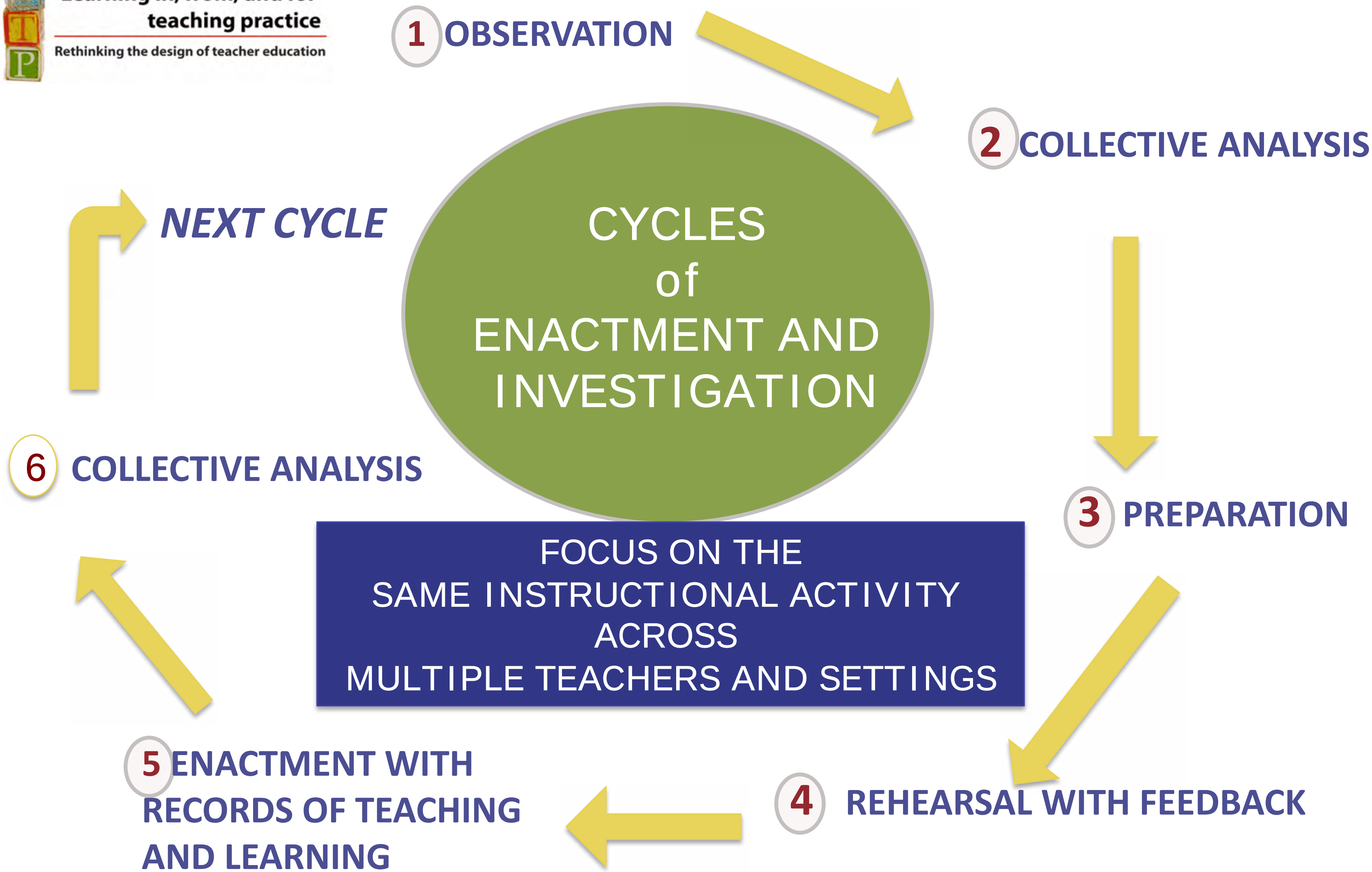
1. Situate teachers' opportunities to learn in the curriculum they teach
   - Work on the actual content and ways to represent and explain it
   - Study students' work and thinking on the specific content
   - Work on detailed lesson design
2. Work on specific instructional practices
   - Choose a small set of high-leverage practices
   - Provide focused time to study, rehearse, analyze, and practice these HLPs

Viewing focus

1. What practices of teaching are being taught in each example of rehearsal and coaching?
2. What learning opportunities are afforded by providing serious opportunity to rehearse in front of colleagues?

<Video Clip>

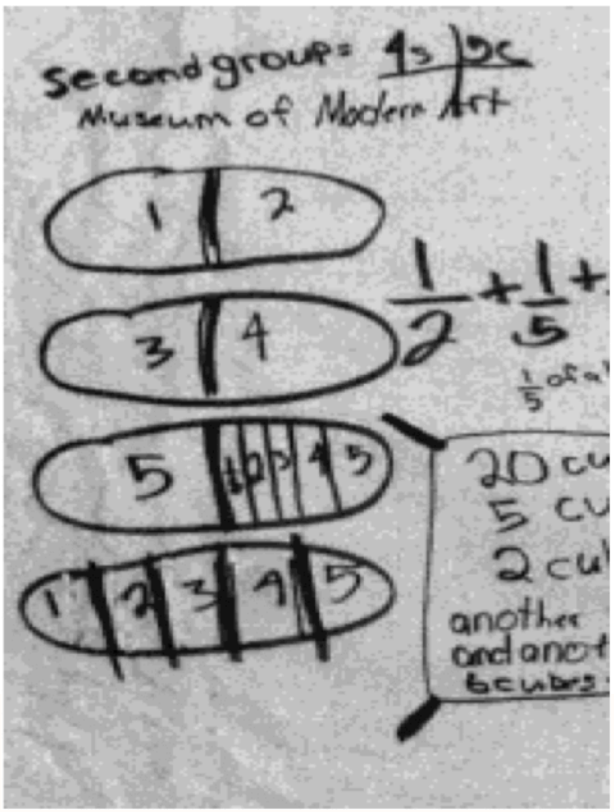
Learning in, from, and for teaching practice
Rethinking the design of teacher education



Learning in, from, and for teaching practice
Rethinking the design of teacher education

Problem being rehearsed

A fifth grade traveled on a field trip in four separate cars. The school provided a lunch of submarine sandwiches for each group. Group 2 had 5 people and shared 4 subs equally. How much did each person in Group 2 get?



$$\frac{1}{2}+\frac{1}{5}+\frac{1}{5}$$

*Image used with permission

Clip highlights

1. Teacher intern rehearses an interaction with a "student" who has made a key error in naming fractional pieces of a whole sub (calling one-fifth of whole sub the same as one-fifth of half a sub)
2. Teacher educator stops exchange to discuss how the intern phrased her question:

> ***"So those pieces would have to be equal if they're both a fifth?"***

3. Other interns comment on the mathematical point
4. Teacher educator asks intern to try again with new phrasing

End-of-session wrap up

View the video segment again:

1. What does Mackenzie seem to know?
2. What practices of teaching are made visible by the segment?

Write your thoughts and then share.

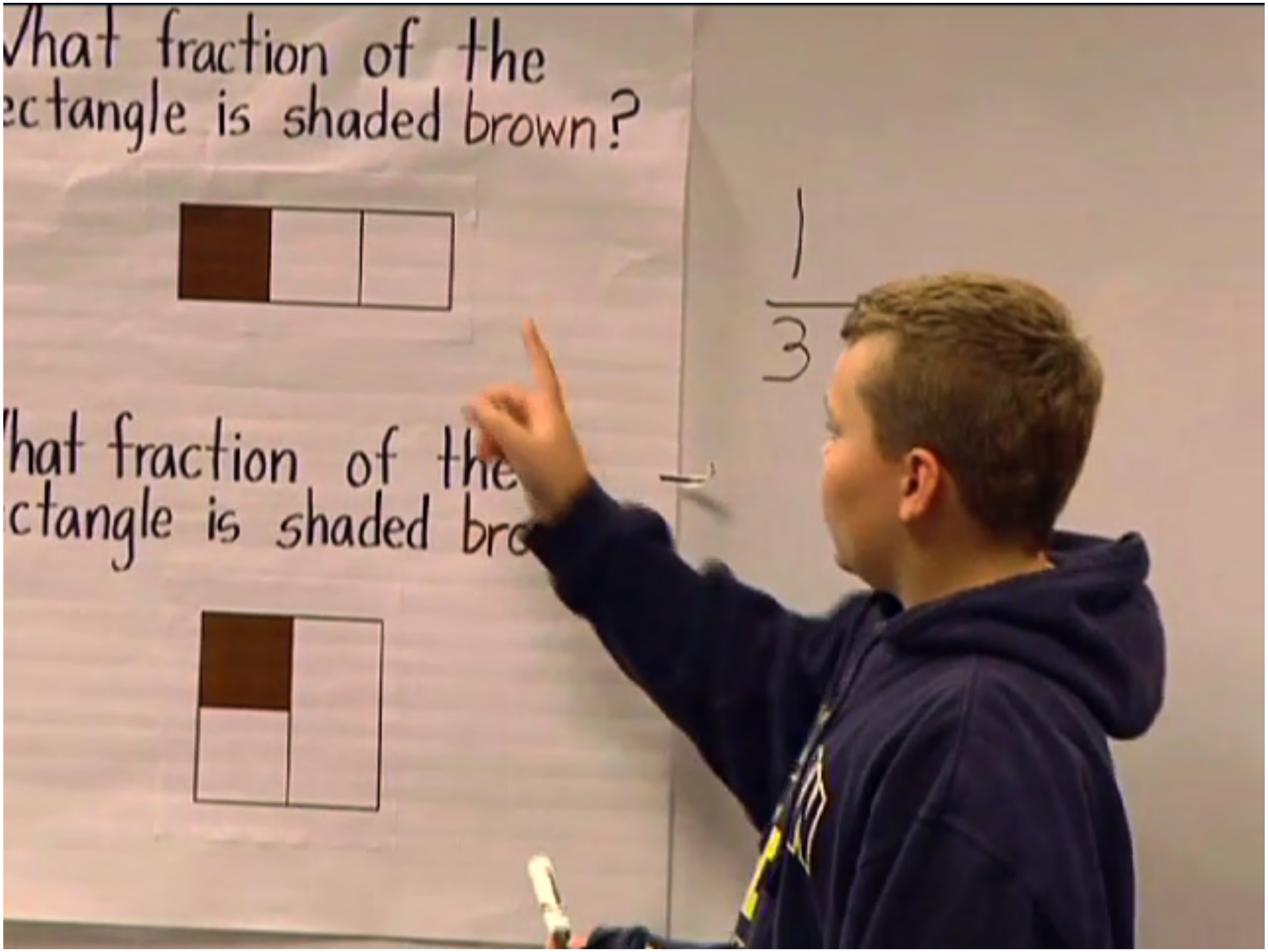
- Opportunities to pilot new elementary teacher development materials focused on mathematics teaching practice
- For information and to sign up, please go to:
- http://www.umich.edu/~devteam/

Establishing a new organization housed at the University of Michigan

SCHOOL OF EDUCATION UNIVERSITY OF MICHIGAN

A national "utility" for research and development on the professional preparation and continuing development of teachers, with four primary branches:

1. Materials and resources for teacher educators and professional developers;
2. Training for teacher educators and developers;
3. Research on approaches to professional training; data and information about teaching quality and development
4. Communications, policy analysis and leadership for the improvement of teaching quality


Vhat fraction of the
ectangle is shaded brown?
1
3
'hat fraction of the
ctangle is shaded bro

THANK YOU!

Slides will be available at Deborah Ball's website

(Google or Bing "Deborah Ball")

SCHOOL OF EDUCATION UNIVERSITY OF MICHIGAN

Credits

Image on slide 10 - "Lavalette Sunrise" by Flickr user willrich
Licensed under a Creative Commons Attribution-NonCommercial 2.0 Generic License
http://creativecommons.org/licenses/by-nc/2.0/deed.en

Image on slide 10 - New York Times Magazine cover art by R. Kikuo Johnson, March 7, 2010