Judith E. Jacobs Lecture

What Kind of Mathematical Work is Teaching, and How Does It Shape a Core Challenge for Teacher Education?

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Association of Mathematics Teacher Educators 2007 Conference

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How is teaching mathematics well related to knowing and being “good at” mathematics?

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The ambitious goal of a more mathematically skilled population

- Goals for mathematical proficiency have increased and broadened over the last 50 years
  - Who needs to be successful with math
  - What mathematical proficiency is
  - What is important to learn
- This is not "reform," but radical renovation

The role of teachers, and the problem

- Teachers are key to this ambition: any other tool (e.g., technology, curriculum, class size, time) depends on teachers' capacity to manage and use the tool and to help students do so

But --

- We have a shortage of teachers, and
- We do not reliably prepare enough people to teach mathematics in ways that meet these aspirations

How can we develop and support the teachers we need?

- One favored approach: recruit mathematically skilled people into teaching

THE ANN ARBOR NEWS

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Pfizer to close here

Ann Arbor campus to be shut down

Pfizer's 2,410 job cuts shock Michigan

January 23, 2007

BY JOHN GALLAGHER
FREE PRESS BUSINESS WRITER

Feeling ambushed by the latest blow to Michigan's economy, state and local leaders scrambled Monday to bounce back from Pfizer Inc.'s surprise decision to eliminate 2,410 jobs in Ann Arbor, Kalamazoo and Plymouth Township.

Michigan is no stranger to job loss, but this one stung particularly hard because the high-paying

Why does recruiting mathematically skilled people to teaching seem to make sense?

1. They know math.
2. They like math and can show students why it matters.
3. They can serve as good role models.
4. They are "smart."

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How is this presumed to work?

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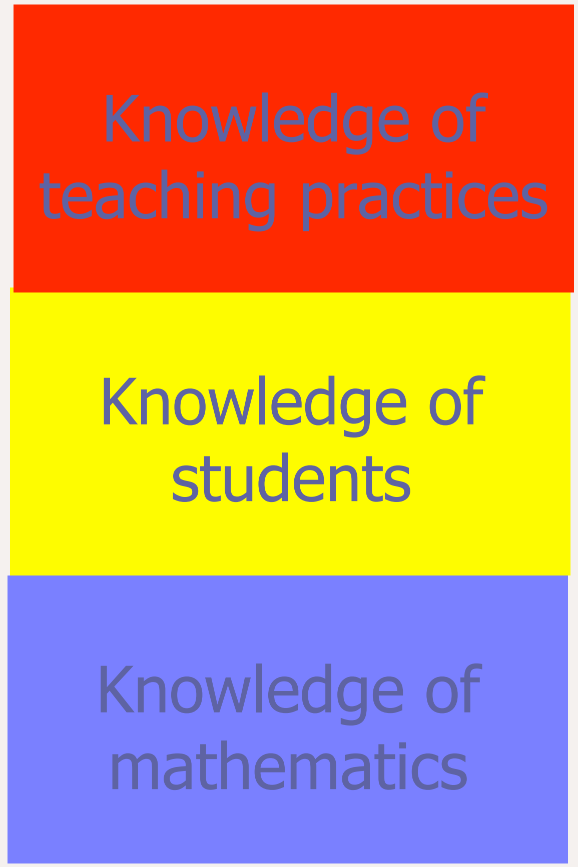
An additive view



How sensible is this view?

Consider some obvious mathematical demands of teaching. What do teachers do?

- Use textbooks
- Present content (either from the textbook or by one's own design)
- Show students how to solve problems
- Answer students' questions
- Assess students' work (responses in class, homework, tests)

These all require knowing mathematics.

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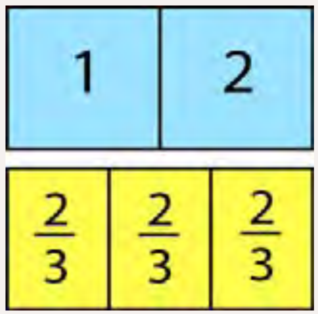
A closer view: Teaching as mathematical work

- Using and analyzing representations, and mapping across different kinds of representations
- Defining terms and attending closely to language
- Using and inventing notation
- Producing and analyzing explanations
- Generating simpler and more complex versions of a problem
- Asking mathematical questions
  - Why does this work? Does this work in all cases? Do we have all the solutions? How are these two representations related?
- Thinking of special cases
  - Boundary cases, or examples that might push an initial idea

Analyzing representations: Which of the following can be used to represent 2 ÷ 2/3?

(a)



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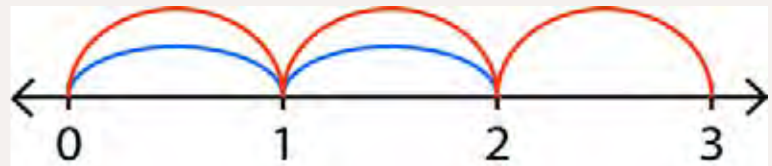

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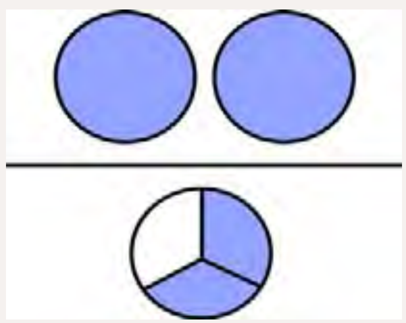

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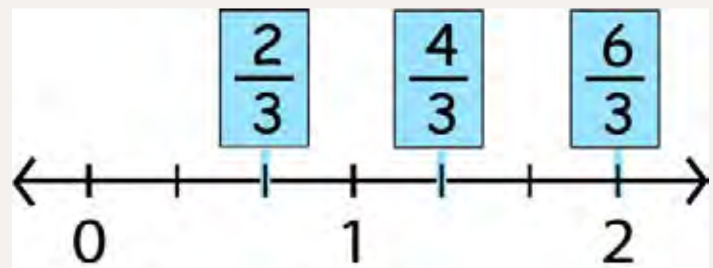
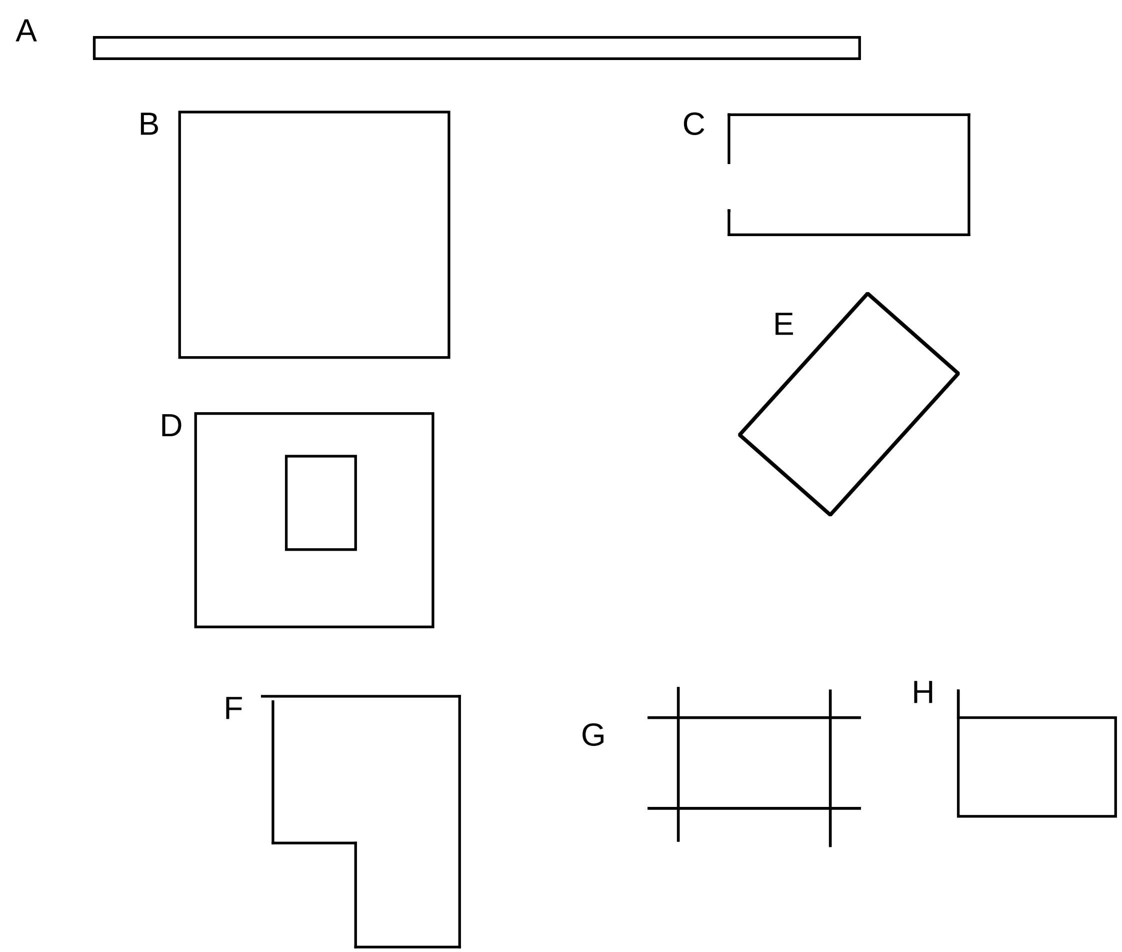


Generating special cases: Which of these are rectangles?



Attending closely to language: Do these correctly define "even number"?

Is 7 even?

1. A number that can be divided in two equal parts with nothing left over is even.
2. A whole number is even if it can be divided into groups of 2 with nothing left over.
3. A number with 0, 2, 4, 6, or 8 in the ones place is even.

Is 32.7 even?

Teaching as mathematically "natural" work, and the limits of this perspective

- Some aspects of teaching depend on mathematical instincts, habits of mind, practices
- So the additive view of learning to teach may make sense — add other knowledge to mathematical knowledge and habits

but —

- Teaching mathematics also involves doing things that are mathematically *unnatural*

Teaching is itself unnatural work

Common ways of being

1. Telling and showing others, doing things for people
2. Being "yourself"
3. Assuming you know what others mean
4. Correcting and smoothing over mistakes
5. Assuming others experience things as you do
6. Liking/disliking people

Ways of being in teaching

1. Listening and watching others, help others do
2. Being in professional role
3. Probing others' ideas
4. Provoking disequilibrium and error
5. Not presuming shared identity; seeking to learn others' experiences and perspectives
6. Seeing people more descriptively

The specific case of mathematics teaching as mathematically unnatural work

1. Unpacking mathematical ideas
2. Listening to mathematically imprecise language
3. Not automatically affirming correct statements
4. Hearing what others say, not what you think
5. Surfacing "error"

Example #1: Unpacking mathematical ideas

What makes this unnatural?

Mathematics aims for compression and consolidation.

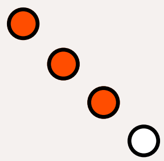
What is a fraction (1)?

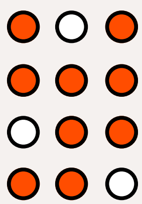
A fraction is a representation of a rational number of the form *a*/*b*, where *a* and *b* are integers and $b \neq 0$.

A rational number is a real number representable as a quotient of two integers, with the denominator $\neq 0$.

What is a fraction (2)? Representations of three-fourths

a) 

b) 

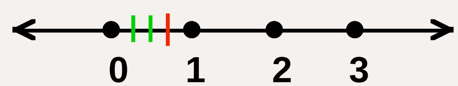
c) 

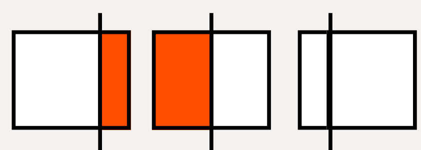
d) How many 4's are there in 3?

e) 18 crayons out of a box of 24

f) .75

g) I want to share 3 bottles of soda equally among 4 people. How much will each person get?

h) 

i) 

Example #1: Unpacking mathematical ideas

- As ideas are learned, the elements of the ideas need to be encountered and developed to get to a more finished understanding
- To teach a concept or a procedure, must see the core constituents in order to stage and sequence learning, to construct representations, to hear

Example #2: Listening to mathematically imprecise language

What makes this unnatural?

Mathematics depends on precision; is a strictly denotative language. Taking things literally and not making inferences is a value.

Problem: Joshua and the peas

Joshua ate 16 peas on Monday and 32 peas on Tuesday. How many more peas did he eat on Tuesday than on Monday?

What is Rania saying?

"I would say something else. I want to prove that his answer is right."

Video clip #1

Example #2: Listening to mathematically imprecise language

- As students learn, they necessarily express themselves informally -– and imprecisely.
- This requires:
  - Listening generously and making inferences to interpret
  - Tolerating imprecision and assuming sense-making
  - Figuring out ways of probing what students do mean

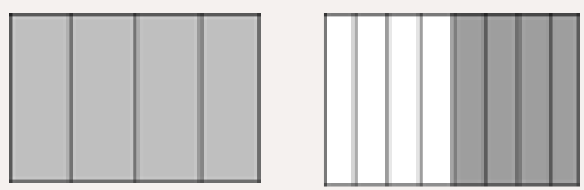
Example #3: Not automatically affirming correct statements

What makes this unnatural?

When correct statements are made, important to mark their truth.

Why not say, “Good, that’s right” to Lin?

“I think this one *(the four fourths)* is bigger because um four eighths, that's only like a half and this *(the four fourths)* is a whole piece.”



Video clip #2

Example #3: Not automatically affirming correct statements

- Just because something correct is said doesn't ensure understanding.
- A culture of mathematical reasoning must consistently place authority on verified explanations.

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Example #4: Hearing what others say, not what you think

What makes this unnatural?

Mathematical frameworks provide a strong orienting instinct; they are difficult to suspend.

What is Shea's idea?

"I was just thinking about six, that it's a . . . I'm just thinking. I'm just thinking it can be an odd number, too."

Video clip #3

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Example #4: Hearing what others say, not what you think

- When students talk, focusing on what they say and what they might mean without your own strong mathematical overlay
- Suspending assumption of "misconception" and attribution of "error"

Example #5: Surfacing and taking up "error"

What makes this unnatural?

Mathematical instinct is to critique and correct and to not allow incorrect or sloppy statements to stand.

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Why take up Kevin's claim that he "did it wrong"?

"First I did something different and then I was wrong."

Video clip #4

Example #5: Surfacing and taking up "error"

- Learning involves sensible efforts that run amuck; discussing and analyzing these can develop stronger understanding
- Some apparent "errors" are not errors at all (e.g., Shea)
- This requires suppressing "natural" desires to:
  - Protect students from "exposure"
  - Correct "wrong" statements

Teaching mathematics is not a natural extension of learning mathematics

- Teaching is not just about being "yourself" mathematically
- Being "smart" in teaching requires some crucial skills and habits that are not just extensions of other mathematics learning
- Major implications for equity and ambitious goals for renovating mathematics education
- Teaching is a role; teaching mathematics means acquiring new instincts and habits, new ways of seeing mathematically that are specialized for the work of teaching

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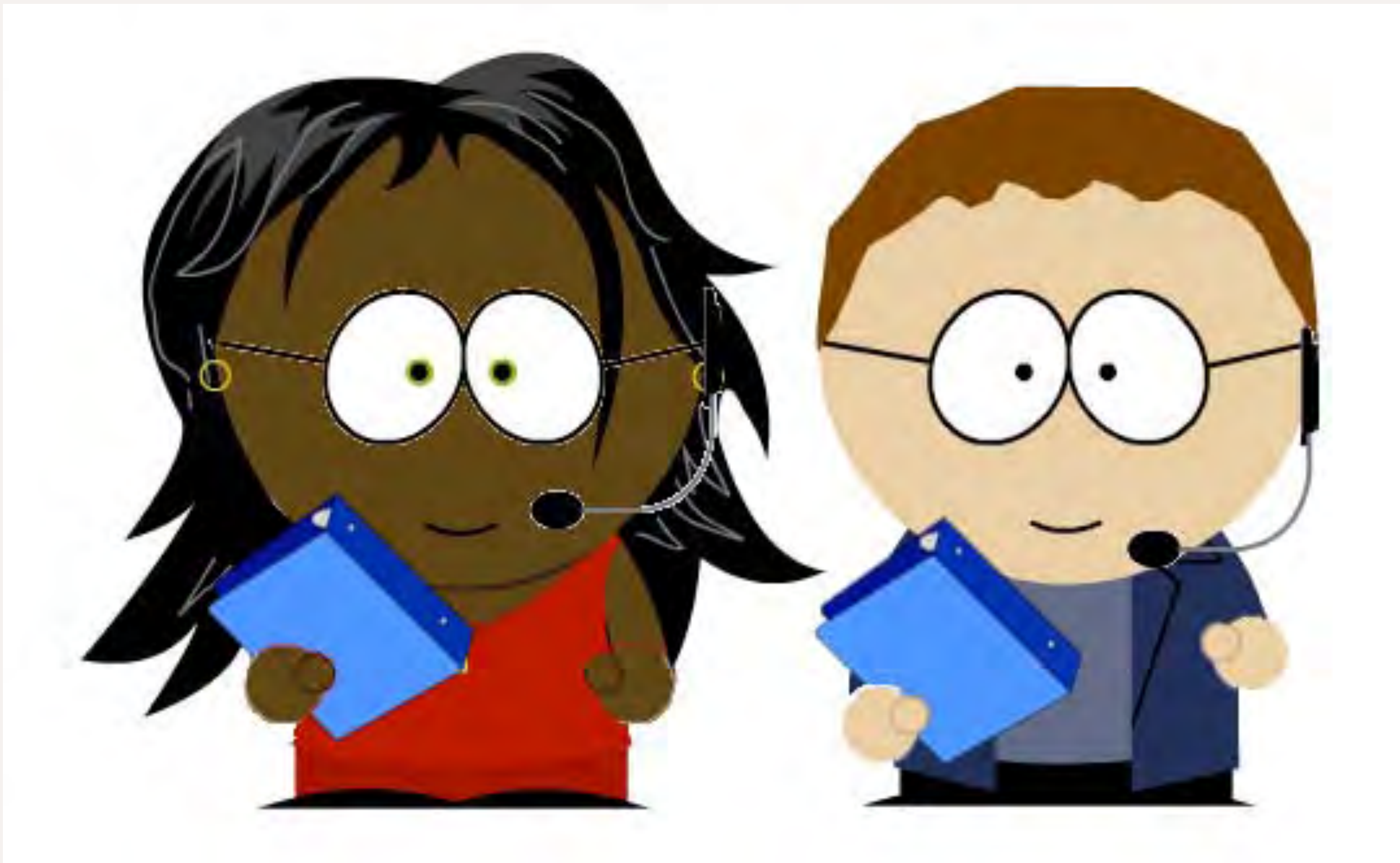
A different view of what it would take



The challenge of learning to teach mathematics

- Although it may seem logical to "add" knowledge and skills to a strong mathematical base, teaching also requires learning to do and think mathematically that are not "natural"
- Professional education must help teachers engage in unnatural activity that is not automatically learned from learning math or gaining experience

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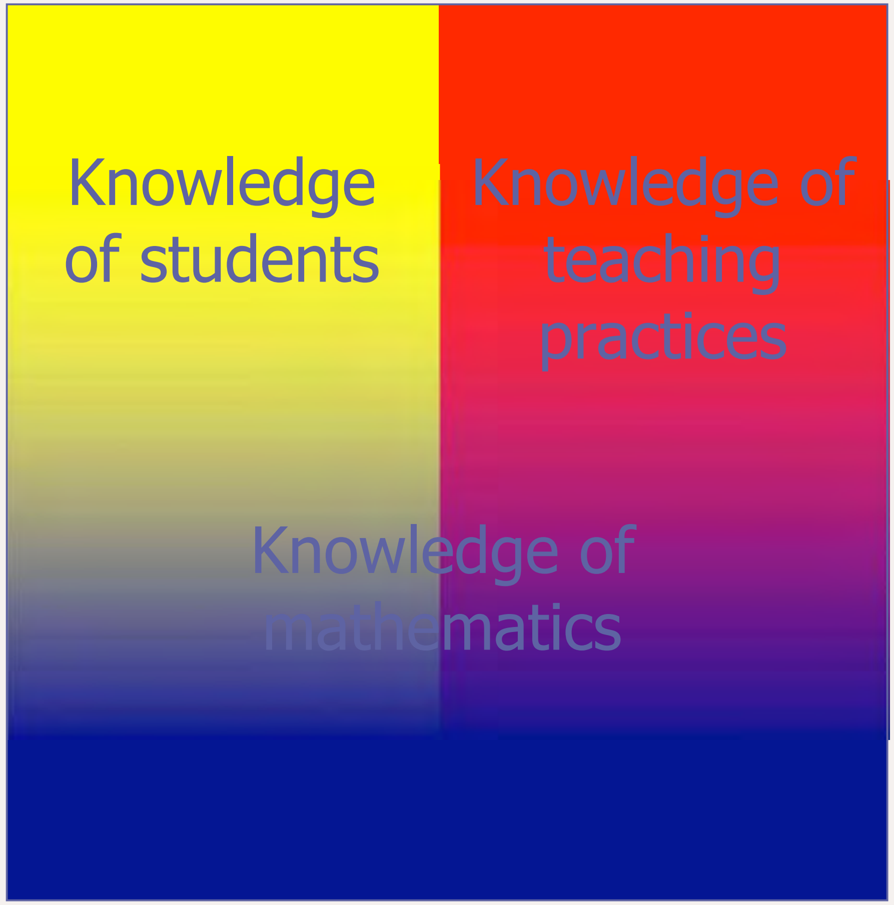
An additive view

Knowledge of teaching practices

Knowledge of students

Knowledge of mathematics

A more transformative view



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Questions

1. How can mathematically skillful people be helped to make this sort of transformation?
2. How do people who enter with strong conventional mathematics backgrounds compare with those whose mathematical skill is grown from being teachers?
3. How can mathematical instincts for teaching be developed?
4. Are there similar phenomena in other fields?
5. How can professional education prepare people for the mathematical work of teaching and how it can it be best tailored for different kinds of entrants?

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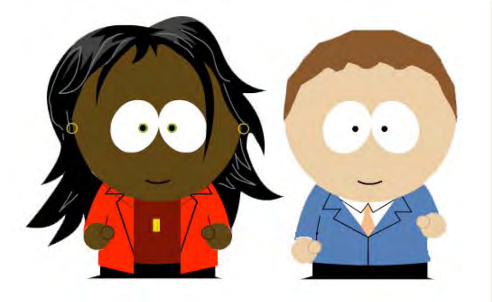
Our challenge



- The policy environment demands “qualified teachers” without good evidence of what constitutes “qualification”
- Common sense triumphs in the absence of solid evidence for alternatives
- Teaching mathematics is not natural; good professional education matters: what are the forms that that can take and what is the evidence of their comparative advantage?
- As professional mathematics teacher education community, we need to step up

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