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BOOK REVIEW

Carolyn Kieran, Ellice Forman, and Anna Sfard (Eds.). *Learning Discourse: Discursive Approaches to Research in Mathematics Education.* Dordrecht, The Netherlands: Kluwer Academic Publishers, 2002, 298 pp. ISBN 1-4020-1024-9 $105 (cloth).
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In their chapter of the book *Learning Discourse* Vicki Zack and Barbara Graves describe an episode in which Ms. Zack's fifth graders decide that the number of diagonals in a polygon with S sides is

$$\frac{(S-3)\times S}{2}$$

Jeff makes a connection between this and an earlier problem about the number of tunnels needed to connect prairie dog burrows, each one to every other. Spurred by Jeff's remark, Ms. Zack asks the class to write a formula for the tunnel problem.

After working a while, Jeff gets an idea and explains it to his partner, Micky. He excitedly explains that the number of tunnels is

$$\frac{(S-3)\times S}{2}+S$$

> Jeff: So what I did is I did point A times sides divided by 2 then plus sides[1] 'cause you get the diagonals plus the sides, and then that's all the lines you can draw.

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[1]Jeff uses A for $(S - 3)$.

Micky asks him to slow down. Together, they walk through the explanation. After some further discussion, Micky agrees, but then wonders why you add *S* but "without the minus 3."

Micky: Okay, so it's basically the same thing but you just add on the sides.
Jeff: Add the sides.
Micky: Except you add it once more without the minus 3.
. . .
Micky: See, once you think of it, the *Z* minus 3 seems pretty weird.[2]

Micky and Jeff go on to puzzle about whether the addition of *S* compensates for the "minus 3" and to wonder whether they "will get that 3 back." The boys listen thoughtfully to each other, periodically refocusing each other on the question they have posed for themselves. Several days later they resume their work.

Micky: My only question is where did the 3 go? Now, that's all I'm wondering about. I understand the rest.
Jeff: Okay. I have no idea where the 3 went. It probab—, but—, the thing is, why do you need the 3? 'Cause it's not 3. It's not 3. From here [*refers to a vertex in their drawing*] it's 3 but then you got this point, that 3—
Micky: But that should actually be minus 1 cause it cannot connect with itself, but in the problem it can connect with others.
Jeff: Exactly.

Having said that it should be "minus 1," they decide to try the formula using minus 1 in place of the minus 3 and without adding on the *S*.

$$\frac{(S-1)\times S}{2}$$

They try out several different values, excited that it seems to work.

Jeff: Try it, try it all you want. We've just figured out two ways to figure out tunnels.
Micky: But that would be the most straightforward, it'll—
Jeff: That'll be the more straightforward because—
Micky: You wouldn't have to do an extra, uh, adding on.
Jeff: And an extra subtracting. That's where you get the 2 back. It wasn't 3 that we were getting back. It was the 2.

[2]In his independent work and throughout the discussion Micky uses *Z* instead of *S*.

Micky: 2. So this is actually better.
Jeff: This is better than before.

Although these boys have not had formal instruction on the equivalence of algebraic expressions, they come to see that the *S* they add on matches two of the *S*s subtracted in the numerator, which accounts for the change from $S - 3$ to $S - 1$ in the formula. While solving a problem posed by the teacher, they raise a question of their own making and work together over an extended period of time to resolve it. What they do is impressive. We get the sense that we have our finger on the pulse of mathematics teaching and learning. But, what are they doing? What here is so impressive? And what happened before to lead to this? These are crucial questions for mathematics education.

Learning Discourse is worth reading because it closely scrutinizes classroom exchanges, such as the one thus described, with an eye for central questions about mathematics teaching and learning. The book offers insights, potential answers, and methodological tools.

OVERVIEW

This book, which first appeared as Volume 46 of *Educational Studies in Mathematics,* includes seven chapters and two commentaries. Of the seven chapters, two make theoretical arguments, two analyze exchanges between students, and three examine whole-class interaction. In *Learning Discourse*, the authors propose a focus for research in mathematics education. As Sfard, Forman, and Kieran describe in the preface, the unifying theme is the conceptualization of mathematical thinking as a form of communication, in which learning mathematics means becoming fluent in mathematical discourse.

Attention to mathematical discourse is not new, but this book argues that it is strategic for studying and improving mathematics teaching and learning. By including transcript, the authors make the rich texture of mathematics teaching and learning palpable. By mobilizing sociocultural theories of learning and tools from discourse analysis, they help us see and hear more of what is happening to produce success and failure in mathematics classrooms. And, by developing their focus on mathematical communication, the authors contend that discursive approaches to mathematics education research provide leverage on fundamental issues of mathematics teaching and learning and thus should hold a central place in research efforts of the field.

Several approaches could be taken to reviewing this book, each associated with a different goal of the book. We have chosen to focus our attention on a bottom-line goal—to improve mathematics teaching and learning. Because the dynamics among students, teachers, and mathematics directly affect student learning (Cohen

& Ball, 2001), we examine what this book helps us see about students, teachers, and mathematics and their interplay in classrooms.

STUDENTS AS LEARNERS OF MATHEMATICS

Improving mathematics teaching and learning does not depend on knowing everything about students. Rather, it depends on knowing those things that bear most on students' learning of mathematics. Unfortunately, it is not always easy to determine what matters. By analyzing what students do and say in classrooms, *Learning Discourse* offers insight into what students are likely to be able to use—and not use—of their classroom experiences.

Anna Sfard's analysis offers one example. She analyzes exchanges between two 12-year-old boys, Ari and Gur, as they extend a table of linear data. Sfard describes the episode as a "failure in the making" (p. 15). Reading the transcript, one gets a sense of what she means. Ari seems mathematically capable and successfully works through the problems. Gur struggles. He seems not to get the problem and unable to follow Ari's progress in solving it. The collaborative instruction meant to contribute to both students' learning instead falls back into a familiar pattern of supporting some students and not others.

Building on a basic idea in discourse analysis—that talk is largely an attempt to make other people act or feel according to one's intentions—Sfard separates the transcript into two parallel columns, one for each boy, drawing downward arrows for requests the student makes in his talk, upward arrows for reactions to things said earlier. The analysis offers footprints of the discursive moves of each boy. Quite strikingly, the picture is one of Ari carrying on a conversation with himself. He neither makes requests to Gur nor responds to Gur. In contrast, Gur comments on Ari's talk, asks for clarification, conveys interest, and works to maintain face—his own and Ari's. Sfard's analysis reverses the picture of success and failure. With regard to the social nature of the exchange, Gur carries the interactional load, demonstrating more interactional competence than Ari. Sfard suggests that Gur's attention to Ari's thinking, with no help from Ari, may have done more to get in his way than it did to help him. She leaves us to wonder whether Gur might have been more successful with the mathematics and in learning, if he had worked alone or with a different partner. Sfard's analysis raises important questions for mathematics education. When do collaboration and group work support learning? What is the relation between social competence and mathematical performance? What do failures such as this one imply about using group work or about what would need to be done to make group work productive (what Ari might need to be taught)?

Consider next Carolyn Kieran's chapter. Kieran has students solve problems involving graphs of rational functions, first working in pairs, then individually. Her aim is to see whether the thinking done in joint work carries over to individual work on similar problems. Using the same analytic tools as Sfard, she describes how, when the mathematical work is advanced in interpersonal exchanges (such as those attempted by Gur), students' joint success carries over to individual work, but when the mathematical solution gets developed predominantly within the personal channel (like those carried out by Ari), only the student who leads the work does well in later individual work. Reflecting on these results, she notes that productive interaction is difficult and that effective group work may be more challenging than reformers suggest.

Sfard's and Kieran's analyses identify features of the talk that may support or disrupt learning. For example, in Kieran's analysis the most noticeable difference between the groups was the degree of explicitness in their talk. In successful groups students made their thinking public, were explicit about the referents of their talk, and checked in with each other about their interpretations of what was being said. In contrast, unsuccessful students talked abundantly, but left important parts of the mathematics unsaid. Their talk was fragmented, opaque, and under-elaborated, especially once the group started down a solution path. It was as if, because they thought they shared a common understanding, they did not fully express ideas. But, by not explicating ideas, partners never developed a full understanding of what was said.

The importance of being explicit is a theme that emerges across the chapters. This is nicely demonstrated in the exchange between Jeff and Micky described at the outset of this review. Early on, Jeff pours out his thinking in a stream of talk that is hard to follow. Micky responds, "Okay, so you're going too fast. Repeat the sentence one more time slowly." Not only does Jeff repeat it again more slowly, but the two boys carefully coordinate their words, the symbolic expression, and the diagrams. They point—with fingers as well as with word emphasis and hesitations. They attend to each other and watch to see that they are making sense, avoiding the far too common situation where mathematical talk passes by listeners without definite shape or form. Micky asks Jeff to be more explicit, Jeff responds, and together the boys successfully develop a fuller understanding of the mathematics.

Although the chapters vary, many bring to the fore students' mathematical talk and work and provide helpful tools for examining it. We could give many more examples of this book's insights into students as learners of mathematics. One thing missing in this work is attention to issues of discourse as they interact with students' social and cultural identities. Although some of the chapters report on school context and on demographics, these details do not seem to figure in the analyses. For example, Mary Catherine O'Connor notes that many of the students in her study came "from communities and cultural backgrounds in which talk and argument as a means of intellectual investigation are not prominently featured" (p. 149). This difference in familiarity with the language of instruction is a significant equity issue for education (Delpit, 1988; Heath, 1991; Lubienski, 2000). However, O'Connor's analysis neither focuses on what resources these students bring to mathematical talk nor investigates what students would need to be taught to support their attempts to develop effective ways of talking and learning mathematics. This lack of attention across the book matters to the extent that stark achievement differences in mathematics are due not just to social background as background, but to social background as it plays out in instructional interactions. Analyses of how race, culture, and class might be playing out in mathematics talk are sorely needed in the field. We do not mean to suggest the authors should have done different studies or written a different book, but that folding a consideration of equity into their analyses would have added to the bearing the book has on questions of improvement.

That said, the chapters provide valuable insights into students as learners of mathematics, insights that are both theoretically important and practically useful. The detail with which the book's contributors examine mathematical discourse yields fine-grained analyses of students' thinking and activity. And, unusual for research in mathematics education, as these authors focus closely on learning, they deliberately attend to teaching and to mathematics. However, and somewhat paradoxically, their examinations of teaching and of mathematics do not yield insights of similar potency. We turn next to see what the chapters reveal about the practice of teaching. What do the authors uncover, and what is left out of view?

THE PRACTICE OF TEACHING

Many of the dynamics of learning that emerge so vividly across the chapters have implications for the work of teaching. For example, the importance of explicitness in students' interactions discussed previously suggests a need for teachers to learn how to help students be more explicit in their talk. And as Sfard and Kieran describe how group work can go awry, their analyses also point the way to identifying what students would need to be taught to make group work productive. Similarly, although Zack and Graves describe student practices, the detail contained in their transcripts suggests ways a teacher might design instruction to teach such practices.

O'Connor's chapter goes the furthest in analyzing teaching. She describes a whole-class discussion of the two questions: *Can any fraction be turned into a decimal?* and *Can any decimal be turned into a fraction?* She describes the discourse structure of the class as "position-driven," in which students make claims supported by evidence and others raise challenges. She says the teacher's role is not to validate, but to clarify students' contributions and to raise her own challenges as needed. The headings of her description of the episode suggest the range and focus of her conceptualization of teaching. Notice that most of the titles describe what the teacher does.

4.1 Challenging a robust conjecture and its proponents
4.2 Managing student counterexamples and alternative conceptions
4.3 Posing another challenge: What about one eighth?
    4.3.1 Interlude: Reviewing definitions and purposes
    4.3.2 Modeling confusion
    4.3.3 Using student insights
4.4 Trying to leverage a new method
4.5 Reintroducing one third, a clearer case
4.6 Posing a new question
4.7 Managing limitations of physical and discursive tools
4.8 Day two: Pushing the exploration of repeating decimals
4.9 Bruno's insight

O'Connor shows how the teacher she studied uses moves, such as introducing a challenge to students' thinking, to direct the development of students' mathematical thinking. Having described the episode, she then identifies problems teachers face in leading class discussions. For instance, according to O'Connor teachers have to deal with unintelligible contributions from students, which requires attending to students' feelings while at the same time preserving the conversational thread. The second problem of teaching she identifies is dealing with incorrect contributions. She claims that incorrect statements may be left uncorrected in exploratory phases, but not during summative phases.

> When we are in the heavy lifting and framing stages of developing new ideas, stopping to correct every flaw is disruptive to the real work. When the ideas are ready for polishing, however, correctness in every respect must be the goal. (p. 177)

She concludes by calling for research that analyzes mathematics teaching in conjunction with discourse formats, as she has done. She advocates for the Lakatos-like exchange of a position-driven discussion and points out that people who do not understand teaching are likely to misconstrue and misjudge the work teachers do—such as when they see teachers leave an error uncorrected without taking into consideration where it occurs in the development of a discussion.

These are significant contributions. O'Connor raises practical questions attuned to the realities of teaching and makes visible and names several common problems of teaching. She also raises important issues for debate.

Several other chapters also address issues of teaching, although more implicitly. Their analyses help us see how a question can ground a mathematical investigation, how the crediting and naming of important ideas can support respectful exchange and conceptual development, how differences can be a resource for learning, and how dispositions, once cultivated, can lead to substantive mathematical discussions. What these analyses do less is to examine the work of the teacher in creating these conditions or carrying out these moves.

In part, this reflects the current state of research in mathematics education. The field is in need of strategic characterizations of the underlying tasks of teaching (such as figuring out what students think) and varied ways teachers can accomplish those tasks (such as asking probing questions or designing assessments that prompt students to document their thinking). And, conceptualizations need to be faithful portrayals of teaching, and they need to be formulated in ways that support teachers in carrying out their work. Even further, they need to be formulated in ways that support teachers in their efforts to learn new ways of teaching mathematics. These criteria for researchers efforts to characterize mathematics teaching are daunting, yet crucial, if researchers hope to contribute to the improvement of mathematics education. This book begins to make inroads into this difficult terrain, and it reveals just how much work lies ahead.

THE MATHEMATICS AT PLAY IN TEACHING AND LEARNING

Having focused on students and on teachers, we now turn to the third ingredient of instruction: mathematics. What do these chapters reveal about how mathematics itself shapes classroom discourse? Some chapters characterize the mathematics at play, and some offer tools to analyze mathematics in talk, but the role of mathematics in shaping the dynamics of teaching and learning remains largely hidden.

For instance, Bert van Oers maintains that mathematics is best thought of as an activity or practice, as a collection of values, rules, tools, and things to say and do, and that the goal of mathematics education is to develop students' ability to participate in a mathematical community of practice. According to van Oers, students learn from the modeling teachers do and from the expectations they set for what students do. In closing he writes:

> For the educational agenda we may conclude now that the further improvement of mathematics education requires that pupils be enticed by the teacher to take part in a mathematical practice and especially in mathematical discourse within that practice. More attention therefore should be given to the development of the mathematical genre. (p. 81)

Although van Oers argues that mathematical practices are a key goal and although other authors echo this position, less explicit treatment is given to what mathematical practices are, which ones matter to teaching and learning, or where they are visible in the discourse.

In *Learning Discourse*, a second approach to analyzing mathematics is the focal analysis developed by Sfard and Kieran (2001). They distinguish between the *pro-*

nounced focus (what is stated explicitly), the *intended focus* (the speaker's private meanings), and the *attended focus* (the public expression of the intended focus). The attended focus is the whole performance, which might include emphasizing certain words or pointing to the shape of a graph (i.e., all of the intonations, gestures, and unsaid words that convey intentions). They argue that we do not have easy access to a person's intended focus, but that analysis of the words and performances gives us a better sense of the mathematical objects about which students are thinking. They use focal analysis to track on the mathematical objects being talked about, which helps them distinguish when students are simply tossing around words of the problem from when their talk suggests a coherent mathematical thread. Useful would be to examine how focal analysis might offer tools for the improvement of teaching and learning (i.e., what teachers and students might need to do or learn to do).

Approaching the mathematics from yet a different angle, O'Connor begins her chapter by discussing the mathematics of the problem students do. She points out that the teacher's instructional goal was for students "to begin to see that 'decimals' and 'fractions' are alternative representational formats, not different types of numbers" (p. 149). With this goal in mind, she discusses a range of mathematical issues that bear on teaching the problem. She contrasts, on the one hand, the concerns a mathematician might have for technical precision in the problem statement and, on the other hand, the efforts a teacher might make to state the problem in a way that is mathematically appropriate yet meaningful for fifth graders. She considers methods for converting numbers into different representations and walks through specific examples, just as a teacher might do to prepare to teach. For example, in converting 0.235, 1/3, or 1/7, what will students encounter and how does it depend on the methods they employ? In her discussion, she points to the demanding, yet crucial, mathematical task a teacher faces in deciding what methods her students have available and recognizing what numbers those methods will effectively transform. Her discussion provides a helpful task analysis of the mathematics of the problem for the purposes of teaching it.

Although a central goal of her chapter is to characterize the complex work of the teacher in conducting whole group discussions, O'Connor's descriptions are also filled with mathematical considerations. She considers the role of examples, counter-examples, and definitions in moving the discussion and students' thinking forward, and these shape her characterization of the tasks of teaching, such as challenging a robust conjecture, managing student counterexamples, and reviewing definitions and purposes. The discourse structure used by the teacher is position-driven, in which positions are mathematically defended claims. And, one of her major categories of the work teachers do is what she calls dealing-with-the-incorrect in which she argues that exploratory work requires less insistence on technical correctness whereas summative work requires more.[3] These mathematical

[3]It would be fascinating to know whether this tendency is empirically visible among research mathematicians as well.

considerations significantly shape O'Connor's analysis of the work of teaching. And, it is mathematical considerations such as these—tied to the work that students and teachers do—that we need to better understand if we are to improve the teaching and learning of mathematics.

Although O'Connor pays close attention to the very interesting mathematics at play in the sequence of lessons she analyzes, some crucial mathematical elements are highlighted, whereas others remain buried. For example, the importance of mathematical conjectures is in the foreground, but the explicit role of definitions in mathematical justification is not. In a position-driven discussion, conjectures would seem to require mathematical justification, yet it is unclear what would count (in this classroom) as adequate justification. For example, at one point the teacher asks: "How come we got a repeating decimal for one third and we didn't get a repeating decimal for one fourth or one fifth or two fifths?" (p. 162).

These fifth graders spot the pattern and conclude that one third repeats because 3 is not a factor of a power of 10, but in what sense does this justify the claim? Justification would seem to require the Fundamental Theorem of Arithmetic, which is beyond the scope of the work in this class. Thus, what might count as a reasonable justification in this context?

Across the analyses in this book, mathematical justification draws little attention, despite the fact that it would seem to be a key discourse challenge in mathematics classrooms. For example, Ellice Forman and Ellen Ansell give extensive evidence that the teacher they studied believed that conventional algorithms were confusing to students and so did not request explanations or devote time as she did for invented algorithms. However, we wonder if the teacher's lack of knowledge about the justification of conventional algorithms may have shaped her actions as much as did her beliefs that they are confusing for students. The justification of a conventional algorithm requires significant understanding and unpacking of the notions of place value and the structure of the base-10 number system. Forman and Ansell's framework helps them look closely at subtle issues that shape teachers' decisions, but a mathematical lens might have contributed additional perspective on the demands that would accompany a decision to request explanations from students.

The classroom interactions described in this book are replete with descriptions of the mathematics and mathematical practices. Still, the tools used to analyze the mathematics of teaching and learning seem still under development. As analyses proceed, the mathematics seems to slip out of view. In many ways, attention to a theory of learning is the culprit—in this book and in the field at large. A theory of learning, sociocultural or otherwise, is less likely to hold onto the mathematics of teaching and learning. Research built solely on theories of learning will repeatedly turn our attention back to the psychological and sociological basis for classroom interaction, rather than the mathematical. And although we agree with Stephen Lerman that frameworks for bringing to light the "social origins of knowledge"

(p. 97) are crucial, mathematics itself, seen as a set of social and intellectual practices engaged in communities, would contribute critically to such frameworks. As van Oers rightly points out, mathematical practices and mathematical discourse are fundamentally mathematical in nature. In other words, researchers need to attend to mathematical talk not just because it is an important resource for learning but also because talk constitutes the mathematics being taught and learned.

Examining what is distinctive about mathematical talk, unpacking the mathematics of it, and considering the ways that it constrains and enables classroom interactions, will add to our understanding of what is happening, and could happen, in mathematics classrooms. The mathematics that drove earlier mathematics education research focused more on the products and content of mathematics, not its social and intellectual practices. It is greater development of this latter perspective on mathematics as practice that would strengthen the mathematical basis for discursive analyses of mathematics teaching and learning.

CONCLUSION

This book offers many insights into the teaching and learning of mathematics. The authors' focus on classroom talk puts the key workings of instruction squarely in view. Although their focus intentionally takes attention away from the environments in which teaching and learning take place, it nonetheless provides important leverage for the improvement of teaching and learning. The authors' analyses of students and their talk help us understand essential dynamics of instruction. They show us where and how breakdowns occur and give us a better sense of what students would need to be able to do so they could productively use and contribute to classroom instruction.

Additionally, these authors concern themselves with the practice of teaching and with the mathematics at play in instruction. By conceptualizing mathematics learning as increased participation in mathematical discourse and mathematical practices and by attending closely to students' mathematical talk, they are positioned to propose work that teachers might do to better shape students' engagement in mathematics. And, although not as robust as their analyses of learning, a sensitivity to mathematical issues is evident in their choices of episodes and in the tenor of their discussions.

At the same time, the book's attention to the three key components (students, teachers, and mathematics) is uneven. Its analyses of learning are abundant and compelling, but less developed for teaching and for mathematics. Although the book sets out to use communication as a lens for studying learning, teaching, and mathematics, many of the studies shift instead to advancing sociocultural theories of learning. This creates an imbalance, obscuring significant aspects of what is important in classroom interactions. Without adequate consideration of what is happening with regard to all three components of instruction, interpretations remain incomplete and lessons tenuous. The reasons for this imbalance are straightforward. Researchers are blessed with robust theories of learning, whereas existing theories of teaching and of mathematics pale by comparison. The primitive state of such theories is evident throughout mathematics education research. *Learning Discourse* steps firmly into this terrain, providing a helpful parley.

At times, we sensed that the authors' attempts to advance sociocultural theories of learning and to introduce new adaptations of discursive methods may inadvertently confound their interest in improving teaching and learning. Why might this be?

Schwab argues that education, because it is concerned first and foremost with solving problems of practice rather than building theory, needs to make eclectic use of theory in its attempts to improve practice (Schwab, 1961/1978). Researchers' overall attention needs to be to problems of teaching and learning, not to theoretical perspectives. They need to choose and combine units of analysis, methodological tools, and theoretical frameworks as they probe practical problems of instruction.

In many ways, *Learning Discourse* follows Schwab's lead. Analyses begin with basic problems of practice. The focus on communication keeps analyses grounded in practice. Likewise, the book seeks to bridge units of analysis and to use theory as a resource. For example, in turning to sociocultural theories of learning and discursive methods, these authors are trying to "bridge the individual and social" in their analyses of learning (p. 9). And, as Sfard points out, "communicational approaches should be seen as a complement rather than as a replacement for the more traditional outlooks" (p. 49). These are important moves.

However, by systematically structuring analyses around a theory of learning, attention is pulled away from both mathematics and teaching. Without more balanced attention to all three components and to their dynamic, we lack resources for improving mathematics education. A next step suggested by this book's accomplishments is to find theories that help bridge the individual, the social, and the mathematical, and simultaneously, to use theory eclectically in our efforts to understand the role of students, teacher, and mathematics in the teaching and learning enterprise. This book gets us close enough to see this. May we use its lessons wisely.

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