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Agent-based and analytical modeling to evaluate the effectiveness of greenbelts

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Abstract

We present several models of residential development at the rural–urban fringe to evaluate the effectiveness of a greenbelt located beside a developed area, for delaying development outside the greenbelt. First, we develop a mathematical model, under two assumptions about the distributions of service centers, that represents the trade-off between greenbelt placement and width, their effects on the rate of development beyond the greenbelt, and how these interact with spatial patterns of aesthetic quality and the locations of services. Next, we present three agent-based models (ABMs) that include agents with the potential for heterogeneous preferences and a landscape with the potential for heterogeneous attributes. Results from experiments run with a one-dimensional ABM agree with the starkest of the results from the mathematical model, strengthening the support for both models. Further, we present two different two-dimensional ABMs and conduct a series of experiments to supplement our mathematical analysis. These include examining the effects of heterogeneous agent preferences, multiple landscape patterns, incomplete or imperfect information available to agents, and a positive aesthetic quality impact of the greenbelt on neighboring locations. These results suggest how width and location of the greenbelt could help determine the effectiveness of greenbelts for slowing sprawl, but that these relationships are sensitive to the patterns of landscape aesthetic quality and assumptions about service center locations.

Keywords: Land-use change; Urban sprawl; Agent-based modeling; Landscape ecology

1. Introduction

Population increase, decreasing household sizes (Liu et al., 2003), and increases in area developed per household (Vesterby and Heimlich, 1991) all contribute to increase in the amount of land converted for development in metropolitan areas throughout the world. Land development for residential, commercial and industrial uses at the urban–rural fringe can have a variety of negative ecosystem impacts, including habitat destruction and fragmentation, loss of biodiversity, and watershed degradation (Alberti, 2000). Landscape ecological theory (Turner et al., 2001) suggests that, in addition to how much development occurs, the extent of these impacts is determined by where the development occurs relative to ecological features and its overall spatial pattern.

A number of approaches have been proposed to minimize the ecological impacts of development, by manipulating the spatial patterns of development to minimize sprawl and excess land usage. These approaches include establishment of greenbelts of preserved lands around cities (Mortberg and Wallentinus, 2000), clustered or "new urbanism" designs (Arendt, 1991), which involve increased use of higher density development and mixtures of land uses within developments, purchase or transfer of development rights (Daniels, 1991), and alteration of tax or investment policies (Boyd and Simpson, 1999), among others. For each of these alternative strategies, the costs of implementation need to be considered (Boyd and Simpson, 1999) along with the long term conservation benefits obtained.

To evaluate the benefits of any given option, the dynamics of development at the urban–rural fringe and

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their linkages to ecological impacts need to be understood. Because the impacts are driven to a large extent by the location and spatial patterning of the development, this understanding needs to be spatially explicit. In order to understand the drivers of urban development and their possible future impacts on land development, and to develop scenarios that can be used to test alternative approaches to minimizing these impacts, a variety of spatial modeling approaches have been employed. The work of Landis and colleagues (Landis, 1994; Landis and Zhang, 1998a, b) illustrates a simulation approach based on discrete choice statistics that focuses on estimating the likely locations of development. Similarly, Pijanowski et al. (2002) used artificial neural networks to identify non-linear interactions between predictor variables and likely locations of development. Alternative modeling approaches have focused on how the patterns of development evolve through spatial interactions and, in many cases, have used analogies with physical systems (e.g. diffusion limited aggregation and correlated percolation) to represent processes of urban growth (Makse et al., 1998; Zanette and Manrubia, 1997). Cellular models (Clarke et al., 1997) represent an approach that is intermediate in realism between statistical location models and physical analog interaction models, combining some of the strengths of both.

These powerful simulation models have been used to evaluate the impacts of a variety of land-use policy instruments. Each of them represents the land-use state at each location and the variables and processes that determine that state. An important next step in the evolution of land-use models, and improving their utility for policy scenarios, is directly representing the heterogeneous set of actors in the land-use change process (Page, 1999), their decision making processes, and the physical manifestation of those changes on the landscape. Agent-based models (ABMs) serve as tools for this purpose. Otter et al. (2001) presented an ABM of land development that includes a reasonable representation of the different types of agents and that makes an initial contribution on which further developments in this area might build. Further, experimentation with this kind of model can improve our understanding of how the interaction between landscape characteristics and the preferences and behaviors of agents might influence ecological diversity and function.

A key challenge in modeling such multi-agent systems with agent-based models is providing confidence in the models' results (Parker et al., 2003). Often establishing confidence in a computer model is divided into two steps: (1) verifying that the computer program is free of "bugs" and correctly implements the conceptual model and (2) validating the model by showing it generates output that matches the relevant aspects of the system being modeled (Kelton and Law, 1991). In practice, carrying out those procedures is not so straightforward. First, verification of program correctness cannot be guaranteed for any but the simplest of programs; thus in practice we can only increase confidence that a program is correct by a combination of software engineering and testing techniques (McConnell, 1993). Second, validation also is a non-trivial exercise, since it involves judgements about how well a particular model meets the modeller's goals, which in turn depends on choices about what aspects of the real system to model and what aspects to ignore. Critical issues that must be considered include what level of detail to try to match (data resolution) and how to handle issues of "deep uncertainty" found in complex adapative systems (Bankes, 2002).

Because of these difficulties, typical practice is to establish confidence in the results of a model through a mix of techniques, most of which contribute to both verifying and validating the model. Sensitivity analysis and other "parameter sweeping" technique can provide support for computer program correctness and model plausibility, by improving understanding of the behavior of a model under a range of plausible conditions (Kelton and Law, 1991; Miller, 1998). In some cases model calibration is carried out, i.e. model parameters are adjusted ("tuned") until the model output matches the real world data of interest. For the calibration to be convincing, we also must show those parameter values are "plausible," e.g. by basing them on empirical data or by arguing that experts support the "face validity" of the parameters chosen. We also can "dock" models to other related models (Axtell et al., 1996), to show the results are common to more than just one model or implementation.

Beyond simple verification and validation of an ABM, we also want to be confident that we have a clear understanding of the agent-based model's processes and of the behavior and results those processes produce. Because agent-based modeling is a new, potentially valuable approach to understanding complex phenomena like settlement patterns, much can be gained from understanding the models themselves. Further, such an understanding of an ABM is a necessary step in using the model to understand the fundamental processes in the (more complex) real world system that the model is meant to represent.

Because an ABM usually is itself a complex system, it can take considerable effort to understand even the simplest of models (Casti, 1997; Axelrod, 1997; Bankes, 2002). Axelrod (1997) argues that simulation is a third way of doing science, combining aspects of deduction (knowledge based on proofs from axioms) and induction (knowledge from observed regularities in empirical data). That is, the ABM can be viewed as a fully specified formal system (like the axiomatic basis for deducing theorem proofs) which, when run, generates data that requires careful analysis (induction) to understand and summarize. For instance, we can induce regularities by analyzing the model output in ways similar to those used on data from a real-world system[1].

In this paper we demonstrate another way to understand the basic processes in an agent-based model and, by extension, to help us understand processes that may be at play in the system being modeled. The approach we use in this paper involves comparing the behavior of an agent-based model to the behavior of a simpler mathematical model of land development. This comparison has a number of benefits, including:

- By having two separate "implementations" which both generate the same fundamental results, we increase our confidence in the veracity of both models;
- The results from the stark mathematical model can be shown to hold in more general contexts which an ABM can represent, e.g. spatial heterogeneity, discrete service center distributions and other extensions not amenable to mathematical analysis; and
- The theorems we are able to prove for the mathematical model give us deeper insights into the processes that generate the fundamental dynamics of the ABM.

In general, agent-based models may be constructed to serve as minimal *realistic* models of real-world complex adaptive systems. However, the fact that we often cannot prove theorems about the agent-based models makes for a shaky foundation. But, if we can both prove theorems about simplifications of the ABMs *and* show that the conclusions of those theorems hold in more general agent-based models, we enrich the scientific enterprise.

The comparison of an ABM to a simpler mathematical model can also be viewed as a kind of "docking" exercise (Axtell et al., 1996). In this case one model is computational and the other is mathematical (instead of comparing two computational models), but the basic goal is the same, i.e. to study the "…troublesome case in which two models incorporating distinctive mechanisms bear on the same class of social phenomena, …" (Axtell et al., 1996, Section 1.1), in part to carry out "…tests of whether one model can subsume another" (Axtell et al., 1996, abstract). As emphasized in Axtell et al. (1996), a key issue is how to assess the "equivalence" of two models. For this paper, we focus on "relational equivalence" between the models, showing that they both generate the same relationships between results, e.g. as analogous parameters are varied. If the models are relationally equivalent, we can be more confident that (1) the mathematical model helps us understand the key processes in the ABM, and (2) the ABM can be viewed as subsuming the mathematical model, allowing us to study a wide variety of cases that are mot mathematically tractable.

In summary, in this paper we present several models of residential development at the rural–urban fringe. In all models, the common conceptual model consists of agents choosing where to locate based on preferences for minimizing distance to services and maximizing aesthetic quality of the chosen location. We use the models to evaluate the effectiveness of a greenbelt, which is adjacent to a developing area, for delaying development outside of the greenbelt. Our one-dimensional mathematical model focuses on the interactions between greenbelt location and width, the spatial distribution of aesthetic quality, and the resultant amount and timing of development beyond the greenbelt. We explore the model under two different assumptions about the spatial pattern of service centers. Next, we implement the same basic mechanisms of the mathematical model in a one-dimensional discrete ABM setting. We then demonstrate the flexibility of the ABM framework by relaxing assumptions and extending the representation of the system to include (1) a two-dimensional landscape and (2) an effect of the greenbelt on the aesthetic quality of the nearby environment.

2. Methods

2.1. Mathematical model

We first construct a one-dimensional mathematical model of resident settlement choices in the presence of a greenbelt. We use this model to derive some basic properties about greenbelts, such as a tradeoff between the width of a greenbelt, its location and the rate of development to its right. These basic principles, then, set the stage for evaluation of dynamics within the agent-based modeling framework, described in Section 2.2

In the basic model, agents care about two features of a location x: its distance to services, and its aesthetic quality, which we denote by q_x. Aesthetic quality is defined as the value that residential agents derive from locations because of their scenic and other natural amenities. We assume that an agent's utility from a location increases in proportion to the location's aesthetic quality and decreases in proportion to its distance to services, and that agents choose to occupy the location that maximizes utility. In this model, we assume that there is a finite number of agents.

Each of M agents chooses a location from the set $\{0, 1, \ldots N\}$ *at which to live. At most, one agent can live at each location. Therefore,* $M \leq N+1$.

F: $\{0, 1, \ldots, N\} \rightarrow \{0, 1\}$ *denotes the locations of the agents.* $F(x) = 1$ *if an agent resides at location x and* 0 *otherwise.*

[1] The key methodological difference between how we analyze output of agent-based models versus real-world data is that for ABMs we have less use for formal statistical measures like t statistics, because we can achieve a trivial kind of statistical significance by running the model an arbitrary number of times.

A greenbelt (g, w) *begins at the location* $g \in \{0, 1, \ldots, N-w+1\}$ *of width* w *with* $g \geq M$. *No agents may live in the locations* $\{g, (g+1), \ldots, (g+w-1)\}$.

The purpose of the greenbelt is to keep all of the agents on one side, in this case to the left. For convenience, we will say that an agent at location x resides left of the greenbelt if $x<g$ and to the right of the greenbelt if $x> (g+w-1)$. Notice that in our definition of a greenbelt, we required that $g \geq M$. Without this constraint, the greenbelt cannot prevent sprawl.

Given a distribution F, the utility to an agent living at location x is given by:

$$U(x,F) = q_x - s(x,F) \tag{1}$$

where $s(x, F)$ *is the distance from x to services, which can be a function both of the location of the agent and of the distribution of all agents.*

In our two-dimensional (2D) ABMs, we begin with a service center on the left most edge of the grid[2]. Subsequent service centers gradually locate rightward as the population grows (see process description below). To capture these two characteristics of the service centers, their bias to the left and their spread with the population, we consider two distinct cases for the mathematical model. In the first, we assume that there is a single service center at the leftmost edge of the space. This assumption corresponds with the mechanism used in the one-dimensional (1D) ABM. In the second, we assume that the distance to services left of the greenbelt depends only upon the number of agents located there. This second case contains two implicit assumptions. First, the services are evenly distributed relative to agents left of the greenbelt, and second no one left of the greenbelt jumps the greenbelt to obtain services. The first of these implicit assumptions makes sense provided that services are fairly divisible or travel costs left of the greenbelt relatively low or equal[3]. We formalize these assumptions as follows:

Case 1. Left Edge Service Centers (LESC): $s(x, F) = x$

Case 2. Evenly Spaced Service Centers (ESSC): *If K agents live to the left of the greenbelt* $\left(\sum_{y=0}^{g-1} F(y) = K\right)$ *then* $s(x,F) = \frac{\eta q}{K}$ *for* $x<g$, *where* η *is a parameter representing the density of services.*

Under LESC, the utility to an agent at location $x<g$, $U(x, F)$ equals $q_x - x$, under ESSC it equals $q_x - \frac{\eta q}{K}$.

Notice that neither of these assumptions depends much on the particulars of F. Under LESC, distance is independent of F and under ESSC, all that matters is the number of agents to the left of the greenbelt. Nevertheless, we keep the $s(x, F)$ notation because, in our 2D ABMs, the distribution of services and hence the distance to them depends explicitly on where agents locate.

To analyze whether a greenbelt prevents sprawl, we need to compare the utility to the Mth agent living left of the greenbelt with the utility that the agent could obtain if it jumped the greenbelt. We assume that if a single agent lives right of the greenbelt it must cross the greenbelt to get services. Once the agent crosses the greenbelt, which is width w, the agent has the same distance to services as someone living on the left edge of the greenbelt.

If a single agent chooses a location $y \geq g + w$, *then we can write that agent's utility as*

$$U(y,F) = q_y - (y - g) - s(g - 1,F) \tag{2}$$

We will say that a greenbelt of width w beginning at g *prevents sprawl* if it is the case that if $M-1$ agents locate left of the greenbelt, then the Mth agent will strictly prefer to locate on the left side of the greenbelt as well. This definition does not imply that the greenbelt will always prevent sprawl, only that it *could* prevent sprawl. If a developer provided services right of the greenbelt, settlement might occur there. Our definition says that if no such development occurred right of the greenbelt, then an individual would have less incentive to live there.

Building on this framework, we develop proofs for a number of claims with respect to the interactions between greenbelt placement, width, and effectiveness. These results are compared with results from the agent-based models.

2.2. Agent-based models

We describe three agent-based models in this paper. The ABM approach allows us to evaluate dynamics similar to those of the mathematical model, but also to relax assumptions and include the effects of alternative location preferences of the residential population, incomplete or imperfect information available to residents, spatial variations in the aesthetic quality of the landscape, and the locations of services provided to the residential population (e.g. including jobs, retail, and schools).

The models are kept as simple and stark as possible to allow comparison with the mathematical model and experimentation with critical aspects of the dynamics of the system. Agents in the model include both residents and service centers. Their function is to locate themselves on a one- or two-dimensional lattice that has a set of heterogeneous attributes. Residential agents choose their locations on the lattice by examining the environmental and location attributes, including distance to service centers, of multiple locations.

[2] Service centers are assumed to take no space in the mathematical model.

[3] Travel costs would be relatively equal if everyone were taking public transportation.

The models are modular. In other words, certain functions in the models can be controlled while others are examined. This allows the introduction of additional agents, attributes, and behaviors as needed. The models were developed using Swarm[4] and are available on-line[5]. We describe the three major elements of the models in turn: the environment, the agents that locate themselves within that environment, and the ways the agents interact with the environment and each other. Next, the differences between the three models are described. We developed a 1D model (ABM 1D) for direct comparison with the mathematical model, then two 2D models (ABM 2D and ABM 2Dq) that demonstrate extensions of the simpler model.

2.2.1. The Landscape

Each cell in a lattice (representing a location on the landscape where a resident can locate) is described by attributes that affect agent behavior. These can include soil quality, ecological sensitivity and other factors. The single environmental attribute we use in this paper is termed aesthetic quality (q_{xy}), which is defined in the same way as in the mathematical model. We implemented the attribute as a score in the range [0, 1]. In the models presented here, the score is set according to an assumed spatial distribution at the beginning of a model run and is not changed by development that occurs during the run.

The greenbelt is represented in the models by identifying certain cells as "preserve," which can not be developed. Neither residents nor service centers can locate in these areas. The greenbelt is described by two parameters: (1) preserve start (g), the x-location that is the start of the greenbelt, assuming that the far left is 0; and (2) preserve width (w), the width of the greenbelt. In the 2D models, the greenbelt is assumed to be a continuous rectangle from the top of the lattice to the bottom.

The width of the landscape, *X Size*, is increased by the value of w to allow comparison between runs. Therefore, in all ABM experiments the total number of sites available for development remains constant.

Another attribute assigned to each cell on the lattice described the location of each cell in the lattice relative to service centers, called Service Center Distance (sd_{xy}). This variable describes how accessible each cell is to service centers and is recalculated each time a new service center is added. sd_{xy} is measured by summing the inverse of Euclidean distances to the nearest eight service center locations from that cell. Using that formula alone, a cell in a 2D landscape that is surrounded by service centers would receive a score of eight. Because it seems reasonable that the residents of a cell would not receive additional benefit from more than about two immediately adjacent service centers, we set maximum contribution to utility from service centers to be two. Thus,

$$sd_{xy} = 0.5 * \max\left[2, \left(\frac{1}{\|sc_1\|} + \cdots + \frac{1}{\|sc_8\|}\right)\right] \tag{3}$$

where $\|sc_i\|$ is the Euclidean distance to the ith nearest service center from x, y. Thus, a cell adjacent to two or more service centers receives the maximum sd_{xy}= 1.0. We use Euclidean distance in these models for simplicity, but later versions include options for Manhattan and road network distances.

2.2.2. Agents

The two basic agent types in the models are residents and service centers. When a resident or service center enters the landscape, it takes up one cell in the lattice. Once a cell is occupied, it is unavailable for new residents or service centers. Although residents have multiple attributes that affect how they evaluate locations and that can be used to distinguish among different types of residents, service centers do not have any attributes. At present service centers are merely protoagents, designed to represent the range of commercial and industrial concerns to which residents need access for goods and employment. Their behavior is relatively automatic and simple, but their presence greatly affects how residents determine where to live.

Residents have two important attributes: (1) *aesthetic preference* ($\alpha_q \in [0, 1]$), the weight that an agent gives to aesthetic quality in deciding where to locate; and (2) *service center preference* ($\alpha_{sd} \in [0, 1]$), the weight that an agent gives to the nearness of an area to service centers. Though the distribution of preferences can be set in a variety of ways, we use only three different combinations of settings for the agent preferences, all of which result in all residents in a given run having identical preferences. Preferences were either (a) all set to 0.0, meaning that the agents locate themselves in the world randomly, (b) set such that $\alpha_{sd} = 0.5$ and $\alpha_q = 0.0$, or (c) $\alpha_{sd} = 0.5$ and $\alpha_q = 0.5$.

2.2.3. Agent behavior

The agent behavior of interest is how new residents locate themselves on the lattice. Each model run begins with an initial service center located on the left edge of the lattice. During each step of a model run, a number of new residents enters the map. The rate of residents moving into the landscape is determined exogenously. Residents then choose their locations based on the set of defined preferences and landscape attributes.

[4] Available from http://www.swarm.org.

[5] Go to http://cscs.umich.edu/sluce under models. All models in this paper used the same code base.

To select a location, a new resident T looks at some number of randomly selected cells and moves into the cell that has the highest utility for T (with ties broken randomly). Utility is calculated slightly differently in the 1D (Section 2.2.4) and 2D (Section 2.2.5) models.

2.2.4. *ABM 1D*

The first of our ABMs (ABM 1D) was designed to dock to the LESC case of the one-dimensional mathematical model. The landscape size is defined by its width *X Size* (the height, or *Y Size* is always one). *X Size* has a minimum of 80 but is variable, depending on the width of the greenbelt (see Section 2.2.1).

For the 1D model runs, a single service center is initially placed on the left side and, like the LESC case but in contrast to the 2D models (Section 2.2.5), no others are created during the runs. The number of residents entering the landscape was set to 1 per step. The number of cells a resident samples before selecting a location was set to an arbitrarily large number to allow residents to sample all available locations (equivalent to the perfect-information assumption of the mathematical model). The utility of a cell to the agent in this simple model is $\alpha_{sd} * sd_{xy}$ (where $\alpha_{sd} = 0.5$).

One experiment was run with ABM 1D to match as closely as possible the assumptions of the simplest of the LESC case of the mathematical model (Table 1). All residents had identical preference for distance to services and zero preference for aesthetic quality (i.e. aesthetic quality was constant). Through this very restricted case, we were able to duplicate a specific instance of the mathematical modeling results, specifically Claim 1 described in Section 3.1, within the ABM framework.

2.2.5. *ABM 2D*

The landscape of our two-dimensional ABMs (i.e. ABM 2D and ABM 2Dq) is a two-dimensional lattice of size *X Size* by *Y Size* (illustrated in Fig. 1).

Table 1
Parameter settings for agent-based model experiments

Experiment	Model	α_{sd}	α_q	Aesthetic quality distribution
1	ABM 1D	0.5	0.0	uniform
2	ABM 2D	0.0	0.0	uniform
3	ABM 2D	0.5	0.0	uniform
4	ABM 2D	0.5	0.5	random
5	ABM 2D	0.5	0.5	left high
6	ABM 2D	0.5	0.5	right high
7	ABM 2D	0.5	0.5	tent
8	ABM 2D	0.5	0.5	valley
9	ABM 2Dq	0.5	0.5	left high
10	ABM 2Dq	0.5	0.5	right high
11	ABM 2Dq	0.5	0.5	tent
12	ABM 2Dq	0.5	0.5	valley

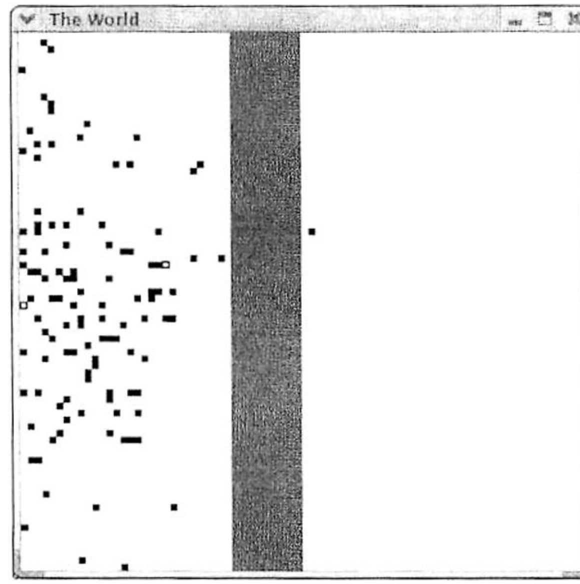

Fig. 1. Graphic output from one run of our agent-based model. Cells with residential agents are black, those with service centers are white with black outlines, and those in the greenbelt are gray.

2D model runs use a constant *Y Size* of 80 (*Y Size*= 1 for the one-dimensional case) and a variable *X Size*, as described in Section 2.2.4. The initial service center is placed in the middle of the left edge of the landscape.

The rate of new residents entering the landscape was set to 10 per step. Residents sample only 15 cells from the landscape before selecting a location. This selection process is intended to reflect the effects of incomplete or imperfect information available to the residents as they select a location. The utility of the cell at location x, y for a given agent with specified α values is determined in the following way:

$$u_{xy} = 0.5 * \left(\alpha_q * q_{xy} * sd_{xy} + \alpha_{sd} * sd_{xy}^2\right) \quad (4)$$

This equation captures the empirical observation that, although aesthetic quality is an important determinant of utility, it is not generally considered independently of distance to services, which provides access to jobs, health care, entertainment, etc. With this utility function, residents consider the tradeoffs between aesthetic quality and distance to services, and weight near locations much higher using squared distance.

After some number of residents is created (arbitrarily set to 100), a service center is created near the last resident to enter the model[6]. This process, which we believe to be reasonable, introduces an important positive feedback to the system that can result in path dependent behavior. Because the initial service center

[6] This process is included in both 2D models and approximates the entry of service centers near areas of new residential development, i.e. in response to or in anticipation of demand for services.

was located on the left edge and because service centers fan out from the left as development increases, we expected the 2D ABMs to behave somewhere in between the LESC and ESSC cases described in Section 2.1.

Experiments conducted with ABM 2D evaluated the effects of five different idealized patterns of aesthetic quality for evaluating model dynamics (Table 1): random; "left-high"-high values on the left of the map, decreasing linearly to low values on the right; "right-high"-the opposite of left-high; "tent"-a ridge of high values along the center two rows of the landscape, with values decreasing linearly to the top and bottom; and "valley"-similar to tent, but with high values on the top and bottom edges and decreasing towards the center.

2.2.6. ABM 2Dq, with greenbelt affecting quality

The third ABM, which we call ABM 2Dq, is identical to ABM 2D (Section 2.2.5), but it includes a modification in which the greenbelt results in higher values of aesthetic quality at neighboring cells. The effect falls quickly and linearly with distance from the greenbelt, such that adjacent cells have an aesthetic quality score of one, cells that are three cells from greenbelt have a score of 1/3 and cells more than three cells from the greenbelt are unaffected by the greenbelt. The aesthetic quality near the greenbelt is the maximum of (a) the score based on the predefined pattern and (b) the score based on proximity to the greenbelt. The ABM 2Dq experiments evaluated the effects of the greenbelt affecting neighboring quality for four of the different initial patterns of quality described in Section 2.2.5 and listed in Table 1.

2.2.7. Measuring model outcomes

The outcomes of ABM 1D were evaluated by running the model until the total number of agents equals the total number of cells to the left of the greenbelt. This is equivalent to the constraint on the mathematical model that $g \geq M$ (Section 2.1). We then recorded whether or not any of the agents located to the right of the greenbelt.

To measure the degree to which the greenbelt served to forestall development beyond the greenbelt in both of our 2D ABMs, we recorded the number of developments beyond the preserve (*dbp*) at each time step. This is the number of residents and service centers that have an x value greater than $w+g$. We then calculated $T(dbp=300)$, the average number of time steps that it took for 300 cells on the right side of the greenbelt to be developed. The threshold is arbitrary, but selected as a reasonable indicator to allow comparison among runs and experiments. This measure gives an indication of how effective the greenbelt is at delaying development beyond the greenbelt.

3. Results

The results presented below describe the effects that greenbelts have on the locations of development, taking mathematical and agent-based approaches in turn.

3.1. Mathematical modeling results

The results of the mathematical model are presented as a series of claims with corresponding proofs. Here we show how increasing the width of a greenbelt necessarily increases the probability it prevents sprawl, but pushing the greenbelt further out need not, depending upon the assumption we make about the placement of services. We also show how the correlation between aesthetic quality and the location of the greenbelt can impact greenbelt efficacy.

Our first claim states that if the aesthetic quality is the same for all locations then any greenbelt prevents sprawl.

Claim 1. *Under either LESC or ESSC if $q_x=q$ for all x, then any greenbelt prevents sprawl.*

Proof. The utility to the Mth agent if it locates at $x<g$ to the left of the greenbelt equals $q - s(x,F)$, but if the Mth agent locates at $y \geq g+w$ right of the greenbelt its utility equals $q-(y-g)-s(g-1,F)$. Under either LESC or ESSC, $s(g-1,F) \geq s(x,F)$ if $x<g$. Since the agent living to the right of the greenbelt must also subtract the distance $(y-g)$ from its utility, the result follows.

The previous claim may seem rather obvious but it hints at an important insight. In our model, the pursuit of aesthetic quality compels agents to jump the greenbelt. Greenbelts will have a rougher time preventing sprawl if the area right of the greenbelt has high aesthetic quality. Therefore, if the greenbelt, can encompass the regions of highest aesthetic quality, it stands a better chance of preventing sprawl.

Prior to stating our next claim, we introduce two new variables. We define the *best location right of the greenbelt*, $\rho(g, w)$ to be the location $y \geq g+w$ that maximizes $q_y-(y-g)-s(g-1,F)$. Similarly, let ℓ_g denote the *location left of the greenbelt that gives the Mth highest utility*. Claim 2 states that the greenbelt prevents sprawl so long as the loss in distance to services exceeds the gain in aesthetic quality from jumping the greenbelt.

Claim 2. *Under either LESC or ESSC, a greenbelt (g, w) prevents sprawl if $(\rho(g,w)-g) > (q_{\rho(g,w)}-q_{lg})$.*

Proof. The utility to the Mth agent if it locates at $\ell_g < g$ equals $q_{\ell_g} - s(\ell_g,F)$. If the agent locates right of the greenbelt, the highest utility it can obtain is $q_{\rho(g,w)} - (\rho(g,w) - g) - s(g-1,F)$. Again under either LESC or ESSC, $s(g-1,F) \geq s(\ell_g,F)$. It therefore follows that the utility is higher left of the greenbelt if $(\rho(g,w) - g) > (q_{\rho(g,w)} - q_{\ell_g})$.

Several corollaries follow from this claim. The first states that if a greenbelt prevents sprawl and is made wider, then it will continue to prevent sprawl.

Corollary 1. *Under either LESC or ESSC, if the greenbelt* (g, w) *prevents sprawl, then so does any greenbelt* (g, w')*, where* $w' > w$.

Proof. If $w' > w$, then there are two possibilities. First suppose that $\rho(g,w) = \rho(g,w')$ in which case the result follows because the utilities are unchanged. Second, suppose that $\rho(g,w) \neq \rho(g,w')$. By assumption, $\rho(g, w)$ was the best location right of the greenbelt (g, w). Therefore, the utility $U(\rho(g, w), F)$ is greater than or equal to the utility of any other location right of $\rho(g, w)$, including $\rho(g, w')$.

$$q_{\rho(g,w)} - (\rho(g,w) - g) - s(g-1,F) \geq q_{\rho(g,w')} - (\rho(g,w') - g) - s(g-1,F) \quad (5)$$

Next, using the same notation as the previous claim, since by assumption (g, w) prevented sprawl, the utility of the Mth best location left of g is greater than the best location right of (g, w):

$$q_{\ell_g} - s(\ell_g,F) > q_{\rho(g,w)} - (\rho(g,w) - g) - s(g-1,F) \quad (6)$$

which in turn implies that

$$q_{\ell_g} - s(\ell_g,F) > q_{\rho(g,w')} - (\rho(g,w') - g) - s(g-1,F) \quad (7)$$

which completes the proof.

The second corollary states that the same is true for pushing the start of the greenbelt further to the right provided that all service centers are on the left edge (LESC).

Corollary 2. *Under LESC, if the greenbelt* (g, w) *prevents sprawl then so does the greenbelt* (g', w) *if* $g' > g$.

Proof. Note that increasing g cannot lower the utility to the Mth agent living to the left of the greenbelt. If $\ell_g = \ell_{g'}$, utility is unchanged. If not, $\ell_{g'} \geq g$ and utility weakly increases. Therefore, it suffices to show that the utility to the first agent moving to the right of the greenbelt cannot increase when the greenbelt moves to the right. As in the previous corollary, there are two possibilities. First suppose that $\rho(g,w) = \rho(g',w)$, in which case the result follows immediately because the utilities are unchanged. Second, suppose that $\rho(g,w) \neq \rho(g',w)$. By assumption, $\rho(g, w)$ was the best location right of the greenbelt (g, w). Given LESC, the utility from locations $\rho(g, w)$ and $\rho(g', w)$ do not change when the start of the greenbelt moves from g to g'. Therefore, it must be the case $\rho(g, w)$ now lies in the interior of the greenbelt. Therefore, the new best location to the right of the greenbelt, $\rho(g', w)$, cannot give higher utility than $\rho(g, w)$.

The third corollary states that a similar result need not hold under ESSC. The intuition behind this finding is that the distance from the best location right of the greenbelt (g, w) to the start of the greenbelt will decrease if that location does not become part of the new greenbelt (g', w). Therefore, if we increase g we implicitly move service centers further to the right and that may make a location right of the original greenbelt relatively more attractive.

Corollary 3. *Under ESSC, if the greenbelt* (g, w) *prevents sprawl it does not necessarily imply that the greenbelt* (g', w) *prevents sprawl for* $g' > g$.

Proof. The proof is by construction of a sufficient condition under which increasing g by one makes preventing sprawl more difficult. Let $g' = g+1$. Assume that $\ell_g = \ell_{g+1}$ and that $\rho(g,w) - \rho(g+1,w) = g+w+2$, so that the best locations right and left of the greenbelt do not change. Further, assume that the greenbelt (g, w) prevents sprawl, i.e. the Mth agent obtains higher utility moving to ℓ_g than moving to $\rho(g, w)$, leaving $M-1$ agents to the left of g:

$$q_{\ell_g} - \frac{\eta g}{M} > q_{\rho(g,w)} - (w+2) - \frac{\eta g}{M-1} \quad (8)$$

The condition for the greenbelt $(g+1,w)$ to *not* prevent sprawl can be written as:

$$q_{\rho(g+1,w)} - (w+1) - \frac{\eta(g+1)}{M-1} > q_{\ell_{g+1}} - \frac{\eta(g+1)}{M} \quad (9)$$

Given that $q_{\ell_g} = q_{\ell_{g+1}}$ and $q_{\rho(g+1,w)} = q_{\rho(g,w)}$, we can rewrite these inequalities as

$$\frac{\eta g}{M(M-1)} + w + 2 > q_{\rho(g,w)} - q_{\ell_g} \quad (10)$$

and

$$\frac{\eta(g+1)}{M(M-1)} + w + 1 < q_{\rho(g,w)} - q_{\ell_g} \quad (11)$$

Therefore, increasing g by one makes preventing sprawl more difficult provided that

$$\frac{\eta g}{M(M-1)} + 1 > \frac{\eta(g+1)}{M(M-1)} \quad (12)$$

This can be written as $M(M-1) > \eta$ which is easily satisfied for large M.

To summarize these three corollaries, pushing a greenbelt further out does not necessarily mean that it will be more likely to prevent sprawl, but making the greenbelt wider will. Under LESC, pushing the greenbelt further right does have the expected effect. The proof under ESSC relied on a counterexample. This suggests the question of whether the result that holds for LESC holds for ESSC in expectation given some distribution of aesthetic quality. As we shall now show, demonstrating that the probability that a greenbelt (g, w) prevents sprawl increases in g is problematic.

Recall that ℓ_g is the location with the Mth highest aesthetic quality among those locations left of the greenbelt. Let U^{left} be the random variable that equals the utility to the agent residing at ℓ_g and let U^{right} be the random variable that equals the utility to an agent living at $\rho(g, w)$ given that $M-1$ agents live left of g.

The probability that a greenbelt (g, w) prevents sprawl equals the probability that U^{left} is greater that U^{right}. This is equivalent to the following inequality

$$q_{\ell_g} - \frac{\eta g}{M} > q_{\rho(g,w)} - (\rho(g,w) - g) - \frac{\eta g}{M-1} \tag{13}$$

It suffices to show that as g increases, this inequality becomes easier to satisfy for a fixed w. There are three effects to consider. First, though increasing g increases both $\frac{\eta g}{M}$ and $\frac{\eta g}{M-1}$, it increases the latter by more. Therefore, the net effect is a relative decreases in the right hand side of the inequality as g increases. Second, q_g is weakly increasing in g because there are more locations from which to draw the Mth best. Therefore, the left hand side of the inequality gets larger. The third effect depends on whether increasing g to g' places the location $\rho(g, w)$ left of the new greenbelt (g', w). If so, a new best location right of the greenbelt would have to be located. This decreases the right hand side of the inequality. But, if not (if $\rho(g, w)$ is unchanged), then the term $(g-\rho(g,w))$ increases by one and the greenbelt is likely to be less effective.

Suppose that we increase g by one. There are two cases to consider. First, suppose that $\rho(g,w) = g+w$, then increasing g by one increases the probability that the greenbelt prevents sprawl. Second, if $\rho(g,w) > g+w$, then the probability that the greenbelt prevents sprawl increases if and only if

$$q_{g+1} - q_g + \frac{\eta g}{M(M-1)} > 1 \tag{14}$$

This inequality may hold for some M, g and for some distributions of aesthetic quality, but for large M the result is not likely to hold unless aesthetic quality increases in g at least linearly, a case we analyze next.

This analysis shows that we cannot say for certain or even probabilistically that increasing g helps to prevent sprawl under ESSC, but it does suggest that, holding w constant, g should be increased so that the locations just right of the new greenbelt are of relatively low aesthetic quality. Further, if g gets especially large then our assumption about uniform distance to services becomes unlikely to hold and the probability of jumping the greenbelt decreases accordingly.

As we mentioned, these results were proven without any assumptions about the distribution of aesthetic quality. With the 2D agent-based models, we run experiments with particular patterns of aesthetic quality. Under these scenarios, the results for LESC will be unchanged, but it could be that the results for ESSC, which relied on the construction of a counterexample, do change, so they are worth exploring in each context. In the first scenario, we assume that aesthetic quality increases linearly from the left side. To capture this formally, let the aesthetic quality of location x equal θx, where $\theta < 1$. It follows then that if M agents live left of the greenbelt then they will live at locations $g-1$ to $g-M$. Given that $\theta < 1$, it follows that the best location right of the greenbelt will be at location $g+w$. We can now state the following claim.

Claim 3. *Under ESSC, if $q_x = \theta x$, with $\theta < 1$, then a greenbelt (g, w) prevents sprawl if and only if $w > \frac{\theta M}{1-\theta}$.*

Proof. The utility to the Mth agent living left of the greenbelt equals $\theta(g - M) - \frac{\eta g}{M}$. The utility to the agent if it moves to the best location right of the greenbelt will equal $\theta(g + w) - w - \frac{\eta g}{M}$. Therefore, the greenbelt prevents sprawl if and only if $\theta(g - M) > \theta(g + w) - w$ which reduces to $(w - \theta w) \geq \theta M$. The result follows.

Notice that this result implies that the width of the greenbelt matters but not its starting point. However, this result is partially an artifact of the linearity assumption about aesthetic quality. If we allowed aesthetic quality to have a different functional form then g could matter.

In our second special case, we assume that the aesthetic quality of a location depends upon the location and width of the greenbelt. This means that we must now write q_x as $q_x(g, w)$. We assume that the aesthetic quality is highest adjacent to the greenbelt. Formally this means that $q_{g+w}(g,w) \geq q_x$ for all x. In this special case, it can be shown that increasing g makes preventing sprawl easier even under ESSC.

Claim 4. *Assume $q_{g+w}(g,w) = q^* \geq q_z(g,w)$ for all z, g, and w. Under ESSC or LESC, if the greenbelt (g, w) prevents sprawl then so does the greenbelt (g', w) if $g' > g$.*

Proof. By above the claim holds for LESC, so it suffices to show that it is true for ESSC. Since under ESSC, $q_{g+w} \geq q_y(g,w)$ for all $y \geq g + w$, it follows that $\rho(g,w) = g + w$ for all g and w.

The greenbelt (g, w) prevents sprawl implying that

$$q^* - (g + w - g) - \frac{\eta g}{M-1} < q_{\ell_g} - \frac{\eta g}{M} \quad (15)$$

Since $g' - (g' + w) = w = g - (g + w)$ this implies that

$$q^* - ((g' + w) - g') - \frac{\eta g}{M-1} < q_{\ell_g} - \frac{\eta g}{M} \quad (16)$$

Since $q_{\ell_g} \leq q_{\ell_{g'}}$, it follows that

$$q^* - ((g' + w) - g') - \frac{\eta g}{M-1} < q_{l_{g'}} - \frac{\eta g}{M} \quad (17)$$

And since $g' > g$, it follows that

$$q^* - ((g' + w) - g') - \frac{\eta g'}{M-1} < q_{\ell_{g'}} - \frac{\eta g'}{M} \quad (18)$$

which completes the proof.

Therefore, in the case where the aesthetic quality is highest near the greenbelt we should see a stronger benefit from increasing g than under the other scenarios.

3.2. Agent-based modeling results

3.2.1. ABM 1D experiment

The results for Experiment 1, run with w= 1 and 15 and g= 20 and 40, are not reported in table form because they were identical for each run. Specifically, all sites left of the greenbelt were occupied before any sites right of the greenbelt were developed every time the model was run (for a total of 30 runs for each case). Thus, with parameter settings that matched the implementation of LESC case of the mathematical model (Section 2.2.4 and Table 1), reproduced exactly the results described in Claim 1 (Section 3.1), regardless of the location (g) and width (w). This simplest case represents a strict, but limited, verification of the models, in the sense that the two models were as similar as possible and produced the same results.

3.2.2. ABM 2D experiments

The remaining results use ABM 2D and ABM 2Dq, which incorporate interacting preferences in the utility function, and incomplete or imperfect information to the agents (i.e. which introduces stochasticity). These models allow us to explore the relational equivalence of the dynamics with those found in the starker mathematical model.

The results for the 2D ABM experiments are presented using our measure of the number of developments outside the greenbelt and how quickly a critical mass (defined as 300 developed cells) is reached, $T(dbp= 300)$. A more effective greenbelt, by this second measure, is one that has a longer time until 300 cells right of the greenbelt are developed.

To explore the interacting effects of placement and width of the greenbelt, we compare results with two different values of g (20 and 40) and of w (1 and 15) for each experiment. The results obtained from 30 runs of the model for each experiment are presented in Table 2.

Table 2
Results from ABM 2D experiments. Average time to 300 developments beyond preserve, $T(dbp= 300)$. The mean and standard deviation (in parentheses) were calculated across 30 runs of the model. Parameter settings for experiments are described in Table 1

Experiment	w= 1		w= 15	
	g= 20	g= 40	g= 20	g= 40
2	39 (1)	61 (2)	39 (1)	60 (2)
3	113 (23)	275 (47)	151 (26)	337 (19)
4	86 (19)	194 (52)	103 (29)	278 (39)
5	131 (21)	320 (25)	167 (15)	344 (3)
6	44 (7)	71 (30)	47 (14)	99 (62)
7	77 (12)	171 (33)	93 (20)	221 (39)
8	90 (15)	160 (37)	115 (29)	218 (70)

Using random placement, a g of 20 and a w of 1, we calculate that it should take 39 time steps to reach dbp= 300. Changing g to 40 gives 59 time steps. The results from Experiment 2, in which resident location is determined randomly, indicate that the ABM 2D results are within one standard deviation of those expectations, for both w= 1 and w= 15, though the agent-based model tends to be slightly late in reaching the threshold level of development (Table 2). This simple result is evidence that the two-dimensional ABM is working properly (though we can never be absolutely certain that there are no programming errors).

Because of the location of the initial service center on the left edge of the landscape, setting only α_{sd} to 0.5 (i.e. Experiment 3) increased the amount of time it took for development to reach critical mass on the right side. The results show a significant increase in $T(dbp= 300)$ (Table 2). The effect is non-linear, with increasing delays accompanying increasing w and g. The relatively high number of steps before $T(dbp= 300)$ remains consistent with the findings in Section 3.1 that greenbelts prevent sprawl when decisions are influenced by location relative to service centers and not by aesthetic quality.

When we also set α_q to 0.5 (Experiment 4), the spatial pattern of aesthetic quality had an effect on the process. This case is most similar to that in Claim 2 (Section 3.1), in which the pattern of aesthetic quality affects the greenbelt effectiveness, though strict comparison is limited by a more realistic set of assumptions in ABM 2D. Setting the distribution of aesthetic quality to a random pattern causes some of the most desirable cells to lie to the right of the greenbelt. These then are selected by residents (Table 2). The inclusion of a random aesthetic quality pattern reduces the time to cross the greenbelt. For a variety of values of w and g, we found $T(dbp = 300)$ was about 75% lower in Experiment 4 than in Experiment 3.

Further results indicate that increasing the width of the area to the left of the greenbelt (i.e. increasing g) allows one to decrease the width of the greenbelt while achieving the same delay of sprawl. For instance, to achieve $T(dbp = 300) = 180$, increasing g from about 30 to 40 enables a drop of w from 15 to about 1. Because the service centers in ABM 2D tend to stay to the left of the landscape with the residents, this finding is consistent with the basic finding in Corollary 2 of Claim 2 in Section 3.1 (i.e. the LESC case), which shows that increases in g result in a more effective greenbelt.

3.2.3. Patterns of aesthetic quality

As the patterns of aesthetic quality are made more realistic, specific mathematical claims become more difficult to prove, as Corollary 3 in Section 3.1 demonstrates. However, the ABM permits evaluation of performance for any given pattern of aesthetic quality (Experiments 5 through 8).

The longest $T(dbp = 300)$ measured across all patterns of aesthetic quality were obtained with aesthetic quality decreasing from the left (Experiment 5, Table 1). Agents tended to stay to the left to be near services and to access the most high-quality sites. The increase in $T(dbp = 300)$ is about 1.5 times that for the case of random aesthetic quality. For the case of $w = 15$ and $g = 40$, the increase is slightly lower, because we only ran the model to 401 steps and runs that did not reach $dbp = 300$ by then were assigned a value of 401.

Reversing the pattern of aesthetic quality (i.e. increasing to the right) drops $T(dbp = 300)$ by one-third to one-half compared with random aesthetic quality (Experiment 6, Table 1). The logic is the reverse of the above.

The results using the "tent" and "valley" patterns of aesthetic quality (Experiments 7 and 8) reflect the more complex interactions between the location of the initial service center, the patterns of aesthetic quality and the feedback resulting from creation of service centers. At $g = 20$ the valley pattern results in consistently higher $T(dbp = 300)$, though not outside the standard deviations of either trial, than does the tent pattern (Table 2). This is because the location of the seed service center in the middle of the left edge coincides with the top of the ridge of the aesthetic quality surface for the tent case. At $g = 40$, however, $T(dbp = 300)$ is not as different. In fact the mean with the tent pattern is slightly higher than that with the valley pattern. This convergence might be explained by the greater amount of time, at $g = 40$, the clusters of development have to align themselves with the ridges of the aesthetic quality surface and, with the help of the new service centers, develop along the top and bottom edges.

3.2.4. ABM 2Dq experiments

The ABM 2Dq results illustrate the effects of a positive influence of the greenbelt on the aesthetic quality of cells in its vicinity (Table 3). We indicated some of these effects using the mathematical model, as described in Claim 4 in Section 3.1. Though the effect is small, there is a consistent delay in the time to development on the right. The delay is most substantial for the situation in Experiment 10 (with the right-high pattern of aesthetic quality) and with $g = 20$ and $w = 1$. Intuitively, by increasing the aesthetic quality for some cells to the left of the greenbelt, i.e. those immediately adjacent to it, the rate at which the residents jump the greenbelt is slowed. A smaller effect is observed for the tent and valley patterns, because not as many cells to the left of the greenbelt have their aesthetic quality raised by the greenbelt. There is no effect in the left-high case, because the left is already rich in aesthetic quality.

4. Discussion and conclusions

We have focused on the effectiveness of greenbelts to illustrate the value of these modeling frameworks for evaluating policies to minimize the ecological impacts of land-use change. Some of the results presented here were generated within a mathematical and some within an agent-based modeling framework. In addition to the insights they provide, the use of the two models in tandem has several other advantages. At the most basic level, the fact that the results are in general agreement, and in specific agreement when the implementations were most similar, reduces the possibility of mathematical or programming errors. Second, the fact that the agent-based model was dynamic and in a higher dimensional space suggests that the fundamental forces described in the mathematical model holds in more general contexts. Finally, the mathematical underpinning places the agent-based model on firmer footing—we have a deeper understanding of why we see what we see in the multi-agent simulation.

Table 3
Results from ABM 2Dq experiments. Average time to 300 developments beyond preserve, $T(dbp = 300)$ when greenbelt affects aesthetic quality in its vicinity. The mean and standard deviation (in parentheses) were calculated across 30 runs of the model. Parameter settings for experiments are describe in Table 1

Experiment	$w = 1$		$w = 15$	
	$g = 20$	$g = 40$	$g = 20$	$g = 40$
9	131 (16)	313 (26)	168 (11)	344 (5)
10	55 (8)	69 (31)	54 (14)	102 (62)
11	82 (16)	179 (30)	107 (16)	230 (37)
12	101 (26)	180 (47)	121 (34)	217 (65)

A good example of the second point was illustrated when we incorporated the effect of the greenbelt on the aesthetic quality of neighboring cells. The mathematical model (Claim 4, Section 3.1) shows that, when the greenbelt increases nearby aesthetic quality, it is more likely to prevent sprawl across the greenbelt. This same general relationship was observed in the slowing of development outside the greenbelt using the ABM 2Dq model (Table 3), in which the assumptions of the mathematical model (e.g. perfect information to residents) were relaxed.

An example of the deeper understanding afforded by the two models is illustrated in the ESSC case of the mathematical model, in which Corollary 3 states that moving the greenbelt away from the developing region (i.e. to the right) does not always prevent sprawl. Though sprawl was always slower when the greenbelt was moved to the right in the ABM 2D model, the mathematical model identifies instances where this need not be the case. In particular, the mathematical model shows that, if the pattern of aesthetic quality is such that moving the greenbelt to just to the left of an area of high aesthetic quality might make such an area particularly attractive for development. Though none of our ABM 2D experiments illustrated this case, the mathematical model identifies it as a possible exception to the general relationships observed with the ABM 2D model.

It is important to recall that the two cases presented for the mathematical model, a single service center (LESC) and evenly spread services (ESSC), differ from the way the 2D agent-based models handle service centers. However, by comparing the two approaches we can see what characteristics influence greenbelt efficacy. The flexibility of the agent-based model offers advantages. The two-dimensional ABM, as constructed, lies between the two simple cases of the mathematical model. The ability of agent-based models to explore the interesting cases between the starker models is one of its many strengths, especially since reality is more likely to be represented by those intermediate cases than by the starker models.

On the basis of the mathematical model, we concluded that increasing the width, w, of the greenbelt increases its effectiveness at slowing sprawl. The effect of increasing the location of the greenbelt, g, has differing effects, depending on the behavior assumed for service centers. To the extent that the service center locations are not changed as g moves further out, increasing g will slow the rate of settlement outside the greenbelt. If, though, service centers also sprawl as g increases, then increasing g will be less able to prevent sprawl. The net result is intuitive and powerful. If sprawl is such that it proceeds in isolated pockets of agents moving further out who do not have sufficient demand to take services with them, then increasing g will make the greenbelt more likely to prevent sprawl. But, if services are creeping toward the inner border of the greenbelt, then increasing g could have the opposite effect by bringing locations of high aesthetic quality closer to services.

The results from the ABMs illustrate the value of agent-based models for evaluating policies in situations where multiple agents interact to produce collective outcomes that might need to be managed in some way. The mathematical modeling framework is limited by the necessity of making relatively simple assumptions that fail to capture all the complex dynamics of the real system. The ABM, on the other hand, can be extended to include a two-dimensional landscape representation, agents with heterogeneous preferences and incomplete information, real or designed patterns of landscape properties, and complex interactions like the effect of the greenbelt on aesthetic quality of neighboring cells. These extensions all improve the realism of the model and its applicability for evaluating alternative mechanisms to achieve desired urban growth patterns.

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