# Bonds, Interest Rates, and Present Discounted Value

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## I. Review of Present Discounted Value

The issue of bonds will revolve around one key factor – present discounted value. Present discount value is fundamental to economics and if you are familiar with the concept, you will greatly benefit in your everyday transactions.

The present discounted value is a way of measuring the value of future income in today's dollars. The formula also works in the reverse, enabling us to find the future value of a current investment.

1. Calculating future value

Let us suppose that you open up a savings account at the campus credit union. Into this savings account, you place $100 in savings. The interest rate is 5%. What is the value of the account in 1 year, 2 years, and n years?

To do this we will use the following formula: $F = P(1+R)^t$ where F is the future value, P is the present value, R is the interest rate, and t is the number of years from today.

In one year, the account is worth $F = 100(1+.05)^1$ = $105.

In two years, the account is worth $F = 100(1+.05)^2$ = $110.25.

In n years, the account is worth $F = 100(1+.05)^n$.

2. Calculating present discounted value

Let us suppose that you are opening up a savings account at the campus credit union. Before deciding how much money to place in the savings account, you determine that you want to have $100 in a certain number of years (1 year, 2 years, and n years) at a given interest rate of 5%.

To do this, we will simply solve the above formula for P. We get $P = \frac{F}{(1+R)^t}$ where P now equals the present discounted value.

If you want 100 dollars one year from now, you should invest $P = \frac{100}{(1+.05)^1}$ = $95.24.

Therefore, the present value (of $100 in one year) is $95.24.

If you want 100 dollars in two years, you should invest $P = \frac{100}{(1+.05)^2}$ = \$90.70.

Therefore, the present value (of \$100 two years from now) is \$90.70.

If you want 100 dollars in n years, you should invest $P = \frac{100}{(1+.05)^n}$.

<u>3. Calculating present discounted value when F is added for an infinite number of years</u>

Let us look at calculating the value of a stock. Let us define the value of a stock as the sum of the present values of all the future dividends. Let us suppose that the value of the dividend can vary each year and is denoted by the value $d_t$.

Therefore, the present value of the dividends (or the value of the stock) is denoted by the following formula, where the variables are defined as above:

$$P = d_\theta + \frac{d_1}{(1+R)} + \frac{d_2}{(1+R)^2} + \frac{d_3}{(1+R)^3} + \dots .$$

Please note that in this case, it is an infinite series. In reality, the values of the dividends are never known, so this equation is only useful to show change. However, let us suppose that every period's dividends will pay \$50 and the interest rate is 5%.

We can calculate the sum of this series by using the formula for an infinite geometric series.

Here, the formula for the present value is $P = 50 + \frac{50}{(1+.05)} + \frac{50}{(1+.05)^2} + \frac{50}{(1+.05)^3} + \dots$.

To convert this to a geometric series, we must factor out the 50 so that the first period starts with a value of one.

Thus, $P = 50 * \left[ 1 + \frac{1}{(1+.05)} + \frac{1}{(1+.05)^2} + \frac{1}{(1+.05)^3} + \dots \right] =$

$$50 * \left[ 1 + \frac{1}{(1+.05)} + \left( \frac{1}{(1+.05)} \right)^2 + \left( \frac{1}{(1+.05)} \right)^3 + \dots \right].$$

You can now recognize the geometric series in this formula. The sum of this geometric series simply is as follows.

$$P = F * \left( \frac{1}{1 - \frac{1}{1+R}} \right) = 50 * \left( \frac{1}{1 - \frac{1}{1+.05}} \right) = 50*21 = \$1050.$$

Thus, the present value of an infinite number of dividends is \$1050.

4. Calculating present discounted value when F is added for a finite series of years

Now, let us do the same problem and find the present value of the series of dividends over the course of 5 years, where the interest rate and dividends are the same as above.

Therefore, the present value of the dividends is denoted by the following formula, where the variables are defined as above:

$$P = d_\theta + \frac{d_1}{(1+R)} + \frac{d_2}{(1+R)^2} + \frac{d_3}{(1+R)^3} + \ldots + \frac{d_n}{(1+R)^n}$$

Substituting in our values, we get $P = 50 + \frac{50}{(1+.05)} + \frac{50}{(1+.05)^2} + \frac{50}{(1+.05)^3} + \frac{50}{(1+.05)^4}$ .

Notice that the value $d_0$ counts as one period.

We can calculate the sum of this series by using the formula for a finite geometric series. To convert this to a geometric series, we must factor the 50 out so that the first period starts with a value of one.

Thus, $P = 50 * \left[1 + \frac{1}{(1+.05)} + \frac{1}{(1+.05)^2} + \frac{1}{(1+.05)^3} + \frac{1}{(1+.05)^4}\right] =$

$$50 * \left[1 + \frac{1}{(1+.05)} + \left(\frac{1}{(1+.05)}\right)^2 + \left(\frac{1}{(1+.05)}\right)^3 + \left(\frac{1}{(1+.05)}\right)^4\right].$$

You can now recognize the geometric series in this formula. The sum of this finite geometric series is as follows.

$$P = F * \left[\frac{1 - \left(\frac{1}{1+R}\right)^n}{1 - \frac{1}{1+R}}\right] = 50 * \left[\frac{1 - \left(\frac{1}{1+.05}\right)^5}{1 - \frac{1}{1+.05}}\right] = 50*4.55 = \$227.30.$$

Thus, the present value of the dividends over five years is $227.30.

## II. An Introduction to Bonds

The following information (Section II and III) is modified from Chapter 11, "Financial Markets and International Capital Flows" from the Bernanke text. Thanks to Professor Lange at Drexel, who has made PowerPoint slides available in the public domain.

Before looking at how bonds will influence the economy, let us define a bond. A bond is an IOU. It is a legal promise to repay debt – inclusive of what was issued up front and the promised interest. The amount originally lent is the principal and it is paid back at a specific time in the future. When the bond principal is paid, the bond is said to have matured. The regular interest payments on the bond are called the coupon payment. The interest rate paid for a given time period on the bond is called the coupon rate. A bond owner is not required to hold the bond until it matures. An owner of a bond can sell it in the bond market at any time, and the price of the bond is the market value of the bond at any given time. Note, the price of a bond is not equal to the principal in this case. The prevailing interest rate will determine whether the price of the bond is greater than, less than, or equal to the principal. As you can already anticipate, such a definition of bond prices hints at the requirement of PDV to calculate bond prices! Exciting!

## III. Bond Prices and Interest Rates

Bond prices and interest rates are *inversely* or negatively related – in other words, they move in opposite directions. When the interest rate on *newly* paid bonds goes up, the price of existing bonds fall. The price of an existing bond is an indicator of the willingness to pay of financial investors.

Let us take an example to demonstrate why bond prices and interest rates are inversely related. Let us suppose that on March 15, 2006 (today) I purchase a two-year government-issued bond that has a principal of 10,000 dollars (much more than my GSI salary!). The coupon rate on the bond is 10% (a great deal!) and the coupon payments will be paid annually. In other words, the prevailing interest rate this afternoon was 10%. If this is the case, I will receive a coupon payment of 10% * $10,000 = $1000 on March 15, 2007. If I still own the bond when it matures in 2008, I will reactive the principal plus the annual coupon payment (10,000 + 1000 = 11,000 to be paid back on March 15, 2008, just in time to pay back my student loans).

On March 15, 2007, I receive my coupon payment. Once I have the payment in hand, I decide I want to sell my bond in the bond market in order to get some cash. The prevailing interest rate in the bond market is now 20%. Nevertheless, the 10% interest rate still applies to my bond and thus when someone buys it, they will receive $11,000 one year later. Having forgotten my macroeconomics from the previous year, I try selling the bond for a value equal to the principal. No one buys! Why is this the case?

My bond is an old bond. It is not covered under the new interest rates. A financial investor could purchase a newly issued bond at the prevailing interest rate of 20%. Suppose a financial investor, Roger Kaufman, buys a one-year newly issued $10,000 bond. In one year, Professor Kaufman will receive 10,000 + .20*10,000 = 12,000. So, why would he buy my bond and receive only half the interest payments? No rational investor would do so; hence, I need to discount the value of my bond to determine its price in the bond market. Clearly, I must sell the bond at a price such that it guarantees Professor Kaufman a 20% return over the next year. In other words, the price of the bond is determined by the prevailing interest rate and not the old interest rate.

To figure out the price of my bond, I must calculate the present discounted value of the bond. The bond is worth what it will pay in one year discounted by 1 + R. In other words, the $Bond\,price = \frac{principal + coupon}{(1+R)^t}$, where R is the prevailing interest rate and t is the number of years for which the interest rate is expected to be known. Thus, I can sell my bond to

Professor Kaufman for $Bond\,price = \frac{10{,}000 + 1000}{(1+.20)^1} = \$9166.66$. If Professor Kaufman buys the bond at this price, he will receive a 20% return.

Now, suppose the interest rate on March 15, 2007 had fallen to 1%. As you can tell, I can now sell my bond for more that $10,000. Precisely, I can sell my bond for

$$Bond\,price = \frac{principal + coupon}{(1+R)^t} = \frac{10{,}000 + 1000}{(1+.01)^1} = \$10{,}891.08.$$

The only case where the bond price will equal the original bond price that I paid is if the prevailing interest rate is the same as the original interest rate. As you can tell, if the interest rate remains the same, bond prices remain the same. The price of bonds will increase when the interest rate falls – because the interest rate is in the denominator of the PDV formula. However, if the interest rate rises, the bond price will fall. In summary, when the interest payment on new bonds rises, the price of existing bonds falls.

## IV. Practice Problems

We can now use the more complex formulas for present discounted value to figure out the price of multi-period bonds. You should solve these problems on your own.

1. Suppose I purchase a ten-year $10,000 government bond with a coupon rate of 5%. After exactly one year, I decide to sell the bond. The prevailing interest rate is now 10% and it is fixed at 10% for the long-run. What is the price of the bond? (Hint: Remember that the re-payment of the principal will only occur in the final period.)

2. Suppose I purchase a ten-year $10,000 government bond with a coupon rate of 1%. After exactly one year, I decide to sell the bond. The prevailing interest rate is now 2%, but I also know that the interest rate will go up one percent every year. (Two years from now, the interest rate will be 3%; three years from now, it will be 4%, …). What is the price of the bond? (Do not solve this mathematically; only set up the problem.)

3. Suppose I purchase an infinite-year $10,000 government bond with a coupon rate of 3% (the principal never is paid back). After exactly one year, I decide to sell the bond. The prevailing interest rate will always be 3% and I know that the person buying the bond has discovered the fountain of youth, thus living forever. What is the price of the bond? (Food for thought: Is the bond price infinite? Why or why not?).