Prediction of Water Vapor Scale Height from Integrated Water Vapor Measurements

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Abstract -- A method for estimating the 10% height of water vapor in the atmosphere is presented. This 10% height is the altitude below which 10% of the atmospheric water vapor vertical distribution occurs. The estimation is based on a time series of ground-based measurements of integrated water vapor, and results from the behavior of the turbulence structure function for water vapor outside of the outer scale of tropospheric turbulence. This behavior is demonstrated by a model which expands on earlier work by Treuhaft and Lanyi on the behavior of atmospheric turbulence in a "frozen" atmosphere. The model integrates statistics of the climatological average water vapor profile with wind and integrated water vapor measurements. Microwave water vapor radiometer, radar wind profiler, and radiosonde data from the Continental Stratus campaign have been processed and the results are presented in support of the model. It is shown that, through the combination of integrated water vapor data from a microwave radiometer, wind profile data from a tropospheric radar, and a realistic model created with local radiosonde launches, the water vapor 10% height in the atmosphere can be successfully predicted with a simple algorithm.

INTRODUCTION

Treuhaft and Lanyi Model

Much work has been done towards the modeling of turbulence in atmospheric water vapor with varying degrees of success. In particular, the work of Treuhaft and Lanyi [1] served as a basis for the ideas presented here.

Assumptions of the Treuhaft and Lanyi Model: The model used by Treuhaft and Lanyi had several important assumptions. Treuhaft and Lanyi modeled atmospheric water vapor as a flat slab, which had a constant mean density of water vapor from the ground to some height, *h*, and no water vapor for altitudes greater than *h*. Their model assumed a "frozen" atmosphere, which is one in which the wind speed is high enough that no appreciable change occurs in the turbulence structure in the time necessary for the turbulence structure to flow over some fixed point on the ground. This assumption allows the transformation of a temporal series of measurements into an apparent spatial series if an effective wind, which can be seen as driving the turbulence structure over the ground site, can be determined. Treuhaft and Lanyi also assumed perfect Kolmogorov turbulence within their slab.

Kolmogorov turbulence: This model of turbulence is characterized by the behavior of a structure function

$$D_f(\bar{r}) = \left\langle \left(f(\bar{x}+\bar{r}) - f(\bar{x})\right)^2 \right\rangle \tag{1}$$

of some constituent present in the turbulent field. This structure function will have a power law behavior [2] in a range of separations called the inertial subrange

$$D_f(\bar{r}) = C^2\left(|\bar{r}|\right)^{2/3} \tag{2}$$

which divides the microscale of turbulence from the outer scale of turbulence. Equation (2) assumes local isotropy in the turbulent structure, which leads to the structure function being only a function of the magnitude of the separation. For atmospheric turbulence, the inner scale of turbulence is on the order of magnitude of a few millimeters, while the outer scale is in the tens of meters [3].

Conclusions of Treuhaft and Lanyi: Treuhaft and Lanyi found that the power law exponent in (2) varied when a local fit to the structure function of integrated water vapor with separation distance was performed. This exponent assumed a value of 1.67 for small separations and decreased to 0.67 for separations much greater than the thickness of the water vapor slab. When the separation was approximately equal to the thickness of the water vapor slab, the exponent was approximately 1, and the rate of change of the power law exponent with separation was greatest there.

Our Improved Model

Model Description: A new model for modeling the effects of turbulence on atmospheric water vapor has been created

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based on this work. This improved model includes a more realistic water vapor profile as well as some measure of the cross-correlation of water vapor variability at different altitudes. The water vapor profile used in the improved model is taken directly from radiosonde measurements. This should provide more realistic results than a slab model. The cross-correlations between heights, which were calculated from 18 radiosonde launches over a two month period, were included in the model to more accurately portray the apparent lack of interaction between the boundary layer, which is the well-mixed portion of the atmosphere extending from the ground to a few kilometers, and the free atmosphere, which is the portion of the atmosphere above the boundary layer. The free atmosphere is characterized as a more laminar flow than the boundary layer. The variability in the free atmosphere is relatively independent of those in the boundary layer. The cross-correlation we calculated is fairly strong within the boundary layer and the free atmosphere, but is weak between these two regimes. The equation describing the model is included in the next section.

Method of estimating the 10% water vapor height: In the new model, the 10% water vapor height can be estimated by noting that the power law exponent in (2) appears to be related to some measure of the thickness of the water vapor distribution. Based on this, we sought a relationship between the height of the water vapor and the power law exponent in the structure function at a specific separation. The separation that we picked was the 10% water vapor height of the average profile created by averaging the water vapor density of each of the 18 radiosondes.

MODEL

The model used to calculate the integrated water vapor structure function follows Treuhaft and Lanyi's derivation, with two major changes. The first change is the use of real water vapor profiles from radiosonde observations in place of the slab profile. The second modification is the inclusion of cross-correlations between water vapor densities at different altitudes. These cross correlations are used to modify the turbulence model used from one which is purely Kolmogorov to one which closely approximates Kolmogorov within either the boundary layer or the free atmosphere (where the cross correlations were relatively high) and approximates independence when the heights are on opposite sides of the dividing line between these two regimes (where the cross correlations were relatively low). Factoring in the cross-correlation effectively makes our model into two slabs, one representing the boundary layer and one representing the rest of the atmosphere, but gives us a mathematical way to calculate a structure function across the boundary layer and into the free atmosphere.

The final improved model is

$$D_{IWV}(r) = C^2 \int_0^\infty \int_0^\infty w(z) w(z') r^2(z,z') \cdot \left\{ \left(\sqrt{r^2 + (z-z')^2} \right)^{2/3} - (z-z')^{2/3} \right\} dz dz' \quad (3)$$

where C is a structure function constant, w(z) is the water vapor at height z, r^2(z, z') is the correlation coefficient between heights z and z', and r is the horizontal separation. The structure functions and local power law exponents from this model are in similar to the results in Treuhaft and Lanyi.

DATA

The data used here were collected during spring 1995 in State College, Pennsylvania, which has a continental mid-latitude climate. Integrated water vapor measurements were made every 5 seconds by the Penn State University Meteorology Department's microwave radiometer. The water vapor profiles were measured by Vaisala RS-80 radiosondes, which were launched from the radiometer location. The wind profiles were from the Penn State stratosphere-troposphere radar, which is located approximately 20 km south of the radiometer location. These measurements were a part of the Continental Stratus intensive field operations Phase II.

PROCESSING

Using the "frozen" atmosphere assumption, profiler measured winds at an altitude of 1 km above ground level were used to create a spatial series of integrated water vapor from a 2 hour temporal series of radiometer data around each radiosonde launch. The structure functions for each period were calculated, and for each period the structure function value at a separation of one sample (5 seconds) was subtracted from the structure function at all separations. This value was assumed to primarily result from radiometer ΔT noise. This noise can be shown to corrupt the structure function calculation. Some question as to the correct value to remove remains as the value subtracted is somewhat larger than the ΔT noise calculated for the radiometer. The resulting structure function was smoothed by a 5-point triangular filter.

From the resulting structure functions, the respective local power law exponents were calculated by doing a 3-

point linear fit to the common logarithm of the structure function versus the common logarithm of the separation distance. The analytical derivative of the linear fit at each point was taken to find the slope which is the local power law exponent.

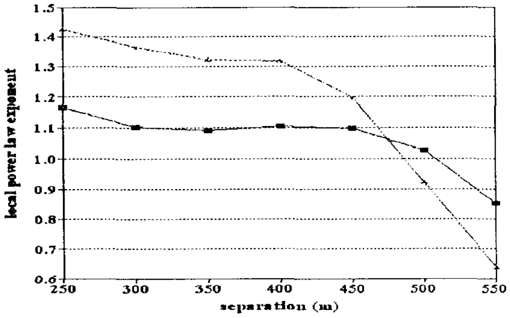

Figure 1. Power law exponents for two sample structure functions from radiometer data.

The power law exponents of two sample structure functions are shown in Fig.1. The data indicated by the filled squares is from a period with a 10% height of 132 meters, while the data represented by filled triangles is from a period with a 10% height of 280 meters. It is because of the difference in power law exponent, which is evident in Fig.1, that we are able to estimate the 10% height. As shown in Fig.1, the exponent for the 132 meter data should be smaller than that for the 280 meter data due to the decrease of exponent with increasing separation to height ratio.

RESULTS

The 10% height of the average water vapor profile is 335 meters. In order to estimate the 10% height of the individual profiles, we calculate the power law exponent at 335 meters. The power law exponent of each structure function calculated from the model (using the individual radiosonde profiles rather than the average profile) is shown in Fig.2 versus the individual 10% water vapor heights. The power law exponent of each structure function calculated from the radiometer data was then compared to the 10% water vapor height, as shown in Fig.3. The smaller number of points in Fig.3 is due to equipment dropout and sampling problems due to the variations in wind speed while the sampling rate of the radiometer remained constant.

The match between Fig. 2 and Fig.3 appears to be fairly good, and leads one to suspect that one could estimate 10% water vapor height from only integrated liquid water vapor and wind data.

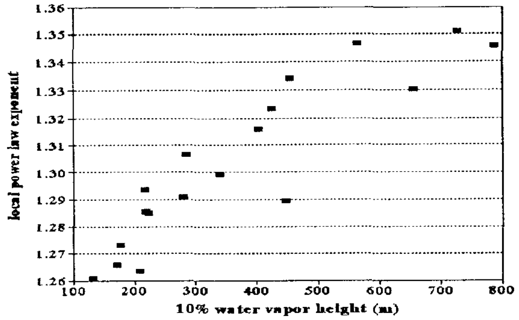

Figure 2. Comparison of exponent at fixed separation with 10% water vapor height. Results from the model.

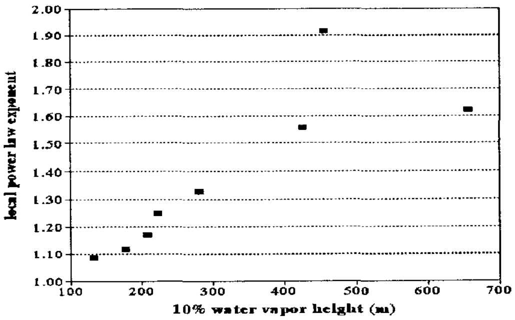

Figure 3. Comparison of exponent at fixed separation with 10% water vapor height. Results from radiometer data.

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