Using Asymmetry to Estimate Potential

Waldo Tobler, Geographer
University of California
Santa Barbara, CA 93106-4060
http://www.geog.ucsb.edu/~tobler
The concern is with complete, square, asymmetric, valued tables, though the procedure may also work with two mode tables.

For this demonstration I have used only small examples.

One example is based on geographic data, the other on journal-to-journal citations.
The form of a movement table $M_{ij}$

In a non-geographic, network, environment the ‘places’ are sometimes called ‘actors’.
Let $M_{ij}$ represent the movement table, with $i$ rows and $j$ columns. It can be separated into two parts, as follows.

\[ M_{ij} = M^+ + M^- \]

where

\[ M^+ = (M_{ij} + M_{ji})/2 \quad \text{symmetric} \]
\[ M^- = (M_{ij} - M_{ji})/2 \quad \text{skew symmetric} \]

The variance can also be computed for each component, and the degree of asymmetry can be computed.
How the two parts are used

I consider the symmetric component as a type of background. The real interest is in the asymmetric part.

In the geographic case the position of the places is known.

But if locations are not given then the symmetric part may be used to make an estimate of these positions.

This estimate is made using an ordination, trilateration, or multidimensional scaling algorithm.
The first example uses a 33 by 33 matrix of commuting in the vicinity of Munich, Germany.

The matrix is shown next.

A map of the regions is given in:

A geographic example

**Munich Commuting 1939**

Between 33 districts of known location

| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|

...continued
Adding across the table, the column marginals give the outsums (a.k.a. outdegree). Summing down the rows gives the insums (a.k.a. indegree).

The ‘sending’ places (rows) are known as ‘sources’, and are shown on the map as negative signs.

The ‘receiving’ places (columns) are the ‘sinks’ and are shown as plus signs.

The size of the symbol represents the magnitude of the movement volume.
The movement from source to sink can be computed to show the direction and magnitude of the movement.

The computation is based on the asymmetry of the movement table.

Small directed vectors represent this movement on the next map.
Munich Commuting
Displacement vectors
An interpolation is then performed to obtain a vector field from the isolated individual vectors.

This is done to simplify the mathematical integration needed to obtain the forcing function.
Interpolated Field
of displacement vectors
Computed Potential

based on the displacement vectors
The computed potential should have the vector field as its gradient.

This is a hypothesis that can be tested.

The base level of the potential is determined only up to a constant of integration.

The vector field, to be a gradient field, must be curl free. This can also be tested.
The attempt is now made to apply these ideas in a social space.

This can be considered a development of Lewin’s *Topological Psychology* or his *Field Theory in the Social Sciences*.

The data represent citations between a small set of psychological journals. Larger citation tables are now also available.
## Citations among psychology journals

Coombs et al 1970  
Data from 1964

<table>
<thead>
<tr>
<th>Journal</th>
<th>AJP</th>
<th>JASP</th>
<th>JAP</th>
<th>JCPP</th>
<th>JCP</th>
<th>JEdP</th>
<th>JExP</th>
<th>Pka</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>American Journal of Psychology</td>
<td>119</td>
<td>-</td>
<td>8</td>
<td>4</td>
<td>21</td>
<td>0</td>
<td>1</td>
<td>85</td>
<td>2</td>
</tr>
<tr>
<td>Journal of Abnormal and Social Psychology</td>
<td>32</td>
<td>510</td>
<td>16</td>
<td>11</td>
<td>73</td>
<td>9</td>
<td>119</td>
<td>4</td>
<td>774</td>
</tr>
<tr>
<td>Journal of Applied Psychology</td>
<td>2</td>
<td>8</td>
<td>84</td>
<td>1</td>
<td>7</td>
<td>8</td>
<td>16</td>
<td>10</td>
<td>136</td>
</tr>
<tr>
<td>Journal of Comparative and Physiological Psychology</td>
<td>35</td>
<td>8</td>
<td>0</td>
<td>533</td>
<td>0</td>
<td>1</td>
<td>126</td>
<td>1</td>
<td>704</td>
</tr>
<tr>
<td>Journal of Consulting Psychology</td>
<td>6</td>
<td>116</td>
<td>11</td>
<td>1</td>
<td>225</td>
<td>7</td>
<td>12</td>
<td>7</td>
<td>385</td>
</tr>
<tr>
<td>Journal of Educational Psychology</td>
<td>4</td>
<td>9</td>
<td>7</td>
<td>0</td>
<td>3</td>
<td>52</td>
<td>27</td>
<td>5</td>
<td>107</td>
</tr>
<tr>
<td>Journal of Experimental Psychology</td>
<td>125</td>
<td>19</td>
<td>6</td>
<td>70</td>
<td>0</td>
<td>0</td>
<td>586</td>
<td>15</td>
<td>821</td>
</tr>
<tr>
<td>Psychometrika</td>
<td>2</td>
<td>5</td>
<td>5</td>
<td>0</td>
<td>13</td>
<td>2</td>
<td>13</td>
<td>58</td>
<td>98</td>
</tr>
<tr>
<td>Total</td>
<td>325</td>
<td>683</td>
<td>133</td>
<td>637</td>
<td>321</td>
<td>80</td>
<td>984</td>
<td>102</td>
<td>3,265</td>
</tr>
<tr>
<td>To Journal</td>
<td>Journal to Journal Citations</td>
<td>Net</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>------------</td>
<td>-----------------------------</td>
<td>-----</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>From</td>
<td>X</td>
<td>Y</td>
<td>Flow</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AJP</td>
<td>119</td>
<td>8</td>
<td>4</td>
<td>21</td>
<td>0</td>
<td>1</td>
<td>85</td>
<td>2</td>
<td>125</td>
</tr>
<tr>
<td>JASP</td>
<td>32</td>
<td>510</td>
<td>16</td>
<td>11</td>
<td>73</td>
<td>9</td>
<td>19</td>
<td>4</td>
<td>-1382</td>
</tr>
<tr>
<td>JAP</td>
<td>2</td>
<td>8</td>
<td>84</td>
<td>1</td>
<td>7</td>
<td>8</td>
<td>16</td>
<td>10</td>
<td>-261</td>
</tr>
<tr>
<td>JCPP</td>
<td>35</td>
<td>8</td>
<td>0</td>
<td>533</td>
<td>0</td>
<td>1</td>
<td>126</td>
<td>1</td>
<td>1302</td>
</tr>
<tr>
<td>JCP</td>
<td>6</td>
<td>116</td>
<td>11</td>
<td>225</td>
<td>7</td>
<td>12</td>
<td>7</td>
<td>-924</td>
<td>-2</td>
</tr>
<tr>
<td>JEdP</td>
<td>4</td>
<td>9</td>
<td>7</td>
<td>0</td>
<td>3</td>
<td>52</td>
<td>27</td>
<td>5</td>
<td>-180</td>
</tr>
<tr>
<td>JExP</td>
<td>125</td>
<td>19</td>
<td>6</td>
<td>70</td>
<td>0</td>
<td>0</td>
<td>586</td>
<td>15</td>
<td>904</td>
</tr>
<tr>
<td>Pka</td>
<td>2</td>
<td>5</td>
<td>5</td>
<td>0</td>
<td>13</td>
<td>2</td>
<td>13</td>
<td>58</td>
<td>416</td>
</tr>
</tbody>
</table>

AJP  Am J of Psychology  
JASP J of Abnormal & Social Psychology  
JAP  J of Applied Psychology  
JCPP J of Comparative & Physiological Psychology  
JCP  J of Consulting Psychology  
JEdP J of Educational Psychology  
JExP J of Experimental Psychology  
Pka  Pschometrika

Engelwood Cliffs, NY, Pages 73-75
The table gives the being-cited journal across the columns. But the information can be considered to move from that journal to the citing journal.

Therefore the transpose is used to produce the source to sink map.
Journal Sources and Sinks
We now have an assignment problem. How to get 163 citations from JExP, 85 from AJP, & 4 from Pka to the 5 receiving journals. There are obviously many possibilities.

One solution is to use the “Transportation Problem” (Koopmans, Kantorovich, ~1949): Minimize $M \cdot d..$, subject to $M \cdot J = O_I$, $M_I = I_J$, $M_{IJ} >= 0$, given the distances computed from the coordinates and using the simplex method for the solution.

A more realistic solution is given by the quadratic transportation problem: Minimize $M^2 \cdot d..$, subject to the same constraints.

Both of these solutions result in discrete answers, and ‘shadow prices’. We are looking for a spatially continuous solution that allows vectors and streamlines, in order to determine spatial flow fields and a continuous potential.
The observed two-way flow between the journals

Journal Flow Map
Coombs et al 1970
The total flow between the journals

Journal Flow Map
Coombs et al data
The net flow between the journals

Journal Flow Map
Coombs et al data
The next step is to compute the displacements between the cited journals.

This is based on the asymmetry of the citations table.

The fundamental idea being that there exists a ‘wind’ making movement easier in some directions.

The mathematical details are given in a published paper.

Displacement between Journal Citations
Then the potential is computed by integration.

This potential should be such that its gradient coincides with the displacement vectors.

It may be necessary to use an iteration to obtain this result.
Journal Potential Function
Flow and Potential between Psychological Journals
Some questions

Suppose a new psychological journal were started. Where should it be inserted into in this space?

Does it make sense to treat journal citations as being located in a continuous two-dimensional social space?

Can other social data be treated in a similar fashion, for example social mobility tables?

And more general network data?
CONCLUSION

I have given some speculative thoughts on how one might represent network relations with vectors fields and scalar potentials in a continuous social space.

Still needed are error estimates.

Your comments are desired.
Thank you for your attention.

http://www.geog.ucsb.edu/~tobler
References


