MODELLING LOCATIONAL DECISION MAKING OF FIRMS USING MULTIDIMENSIONAL FUZZY DECISION TABLES: AN ILLUSTRATION

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1 INTRODUCTION

The locational decision making of firms can be modelled from different perspectives. Especially in economic research, several authors have suggested to use stated or revealed preference choice models to predict the probability that a particular location will be chosen as a function of its locational and non-locational attributes (Timmermans, 1986; Moore, 1988; Henley *et al.,* 1989; Friedman *et al.,*1992). Such algebraic models do have the appeal of theoretical rigour, mathematical sophistication, and an associated error theory. However, the application of such models, in particular when revealed preference is used, is characterised by many problems, including high multi-collinearity among explanatory variables, complexity in the sense of a large number of influential attributes, and the fact that algebraic equations by definition cannot capture all theoretical notions. For example, the situation that at one level a locational requirement serves as a veto criterion, whereas at another level compensation is allowed, is difficult to represent using an algebraic equation.

A modelling approach that avoids these problems is qualitative modelling. The quintessence of this approach is to represent the locational decision-making process in terms of a set of IF, THEN … ELSE expressions. These logical expressions have sufficient flexibility to represent a wider variety of decision rules. On the other hand, their "crisp" (or exact) nature implies the lack of an error theory, limiting in some cases the realism of such systems. In previous papers, Witlox *et al.* (1997) and Witlox and Timmermans (2000, 2002) therefore argued for the development of multidimensional *fuzzy* systems.

The current paper reports on the application of such a model to represent the locational decision making behaviour of firms, taking the petrochemical industry as an example. The article is organised as follows. In the second section, the problem of membership value measurement in a decision table environment is introduced. The aim is to introduce the technique that will enable us to estimate membership values of the fuzzy sets used in the condition part of a fuzzy decision table. The third section of this article discusses the process of membership value estimation. The fourth section reports on the application. Finally, in the fifth section, the results of this study are summarized and some issues for future research are discussed.

2 MEMBERSHIP VALUE MEASUREMENT IN A FUZZY DECISION TABLE

2.1 Fuzzy decision tables

A decision table (DT) consists of an exhaustive set of mutually exclusive conditions, leading to particular actions. Each DT consists of four quadrants: condition set [C_i], action set [A_j], condition space [SPACE (C_i)], and action space [SPACE (A_j)]. The *condition set* consists of all the relevant conditions or attributes (inputs, premises or causes) that have an influence on the decision-making process. The *condition space* specifies all possible combinations of *condition states* of a condition. The *action set* contains all the possible actions (outputs, conclusions or consequences) a decision-maker is able to take. This is, the action set points to the possible choice outcome if (for instance) an existing location with a number of specific characteristics is processed through the DT. Finally, the *action space* contains the categorizations of all the possible *action states* of an action. Any vertical linking of an element from the condition space with an element from the action space produces a decision rule (Figure 1).

Figure 1: The general structure of a decision table

Problem area	
CONDITION SET	CONDITION SPACE
ACTION SET	ACTION SPACE

Traditionally, decision tables (DTs) are crisp, indicating that the conditions are specified in an exact manner. A potential problem of such DTs is that any measurement error is not taken into account. Fuzzy decision tables (FDTs) offer a solution to this problem. A fuzzy decision table (FDT) is an extended version of a crisp DT in order to deal with imprecise and vague decision situations (Francioni and Kandel, 1988; Vanthienen *et al.*, 1996). The extension amounts to the introduction of fuzzy sets in the condition and action space of the crisp DT; the crisp condition and action states are replaced with fuzzy conditions and actions. The latter two are a combination of fuzzy sets. A membership function needs to be specified which represents the extent to which a particular attribute level meets a particular condition

.2.2 **Membership value measurement**

To discuss the issue of membership value measurement in a decision table environment, the problem is formally defined as follows. Assume that a decision table is characterised by I conditions (i = 1, 2, ..., I) with n_i states and A action states. In the case of a fuzzy table, the n_i states for each condition are fuzzy, and the table has M fuzzy action states, FAS_m (m = 1, 2, ..., M). FAS_m (m = 1, 2, ..., M) is carried out, if and only if, the fuzzy condition states $FCS_{1?}$ & $FCS_{2?}$ & $FCS_{3?}$ & $FCS_{4?}$ & $FCS_{5?}$ & ... & $FCS_{I?}$ are simultaneously satisfied. The question marks in the indices refer to the fuzzy condition states associated with each particular condition that, simultaneously combined, will result in the execution of fuzzy action state FAS_m. Note that FAS_m may be carried out by means of various combinations of fuzzy condition states. An important point is that in an FDT all interpretations should be made at the individual decision rule level (Wets, 1998). This point implies that we need to formalize our problem at the level of rules. To this end, three new notations are introduced: S_r, x_{ir}, and μ_{RULE_r}. Let S_r define the crisp set of all combinations (x_{1r}, x_{2r}, ..., x_{Ir}) resulting in the execution of $RULE_r$ (r = 1, 2, ..., R). Let x_{ir} define the identifier or index (one of the numbers 1, 2, ..., n_i) of the fuzzy condition state of C_i, in a way that $RULE_r$ is carried out, if fuzzy condition states $FCS_{ix_{ir}}$ for all i (i = 1, 2, ..., I) are satisfied. More formally: if $\exists\ r \in \{1,..., R\}$ and $\forall\ i \in \{1,..., I\}$: $FCS_{ix_{ir}}$ is satisfied, then $RULE_r$ is carried out. Finally, μ_{RULE_r} represents the membership value for RULE$_r$. The membership value for a single fuzzy action state (i.e. μ_{FAS_m}) can be achieved by adding the membeship values for the corresponding rules. From now on, it is assumed that when action $RULE_r$ (r = 1, 2, ..., R) is executed, all indexes x_{ir} (i = 1, 2, ..., I; r = 1, 2, ..., R) are known.

The finite set of fuzzy decision rules may then be defined as follows:

if $FCS_{1x_{1r}}$ and $FCS_{2x_{2r}}$ and ... and $FCS_{Ix_{Ir}} \rightarrow RULE_r$; $\forall\ r \in \{1,..., R\}$. [1]

Assume further that for I conditions with n_i fuzzy condition states for each i, there exists a membership value (or truth value) $\boldsymbol{\mu} \in [0,1]$. The problem then is to estimate these membership values according to the following model specification:

$$\mu_{RULE_1} = \mu_{1x_{11}}(C_1)\mu_{2x_{21}}(C_2)\ldots\mu_{Ix_{I1}}(C_I) \quad [2a]$$

$$\mu_{RULE_2} = \mu_{1x_{12}}(C_1)\mu_{2x_{22}}(C_2)\ldots\mu_{Ix_{I2}}(C_I) \quad [2b]$$

... ...

$$\mu_{RULE_R} = \mu_{1x_{1R}}(C_1)\mu_{2x_{2R}}(C_2)\ldots\mu_{Ix_{IR}}(C_I) \qquad [2c]$$

Or, equivalently,

$$\mu_{RULE_r} = \mu_{1x_{1r}}(C_1)\mu_{2x_{2r}}(C_2)\ldots\mu_{Ix_{Ir}}(C_I)\,;\ \forall\, r \in \{1,\ldots, R\}. \qquad [2d]$$

Subject to:

$$\sum_{k=1}^{n_i} \boldsymbol{\mu}_{ik}(C_i) = 1;\ \forall\, i \in \{1,\ldots, I\}. \qquad [3]$$

Relation [3] is called the fuzzy condition state partition constraint. It states that the membership function values associated with the fuzzy condition states of a particular condition in the FDT need to sum up to unity. Hence, a probabilistic approach to fuzzy set theory is advocated here.

Suppose that an estimate of $\mu_{ix_{ir}}(C_i)$ can be represented by the parameters, $\alpha_{ix_{ir}}$, leading to an estimate of $\hat{\mu}_{RULE_r}$, the model to be estimated then becomes:

$$\hat{\mu}_{RULE_r} = \alpha_{1x_{1r}}\alpha_{2x_{2r}}\alpha_{3x_{3r}}\ \ldots\ \alpha_{Ix_{Ir}}\,;\ \forall\, r \in \{1,\ldots, R\} \qquad [4]$$

Subject to

$$0 \leq \alpha_{ix_{ir}} \leq 1;\ \forall\, r \in \{1,\ldots, R\};\ \forall\, i \in \{1,\ldots, I\}. \qquad [5a]$$

$$\sum_{k=1}^{n_i} \alpha_{ik} = 1;\ \forall\, i \in \{1,\ldots, I\}. \qquad [5b]$$

For the model to be consistent, the following relation should also be satisfied:

$$\sum_{r=1}^{R} \hat{\mu}_{RULE_r} = 1. \qquad [6]$$

Finally, the number of parameters to be estimated can be derived from the number of conditions and the number of associated fuzzy condition states. For instance, to estimate C_1 with n_1 fuzzy condition states, (n_1-1) parameters are required. This complies with the imposed constraints on the model specification. Thus, generalizing for I conditions with n_i (i = 1, 2, ..., I) fuzzy condition states, the number of parameters, denoted by J, to estimate is equal to:

$$\sum_{j=1}^{I} (n_j\text{-}1), \text{ or rearranged: } (\sum_{j=1}^{I} n_j) - I. \qquad [7]$$

3 MEMBERSHIP VALUE ESTIMATION

The goal of the membership estimation is to find parameters that lie within the [0,1] interval. Calibration is done using maximum-likelihood (ML) estimation. To briefly describe the ML procedure, a likelihood function, L: $\mathbf{Q} \rightarrow \mathrm{R}^+$ is introduced. This is a function of the unknown parameters that is denoted $L(\mathbf{Q})$, where $\mathbf{Q}$ denotes the collection of unknown parameters being estimated in the model. The basic principle of the ML estimation is to find the value that maximises the likelihood of the observed sample. The maximum likelihood function for I independent conditions can then be written as follows:

$$L(\mathbf{Q}) = (\hat{\mu}_{RULE_1})^{f_{RULE_1}} (\hat{\mu}_{RULE_2})^{f_{RULE_2}} \ldots (\hat{\mu}_{RULE_Z})^{f_{RULE_Z}} \qquad [8]$$

which is equal to,

$$L(\mathbf{Q}) = \prod_{z=1}^{Z} (\hat{\mu}_{RULE_z})^{f_{RULE_z}} \qquad [9]$$

where f_{RULE_z} denotes the number of observations (or frequencies) for each of the $z \in \{1,..., Z\}$ associated fuzzy decision rules $RULE_z$.

The log-likelihood function may then be written as:

$$L^*(\mathbf{Q}) = \sum_{z=1}^{Z} f_{RULE_z} \ln (\hat{\mu}_{RULE_z}). \qquad [10]$$

By substituting $\hat{\mu}_{RULE_z}$ with their associated, expanded fuzzy condition state paths, the result is the following:

$$L^*(\mathbf{Q}) = \sum_{z=1}^{Z} f_{RULE_z} \ln\Big[\sum_{S_z} \alpha_{1x_{1z}} \alpha_{2x_{2z}} \alpha_{3x_{3z}} \dots \alpha_{Ix_{Iz}} \Big]. ; \forall\, z \in \{1,\dots, Z\} \qquad [11]$$

The log-likelihood function $L^*(\mathbf{Q})$ is maximized with respect to the parameters $\alpha_{ix_{iz}}$ subject to the imposed constraints stated in relations [5a] and [5b] and [6].

4 APPLICATION

4.1 The case

In this sub-section, we will illustrate the use of the multi-dimensional membership value estimation procedure applied to the data collected. In particular, we intend to create a fuzzy equivalent of the following DT representing locational requirements.

Table 1: DT representing the locational requirements

C_1 Site within port zone?	yes					no
C_2 Site *near* residential area?	yes	no				-
C_3 Site *near* school/hospital?	-	yes	no			-
C_4 Site *near* recreational area?	-	-	yes	no		-
C_5 Site *near* scenic area?	-	-	-	yes	no	-
A_1 Suitable	.	.	.	.	x	.
A_2 Non-suitable	x	x	x	x	.	x
	$RULE_1$	$RULE_2$	$RULE_3$	$RULE_4$	$RULE_5$	$RULE_6$

Table 1 depicts a decision table that specifies a number of essential locational requirements that have to be fulfilled to consider a particular site suitable for the economic activity under investigation. Only if a site meets all five conditions, is it classified as being suitable (i.e. rule $RULE_5$). In all other cases, a site is evaluated as "non-suitable". Note that condition C_1 is a strictly crisp condition. It deals with the issue of whether or not a site is situated somewhere within the legal boundaries demarcating the port region. Given that a site is either located within this zone or is not, only a crisp evaluation is possible. In contrast, the remaining four conditions all contain the notion of *nearness*, which is fuzzy and context-related. One way to avoid this type of vagueness is to redefine the conditions in the table in non-fuzzy, crisp terms. For example, instead of asking a decision-maker whether or not a potential site is "near" a residential area, school/hospital, recreational area and scenic area, the decision-maker can be asked to indicate minimum separating distances. By way of illustration, Table 2 displays the resulting values.

Table 2: DT representing the locational redefined requirements

C_1 Site within port zone?	yes					no
C_2 Distance to residential area (km)?	$X \leq 5$ (short)	$X > 5$ (long)				-
C_3 Distance to school or hospital (km)?	-	$X \leq 8$ (short)	$X > 8$ (long)			-
C_4 Distance to recreational area (km)?	-	-	$X \leq 10$ (short)	$X > 10$ (long)		-
C_5 Distance to scenic area (km)?	-	-	-	$X \leq 15$ (short)	$X > 15$ (long)	-
A_1 Suitable	.	.	.	.	x	.
A_2 Not satisfied	x	x	x	x	.	x
	RULE$_1$	*RULE*$_2$	*RULE*$_3$	*RULE*$_4$	*RULE*$_5$	*RULE*$_6$

An alternative approach would be to construct a fuzzy decision table by replacing all crisp condition states that have to be fuzzified with associated fuzzy membership values. In the present context, only condition C_1 remains crisp which implies that its two states are each assigned a crisp value: i.e. CS_{11}= 1 (denoting "yes") and $CS_{12} = 0$ (denoting "no"). The condition states of the four remaining conditions are each assigned an unknown fuzzy value (denoted by α). In what follows, we assume that α refers to the fuzzy set "long", whereas (1 - α) points to the fuzzy set "short". Note that the degree of accepting the alternative increases as distance increases. Consequently, we are concerned with the estimation of the membership values of the fuzzy set "long", and deduce from it the membership values of the fuzzy set "short". Hence, $FCS_{22} = \alpha_1$, $FCS_{32} = \alpha_2$, $FCS_{42} = \alpha_3$ and $FCS_{52} = \alpha_4$. Because the membership function values over the domain of a fuzzy set need to add to one, the corresponding fuzzy condition state values for the notion "short" are equal to $FCS_{21} = (1\text{-}\alpha_1)$, $FCS_{31} = (1\text{-}\alpha_2)$, $FCS_{41} = (1\text{-}\alpha_3)$ and $FCS_{51} = (1\text{-}\alpha_4)$. The result is shown in Table 3.

Table 3: Contracted FDT representing the locational prerequisites

<table>
<tr><td>C_1 Site within port zone?</td><td colspan="5">1</td><td>0</td><td>crisp</td></tr>
<tr><td>C_2 Distance to residential area?</td><td>$(1-\alpha_1)$</td><td colspan="4">α_1</td><td>-</td><td>fuzzy</td></tr>
<tr><td>C_3 Distance to school or hospital?</td><td>-</td><td>$(1-\alpha_2)$</td><td colspan="3">α_2</td><td>-</td><td>fuzzy</td></tr>
<tr><td>C_4 Distance to recreational area?</td><td>-</td><td>-</td><td>$(1-\alpha_3)$</td><td colspan="2">α_3</td><td>-</td><td>fuzzy</td></tr>
<tr><td>C_5 Distance to scenic area?</td><td>-</td><td>-</td><td>-</td><td>$(1-\alpha_4)$</td><td>α_4</td><td>-</td><td>fuzzy</td></tr>
<tr><td>FAS_1 Suitable</td><td>.</td><td>.</td><td>.</td><td>.</td><td>x</td><td>.</td><td>fuzzy</td></tr>
<tr><td>FAS_2 Non-suitable</td><td>x</td><td>x</td><td>x</td><td>x</td><td>.</td><td>x</td><td>fuzzy</td></tr>
<tr><td></td><td>$RULE_1$</td><td>$RULE_2$</td><td>$RULE_3$</td><td>$RULE_4$</td><td>$RULE_5$</td><td>$RULE_6$</td><td></td></tr>
</table>

With the exception of condition C_1, all conditions in Table 3 have been assigned an unknown membership value that needs to be estimated. As a result of this membership value substitution, the action states of the table (i.e. FAS_1 and FAS_2) also become fuzzy since they combine different fuzzy condition states. The fuzziness in the table is visualised by the shades in the table.

In order to obtain valid parameter estimates, the estimation should take place at the individual decision rule level of the expanded FDT. Hence, the log-likelihood function, $L^*(\mathbf{Q})$ with $\mathbf{Q} = (\alpha_1,\alpha_2,\alpha_3,\alpha_4)$, for the expanded FDT requirements is:

$$L^*(\mathbf{Q}) = \sum_{z=1}^{16} f_{RULE_z} \ \ln(\hat{\mu}_{RULE_z}) \qquad [12]$$

The value of f_{RULE_z} referring to the number of observations for $RULE_z$ (z = 1, 2, …, I), should then be maximised with respect to the four unknown α-values, subject to the constraint: $0 \leq \alpha_1,\alpha_2,\alpha_3,\alpha_4 \leq 1$. Note again that all interpretations are done on the expanded version of the FDT. Thus, all columns of the table contain only simple states (no combination of states). The relation between the expanded (with I = 16) and contracted (with I = 5) FDT is as follows:

Decision rules expanded FDT	Corresponding decision rule contracted FDT (Table 3)
$RULE_1 = (1 - \alpha_1)(1 - \alpha_2)(1 - \alpha_3)(1 - \alpha_4)$; $RULE_2 = (1 - \alpha_1)(1 - \alpha_2)(1 - \alpha_3)\alpha_4$; $RULE_3 = (1 - \alpha_1)(1 - \alpha_2)\alpha_3(1 - \alpha_4)$; $RULE_4 = (1 - \alpha_1)(1 - \alpha_2)\alpha_3\alpha_4$; $RULE_5 = (1 - \alpha_1)\alpha_2(1 - \alpha_3)(1 - \alpha_4)$; $RULE_6 = (1 - \alpha_1)\alpha_2(1 - \alpha_3)\alpha_4$; $RULE_7 = (1 - \alpha_1)\alpha_2\alpha_3(1 - \alpha_4)$; $RULE_8 = (1 - \alpha_1)\alpha_2\alpha_3\alpha_4$;	$\rightarrow$ $RULE_1$;
$RULE_9 = \alpha_1(1 - \alpha_2)(1 - \alpha_3)(1 - \alpha_4)$; $RULE_{10} = \alpha_1(1 - \alpha_2)(1 - \alpha_3)\alpha_4$; $RULE_{11} = \alpha_1(1 - \alpha_2)\alpha_3(1 - \alpha_4)$; $RULE_{12} = \alpha_1(1 - \alpha_2)\alpha_3\alpha_4$;	$\rightarrow$ $RULE_2$;
$RULE_{13} = \alpha_1\alpha_2(1 - \alpha_3)(1 - \alpha_4)$; $RULE_{14} = \alpha_1\alpha_2(1 - \alpha_3)\alpha_4$;	$\rightarrow$ $RULE_3$;
$RULE_{15} = \alpha_1\alpha_2\alpha_3(1 - \alpha_4)$;	$\rightarrow$ $RULE_4$;
$RULE_{16} = \alpha_1\alpha_2\alpha_3\alpha_4$.	$\rightarrow$ $RULE_5$.

The expanded part of the FDT contains 16 decision rules. In the crisp case, only one decision rule would match, namely that rule where all ones are found for a particular combination of α_i and $(1 - \alpha_i)$. In all other rules, at least one zero-value would be found which, using the product operator, results in a zero match. In the fuzzy case, however, more than one decision rule can match a given combination of condition values. Hence, each of the 16 decision rules will influence the decision to be made. The possibility of accepting a rule is found again by applying the product operator.

4.2 Data collection

To estimate the different membership values of the FDT, data on the relationship between actual distances and their classifications into long versus short are required. We propose to obtain these data by conducting an experiment in which the experts are presented with a number of profiles (combinations of conditions) and asked to evaluate these profiles in terms of the action states of the table. The profiles are derived from the expanded DT. The total number of profiles in the design is a function of the number of conditions to be fuzzified and the number of associated condition states making up the DT.

The DT shown in Table 2 contains four conditions, each having only two associated condition states. These two different states reflect the fact that only one crisp assessment point is used as a limiting value. For example, in the case of condition C_2, this single measurement point is equal to 5 km. The use of only one measurement point implies that no distinction can be made between values that fall within the same condition state. Hence, if the aim is to fuzzify a specified measurement point so that a gradual transition between crisp states becomes possible, it is necessary to introduce additional evaluation points. These additional measurement values should be centred on the initial crisp boundary point resulting in a fuzzy range.

Note that, if we do not introduce additional measurement points and simply present the experts with different profiles in which conditions can only take on two different values (e.g. $X \leq 5$ and $X > 5$), then this would not lead to a fuzzification of the initial measurement point in question. Instead, we would be measuring the degree of heterogeneity present in the sample. By contrast, the introduction of additional measurement points, subdividing the domain of the condition in more condition states, allows for subtler decision-making and is also indicative of the degree of crispness of the different boundary values.

The introduction of additional measurement points implies that the specified log-likelihood function, $L^*(\mathbf{Q})$ with $\mathbf{Q} = (\alpha_1,\alpha_2,\alpha_3,\alpha_4)$, or in short, $\mathbf{Q} = (\alpha_i)$ needs to be respecified. Hence, the following substitution is made:

$$\mathbf{Q} \rightarrow \mathbf{Q}^*, \text{ with } \mathbf{Q}^* = (\alpha^*_i), \text{ where } \alpha^*_i = mp^0_i \pm c_i. \qquad [13]$$

where mp^0_i is the initial boundary measurement point.

As a result of this substitution, the following log-likelihood function specification, $L^*(\mathbf{Q}^*)$, is obtained which should be maximised subject to $0 \leq \alpha^*_i \leq 1$. Note that c_i determines the spread of the fuzzy interval around the initial crisp measurement point. It specifies a lower (i.e. $mp^0_i - c_i = mp^-_i$) and an upper (i.e. $mp^0_i + c_i = mp^+_i$) measurement point. The membership values for each introduced measurement point for each condition are represented by α^*_i. Consequently, depending on the value of c_i, α^*_i denotes the estimated membership value for the:

(i) lower mp ($mp^0_i - c_i = mp^-_i$) : $\alpha^*_i = \alpha^-_i$ subject to $0 \leq \alpha^-_i \leq 1$;

(ii) initial mp (mp^0_i) : $\alpha^*_i = \alpha^0_i$ subject to $0 \leq \alpha^0_i \leq 1$;

(iii) upper mp ($mp^0_i + c_i = mp^+_i$) : $\alpha^*_i = \alpha^+_i$ subject to $0 \leq \alpha^+_i \leq 1$.

It logically follows from the interpretation given to the estimated membership values that the following relation should also hold: $0 \leq \alpha^-_i \leq \alpha^0_i \leq \alpha^+_i \leq 1$ for the results to have face validity. When the distance increases, the membership function should increase as well.

In the present paper, c_i was taken as 2 km. Thus, for condition C_2 representing the distance to a residential area the three measurement points with their associated to be estimated membership values are equivalent to:

$mp^{-}_{1} = 5 - 2 = "3" \quad \rightarrow \alpha^{-}_{1};$

$mp^{0}_{1} = "5" \quad \rightarrow \alpha^{0}_{1};$

$mp^{+}_{1} = 5 + 2 = "7" \quad \rightarrow \alpha^{+}_{1}.$

The specification of the measurement points used for each of the four conditions that need to be fuzzified can be found in Table 4.

Table 4: Specification and encoding of the measurement points

Condition	Specified crisp condition states	Measurement points (mp) used
C_1	"yes", "no"	—
C_2	"X ≤ 5", "X > 5"	mp^{-}_{1} = "3"
		mp^{0}_{1} = "5"
		mp^{+}_{1} = "7"
C_3	"X ≤ 8", "X > 8"	mp^{-}_{2} = "6"
		mp^{0}_{2} = "8"
		mp^{+}_{2} = "10"
C_4	"X ≤ 10", "X > 10"	mp^{-}_{3} = "8"
		mp^{0}_{3} = "10"
		mp^{+}_{3} = "12"
C_5	"X ≤ 15", "X > 15"	mp^{-}_{4} = "13"
		mp^{0}_{4} = "15"
		mp^{+}_{4} = "17"

Combining these 12 different measurement points in all possible ways would yield $3^4 = 81$ experimental profiles (Addelman, 1962). Obviously, evaluation of this number of profiles would be too demanding a task for the experts and hence it was decided to use an orthogonal fractional factorial design. There are several possible basic plans that can be used to construct such a design, whereby the number of profiles ranges from 9 to 32. In the present paper, it was decided to use a plan involving 16 profiles, to construct the orthogonal fractional factorial design. Each profile consists of a combination of distances on the four conditions.

4.3 Tasks

Next, experts were asked to evaluate each profile in terms of the decision table's action states. By way of an example, Table 5 illustrates how the first profile specified in Table 4 is transformed in a DT (split in two parts to fit the size of the page). This DT, depicting 16 different rules, was written on an option card and presented to the respondents. The respondents were then asked to appraise each individual decision rule by filling in the action part of the DT (FAS_1 = 'satisfied'; FAS_2 = 'not satisfied'). Clearly, an identical approach was followed with respect to the remaining 15 profiles in the experiment. Note that condition C_1 is not included in the option card, given its non-fuzzy character.

Table 5: Illustration of an option card (profile 0 0 0 0)

1. C2 Dist. to residential area?	X =< 3							
2. C3 Dist. to school/hospital?	X =< 6				X > 6			
3. C4 Dist. to recreational area?	X =< 8		X > 8		X =< 8		X > 8	
4. C5 Dist. to scenic area?	X =< 13	X > 13	X =< 13	X > 13	X =< 13	X > 13	X =< 13	X > 13
1. FAS1 Satisfied	.	.	.	.	.	.	.	.
2. FAS2 Not satisfied	.	.	.	.	.	.	.	.
	1	2	3	4	5	6	7	8

1. C2 Dist. to residential area?	X > 3							
2. C3 Dist. to school/hospital?	X =< 6				X > 6			
3. C4 Dist. to recreational area?	X =< 8		X > 8		X =< 8		X > 8	
4. C5 Dist. to scenic area?	X =< 13	X > 13	X =< 13	X > 13	X =< 13	X > 13	X =< 13	X > 13
1. FAS1 Satisfied	.	.	.	.	.	.	.	.
2. FAS2 Not satisfied	.	.	.	.	.	.	.	.
	9	10	11	12	13	14	15	16

In total, useful answers of 16 experts were obtained. These experts were all CEO's of the companies included in the sample and had experience in site selection. The obtained answers are in fact frequency data for each action state of each decision rule of each DT in the experiment. In total, 4,096 observations (i.e. 16 profiles or DTs, each consisting of 16 decision rules evaluated by 16 different respondents) were obtained. These observations were placed in a matrix in the form of aggregated frequency data. This data matrix was used as input file to estimate the membership values for the 12 measurement points.

The membership values were estimated using an optimisation routine that was especially developed for our estimation problem. The threshold value suggests that the iterations ended if the goodness-of-fit of the model did not improve more than 0.0001. The minimum and maximum values for the estimates were set — logically — at 0.0 and 1.0, respectively. The starting values were all set equal to 0.5; but these values were later changed in order ensure that no local optimum was found (ML estimation implies having a unimodal function). The results of the optimisation routine are shown in Table 6.

Table 6 depicts the membership value estimation results for the 12 measurement points (lower, initial and upper measurement points) for the four conditions in the FDT that were fuzzified. In addition, the increment in these estimated membership values is also given. Note that convergence was achieved after 4 iterations.

Table 6: Estimation results

	membership value estimation results lower mp_i	initial mp_i	upper mp_i	Increment in membership values		
	α^-_i	α^0_i	α^+_i	$\alpha^-_i \rightarrow \alpha^0_i$	$\alpha^0_i \rightarrow \alpha^+_i$	$\alpha^-_i \rightarrow \alpha^+_i$
C_2	.088	.145	.241	.057	.096	.153
C_3	.285	.318	.410	.033	.092	.125
C_4	.353	.392	.416	.039	.024	.063
C_5	.412	.438	.461	.026	.023	.049
Log-likelihood: - 3605.43400 (4 iterations); $N = 4096$						

The overall estimation results of the different membership values may be considered very satisfactory. First, the estimation results are consistent with theoretical and logical expectations giving face validity to the results. The membership values increase as a higher measurement point is used. Fulfilling this property of *monotonicity* in the present context is important since the following relation, $0 \leq \alpha^-_i \leq \alpha^0_i \leq \alpha^+_i \leq 1$, should hold.

Second, the model produces identical estimation results if other starting values are used, suggesting that the true optimum was reached. This means that our optimum found is stable and independent from the selection of the starting values. In each case, the model returned our initial optimum found; only the number of iterations needed differed.

Third, our approach demonstrates that a multi-dimensional approach to estimate membership values and functions is possible and also desirable. Instead of selecting *ad hoc* four separate membership functions, they are now estimated simultaneously from the input data.

Analysed in more detail, a number of additional interesting conclusions can be drawn. First, it can be noted that for condition C_2 (representing the distance to a residential area), a distance of 3 km (i.e. lower measurement, mp^-_1) results in an almost zero membership value (α^-_1 = 0.088). Hence, all potential industrial locations that are situated at less than 3 km from a residential area will be evaluated as not satisfactory. In the data matrix used as input file, it can also be seen that nearly all decision rules that have $X \leq 3$ result in a rejection of the alternative. Given that the associated membership value in the fuzzy set "*long* distance to a residential area" is (almost) equal to zero, the possibility of accepting this choice alternative will, as a result of the use of the product operator, also be (almost) equal to zero. Consequently, 3 km is considered (almost) a full member of the fuzzy set "*short* distance to a residential area" which implies that the possibility of *not* accepting this choice alternative will be (almost) equal to one. As the distance between a location site and a residential area increases (use of mp^0_1 and mp^+_1), the membership values to the fuzzy set "*long* distance to a residential area" also increase to .145 and .241, respectively. The increase is not strictly linear since the increments are .057 and .096. If the membership values found are set out against the three measurement points in a two-dimensional plane, and a curve is fitted through it, then the corresponding membership function for condition C_2 is found. Once this function has been determined, it is also possible to calculate the corresponding membership values of other distances than those used as measurement points.

Second, the results found for condition C_3 (distance to school or hospital) are equally satisfactory. Note, however, that the lower measurement point used does not result in an almost zero membership value. It is equal to .285. As one moves along the distance domain, the membership values increase.

Third, the membership value estimates for the measurement point of the remaining two conditions C_4 and C_5 show only little variation. The increment in membership value with changing distances is very small. Hence, their influence on the choice problem may be considered of minor importance. In particular, condition C_5 (distance to a scenic area) has little influence on the choice outcome. The experts' behaviour was little influenced by altering distances to a scenic area. Besides showing little variation, it can be seen that the membership values found are also close to 0.5, which may point to a situation of indecision.

5 CONCLUSION AND DISCUSSION

In this paper, a method proposed to estimate membership values of the fuzzy sets used in the condition and action part of a fuzzy decision table (FDT) was applied to the locational preference of decision makers in the petro-chemical industry. Although, only 16 valuable results from 19 respondents were obtained for the purpose of fuzzification, it can be stated that the constrained maximum likelihood estimation procedure performs adequately with real data. The model produces valid estimation results that fulfil all desired properties (face validity, unimodality, partition constraint). Furthermore, the approach is multi-dimensional and the method is also direct. The latter characteristic refers to the fact that the membership functions can be directly deduced from the input data. The developed approach also concurs very well with the introduced concept of an FDT.

A (more or less disadvantageous) point of the modelling approach that should also be stressed is the problem of data collection. It is no coincidence that this particular problem is closely linked with the table contraction-expansion problem in an FDT. It is only when an FDT is specified and estimated in its expanded form (i.e. at the individual decision rule level) that valuable estimation results can be obtained. To a certain extent, this problem restricts the application of our approach in that the multidimensional fuzzification of large FDTs is rather problematic. Working with expanded FDTs has an effect on the data that are needed to estimate the model; these data should be collected at the individual decision rule level. Hence, the more condition and action states in the FDT, the larger the set of fuzzy decision rules, and thus the more difficult the task will be for the participating respondents.

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