Viewing the relative importance of some surface parameters associated with pre-monsoon thunderstorms through Ampliative Reasoning

Sutapa Chaudhuri and Surajit Chattopadhyay
Department of Atmospheric Sciences
University of Calcutta, India

Address for correspondence: 92, APC Road,Calcutta-700 009

Abstract

Instead of going into the physical detail of the pre-monsoon thunderstorms of north eastern India, a mathematical study has been done to discern the relative importance of some prominent surface parameters namely, surface temperature, relative humidity and air-pressure, in creating severe thunderstorms over the aforesaid region. The dataset associated with this weather phenomenon has been explored through the technique of Ampliative Reasoning. It has been finally found that surface temperature has the most important role in creating pre-monsoon thunderstorms. Relative humidity is less important and air-pressure is the least important.

Key words: Ampliative reasoning, Entropy, thunderstorms

Introduction:

The pre-monsoon thunderstorm, locally known as a Nor'wester, represents a mesoscale phenomenon. This kind of severe storm happens over the Northeastern part of India during the period of pre-monsoon (March-May). Since the pre-monsoon thunderstorms are generally accompanied by torrential rain, high wind speed, hail and so forth, an appropriate prediction with sufficient lead-time has continued to be a challenge to atmospheric scientists. Almost all experiments related to prediction of these storms have been based either upon statistical or numerical techniques (Murphy et al. 1989 [4], Wilks 1997 [5], Kumar et al. 1996 [3]). The complexity of the meteorological system and insufficient data has recurrently led to flawed results. Consequently, no method to date has proved sufficient to predict pre-monsoon thunderstorms over the Northeastern part of India.

The present paper uses the method of "Ampliative" reasoning (Klir and Folger, 2000) [2] to arrange, according to importance, some prominent surface parameters associated with this kind of thunderstorm. The percentage changes in the magnitudes of the corresponding parameters have been taken as the inputs for the study. Ampliative reasoning has been applied to discern the variation in the entropy associated with the probability distributions corresponding to the expected changes (%) in the magnitudes of the parameters under study. The parameter with maximum fluctuation in the entropy with change in the expected change in the magnitude (%) has been identified as the most important parameter associated with the pre-monsoon thunderstorm of the region. Surface parameters tested in this paper are: surface temperature, relative humidity, and air pressure.

Ampliative Reasoning:

Ampliative Reasoning is a probabilistic adaptation of a more general principle of reasoning in which the conclusions are not entailed in the given premises. This principle is based on two statements:

Knowing ignorance is strength.

Ignoring knowledge is sickness.

When applied within the framework of probability theory, this principle is made operational by employing Shannon entropy as the unique measure of information. Here, among all probability distributions that conform to the evidence, the chosen distribution needs to be ensured to have maximum uncertainty (i.e. minimal information) (Burg, 1967) [1]

Thus, the problem is to determine a probability distribution that maximizes the function:

$$H(p_1, p_2, \ldots, p_n) = -\sum_{i=1}^{n} p_i \ln p_i \quad \ldots\ldots(1)$$

The constraints are:

i) $p_i \geq 0 \forall i \in N$

ii) $\sum_{i=1}^{n} p_i = 1$

iii) $E(x) = \sum_{i=1}^{n} p_i x_i$

We construct the Lagrangian,

$$L = -\sum_{i=1}^{n} p_i \ln p_i - \boldsymbol{a}\left(\sum_{i=1}^{n} p_i - 1\right) - \boldsymbol{b}\left(\sum_{i=1}^{n} p_i x_i - \mathrm{E}(x)\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots \ (2)$$

Where $\boldsymbol{a}$ and $\boldsymbol{b}$ are Lagrange Multipliers.

Partial differentiation of equation (2) yields:

$$\frac{\partial L}{\partial p_i} = -\lambda n p_i - 1 - \boldsymbol{a} - \boldsymbol{b}x_i = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \ (3)$$

$$\frac{\partial L}{\partial \boldsymbol{a}} = \quad 1 - \sum_{i=1}^{n} p_i \quad \ldots\ldots\ldots\ldots\ldots\ldots \ (4)$$

$$\frac{\partial L}{\partial \boldsymbol{b}} = \mathrm{E}(x) - \sum_{i=1}^{n} p_i x_i \quad \ldots\ldots\ldots\ldots\ldots\ldots \ (5)$$

Using (3) and i = 1,2,3,…,n

$$\left.\begin{array}{l} p_1 = \exp(-1 - \boldsymbol{a} - \boldsymbol{b}x_1) \\ p_2 = \exp(-1 - \boldsymbol{a} - \boldsymbol{b}x_2) \\ \\ p_n = \exp(-1 - \boldsymbol{a} - \boldsymbol{b}x_n) \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

So, $p_i = \dfrac{\exp(-\boldsymbol{b}x_i)}{\sum_{k=1}^{n} \exp(-\boldsymbol{b}x_k)}$ ……………………………………………………………..(7)

Therefore, $E(x) = \dfrac{\sum_{i=1}^{n} x_i \exp(-\boldsymbol{b}x_i)}{\sum_{i=1}^{n} \exp(-\boldsymbol{b}x_i)}$

$$\Rightarrow \sum_{i=1}^{n} \left[x_i - E(x)\right] \exp(-\boldsymbol{b}x_i) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

When (8) is solved for b and the solution is substituted in (7), maximum entropy probabilities are obtained and thus, maximum H ($p_1, p_2, ..., p_h$) is achieved.

Data and Analysis:

In the present study, thunderstorms occurring over Calcutta (Kolkata), Bhubaneswar, Agartala, Gopalpur have been considered. The number of thunderstorms considered in this study is 65. Values of the previously mentioned parameters before and after thunderstorms have been taken and percentage changes in the values due to thunderstorms have been calculated.

Results, Discussion, and Conclusion:

Equation (8) has been framed by varying n from 1 to 65 for each of the parameters. The expected changes (%) in the magnitudes of the parameters have been put in the place of $E(x)$. Each equation framed this way has a ‘ b ’ that has been found by using the Newton/Raphson method. Each solution for b has produced a maximum entropy probability distribution. Using these probability distributions, entropies as defined in (1) have been calculated for each equation. The summarized results have been displayed in Table-1. From this Table it follows that maximum fluctuation in the entropy value has occurred for surface temperature and minimum fluctuation has occurred in case of air pressure. Thus, as a consequence of severe thunderstorms of the pre-monsoon season, change in the value of surface temperature is more probable than change in the value of relative humidity and air-pressure. Feedback from these parameters into thunderstorm creation therefore suggests that surface temperature has the largest contribution (of the three parameters considered) in creating new severe thunderstorms.

Expected change in the magnitude of the parameter due to thunderstorm (%)	Entropy associated with surface temperature.	Entropy associated with relative humidity.	Entropy associated with air-pressure.
5%	13.0756	10.7634	16.0172
6%	11.0832	10.3211	15.9875
7%	10.0123	9.5674	15.0011
8%	9.1745	8.9921	14.9324
9%	7.2214	7.8764	14.1352
10%	5.3124	6.9342	14.0021
11%	3.1437	5.9873	13.5683

Table-1. A tabular presentation of the entropies associated with different expected magnitudes of changes (%) in some prominent parameters due to thunderstorms.

References:

[1] Burg, J.P.1967, Maximum entropy spectral analysis, in 37^{th} annual international meeting, Soc.of explor. *Geophysics*, Oklahoma City, OK

[2] Klir, G.J. and T.A.Folger, 2000: *Fuzzy-sets, Uncertainty and Information*. Prentice- Hall, USA.

[3] Kumar, A., M.Horeling, A. Leetama, and P. Sardeshmukh, 1996: Assessing a GCM's suitability for making seasonal predictions, *J. Climate*, **9,** pp115-129.

[4] Murphy, A., B.G.Brown, and Y.S.Chen, 1989: Diagnostic verifications and temperature forecast, *Weather and Forecasting*, **4,** pp. 485-501.

[5] Wilks, D. S., 1997: Resampling hypothesis test for autocorrelated fields, *J. Climate*, **10**, pp. 65-82.