Earth: with 23.5 degrees north latitude as the central parallel.

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Frank E. Barmore (reprinted, in part, from The Wisconsin Geographer (with permission)).
Spatial Analysis, the Wisconsin Idea and the UW-System. The Use and Abuse of Dispersion Statistics
Sandra L. Arlinghaus, Ruben De la Sierra. Revitalizing Maps or Images?
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The purpose of Solstice is to promote interaction between geography and mathematics. Articles in which elements of one discipline are used to shed light on the other are particularly sought. Also welcome are original contributions that are purely geographical or purely mathematical. These may be prefaced (by editor or author) with commentary suggesting directions that might lead toward the desired interactions.
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ANIMAPS
Sandra L. Arlinghaus, William D. Drake, and John D. Nystuen
all of both University of Michigan and Community Systems Foundation
with data and other input from Audra Laug, Kris S. Oswalt, Diana Sammataro
University of Michigan; Community Systems Foundation; Pennsylvania State University
(respectively).

Introduction
Spatial analysis is often conducted in a single time slice. One sees thematic maps showing
per capita Gross National Product by county for a single year, yet others showing pH level
from soil samples interpolated across an agricultural surface, and still others showing, at
the global scale, the association between total fertility rates and level of women's
education in developing nations. To be sure, each of these is useful and each offers
directions for further analysis that might otherwise go undetected by a student of the data
set alone; each can also serve as a baseline map from which to build over time.
How much more useful these, and other maps, can become when change over time and
diffusion processes can be tracked in the underlying variables being mapped. A sequence
of static maps can be grouped as a single animated file, flipping from one map to the next,
with exact overlays. When the animated file is uploaded to the Internet, the reader is
offered a unique opportunity, not possible in conventional hardcopy publishing: to have
color animated maps at virtually no cost.
Animated maps (“animaps”) can be made in specific software packages, such as Microsoft
PowerPoint; however, conversion of the file format to one that is readily uploadable to the
web will be required (current versions of PowerPoint offer such an option). Another route
is simply to use animation software that makes animated files from files in .gif or .jpg
format. These packages have the advantage of universality. Any map in a GIS can be
exported, for example, to Adobe PhotoShop and saved there in either .gif or .jpg format.
The manner of exporting the map from the GIS in which it was created may not always be
straightforward (although in most modern GISs it is). When the export process is not
obvious, it is always possible to use the command that is universal in Windows (3.0, 3.1,
95, etc.), alt+PrintScreen, to capture an image of the map on the Windows clipboard and
paste it into a blank canvas in PhotoShop (cropping the pasted image as required).
Universality has many advantages: it offers flexibility to users with many state-of-the-art
options and it offers perhaps unforeseen opportunity to users who are still several
generations removed from state of the art.

Long-range Planning
From 1992 to 1996, Community Systems Foundation (501(c)3 corporation) in Ann Arbor
developed a Management Information System (directed by Drake and Oswalt) for the
country of Syria for UNFPA (United Nations Family Planning Agency). During the course
of that project, there were five missions (MCH/FP: Maternal and Child Health/Family
Planning) to develop a system for the monitoring and evaluation of health care and family
planning at Syrian Ministry of Health facilities (both urban and rural). Among other things,
sequences of GIS maps were developed (Oswalt and Arlinghaus) to show distribution
patterns of acceptance of various health care practices at various geographic scales: from
the highly local to country-wide. Considerable effort was expended in getting best-possible spatial information from field sources. Further effort involved training and education on MIS/GIS use of indigenous personnel, both in-country and in Ann Arbor, with the eventual goal that with mastery of mapping and other software would come independence and the capability for the Syrians, themselves, to continue the process begun with our assistance.

Even in a situation of sparse spatial data, thousands of maps were generated over a period of five years. For the maps to be useful, they needed to become available to a widely scattered audience. What was possible given the state of technology at the time in Syria, was to make a series of wall-map sized posters showing various GIS maps and offering a brief comment next to the map on the poster. When laminated and mounted on the walls in selected Syrian health centers, these maps told a story that the mounds of data collected at those centers never revealed to the workers in the centers. They served as a continuing source of motivation (as they, for the most part, transcended language barriers) and as a glimpse of what the future might bring. It was for related purposes that data accumulated over a fourteen month period was made into an animated map (Arlinghaus, February, 1997). This map compressed volumes of data, of Syrian Ministry of Health data mapped by health center, into a single 20 second file.

Animated map: the small red dots represent Syrian Ministry of Health Centers. The red triangles represent the total number of women visiting the health center in a one-month period. Data sets were from November, 1994 to December, 1995. Because there is no animated legend on this animated map, the single maps are enumerated below by month.
Single maps, by month:

**November, 1994**

**December, 1994**
In order to analyze an animated map it can be helpful also to have the individual maps, from which the animation was made, available as well. In the case of the Syrian animap, it
appears that, when viewed at the country scale, much of the variation occurs in the province of Aleppo (see reference map for place-name information).

Overall, there appeared to be general expansion of acceptance of MCH/FP over time; however, more fluctuation in pattern, from month to month appeared than one might expect. If the expansion is real, certainly the trend as measured by the data is NOT one of steady increase. This observation might lead one to consider the level of steadiness and local cultural preferences in reporting data on a month by month basis. Accumulated data could easily account for the high level of variability. The animation points to one direction for further investigation: that of tracking timeliness of reporting of information.

Global Animap
In the case of the Syrian animap, more information might suggest level of significance of observed patterns. In some cases, diffusion, be it of the "infill" or "sprawl" sort, is clearer than it is in others. A few months ago, Nystuen discussed the concern of his colleague in Entomology, Sammataro, in tracking the global diffusion of the Varroa Mite, a pest which threatens the honeybee population. This seemed a good opportunity to press into service the technique developed for Syria of using animaps.

Data was provided for most of the world's countries for much of the 20th century. The same basic strategy as was used for the Syrian data was to be used for the mite data, as well. One additional suggestion (Nystuen) was to alter the interval between images to reflect the uneveness in time points for collecting the data (longer spacing between frames show a wider gap between data observations). Yet another was to have previous data in one color and data relating to the current frame in a different color (Nystuen). We used a simple color selection to track the advancing wave of the varroa mite across the nations of
the world. Future work with these maps might involve deeper numerical analysis of the data, as percentages or other, and subsequent remapping (Drake) as the data quality permits.

**Varroa Mite Animap**

![Global Distribution of Varroa Mites, 1904](image)

In this case, the pattern of advance is clear and, unlike the Syrian case, is one of steady increase. Still, one might wonder about variability in the timing of reporting. It is interesting that from early beginnings in Southeast Asia, the initial spread was quite slow. With air travel becoming more frequent, post World War II, the spread of the mite accelerated; indeed, with the more generally interconnected world, the pace of mite diffusion has also accelerated. Whether or not there is a causal connection would need verification. There is, however, an obvious spatial association that is enough to suggest such additional study.

**Surrogates for lack of data**

Sometimes it is difficult to acquire data over a period of time. With a bit of imagination, one may instead be able to use a surrogate variable to capture easily what might otherwise have been difficult to capture. To illustrate this sort of technique, in a time-dependent framework, consider the following animap. When African-Americans first came to North America, they entered often along the south and southeastern shores of modern-day U.S.A. Over time, population migrated and moved throughout the country. If one considers as a surrogate to having year by year data for that movement, the fact that not many people move over time all that far from their point of entry to the country, then one
might capture the temporal movement pattern over centuries by the spatial density pattern at a single time slice. To test this idea from the standpoint of simple mapping, the U.S. was mapped by county according to density of African-American population (1990 Census data) (S. Arlinghaus and A. Laug). The mapping of this initial test-run was kept simple: the default lat/long framework, rather than a conventional projection, were used in the GIS (Atlas GIS, version 3.03) for eventual ease in switching to other projections. As had Nystuen, Laug wished to color percentages from previous frames all in one color, with percentages in the current frame colored in a different set of colors. She also wanted to track the advancing edge, as had Nystuen, but in addition wanted to see gradations in that edge. There is a tradeoff in clarity; how many categories should one use on the edge?


The pattern that emerges at this one time slice does appear to mimic the general history of African-American migration in the U.S., over centuries. To have a firmer idea as to the extent to which the internal U.S.migration/density assumptions actually migration to the U.S. over time, a number of additional steps would be required. The animap guides the research direction. Indeed, some of the issues one might reasonably examine involve (but are not limited to)

• confronting the animap based on internal density patterns with an animap based on movement patters from historical data;
• consideration of migration surges based on major political and other events (Detroit in World War II, for example);
• consideration of reverse migration
• consideration of nature of surrogate function (is there an underlying assumption of monotonicity, for example).

All of these might be captured within a broader fractal/chaos framework. Self-similarity is at the heart of this transformation from time to space: migration frequency is similar to density patterns within counties.

Animaps display spatial and temporal pattern together in a single .gif file.

NOTE: TO ACTUALLY SEE THE ANIMATED MAPS, ONE MUST LOOK AT THEM ONLINE.
I have long been impressed with the power of the simplest statistical measures (population size, average and standard deviation) to effectively characterize the distribution of some property of a collection of things. These simple statistics for describing distributions are widely recognized, widely understood and, when properly used, possess significant descriptive and analytical power. These simple statistics have proven to be very useful in description and analysis in other disciplines. For example, the results of polls reported in the popular press often give the sample size, the average result and some measure of the reproducibility based on the standard deviation or its estimated value. In the study of dynamics of rigid bodies, these three measures of the spatial distribution of mass are equivalent to the total mass, the center of mass and the moment of inertia [see note 1]. No other spatial characteristics of the mass distribution are needed for the rigid body equations of motion. When describing the spatial distribution of electrical charge, the equivalent three simplest measures of the distribution -- the total charge, the dipole moment and the quadrupole moment -- are often capable of characterizing all the important features of the distribution’s interaction with other collections of charge. And there are many other examples.

"The distinctively geographical question is why are spatial distributions structured the way they are?" (Abler, et al., 1971). Clearly, such a question can not be answered until the distributions of interest can be described. Geographical spatial description and analysis have become quite sophisticated and quite complex. In spite of this it is still often desirable to describe and make comparisons between spatial distributions in the simplest possible terms -- size, average location and dispersion. Why not use the simple, widely used and widely understood statistical moments of a distribution for describing spatial distributions?

The following discussion will: a) demonstrate the adaptation of the idea of location average and location dispersion to distributions in two (and higher) dimensional spaces; b) point out that the misconceived "ellipse of dispersion" is an inappropriate description of dispersion; c) describe the adaptation of appropriate measures of location and dispersion to non-Euclidean spaces (the curved surface of the earth) and d) as an example, use these measures to describe and comment on some features of the University of Wisconsin System.

II. Definitions

For describing spatial distributions of a collection of things scattered in one dimension, an appropriate minimal set of statistics would be the following: i) the size of the population of the things; ii) their average location; and iii) the standard deviation of their location. The location is usually given as some distance from an arbitrarily chosen marker. For distributions in two or three dimensions these concepts must be appropriately extended. In the case of the size of the population of things, there is no difficulty. The concept of average location is easily extended into two or three dimensions by simply taking the location of each member of the population as a location vector. These location vectors are vectors whose magnitudes and directions are taken as the distances and directions from an arbitrary marker of the various members of the population. Then the power and convenience of vector algebra can be used to calculate the average location or center. The result is a location that corresponds to the "center of gravity" or balance point of the distribution (Barmore, 1991). The extension of the concept of standard deviation into two or three dimensions is more complex and merits more discussion.
The standard deviation in one dimension is \( S \), where

\[
\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} d(i)^2
\]

Eq. (1)

The \( d(i) \)s are the distances from the average location of the various members of the distribution and \( n \) is the number of things in the distribution. If this definition is to be extended to two or three dimensions by replacing the distances, \( d(i) \), with location vectors, as is done in determining the average location, then it must be decided how the "square" of the location vector will be calculated. In order to square a vector, it must be "multiplied" by itself. There are three forms of vector multiplication: i) the scalar product (or "dot" product or inner product); ii) the vector product (or "cross" product or outer product); and iii) the tensor product (or matrix or dyadic product). [See note 2].

If the scalar product is chosen, the result is a scalar -- a single number. This single number corresponds to the unnamed index of dispersion described by Furfey (1927). It is equal to the dynamical radius defined by Stewart and Warntz (1958, p.182). It is equal to the standard distance deviation defined by Neft (1981, p.55). It is also the same as the standard distance and is equal to 0.707 (the square root of 1/2) times \( DS \), the root-mean-square of the \( d(i) \)s, where the \( d(i) \)s are the distances between the various pairs of members of the distribution -- both of which are discussed by Kellerman (1981, p.15, 16). But, Furfey (1927, p.97-98), who introduced the idea, was of the opinion that there were better ways of representing areal distributions. Also, there is another single number that represents dispersion -- average population density. But it has also been found wanting (Day and Day, 1973). A single number is too simple a statistic for dispersion. A statistic is needed that is capable of carrying a richer array of information. The extension of \( S \) into two or three dimensions using the scalar product would, at best, be incomplete.

If the vector product is chosen for the product of the \( d(i) \)s with themselves the result is always zero. This is not useful.

If the tensor product is chosen for the product of the \( d(i) \)s the summed result is a multiple component statistic. These components can be displayed in a variety of ways [again, refer to note 2]. I choose to simply arrange the components in a square array. Then,

\[
\mathbf{S}^2 = \begin{pmatrix}
S_{xx}^2 & S_{xy}^2 & S_{xz}^2 \\
S_{yx}^2 & S_{yy}^2 & S_{yz}^2 \\
S_{zx}^2 & S_{zy}^2 & S_{zz}^2
\end{pmatrix}
\]

Eq. (2)

for the case of extension of \( S \) into three dimensions. If the extension were into only two dimensions it would be a 2x2 array. The various components are given, for example, by

\[
S_{xy}^2 = \frac{1}{n} \sum_{i=1}^{n} [d_x(i) d_y(i)]
\]

Eq. (3)

The diagonal terms of \( \mathbf{S}^2 \) are the squares of the standard deviations in the directions of the three coordinate axes. The off-diagonal or cross-terms are invariant under an interchange of the subscripts so that, for example, \( S_{xy}^2 = S_{yx}^2 \). The square of the standard distance (defined and used by others as noted above) is equal to the sum of the diagonal elements of the array. This sum is invariant under rotation of the coordinate system used in the computations. Hence, the standard distance does not depend on the orientation of the coordinate system chosen. Formal procedures exist for determining the square of the standard deviation in any chosen direction. More useful for our purposes here, is the possibility of finding a coordinate system, rotated relative to the initial one, for which the array appears in "reduced form.”

In this reduced form all the off-diagonal terms are zero and the diagonal terms are: i) the value of the standard deviation squared in the direction for which it has its maximum possible value; ii) the value of the standard deviation squared in an orthogonal direction for which it has its
minimum possible value and iii) the value of the standard deviation squared in a direction orthogonal to the previous two and which has a value between the maximum and minimum value. In the two-dimensional case there are two diagonal terms: i) and ii) above. Then one has:

\[
\begin{pmatrix}
S_{xx}' & 0 & 0 \\
0 & S_{yy}' & 0 \\
0 & 0 & S_{zz}'
\end{pmatrix}, \quad \text{Eq. (4)}
\]

where the diagonal terms are the result of the calculations in directions parallel to the coordinate axes in the new (or rotated) coordinate system, indicated by the primes (').

The computation of these values and directions is an eigenvalue problem. The diagonal elements of \( S^2 \) are the eigenvalues and are found by solving a single cubic equation (or quadratic equation for the two dimensional case). The orientation of the new or primed coordinate system is given by the eigenvectors which are found by solving a triplet of coupled linear equations (or a pair of coupled linear equations in the two dimensional case). The equations are simple and the procedure for solving them is straightforward (Band, 1959, Eqs. 5.18, 5.19).

In addition, as mentioned in note 2, the 3x3 array of Eqs. 2 and 4 can be represented geometrically by a closed surface in three dimensions. If the array is 2x2 then it can be represented by a closed figure in two dimensions. In either case the distance from the center of the figure to the boundary in any particular direction would equal the standard deviation in the same direction. But, this surface in three dimensions is not an ellipsoid and the two-dimensional figure is not an ellipse. This was pointed out by Furfey (1927, p.95) in response to Lefever's proposal of a "standard deviational ellipse" for measuring geographical concentration (Lefever, 1926). Later writers seem to have overlooked or misunderstood this portion of Furfey's paper. For example, Kellerman (1981, p.22) seems to believe that the two dimensional figure will fail to be an ellipse only in some special cases. This is not so and many of the statements by Kellerman about this nonexistent ellipse are incorrect! The only time when the surface or figure will be a simple one is the exceedingly unlikely case when the dispersion shows no directional variation whatsoever. Then the result is a sphere or a circle. An ellipsoid or ellipse does not represent the dispersion in any simple way and is, at best, misleading [see Note 3]. The use of the "ellipse or ellipsoid of dispersion" as usually defined should not continue.

More suitable and proper is a two or three dimensional cross of dispersion. The half-lengths of the arms of the cross are equal to the square root of the elements of the array in its reduced form \((\mathbf{S})\). The arms of the cross extend plus or minus one standard deviation from the center and the center has been taken as the average location. The cross is oriented so that its arms are in the directions for which the standard deviations are maximum and minimum. The standard distance is simply related to the cross, being the square root of the sum of the squares of the half-lengths of the arms.

The two-dimensional cross of dispersion is especially suitable for describing two-dimensional spatial distributions in Geography. It is rich in information, carrying with it all the relevant information about the first and second moments of the distribution. It can be computed in a straightforward way using simple, elementary and non-iterative procedures. The computation is free of mathematical difficulties, provided only that the distributions are finite and dispersed in a finite space. It can be computed for continuous as well as discrete distributions. It can be displayed on a map at the same scale as the map. It is a visually effective and widely recognized symbol. Readers are accustomed to seeing statistical data represented as a point indicating the average and the "error bars" representing the standard deviation. The cross of dispersion is a natural extension of this symbol into two or three dimensions.

III. Statistics for Distributions in Non-Euclidean Spaces

Two limitations of the cross of dispersion must be considered. First, it carries information only about the first and second moments. If the skewness or higher moments of the distribution are
important, then additional statistics must be calculated and methods of displaying the results devised. At least one effort has been made by Monmonier (1992). Second, the preceding discussion has assumed that space is Euclidean or “flat”. But the surface of the Earth is not Euclidean or flat and as a result there are computational difficulties that must be dealt with if statistical concepts are to be extended to two-dimensional distributions on the Earth's surface. Traditionally there have been two different ways of working on problems in non-Euclidean spaces. One way is to find a higher dimensional Euclidean space in which to embed the non-Euclidean space. Then the geometry is familiar and computations can proceed using familiar methods. Thus, one could embed the curved two-dimensional surface of the Earth in a Euclidean three-dimensional space (as indeed it is) and proceed.

The second possibility is to adapt and restrict the computation of statistics of distributions on the non-Euclidean surface of the Earth to the surface. We are largely confined to the Earth's surface and it is appropriate to adopt this provincial point of view when calculating statistics of surficial distributions. The key to the statistical computation on the Earth's non-Euclidean surface is the use of location vectors for specifying position. Two quantities remain well defined in non-Euclidean spaces: lengths of geodesics and the direction of geodesics at a given point. Therefore, a location vector of any particular location is a vector whose magnitude and direction are the length and direction of a geodesic (the arc length and direction of a great circle on a sphere) connecting the particular location and a reference location. The geodesics are curved but the location vectors are "straight". Thus, from the provincial or local point of view of someone at the reference point, the problem appears to be Euclidean -- one can proceed with the second moment computation as outlined in the preceding section. There is a long and honorable tradition in Geography of displaying the curved non-Euclidean surface of the Earth (and distributions on it) on a flat and, of necessity, distorted map. The use of location vectors for position is equivalent to working on an azimuthal-equidistant map centered at the reference location. The chosen reference point is the center (average location) of the distribution. While some distortion remains, distances and directions from the center are "true". This is all that is needed. Standard deviations are dispersions measured from the center.

Thus, the necessary techniques for calculating the simplest three moments of distributions in non-Euclidean spaces are in place. The zeroth moment is simply the population size. The population count is not changed or complicated by the non-Euclidean nature of the surface over which the population is distributed. The first moment is the center (average location) of the population. The computation of the center for distributions on the Earth's surface has been discussed previously (Barmore, 1991, 1992). The second moment (about the mean) is the standard deviation, suitably extended into a two-dimensional non-Euclidean space -- the surface of the Earth. The computation has been outlined above.

IV. The Wisconsin Idea and The University of Wisconsin System

The mission of the University of Wisconsin is often simply stated as serving the people of the state through its teaching, research and service. The University was unusual in its early history by working to serve all citizens of the state in the three areas mentioned. The University has done this by providing a wide range of instruction on and off the campus to a wide variety of citizens, doing research in areas with direct application to problems of the State and providing expert advice to citizens and agencies of the State. Universities were not always so conceived and the particular blend of ideas that drove and described these efforts has come to be called "The Wisconsin Idea." The meaning and the origin of the phrase, The Wisconsin Idea, and an associated phrase, "The Boundaries of the Campus are the boundaries of the State," are not precisely known (Stark, 1995 and note 4). In addition to the export and dispersion of the University activities from the campus to the population of the State, the University (the UW System) now consists of a variety of institutions whose campuses are dispersed about the State. Surely this dispersal is an important part of serving the people of the State and is an important part of The Wisconsin Idea. In what follows, this dispersal of the UW System will be used as an example. The dispersion statistics developed above will be used to describe and illustrate how well the UW System has developed this aspect of the Wisconsin Idea.

If the UW System and the State are to be compared, then each must be defined. I have arbitrarily taken the student population as the significant characteristic of the various components
of the UW System. More specifically, the “fall head counts” of the student populations (UW System, 1994) was used to characterize each campus [see note 5]. The locations of the 13 two-year Center campuses, the 11 four-year Comprehensive University campuses and the two Doctoral University campuses were taken as the location of an arbitrarily chosen “central place” on each campus as shown on the 7.5 min. series, 1:24,000 scale topographical maps published by the U.S. Geological Survey. With equal arbitrariness, I have chosen to characterize the State two different ways -- by its population and by its area within its boundaries. The location of the various campuses of the UW System and the boundaries of the State are displayed on the map that is Figure 1. The display of the third distribution, the population of the State, is more difficult.

Figure 1. A map showing: a) The boundaries of Wisconsin (as distinct from the more familiar mix of some boundaries and some shore line). b) The 13 two-year Center campuses (small open circles), the 11 four-year Comprehensive University campuses (small solid circles) and the two Doctoral University campuses (larger open circles).

When Furfey (1927) introduced what is now called “the standard distance” he was of the opinion that it was not suitable for graphical representation and that it would be “...better to use contour lines.” in order to show how a distribution is dispersed. If the State's area and the State's population are to be compared to (with) the UW System dispersion then all three should be represented the same way. The choice of contour maps for all three representations results in
peculiar maps. A contour map of the areal density of the areas of the State consists of a single contour -- the boundary of the State. A contour map of the areal density of students attending the various campuses would consist of a collection of a large number of nearly coincident contour lines tightly surrounding each campus location. If these two maps were combined it would appear much like Figure 1. A contour map of the general population density of the State would have a more familiar appearance but would be so different from the appearance of the other two maps that simple visual comparison would be difficult. Another possibility for displaying population density would be a “dot” map (where the number and size of the dots represent the population of places) such as those shown in An Atlas of Wisconsin (Collins, 1972). But, while these maps show the population distribution very effectively, it is still not possible to use it for simple visual comparison with the UW System distribution.

In contrast, the average location (center) and standard deviation (two-dimensional cross of dispersion) provide a uniform and consistent way of presenting distributions on a map no matter what the peculiarities of the distributions. I have calculated these statistics for the area of the State, the population of the State and the UW System. They are tabulated in Table 1 and displayed in Figure 2. The centers and crosses of dispersion were calculated as outlined in the previous sections. The computations were done assuming that all three distributions lay on the surface of a sphere whose area equals that of Clarke’s (1866) ellipsoid. The data for computations involving the area of the State are the same as used previously in determining the geographic center of the State (Barmore, 1993). The data for computations involving the general population of the State are the locations and populations of the ca. 2000 sub-county and county units available from the U.S. Census Bureau (1994, 1995). The data sources for the UW System were previously given. The various population counts are for the year 1990.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Distribution Size</th>
<th>Center Location Lat. deg.</th>
<th>Center Location Long. deg.</th>
<th>Cross of Dispersion S(max) km</th>
<th>S(min) km</th>
<th>Azimuth deg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area of State</td>
<td>169 609.8 km²</td>
<td>44.6344</td>
<td>-89.7098</td>
<td>147.37</td>
<td>96.66</td>
<td>-39.31</td>
</tr>
<tr>
<td>Population</td>
<td>4 891 769</td>
<td>43.7280</td>
<td>-88.9829</td>
<td>128.60</td>
<td>69.49</td>
<td>-44.22</td>
</tr>
<tr>
<td>UW Students</td>
<td>159 979</td>
<td>43.6468</td>
<td>-89.4000</td>
<td>125.53</td>
<td>63.21</td>
<td>-48.34</td>
</tr>
</tbody>
</table>

Note: Azimuth is the direction of S(max). It is measured from the North and is taken as positive toward the East.

As Figure 2 shows, the UW System, as I have defined it, is located and dispersed very much like the population of the State. Also, note that, although all three distributions show a strong northwest-southeast dispersion, both populations are well displaced from the location and dispersion of the area of the State. Finally, if the historical trends in these statistics [see the appendix] are reviewed, it is found that the location and dispersion of the UW System are approaching those of the population of the State. From these comparisons it seems that the UW System has been reasonably successful in serving the State by dispersing its facilities throughout the State. The dispersal of the UW System is well matched with the dispersal of the
population it desires to serve.

Figure 2. A map showing the location of the center (average location) and the cross of dispersion (two-dimensional standard deviation) of: a) the area of Wisconsin (labeled A); b) The 1990 population of Wisconsin (labeled P); c) The 1990 student population of the UW-System (labeled S).

V. Summary and Recommendations
The lowest three moments of a distribution (distribution size, average location and standard deviation of the location) are effective statistics for describing and comparing distributions. These three moments can be extended into two- and three-dimensional spaces. Their computation can be adapted to non-Euclidean spaces -- specifically the curved two-dimensional surface of the Earth. When extended into two-dimensional spaces this particular set of statistics
is well suited to being displayed on a map. I have used these statistics to demonstrate that the UW System has fulfilled The Wisconsin Idea in one particular way. Finally, I have called attention to the misbegotten "dispersion ellipse," as usually defined, and recommend that its use not continue.

VI. Appendix.

Historical data are available that allow the statistics discussed in this work to be computed for somewhat more than a century into the past. The statistics are tabulated in Tables 2 and 3. While the quality of the data for 1990, 1980 and 1970 is quite good, the data used for earlier times are less reliable. I have not attempted to reconcile disparities in the data and I have made arbitrary, though reasonable, decisions in the face of historically shifting definitions of the various populations included. For example, I have arbitrarily included UW-Stout in the compilation in 1920, 1930 and 1950 even though it was not part of the system during these years. Because of the various limitations of the data, the statistics for the years prior to 1990 should be viewed as illustrative only.

VII. Notes.

Note 1. As it happens, the moment of inertia tensor is defined as the difference of two quantities [see Goldstein, Cha. 5], one of which is similar to the $S^2$ tensor developed in this work. This is because the moment of inertia tensor is most useful if the distances are measured perpendicular to a specific direction. In contrast, in this work the measure wanted is to be based on distances measured parallel to a specific direction.

<table>
<thead>
<tr>
<th>Year</th>
<th>Distribution Population</th>
<th>0th Moment</th>
<th>1st Moment</th>
<th>2nd Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Center Location Lat. deg.</td>
<td>Long. deg.</td>
<td>Cross of Dispersion S(max) km</td>
</tr>
<tr>
<td>1990</td>
<td>159 979</td>
<td>43.6468</td>
<td>-89.4000</td>
<td>125.53</td>
</tr>
<tr>
<td>1980</td>
<td>155 499</td>
<td>43.6455</td>
<td>-89.4168</td>
<td>126.64</td>
</tr>
<tr>
<td>1970</td>
<td>130 088</td>
<td>43.6537</td>
<td>-89.4017</td>
<td>126.75</td>
</tr>
<tr>
<td>1960</td>
<td>44 587</td>
<td>43.524</td>
<td>-89.402</td>
<td>122.4</td>
</tr>
<tr>
<td>1950</td>
<td>27 498</td>
<td>43.45</td>
<td>-89.49</td>
<td>116.4</td>
</tr>
<tr>
<td>1930</td>
<td>16 455</td>
<td>43.52</td>
<td>-89.58</td>
<td>126.8</td>
</tr>
<tr>
<td>1920</td>
<td>11 531</td>
<td>43.48</td>
<td>-89.57</td>
<td>115.8</td>
</tr>
<tr>
<td>1910</td>
<td>7 167</td>
<td>43.54</td>
<td>-89.52</td>
<td>121.3</td>
</tr>
<tr>
<td>1900</td>
<td>4 451</td>
<td>43.67</td>
<td>-89.45</td>
<td>131.3</td>
</tr>
<tr>
<td>1890</td>
<td>2 043</td>
<td>43.37</td>
<td>-89.37</td>
<td>75.3</td>
</tr>
<tr>
<td>1880</td>
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<td>-89.35</td>
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<td>42.95</td>
<td>-89.48</td>
<td>47.4</td>
</tr>
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<tr>
<td>1850</td>
<td>est. 25</td>
<td>43.07</td>
<td>-89.40</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 2. Historical Distribution Moments for the UW System Student Population
Note 2. Vectors and tensors can be represented in a variety of ways. The components can be simply listed or arranged and displayed in some other suitable way. A vector can be represented as a combination of its components with unit vectors (or pairs of unit vectors in a dyadic notation in the case of tensors). For vectors, the components can be formed into row or column matrices (and for tensor, the components can be represented by a two dimensional matrix of rows and columns). Vectors and tensors can even be represented as geometrical figures or surfaces in some suitable space as is mentioned in the main text of this work. The various representations of vectors and tensors and the algebra needed for working with them in various representations is available in an enormous number of texts. I have found the short summary in Band (1959, Chapter 1) particularly clear and concise.

Note 3. While the figure defined by the various components of the $S^2$ tensor is not elliptical, it is possible to construct an ellipsoid or ellipse that is related to $S^2$. Imagine a vector, $\mathbf{v}$, whose magnitude can have various values when pointed in various directions. If this magnitude is chosen to vary so that

$$\mathbf{v}S^2\mathbf{v} = 1$$

Eq. (5)

no matter what direction the vector, $\mathbf{v}$, points, then the tip of the vector sweeps out an ellipsoid or ellipse as it is allowed to point in various directions. Thus, the ellipsoid or ellipse, "$v"$, is defined in a way that makes it the reciprocal figure of the $S^2$ tensor in the sense that when $v^2$ and $S^2$ are "multiplied" in the appropriate way the result is unity. (The inertial ellipsoid, which is related to the moment of inertia tensor, is constructed in an analogous manner. [See Goldstein, sec. 5-4].) However, the elliptical figure "$v"$ must be visualized in a space where the coordinates are measured in units of the reciprocal of length. How is such a figure to be displayed on a map? On a map of a physical space, it would be desirable to display statistics whose magnitudes are in units of length. Also, such a figure as "$v"" is counter-intuitive -- in directions for which it is small, "$S"" is large and vice versa. The characteristics of the elliptical "$v"" are the inverse of what is
desired and "v" is only related (in a non-intuitive way) to $S^2$. This is not suitable for a quantity that is to be displayed to scale on a map.

Note 4. I am unable to find any use of the phrase, "the Wisconsin Idea", prior to the publication of McCarthy's book of that title (McCarthy, 1912). Also, Jack Stark, author of "The Wisconsin Idea: The University's Service to the State" (Stark, 1995) has informed me (Stark, 1996) that neither he nor other scholars actively interested in the history of The Wisconsin Idea with whom he is in contact are aware of any use of the phrase prior to McCarthy's 1912 book.

Note 5. The UW System is characterized in this work as the "fall head count" for the various campuses for the years shown. The data used do not include the enormous enrollment in the UW-Extension of the UW System. It is somewhat ironic that the component of the system that has contributed so much to The Wisconsin Idea should be excluded. However, location data for the persons enrolled in the various Extension activities was not readily available.

VIII. References.
Concerns that involve the "static" and the "dynamic" elements of any issue are ones that endure, independent of discipline. De la Sierra, in looking at fragmentation patterns involving agricultural land use on either side of the U.S./Mexico border, invokes principles from physics to breathe life back into static satellite photographs of his region of interest (Mapping Entropy, University of Michigan, School of Natural Resources and Environment, Ph.D. Dissertation, 1998). Indeed, there are many ways in which one can revitalize static images and return the dynamic component to them that once was there. Mathematical and scientific models offer one way to do so. Diffusion studies can offer an appealing way to do so when spatial change is measured over time, particularly when the pattern displayed is a simple one in which the eye can grasp changes as one jumps from one image to the next one in time. In the case of satellite photos of agricultural land use patterns, however, the subtle patchwork quilt that fills the entire cathode ray tube does not lend itself well to being captured by the eye to its full extent. Thus, the eye may miss subtle changes that it would grasp in a scatter of dots when the same diffusion display techniques are used for changes in dot scatter and changes in land use pattern.

Thus, we offer here a few visual techniques that offer promising ways of reducing the quilt-like land use pattern to one that is easier to grasp, yet displays the desired information. In the publishing environment of the web, expansive use of color and animation offers opportunities for the display of spatial information previously undreamed of in the conventional publishing environment that traditionally balks even at a few color maps. In the sample below, we exploit various possibilities, beginning with a simple, single black and white Landsat photo.

1. LandSat photo (click on this link) of the U.S./Mexico border region.
De la Sierra partitions the region into urban and rural areas. The urban area is composed of Calexico on the U.S. side of the border, and Mexicali on the Mexico side of the border. The agricultural areas, he separates into U.S. East and U.S. West and Mexico East, Mexico Southeast, Mexico South, and Mexico West. The designations as to direction are in relation to the urban areas. The method of partition is based on a visual determination of pattern of landuse. Different grouping of parcels would result in different patterns and in different numerical measures (below). To suggest the role that the choice of partition might play, and the need for normalization, we include also values for arbitrary regions (with interpretation left to the reader).

2. Table of entropy values for each agricultural region in the partition in the image above with non-arbitrary partition.

Mexico East: 1.86
Mexico Southeast: 1.39
Mexico South: 1.49
Mexico West: 1.72
U.S. West: 1.16
U.S. East: 0.73
Total Mexico: 1.73
Total U.S.: 1.02

For regions selected arbitrarily (but including land on both sides of the border), the values were:

East: 1.76
West: 1.71
Combined total: 1.72

Generally, the measure is based on assessing the amount of disorder that appears in the pattern of agricultural landuse. The Mexico East region displays the greatest amount; the U.S. East region, the least. Readers interested in the detail of definition and calculation are referred to De la Sierra's document. For purposes of display techniques, it is sufficient for
the reader to understand that this measure is a relative one, used to compare one region to another and to understand that higher values represent higher disorder in landuse pattern.

5. A square portion of the LandSat photo was saved, using Adobe PhotoShop 4.0.1 for Windows, as a .bmp file. This bitmap was then used as a backdrop in Visual Explorer 1.1 (WoolleySoft, Scotland) and entropy values from (2) above were entered as "elevation" values for each of the pixels in the six agricultural regions. Thus, a "topographic" map is created from the photographic backdrop in which the topography is determined by the entropy values. Pixels off the topographic surface (at the lower left and in the urbanized areas) were assigned values for "Total U.S." or "Total Mexico" (as appropriate). All values were multiplied by 200 to create a larger spread in the data.

5. When one zooms in on the image in 3, eventually the "elevation" values replace the colored pixels, so that one may check the heights of each pixel to see that all is correct (see linked map).
5. Further visual clarification is obtained by removing the backdrop and zooming in even more, to display the "elevations" to be assigned to each small location (see linked map). In the process, all connection with the original layout is lost but re-registers when put back.
6. With this file of a two-dimensional photo that has been further loaded with elevations, it becomes possible to load this "digitized" file into Visual Animator where the 3-D view of the map comes to life. All of camera position, target position, vertical exaggeration, perspective, camera height, and camera angle have been held at the same setting for this sequence of four views. The international border runs along the cliff. The Mexican border regions show the highest degree of entropy. The U.S. regions exhibit the lowest values.

- The view along the U.S./Mexico border from West to East.
- The view along the border from East to West.
• The view across the border from North to South.

• The view across the border from South to North.
7. A view from above draped with the original photo.

8. A flight through the view above (in 7) appears in the animation below. The flight begins with a view along the western border. The "plane" then moves over to the U.S. Western region and flies eastward, across the red boundary separating U.S. West from U.S. East. The border looms over the U.S. regions, as cliffs over a channel. The gap through is at Calexico.
Book Review: The Universe Below by William J. Broad
(New York: Simon and Schuster, 1997, 432 pages)

Given the dimensions of the earth in comparison with the size of life forms that are spread over its surface, the collective biosphere is no more than a thin film on the earth’s surface, like an oil slick on a puddle, colorful and fascinating but essentially two dimensional and fragmented. The biosphere is thickest near the seashore with lowland forests and swamplands landward and fishing banks and coral reefs seaward. Until recently it was generally thought that most life in the deep sea occurred in the photic zone, which is the upper most 100 meters or so where light still penetrates. Below in the abyssal deep were thought dwelled only a few strange bottom creatures who survived in total darkness and crushing pressure on the debris raining down from the more abundant life above.

William Broad reports on a different, recently evolving spatial model of life’s domains, in which the biosphere extends throughout the volume of the oceans down to the bottom of the deepest trenches. This three dimensional space greatly exceeds the volume occupied by all living things on land and at the ocean surface. And contrary to previous beliefs, this domain is proving richer and more varied of life forms than even Jules Verne imagined. Small, systematic samples are revealing numerous new species with great variation from place to place. The number of species of life in the deep is now thought likely to exceed the number of all land creatures and plant species at the earth’s surface, although this fact remains uncertain because of our sparse knowledge of deep sea life. Some of the newly discovered deep sea species exist in food chains that are independent of the sun’s energy. They live entirely on heat and minerals brought up from the depth of the earth through undersea volcanic fissures. The huge, dark, and largely unknown three-dimensional space of the oceans is the universe below.

In his book, Mr. Broad reports on the human efforts to probe this world beneath. The exploration began tentatively in the Nineteenth Century and has only been vigorously pursued in the second half of the Twentieth Century. The pace of the exploration quickened in the last decade of the century with access to new and powerful technologies. Mr. Broad divides his chronicle into seven chapters each devoted to one facet of the efforts to enter and explore the deep oceans. He begins with a history of early attempts to penetrate the depths. The huge weight of sea water is the problem. Ten cubic feet of sea water equals roughly one cubic foot of lead. The Titanic rests 2 1/2 miles deep, which is about the average depth of the oceans, and where the water pressure is equal to the weight of a tower of lead the height of the Empire State Building. Life forms of the deep, being made primarily of water that is nearly incompressible, are indifferent to the pressure. Anything hollow or containing a cavity, including humans and many of our devices, are disastrously affected by the pressure, hence, the difficulty in exploring the depths. In one of the continuing ironies of our age, the military pioneered the technology that opened the oceans to exploration. They made the deep sea a battlefield in the Cold War. The military were not interested in exploration. Their interest was in being able to operate in deep water to support submarine warfare and undersea espionage. The United States developed a technological advantage over the Soviet Union by investing huge resources...
toward these purposes. Mr. Broad describes several defining events that shaped this effort. For example, in April, 1963, the USS Thresher, the most advanced attack submarine of its day, inexplicably sank, its 129 men lost in water more than a mile and a half deep. The Navy had no way to reach the ship to salvage sensitive equipment or to investigate the mystery of why she was lost. The tragedy led to much greater expenditure on the development of deep submersible craft ostensibly to make possible deep sea rescue operations but also to expand the possibilities for undersea espionage through use of search and salvage capacities to be used to obtain intelligence from sunken Soviet ships, especially nuclear equipment and devices from submarines lost at sea.

Mr. Broad is a Pulitzer prize winning science writer for the New York Times. His investigative powers are evident in this book as he details the political and policy debate that took place in Washington to direct resources into the Navy’s deep submersible operations. Most of the effort was to support espionage which was in line with the tendency of the United States to depend upon technological means for conducting espionage instead of relying on spies and secret agents. After the end of the Cold War much of this military technology was declassified and is now being used in civilian efforts at exploration. Russian equipment is also available for hire and lease as the Soviet Union had also developed deep submersible capabilities during the Cold War.

The chapter on military efforts sets the stage for the remaining chapters by appraising the reader of the difficulties of undersea operations and how the technology addresses them. The rest of the chapters detail the exploration. Chapter three describes a dive in the Pacific Ocean off the Oregon coast in which the author was a passenger of the Alvin, a Navy deep submersible being used by NOAA (National Oceanographic and Atmospheric Administration) to explore undersea volcanic chimneys or smokers around which strange life forms cluster. Chapter four is a report on enterprises that seek to gain fortunes by finding and scavenging lost treasures from shipwrecks buried in deep waters. Deep sea salvaging remains a very expensive activity. Investors want to recover costs by claiming gold and other precious materials or by exploiting public interest in shipwrecks such as with the Titanic. The story reveals that what the technology has now brought within grasp becomes enveloped in controversy over ownership, and moral and ethical issues.

A deep canyon, greater in size than the Grand Canyon, lies beneath Monterey Bay off the central California coast. Because the deep water is very near shore, land-based excursions can frequently be made with much less expense than dives of deep submersibles operating from support ships far out at sea. Chapter Five describes the activities of a research group, whose primary funding comes from David Packard, the billionaire co-founder of the Hewlett-Packard Company. This group, associated with the famous Monterey Bay Aquarium, is using unmanned vehicles to monitor undersea life at all levels to below a mile deep. They are finding an unexpectedly broad range of life forms, which for the first time can be observed in their natural habitat, including many creatures that live at mid-water depths. These life forms have been missed by marine biologist in their rush to the bottom. The Monterey Bay Canyon site is becoming a standard model for deep sea ecology because of the level of exploration but this may be misleading. Other parts of the oceans,
though sparsely sampled, reveal different collections of new species that leaves open the questions of total number of species.

Scientist and fortune hunters are not the only groups interested in deep sea exploration. Commercial interests, encouraged by governments throughout the world, are seeking to exploit deep sea resources in large scale commercial ventures. Chapter Six describes these ventures that include deep sea mining and deep sea fishing. Miners are after petroleum and gas under deep sea beds, manganese nodules on the sea floor, and even the minerals dissolved in sea water. Vast amounts of metals and other minerals exist in seawater but no practical way of extracting them exists. The manganese nodules, which contain traces of other metals as well, are thought to have developed through a bio-concentration process which makes the metal aggregations more accessible than in seawater. The modules lie on the seabed in some places like a vast field of cobblestones. Proposed means for mining them call for robotic machinery to sweep over large territories gathering the nodules and discarding debris. The prospects immediately raised concerns by environmentalist who foresaw likely untold damage to a largely unknown ecology.

The Reagan Administration seized upon the possibility of deep sea mining as a way to develop reliable sources of strategic materials independent of foreign nations. This American stance was in opposition to one of the provisions of the Law of the Sea which proposed that all minerals on or below the ocean’s floor belong to all of the people of the world that should be developed by a United Nations enterprise. On the other hand, the United States readily accepted the Exclusive Economic Zone provision that granted to coastal nations ocean resource development rights out to two hundred miles of adjacent oceans. The long American coast lines plus Pacific island possessions and protectorates created a huge American dominion, by far the largest of any nation. Industrial states have staked out large parts of the Pacific Ocean in anticipation of deep sea mining operations. Actual mining has yet to materialize because deep sea operations have to date proved too expensive to undertake.

Deep sea fishing, that is, fishing at great depths is another matter. Several coastal powers have claimed rights to fish resources found at great depth that are within their two hundred mile exclusive zone. For example, New Zealand commercial fishermen found dense schools of orange roughy about one half to one mile deep over a large sea plateau within their exclusive zone. This led to a rapid commercialization in which factory ships equipped with freezers were employed and the product exported to American and European markets. Huge profits were obtained as fish catches from competitors working established fishing banks were in decline. Unfortunately scientists discovered that the orange roughy and other deep fishes could live for more than one hundred years, some of the oldest living creatures on earth. Their slow growth and reproductive cycle makes them highly vulnerable to over exploitation.

One interesting new resource is exclusively a product of the abyssal environment. The huge pressure at great depths keeps water in liquid form even though water emanating from the volcanic smokers is very hot, well above the boiling point of water at one
atmosphere. Microbes have been found living in this hot environment in which no terrestrial organism could survive. There is a use for these microbes. Microbiologists use enzymes obtained from bacteria to multiply minute bits of DNA (deoxyribonucleic acid) until sufficient quantities become available for analysis and manipulation. Tiny bits of DNA found at crime scenes can be used to identify individuals involved in an incident. Many other amazing applications of genetic material are being discovered and all use microbes as the factories for multiplying the DNA. The best microbes for this purpose are ones that can withstand high temperatures, a fact first found by work with microbes that live in Yellowstone Park hot springs. The heat-loving microbes from the environment of the deep sea hot smokers yield enzymes that do not break down at high temperatures. New levels of purity and efficiency in producing genetic materials are possible because the chemical reactions involved can be carried out at temperatures that destroy any other bacterial contaminants.

The final chapter contains warning of detrimental human impact on the universe below. In the early years of the atomic age radioactive wastes were routinely dumped in deep waters just off-shore from populated coastlines. Most nations ended this practice but the radioactive level in several hot spots around the world remain very high and are not well contained. Other biologically active pollutants are also reaching the abyssal deep in waste streams from coastal industrial and urban sources. Human induced environmental change through contamination and over exploitation has resulted in irreparable damage to other ecosystems. We would be well advised to take care of next wilderness that we are beginning to enter.

Mr. Broad’s book is and informative and interesting. He provides detailed notes and references throughout the book to document his information sources. He also provides a useful glossary, a chronology of important events in deep ocean exploration, and a bibliography. Beyond presenting a well written and structured book, he engages the reader with a sense of wonder that come from exploration of a domain on earth still unknown in modern times.

John D. Nystuen University of Michigan
Earth: with 23.5 degrees south latitude as the central parallel.

VOLUME IX
NUMBER 2
DECEMBER, 1998

Sandra Lach Arlinghaus. Animated Four Color Theorem: Sample Map.
Sandra Lach Arlinghaus. Animaps, II.
The Four Color Problem has a rich history. Readers interested in the history might wish to read appropriate selections in The World of Mathematics. Here, it is simply stated as a theorem and then animated on a U.S. states map.

The Four Color Theorem.

In the plane, four colors are sufficient to color any map and necessary to color some. Note: adjacent regions are to be colored different colors. States that touch at a point only are not considered to be adjacent.

In the coloring scheme below, red was generally used as first choice, green as second, yellow as third, and purple as fourth. The second, third, and fourth choices were used only when required.

On occasion, the general strategy was violated in order to color efficiently; for example, Montana was colored green so that Idaho could be colored red in a vertical alternation pattern of red/green/red. The coloring is not unique. Indeed, one can make inefficient choices so that it appears that one "needs" a fifth color. The ambitious reader might try to improve upon the scheme here. However, there is always a four (or fewer) color solution available although it may not be easy to find. Surprisingly, the solution to coloring requirements on surfaces other the plane were determined well ahead of the solution in the plane.
Thus, it seems suitable that desktop GIS packages should default to four color categories when making thematic maps. Some software does and some does not.

References

TO SEE THE ANIMATION, THE FILE MUST BE VIEWED ON LINE.
Animated maps, "animaps," offer a unique opportunity to visualize changes in spatial pattern over time. Diffusion studies thus offer a natural platform from which to launch animaps (for samples, see "Animaps" in previous issue of Solstice). These maps are dwellers of cyberspace, dependent on it for their existence. To "publish" such a map in a conventional medium, such as a book, would require pages of maps (a costly venture) and still the animation, or time-tracking feature, would be lost (unless of course, as seems to becoming more and more the practice, a CD or similar medium with book files is included with the hard-copy book). Previous work has illustrated the utility of animated maps in a number of diffusion contexts. In this article, animated maps are used as tools to refresh, enliven, and analyze historical maps as well as conceptual models; hopefully, this approach will serve to underscore, as a side issue, the importance of converting historical files and other enduring ideas to an electronic format. These animated maps are presented in the common Euclidean dimensions of point, line, and area. Left to the future is to examine them more generally in Euclidean space as well as in classical non-Euclidean space and then to permit fractional dimension.

An Historical Context: the Berlin Rohrpost

Beginning in 1853, a number of experiments with relatively expensive underground pneumatic communications systems were underway in western Europe. After slowdowns caused by the Seven Weeks War (1866) and the Franco-Prussian War (1870), full-fledged pneumatic communications systems began to appear in major cities in western Europe as a speedy alternative to mail delivery through congested surface streets. Among others, the city of Berlin boasted a substantial pneumatic postal network, known (appropriately) as the "Rohrpost."

By 1901, the "Rohrpost" carried messages under most of Berlin (Figure 1). The heart of the message system was in a central office on Unter den Linden, denoted as the largest circle in Figure 1. Adjacent graphical nodes were linked as underground real-world nodes by "edges" of metal tubing. The real-world nodes had surface housing that could pump and compress air and thus receive and deliver messages.
Figure 1. Static map showing a hierarchy of nodes in the Rohrpost network.

The map of the Rohrpost (Figure 1) shows the linkage pattern of edges joining nodes and reflects, only indirectly as a static map, a hierarchy in the procedure for message transmission. Certain pneumatic stations were designated as having functions of a higher level of service than were other locations. Typically, message containers were pushed, using compressed air, from one higher order office to a handover position intermediate between higher order offices. From this handover position, suction drew the carrier toward the next higher order office. The animated map (Figure 2) shows clearly one handover node, belonging to both black and blue subnetworks. This node is a transfer point, or gate, from compression to suction, as are all other similar offices. This kind of partition was useful in suggesting a graphic code that could be used to track the progress of a message through the system (and thus detect the location of collisions or blockages).
Figure 2. Animated map showing handover position between adjacent subnetworks: this position is a transfer point from compression to suction within the system.

One might wish to consider more than the actual pattern of transmission. Would an analysis of this Rohrpost map, based on existing technique, have yielded an answer that coincided with actual field circumstances as to which node is the hub of the network? Thus, consider the map as a scatter of nodes linked by edges.

The concept of center measures, to some extent, how tight the pattern of connection is around a core of nodes. It measures whether or not there is central symmetry within the structural model: whether or not accessibility within the network is stretched in one direction or another. This sort of broad, intuitive, notion of center does not take into account the idea of volume of traffic; to do so requires looking at more than direct adjacencies in calculating weights for nodes. What happens in a remote part of town may influence traffic patterns across town. The concept of centroid, which rests on the idea of branch weight--the number of edges in the heaviest branch attached to each node, does so. The animated map in Figure 3 shows the branch weight for each node in the Rohrpost. The red node has a heaviest branch with 19 edges in it (shown as the black subnetwork in the animation). Other nodes are color coded according to heaviest branches. Those nodes with higher values are more peripheral in their function to the network: a node with a branch weight of 65 has one route coming from it with 65 edges in it--crossing the entire network from one side to the other. Nodes in a peripheral position have longer heaviest
branches and therefore greater branch weights than do nodes in the interior. Thus, it is reasonable numerically to view the most central, in this context, as the node(s) with the smallest branch weight. In this case, the centroid is the red node and it does coincide with the actual network hub.

**Figure 3.** Example to illustrate branch weight. The heaviest branch from the red node in the Rohrpost has 19 edges, labeled in this figure. All nodes in the Rohrpost graph are labeled with branch weights. Those of lowest value (in this case one node) serve as the centroid.

Beyond the mere calculation of the centroid of the network, one might wonder about using the measure to capture other elements of the network. Thus, the animated map in Figure 4 shows all nodes colored according to branch weight. The first frame of the animation shows the single node of weight 19 as a red node. The next frame shows the node of weight 47 as a red node and shows the node from the previous frame as a black node. Iteration of this coloring strategy, using red for nodes added in a frame and black for nodes accumulated through previous frames, produces the pattern shown in Figure 4. In addition, the time-spacing between successive frames is tied to the numerical distance between branch weights; thus, the time-distance between frames 1 and 2 of the animation (from branch weight 19 to branch weight 47) is substantially longer than is the time distance between any two other successive frames in the animation.
Figure 4. Note in this case the long wait in the animation from the central office to the next tier of offices suggesting the highly dominant central role (in terms of structure) played by the "central" office. Next most dominant is the role of the line of core of offices under Unter den Linden.

What the animation shows is the dominance of the central node and a line of tight control emanating from the center with many peripheral nodes, of roughly equivalent lack of centrality, scattered throughout. When the animation is checked back against the original, this "line" is in fact composed of pneumatic postal offices under Unter den Linden, a central thoroughfare in Berlin during this time period. The fit between model and field is precise at both the point (centroid) and line (street) levels. Thus, one might speculate that the areal pattern of extension/sprawl and infill evident in the Rohrpost animation of Figure 4 functions as a surrogate for actual neighborhood population density patterns in Berlin in 1900 and is thus of significance in studying planning efforts of the time. When this sort of idea is extrapolated to the future, it is not difficult to imagine, instead, satellite positions serving as a similar backdrop against which to test models that can then be extended to offer extra insight about terrain or human conditions.

A Conceptual Model Context: Hagerstrand's Diffusion of an Innovation

The context above suggests one way for considering patterns of spatial extension/sprawl and infill using tools from the mathematics of graph theory. Another, based on probabilistic considerations, employs numerical simulation to speculate or plan.
To follow the mechanics of Torsten Hagerstrand's simulation of the diffusion of an innovation, it is necessary only to understand the concepts of ordering the non-negative integers and of partitioning these numbers into disjoint sets. Indeed, the theoretical material from mathematics of "set" and "function" will underlie the real-world issues of "form" and "process."

Some of Hagerstrand's Basic Assumptions of the Simulation Method (Monte Carlo)
Assumptions to create an unbiased gaming table:
- the surface is uniform in terms of population and transport
- all contacts are equally easy in any direction
- there are an equal number of potential acceptors in each cell

Rules of the game:
- There is a set of carriers at the start (as in Figure 1)
- information is transmitted at constant intervals
- when carrier meets a new person, acceptance is immediate
- the likelihood of a carrier and another meeting depends on the distance between them (distance-decay).

Initial Setup
Figure 5 shows a map of an hypothetical region of the world. After one year, a number of individuals accept a particular innovation. Figure 6 shows a color-coded version of Figure 5; darker colors represent cells with a higher number of initial acceptors.

MAP BASED ON EMPIRICAL EVIDENCE--REGION INTERIOR IS SHADED WHITE; CELLS WITH NUMERALS IN THEM INDICATE NUMBER OF ACCEPTORS IN LOCAL REGION.

Figure 5. Distribution of original acceptors of an innovation--after 1 year--based on empirical evidence. After Hagerstrand, p. 380.
In Figure 7, a map of the same region shows the pattern of acceptors after two years—again, based on actual evidence. Notice that the pattern at a later time shows both spatial expansion and infill. These two latter concepts are enduring ones that appear over and over again in spatial analysis as well as in planning at municipal and other levels. Figure 8 shows a color-coded version of Figure 7.
Might it have been possible to make an educated guess, from Figure 5 alone, as to how the news of the innovation would spread? Could Figure 7 have been generated/predicted from Figure 5? The steps below will use the grid in Figure 9 to assign random numbers to the grid in Figure 5, producing Figure 10 as a simulated distribution of acceptors after two years.

- Construct a "floating" grid (Figure 9) to be placed over the grid on the map of Figure 5, with grid cells scaled suitably so that they match. Center the floating grid on a square in Figure 5 in which there exists an adopter (say 2F)...this is the first cell, working left to right and top to bottom, which contains a numeral. The numbers in the floating grid, used with a set of four digit random numbers, will be used to determine likely location of new adopters. It is assumed that the adopter in F2 (or in any other cell) is more likely to communicate with someone nearby than with someone far away; velocity of diffusion is expressed in terms of probability of contact.
This assumption regarding distance and probability of contact is reflected in the assignment of numerals within the grid--there are the most four digit numbers in the central cell, and the fewest in the corners. The floating grid partitions the set of four digit numbers \( \{0000, 0001, 0002, \ldots, 9998, 9999\} \) into 25 mutually disjoint subsets.

**Figure 9. 5-cell by 5-cell floating grid overlay, partitioning the set of four digit numbers.**

Given a set (or sets) of four digit random numbers--as below. Center the floating grid on F2. Use the first set of random numbers below. The first number is 6248 and it lies in the center square of the overlay. So in the simulation, the previous Figure 5 acceptor in F2 finds another acceptor nearby in F2. Use the map in Figure 10 to record the simulated distribution (a red entry). In cell F2, enter a red +1 to represent the initial adopter. Together with the original adopter, there are now two adopters in this cell.

**RANDOM NUMBERS**

<table>
<thead>
<tr>
<th>SET 1</th>
<th>SET 2</th>
<th>SET 3</th>
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<td>6248</td>
<td>4528</td>
<td>3175</td>
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<td>0925</td>
<td>3492</td>
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Figure 10. Simulated distribution of acceptors, using Set 1 of Random numbers. Original acceptors in black; simulated acceptors in red. Consider edge effect issues.

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Figure 11. A color-coded version of Figure 10.

- Pick up the floating grid and center it on G2. The second random number is 0925, located in the first cell northwest of the center cell of the overlay, cell F1. In Figure 10, a 1 in cell G2 to represent the original adopter, and a 1 in cell F1 represents the later (new) adopter.
- Continue this process, shifting the overlay so that it is centered on every cell with at least one early adopter (from Figure 5). Work from left to right, top to bottom. Use a different random number for each early adopter. If there are 3 early adopters in a given cell, use three different random numbers in entering the results in Figure 10. Just use the random numbers (supposedly already randomized) reading down the column. The enthusiast might wish also to animate the entire movement pattern of the grid.
Figure 12. Two-frame animation. First frame contains actual distribution of adopters after two years. Second frame contains simulated distribution of adopters after two years.

How does the color-coded simulation (Figure 11) compare to the actual distribution of adopters after two years (Figure 8)? Consider the animated map that superimposes actual and simulated distributions as one way to compare pattern (Figure 12). The reader might enjoy using the second and third columns of random numbers to create more simulations and compare them to the first simulation and the actual distribution. From a visual standpoint, one might further imagine subtracting cells from one another and then animating the results. From the standpoint of municipal planning and policy considerations, one might imagine applying this sort of animated model to target key initiators (within a subdivision parcel map, for example) of innovative urban or environmental character that relies on word-of-mouth diffusion throughout a neighborhood. Using a simulation strategy based on location of known neighborhood trend-setters can maximize diffusion of a favored practice while minimizing expenditure of scarce tax-payer funds. Models that can be simply executed have fine potential for actually being used in real-world settings.

References


"To control the Mississippi -- not simply to find a modus vivendi with it, but to control it, to dictate to it, to make it conform -- is a mighty task. It requires more than confidence; it requires hubris." So begins John M. Barry in *Rising Tide*, a tale of the 1927 Mississippi flood and "how it changed America." Barry measures the flood’s effects on political power, race relations, and the land itself. This history is at its best when it describes the personalities and theories that shaped flood control and relief efforts. It also does a good job of integrating natural disaster into political and social history. More typically, academic and popular historians tend to let natural disasters serve as unquestioned, exogenous agents of change.

There are really two stories here, one of the men who had tried to control the river since the early nineteenth century, and one of the men who responded when disaster struck the Mississippi Delta. The characters of the first story are the engineers who debated the "levees only" policy of flood control. Following the theories of the seventeenth-century Italian engineer Giovanni Domenico Guglielmini, some engineers believed that a system of levees would control flood waters not only by damming the banks but also by increasing the velocity of the river’s flow and hence its tendency to scour its own bottom. In effect, the levees-only theory held that the river in flood could be made to dig its own channel. Barry describes bitter rivalries among engineers; some advocating a strict levees-only policy, while others call for creating a system of outlets to divert flood waters. In particular, the author describes a struggle among James Buchanan Eads, famed builder of bridges and Civil War gunboats, Andrew Atkinson Humphreys, the quintessential Army Corps engineer, and Charles Ellet, Jr., a brilliant civilian engineer, to dominate flood control policy. While Eads pursued a system of jetties to increase the speed of the current even at low water, Humphreys and Ellet competed to produce definitive recommendations on river policy.

Mathematically-inclined readers may find much to enjoy here as the author explains how a river flows -- and floods. We learn about declivity, sediment carrying capacity, and dynamic measures such as "second-feet," which describes both the volume and force of a flood. Barry does not share the hubris of nineteenth-century engineers who thought that they could know and therefore control the river. Although engineers could understand the Po, the Rhine, the Missouri, and even the upper Mississippi, the turbulent vicissitude of the Mississippi Delta remains unknowable.

The second story is that of the flood itself, the final futile efforts to contain it, and the relief effort that followed in its wake. The flood, in Barry’s telling, undercut the power of local leaders, spurred black migration to the industrial north, and helped position Herbert Hoover to win the presidency. With their homes flooded, thousands moved into refugee camps on the only dry ground left -- the levees themselves. Things were not easy for anyone, but they were especially difficult for African Americans who were held in refugee
camps at gunpoint. In one instance, armed Boy Scouts were deployed to guard a segregated black camp.

Yet here too the focus is on great men (indeed, women are all but nonexistent). There is LeRoy Percy, the most powerful man in Greenville, Mississippi, and symbol of the Old South. There is Herbert Hoover, "The Great Humanitarian," who is shown here to be far more politically astute and ruthless than that moniker would suggest. And there is the African-American leader Robert Russa Moton, successor to Booker T. Washington at the Tuskegee Institute and chairman of the commission that investigated reports of brutality against flood victims. When the floods came, Percy warned that poor treatment of blacks in the refugee camps would only encourage out-migration to the industrial north. Barry argues convincingly that Percy was right and that this was a more important factor in the black migration than increased mechanization of farming, the more standard explanation. Barry provides such rich and complete detail on his characters that they come to life on the page. But this focus sometimes leads him astray. For example, the choice of a levees-only policy emerges not from the bitter wrangling of the engineers, but from a bias in federal policy toward internal improvements for interstate commerce. As Barry himself notes, since levees promised to deepen the river channel, while outlets would only make it harder to navigate, federal money was available for the former and not the latter. That levees-only had become the dogma of the U.S. Army Corps of Engineers by the 1920s says more about the personality of that institution than that of its leaders.

Many geographers will no doubt find this work a fascinating account; some, however, might be disappointed that Barry’s reverence for the river and the attempt to control its flooding obscures the great waterway’s economic function. He makes mention of the competition between railroad and river traffic, but only indirectly in the context of a Reconstruction-era railroad bridge built at St. Louis. Bridges over rivers are often physical manifestations of power relationships between those who travel and ship by land and those who do the same by water. Barry makes mention of this political dynamic, but the building of the bridge at St. Louis is portrayed as evidence of one man’s iron will rather than as the upshot of transportation politics. More important, the Great Lakes and New York State Barge Canals are absent from his story. Traffic on the Great Lakes outstripped Mississippi River traffic by the middle of the nineteenth century, and efforts to control the river have been as much about making the river a safe and efficient highway as they have been about flood control. Notably absent from the extensive bibliography in this regard are Louis C. Hunter’s classic Steamboats on Western Rivers and William Cronon’s more recent Nature’s Metropolis, which discusses the rivalry between Chicago railroads and St. Louis steam boats in some detail.

--Daniel Albert, University of Michigan
SOLSTICE,

Vol. VIII, No. 1.


Frank Harary. To the Memory of Clyde Tombaugh, 1906-1997.


Part II. Elements of Spatial Planning: Theory.
Merging Maps: Node Labeling Strategies.
Sandra Lach Arlinghaus


Algebraic Aspects of Ratios. Sandra Lach Arlinghaus


Summary description of pattern; Comparison of map pattern; Theoretical model; Point to point order distances; Locus to point order distances; Summary description of pattern; Comparison of map pattern; Theoretical order distances; Analysis of the pattern of urban places in Iowa; Almost periodic disturbance model; Lattice parameters; Disturbance variables; Scale variables; Comparison of M(2) and Iowa; Evaluation; Tables.

Sandra L. Arlinghaus: Construction Zone: The Brakenridge-MacLaurin Construction.


Virginia Ainslie and Jack Licate: Getting Infrastructure Built. Cleveland infrastructure team shares secrets of success;

What difference has the partnership approach made; How process affects products--moving projects faster means getting more public investment; difference has the partnership approach made; How process affects products--moving projects faster means getting more public investment; How can local communities translate these successes to their own settings?

Frank E. Barmore: Center Here; Center There; Center, Center Everywhere.

Abstract; Introduction; Definition of geographic center; Geographic center of a curved surface; Geographic center of Wisconsin; Geographic center of the conterminous U.S.; Geographic center of the U.S.; Summary and recommendations; Appendix A: Calculation of Wisconsin's geographic center; Appendix B: Calculation of the geographical center of the conterminous U.S.; References.

Barton R. Burkhalter: Equal-Area Venn Diagrams of Two Circles: Their Use with Real-World Data

General problem; Definition of the two-circle problem; Analytic strategy; Derivation of B% and AB% as a function of r(B) and d(AB).


Los Angeles, 1994; Policy implications; References; Tables and complicated figures.
William D. Drake, S. Pak, I. Tarwotjo, Muhilal, J. Gorstein, R. Tilden.
Villages in Transition: Elevated Risk of Micronutrient Deficiency.
Abstract; Moving from traditional to modern village life: risks during transition; Testing for elevated risks in transition villages; Testing for risk overlap within the health sector; Conclusions and policy implications

Abstract; Content issues; Production issues; Archival issues; References

John D. Nystuen: Wilderness As Place.
Visual paradoxes; Wilderness defined; Conflict or synthesis; Wilderness as place; Suggested readings; Sources; Visual illusion authors.

Frank E. Barmore: The Earth Isn't Flat. And It Isn't Round Either: Some Significant and Little Known Effects of the Earth's Ellipsoidal Shape.
Abstract; Introduction; The Qibla problem; The geographic center; The center of population; Appendix; References.

Sandra L. Arlinghaus: Micro-cell Hex-nets?
Introduction; Lattices: Microcell hex-nets; References

Sandra L. Arlinghaus, William C. Arlinghaus, Frank Harary:
Sum Graphs and Geographic Information.
Abstract; Sum graphs; Sum graph unification: construction; Cartographic application of sum graph unification; Sum graph unification: theory; Logarithmic sum graphs; Reversed sum graphs; Augmented reversed logarithmic sum graphs; Cartographic application of ARL sum graphs; Summary.
Frank Harary: What Are Mathematical Models and What Should They Be?  
What are they?

Two worlds: abstract and empirical; Two worlds: two levels; Two levels: derivation and selection; Research schema; Sketches of discovery; What should they be?

Frank E. Barmore: Where Are We? Comments on the Concept of Center of Population.

Introduction; Preliminary remarks; Census Bureau center of population formulae; Census Bureau center of population description; Agreement between description and formulae; Proposed definition of the center of population; Summary; Appendix A; Appendix B; References.


Pattern formation: global views; Pattern formation: local views; References cited; Literature of apparent related interest.

Harry L. Stern: Computing Areas of Regions with Discretely Defined Boundaries.

Introduction; General formulation; The plane; The sphere; Numerical examples and remarks; Appendix--Fortran program.

Sandra L. Arlinghaus, John D. Nystuen, Michael J. Woldenberg: The Quadratic World of Kinematic Waves.


The fit of ideas; Truth and proof; Ideas and theorems; Sets and functions; Confusion via surveys; Cost-benefit and regression; Projection, extrapolation, and risk; Fuzzy sets and fuzzy thoughts; Compromise is confusing.
Robert F. Austin: Digital Maps and Data Bases: Aesthetics versus accuracy.
   Introduction; Basic issues; Map production; Digital maps; Computerized data bases; User community.


Sandra L. Arlinghaus, David Barr, John D. Nystuen:
The Spatial Shadow: Light and Dark -- Whole and Part.
   This account of some of the projects of sculptor David Barr attempts to place them in a formal systematic, spatial setting based on the postulates of the science of space of William Kingdon Clifford (reprinted in Solstice, Vol. I, No. 1.).

Sandra L. Arlinghaus: Construction Zone--The Logistic Curve.

Educational feature--Lectures on Spatial Theory.


   This paper examines the urban shift from "people space" to "machine space" (see R. Horvath, Geographical Review, April, 1974) in the Detroit metropolitan regions of 1974. As with Clifford's Postulates, reprinted in the last issue of Solstice, note the timely quality of many of the observations.

Sandra Lach Arlinghaus: Scale and Dimension: Their Logical Harmony.
   Linkage between scale and dimension is made using the Fallacy of Division and the Fallacy of Composition in a fractal setting.

Sandra Lach Arlinghaus: Parallels Between Parallels.
   The earth's sun introduces a symmetry in the perception of its trajectory in the sky that naturally partitions the earth's surface into zones of affine and hyperbolic geometry. The affine zones, with single geometric parallels, are located north and south of the geographic parallels. The hyperbolic zone, with multiple geometric parallels,
located between the geographic tropical parallels. Evidence of this geometric partition is suggested in the geographic environment—in the design of houses and of gameboards.


In a recent paper, we presented an algorithm for finding the shortest distance between any two nodes in a network of n nodes when given only distances between adjacent nodes (Arlinghaus, Arlinghaus, Nystuen, Geographical Analysis, 1990). In that previous research, we applied the algorithm to the generalized road network graph surrounding San Francisco Bay. Here, we examine consequent changes in matrix entries when the underlying adjacency pattern of the road network was altered by the 1989 earthquake that closed the San Francisco--Oakland Bay Bridge.

Sandra Lach Arlinghaus: Fractal Geometry of Infinite Pixel Sequences: "Super-definition" Resolution?
Comparison of space-filling qualities of square and hexagonal pixels.

Sandra Lach Arlinghaus: Construction Zone--Feigenbaum's number; a triangular coordinatization of the Euclidean plane; A three-axis coordinatization of the plane.


This reprint of a portion of Clifford's lectures to the Royal Institution in the 1870s suggests many geographic topics of concern in the last half of the twentieth century. Look for connections to boundary issues, to scale problems, to self-similarity and fractals, and to non-Euclidean geometries (from those based on denial of Euclid's parallel postulate to those based on a sort of mechanical `polishing'). What else did, or might, this classic essay foreshadow?

Sandra Lach Arlinghaus: Beyond the Fractal.

The fractal notion of self-similarity is useful for characterizing change in scale; the reason fractals are effective in the geometry of central place theory is because that geometry is hierarchical in nature. Thus, a natural place to look for other connections of this sort is to other geographical concepts that are also hierarchical. Within this
fractal context, this article examines the case of spatial diffusion. When the idea of diffusion is extended to see "adopters" of an innovation as "attractors" of new adopters, a Julia set is introduced as a possible axis against which to measure one class of geographic phenomena. Beyond the fractal context, fractal concepts, such as "compression" and "space-filling" are considered in a broader graph-theoretic setting.

William C. Arlinghaus: Groups, Graphs, and God.

Sandra L. Arlinghaus: Theorem Museum--Desargues's Two Triangle Theorem from projective geometry.

Construction Zone--centrally symmetric hexagons.
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OTHER PUBLICATIONS, PRODUCED ON-DEMAND.

Philbrick, Allen K. This Human World. Reprint.