PREAMBLE

Imagination is more important than knowledge.

A. Einstein, **On Science**

There is a particular fascination with scholarly expositions of members of the vanguard, who wander through worlds of the unknown exploring new ideas. Little is known about the intellectual realms into which they journey, their imaginations serving as guiding lights, with their writings sometimes appearing to more conservative members of their disciplines as near fantasy or science fiction. Doreian's translation of spatial autocorrelation concepts and findings into sociological contexts exemplifies this category of pioneering work. His paper derives network autocorrelation models from spatial autocorrelation models. The purpose of this paper is to apply spatial autocorrelation models to the analysis of social phenomena distributed across spatial as well as aspatial social networks. In doing so, Doreian transcribes prominent limitations of spatial autocorrelation models for network autocorrelation models. These same sentiments are expressed in Wartenberg's commentary, in which additional applications of network autocorrelation models are gleaned from evolutionary biology, ecology, and environmental epidemiology. Wartenberg's supplemental examples should help dispell the speculative nature some scholars might associate with Doreian's work.

The Editor

Network Autocorrelation Models: Problems and Prospects

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Overview: Network autocorrelation models draw their inspiration from, and share a common representation with, spatial autocorrelation models. The use of a weight matrix, $\mathbf{W}$, to capture network interdependencies and the statement of linear equations provide the communality. Network autocorrelation models can be used to analyze social phenomena distributed across social structures that need not be rooted in geographical space. These include the diffusion of ideas through the networks linking scientists in an invisible college, analyses of economic development for nation–states, and analyses of inter–organizational networks. Many of the problems (and responses to them) encountered in the spatial autocorrelation model literature are applicable directly to network autocorrelation models. These include discussions of boundary effects, issues of aggregation, and dynamic modeling. The problems associated with the specification and estimation of network autocorrelation models are likely to be more difficult than for the spatial case. The additional complexities stem from having to specify and model a time–dependent weight matrix $\mathbf{W}(t)$ rather than simply use $\mathbf{W}$, the necessity to model coupled processes, and the need to use qualitatively different actors linked by multiple processes.

Social scientists in general, and sociologists in particular, lay claim to the study of social phenomena. In large part, this includes analyses of social structures and social processes: social structure is generated by, and in turn constrains, the operation of social processes. If correct, it is trivial to claim that structure and the interdependence of social actors must be included in the analysis of social action. Trivial, but for the fact that it is ignored in much of contemporary social science—especially when the analysis of empirical information is included. Although this data analytic practice appears to fly in the face of empirical reality, it is straightforward to understand the reasons for it. The invention of the social survey, together with the early use of computers, permitted the creation and analysis of large data sets comprised of individual—such as people, groups or organizations—cases. Although early methods of correlations and cross tabulations have largely (but not completely) been superceded by regression, structural equation models, and log–linear models, the underlying presumption of independent data points remains the majority choice. Alas, in many empirical contexts, it does not survive close scrutiny.

* Prepared for the 1989 Symposium "Spatial Statistics: Past, Present and Future," Department of Geography, Syracuse University. The author acknowledges beneficial interaction with Daniel Griffith, including exposure to the manuscripts of those symposium lecturers preceding him, during his visit to Syracuse University; however, Griffith should not be incriminated by any errors that remain.

1. Spatial autocorrelation models

Social phenomena distributed across geographic space provide one arena for challenging the value and utility of models premised on the assumption of independent data points. Behavior at one geographic location need not be independent of behavior at another. Loftin and Ward (1983) provide an example using sociological data.

Their linear model was the conventional population regression function:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon} \tag{1.1}$$

where a vector, $\mathbf{y}$, is predicted from a set of regressors, $\mathbf{X}$, using a vector of parameters, $\boldsymbol{\beta}$.

The disturbance term, $\boldsymbol{\epsilon}$, was treated in two ways. First, it was specified as independently normally distributed (as for ordinary least squares, OLS) and second, as:

$$\boldsymbol{\epsilon} = \rho\mathbf{W}\boldsymbol{\epsilon} + \boldsymbol{\nu} \tag{1.2}$$

where $\boldsymbol{\nu} \sim N(\mathbf{0}, \sigma^2\mathbf{I})$ with $\boldsymbol{\epsilon}$ spatially autocorrelated via $\mathbf{W}$, which captured the interdependence of contiguous areal units. The conjunction of equations (1.1) and (1.2) has been called the spatial disturbances model (Doreian, 1980) as the interdependencies are considered operative on the disturbances alone. Obviously, if the parameter ρ is zero or if $\mathbf{W}$ is uniformly zero, the disturbances model reduces to the OLS model. The predicted variable for Loftin and Ward was a measure of fertility. The regressors, $\mathbf{X}$, contain a set of population density measures (logarithm transforms of persons per room, rooms per unit, units per structure, and structures per acre) together with a class index and an ethnicity index. The units of analysis were 75 community areas making up Chicago. Using OLS, three of the density variables and the class index were found to be significant predictors of fertility. If the specification of (1.2) is correct, and if ρ is known, equation (1.1) can be rewritten as $\mathbf{Y}^\star = \mathbf{X}^\star\boldsymbol{\beta} + \boldsymbol{\nu}$ where $\mathbf{Y}^\star = (\mathbf{I} - \rho\mathbf{W})^{-1}\mathbf{Y}$ and $\mathbf{X}^\star = (\mathbf{I} - \rho\mathbf{W})^{-1}\mathbf{X}$. Use of OLS, where $\mathbf{Y}^\star$ is regressed on $\mathbf{X}^\star$, provide estimates of $\boldsymbol{\beta}$ and σ^2. In general, ρ is unknown. However, OLS for (1.1) will generate a spatially correlated residual, $\hat{\boldsymbol{\epsilon}}$, from which a crude estimate, $\hat{\rho}$, is obtained from regressing $\hat{\boldsymbol{\epsilon}}$ on $\mathbf{W}\hat{\boldsymbol{\epsilon}}$. This process is iterated until $\hat{\rho}$ converges. Using a procedure such as this, Loftin and Ward (1983) fitted the disturbances model, and found that both the class and ethnicity indexes were significant predictors of fertility together with, at most, a single density variable (depending on how the matrix $\mathbf{W}$ was operationalized). With an identical data set and a common model, the empirical evidence supports quite different substantive accounts depending on whether spatial autocorrelation is considered or not.

The discussion thus far treats spatial autocorrelation as a technical problem incorporated into the specification of the disturbance term. White, Burton and Dow (1981) constructed a model of the sexual division of labor in African agriculture. The core variables were female participation in agriculture, patrilocal residence, and degree of polygamy. Following the estimation of their model via OLS, it was clear that the residuals from their analysis clustered spatially. Rather than leave the analysis with an implicit unestimated $\mathbf{W}$, the authors were able to specify it from a linguistic tree constructed for versions of the Bantu language. The underlying idea was that there had been an expansion of Bantu tribes across geographic space and that the data points were not independent but were linked through similarity with regard to language. More broadly, spatial autocorrelation models have been used to deal with what has been known as Galton's Problem.

Maximum likelihood can be used to estimate the disturbances model. Using the notation $\mathbf{A} = \mathbf{I} - \rho\mathbf{W}$, the log–likelihood function can be written as:

$$\ln(\mathbf{L}) = \text{constant} - (N/2)\ln(\sigma^2) - \frac{1}{2\sigma^2}[\mathbf{y}'\mathbf{A}'\mathbf{A}\mathbf{y} - 2\boldsymbol{\beta}'\mathbf{X}'\mathbf{A}'\mathbf{A}\mathbf{X}\mathbf{y} + \boldsymbol{\beta}'\mathbf{X}'\mathbf{A}'\mathbf{X}\boldsymbol{\beta}] + \ln|\mathbf{A}| \quad (1.3)$$

From this it is straightforward to establish the following estimation equations:

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{A}'\mathbf{A}\mathbf{X})^{-1}\mathbf{X}'\mathbf{A}'\mathbf{A}\mathbf{y} \quad (1.4)$$

and

$$\hat{\sigma}^2 = [\mathbf{y}'\mathbf{A}'\mathbf{A}\mathbf{y} - 2\boldsymbol{\beta}'\mathbf{X}'\mathbf{A}'\mathbf{A}\mathbf{y} + \boldsymbol{\beta}'\mathbf{X}'\mathbf{A}'\mathbf{A}\mathbf{X}\boldsymbol{\beta}]/\mathbf{N} \quad (1.5)$$

Substitution of (1.4) and (1.5) into (1.3) yields the concentrated log–likelihood function from which a value of ρ is obtained: $\hat{\rho}$ minimizes

$$\ln(\mathbf{y}'\mathbf{A}'\mathbf{P}\mathbf{A}\mathbf{y}) - (2/N)\ln|\mathbf{A}|$$

where $\mathbf{P} = \mathbf{I} - (\mathbf{A}\mathbf{X})\{(\mathbf{A}\mathbf{X})'(\mathbf{A}\mathbf{X})\}^{-1}(\mathbf{A}\mathbf{X})'$ and $\ln|\mathbf{A}| = \Sigma\ln(1-\rho\lambda_i)$ with $\{\lambda_i\}$ being the eigenvalues of $\mathbf{W}$. With $\hat{\rho}$ as the estimate of ρ, its value can be substituted into (1.4) to get $\hat{\boldsymbol{\beta}}$, and into (1.5) to get $\hat{\sigma}^2$. Approximate standard errors for $\hat{\rho}$, $\hat{\boldsymbol{\beta}}$, and $\hat{\sigma}^2$ are obtained from the variance–covariance matrix obtained from the second partial derivatives of the log–likelihood function. Details of the procedure can be found in Ord (1975), Doreian (1980), or Upton and Fingleton (1985). An alternative to the spatial disturbances model is the spatial effects model where the spatial interdependence is incorporated directly into the statement of the model. A motivating example is found in Mitchell (1969) in a study of the spatial distribution of rebel control—or government control—for the HUK rebellion in the Philippines. It is clear that control of one area has immediate consequence for those areas contiguous with it or easily reached from it. Equation (1.1) is replaced by:

$$\mathbf{y} = \rho\mathbf{W}\mathbf{y} + \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon} \quad (1.6)$$

where $\boldsymbol{\epsilon} \sim \mathbf{N}/(\mathbf{0}, \sigma^2\mathbf{I})$ and the log–likelihood function is

$$\ln(y) = \text{constant} - (N/2)\ln\sigma^2 - \frac{1}{2\sigma^2}(\mathbf{y}'\mathbf{A}'\mathbf{A}\mathbf{y} - 2\boldsymbol{\beta}'\mathbf{X}'\mathbf{A}'\mathbf{y} + \boldsymbol{\beta}'\mathbf{X}'\mathbf{X}\boldsymbol{\beta}) + \ln|\mathbf{A}| \quad (1.7)$$

It is straightforward to establish

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{z} \quad (1.8)$$

and

$$\hat{\sigma}^2 = (1/N)\mathbf{z}'\mathbf{M}\mathbf{z} \quad (1.9)$$

where $\mathbf{z} = \mathbf{A}\mathbf{y} = (\mathbf{I} - \rho\mathbf{W})\mathbf{y}$ and $\mathbf{M} = \mathbf{I} - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$. The spatial effects parameter, $\hat{\rho}$, minimizes

$$\ln(\mathbf{y}'\mathbf{M}\mathbf{y} - 2\rho\mathbf{y}'\mathbf{M}\mathbf{W}\mathbf{y} + \rho^2\mathbf{y}'\mathbf{W}'\mathbf{W}\mathbf{y}) - (2/N)\Sigma\ln(1-\rho\lambda_i)$$

An approximate variance–covariance matrix, as before, is obtained by use of the second partial derivatives of the log–likelihood function. Details are found in Ord (1975) and Doreian (1981).

2. Network autocorrelation models

Network autocorrelation models are, in essence, an extension of spatial autocorrelation models to phenomena where the interdependence among the structural units is generated directly through the operation of some social process (instead of some geographically distributed process). This definition is not intended as a slight upon spatial autocorrelation models. Indeed, the spatially distributed examples of Loftin and Ward, White, *et al.*, and Mitchell all provide a powerful motivation for considering and incorporating more general interdependencies between social actors. The connecting link between network and spatial autocorrelation models is found in the representation of the matrix $\mathbf{W}$. The ways in which $\mathbf{W}$ is constructed in terms of contiguity, accessibility, or common boundaries can be seen as variations of a sociometric scheme where interdependence need not rest directly on geographical, or even physical, characteristics.

2.1. Example 1: scientific values

Science produces empirically validated knowledge. This knowledge, together with guesses, conjectures, and other ideas is distributed across disciplines and specialties. While scientists are, in the main, geographically dispersed they do work within "invisible colleges" which have an internal and stratified structure. The stratification is determined in large part by the publication of scientists in reputable journals. Although journals are among the central institutions of science, there is nothing that guarantees a journal's reputation. Minimally, it depends on the level of interest maintained in it within a scientific community. Further, interest in a journal rests on the extent to which it is seen as publishing significant work. Such a chain of reasoning verges on the circular as concepts like 'reputable,' 'interest' and 'significant' rest on communal standards within a scientific community. Burt and Doreian (1982) argue that these characteristics are maintained by one or more social psychological processes whereby scientists socialize each other. Moreover, these two researchers show that these processes are mediated by the internal structure of a scientific community. If this argument is correct, then a research strategy whereby scientists are sampled from an invisible college and solicited for their views concerning the important journals of their field without taking into account the social relations among those scientists is problematic. Values, in addition to knowledge, are transmitted over a social structure.

2.2. Example 2: dependency theory

Following World War II there was considerable interest in patterns of economic development among Third World countries. Within the sociological literature, economic development models have been seen as inadequate because they "are based on the implicit assumption that countries represent separate systems of economic production" (Rubinson, 1976). Dependency theory models have been constructed as a way of overcoming this limitation, and most variants of this approach assume further that all countries are part of a single system of production that contains multiple political units within it. Dependent variables such as rate of economic growth, level of economic development, and societal inequality have been linked to variables measuring First World penetration of Third and Fourth World countries. Various mechanisms and models have been specified and, when estimated, appear to support arguments that the receipt of developmental aid and the receipt of foreign capital are inimical to the interests of most Third and Fourth World nations in the world system of nations. Of course, this claim has been challenged. It is rather odd that the proponents of these theories, at least in the version of American quantitative sociology, fall back on regression models to sift the empirical information. By all of the arguments of the dependency theorists, the world is an interdependent system and, one would presume, ought to be modeled as such.

As examples accumulate whereby linear models can lead to mistaken inference if spatial autocorrelation is omitted, then in a more general network context there is the serious risk that classical regression models estimated with data depicting nation states (in an interdependent system) are vulnerable to the same kinds of mistaken inference. While the theory is inherently structural, the procedures for estimating model parameters are not. As most social phenomena occur in structural contexts, the problem may be more general. The received wisdom among social network analysts is that social structure makes a difference and must be included in the analysis of most social phenomena. But this claim may be little more than received dogma and it behooves network analysts to spell out the way in which network autocorrelation models could be constructed and estimated.

2.3. Defining W for network autocorrelation models

In the context of spatial autocorrelation, Upton and Fingleton (1985) remark "As ever, the choice of $\mathbf{W}$ is essentially arbitrary ..." The remark is as daunting as it is frank. One of the more common forms of defining the weight matrix, $\mathbf{W}$, for spatial autocorrelation models is to start with a matrix, $\mathbf{C}$, that represents whether or not areas are contiguous. This binary $\mathbf{C}$ is often made row stochastic to form the matrix $\mathbf{W}$ (which then has 1 as the largest eigenvalue). Formally, this is no different to using the conventional sociometric representation of the structure of a group and turning it into a row stochastic matrix. Initial explorations of network autocorrelation models have tended to do this.

In the spatial autocorrelation literature, distances between the centroids of the areal units, together with the specification of a distance–decay model have been used. Similarly, for strongly connected graphs, it would be possible to use graph theoretic distances (of geodesics) in the specification of network autocorrelation models.

These suggested examples, motivated by successes found in spatial autocorrelation models, are imitations that stay very close to the spatial case. While network autocorrelation models imitating spatial models have had some success, it is clear that they need to draw their inspiration from social network ideas.

In geographical examples, contiguity and accessibility are frequently mutually redundant and change slowly. This is not true for most social networks. In social networks, reachability within a graph may provide a genuinely new basis for measuring interdependence. This may be especially true for valued graphs where the matrix elements represent the strength of a link between two actors. Reachability at level n (Doreian, 1974) considers all paths between pairs of actors with a view to finding the path with the largest minimal element n in the path. In essence, a threshold filter is put over the sociomatrix to restrict attention to only those links above a certain level. Actors reachable at one level are not be reachable at another higher level unless there is a path between them whose links are above the threshold value. However, this is only a tiny step from the spatial foundations.

2.4. Equivalence

Of the many ideas that emerged within network analysis during the 1970s and 1980s, the notion of equivalence has captured most of the attention. The sociometric origins of network analysis are seen in the study of small groups. As the computing technology available to social analysts enabled the study of larger systems, it became clear that very large networks verge on the incomprehensible.

This fueled the desire for simpler representations. Of more interest was the idea that networks among individuals (be they people, groups, organizations, or states) could be seen as empirical instantiations of simpler and more fundamental structures. Thus, if it were possible to lay out stringent criteria, large networks could be distilled for their structural essence. The first concept of equivalence, in intuitive terms, was one were two actors are equivalent if they are connected in exactly the same fashion to the rest of the network. Structurally, two such actors are indistinguishable and can be merged to a common position. Formally, the specification of structural equivalence is:

In a graph $< \mathbf{P}, \mathbf{R} >$ made up of a set of actors, $\mathbf{P}$, and a social relation, $\mathbf{R}$, an equivalence, $\mathbf{E}$, is a structural equivalence if and only if for all distinct actors $\mathbf{a}, \mathbf{b}, \mathbf{c} \,\epsilon \mathbf{P}$, $\mathbf{aEb}$ implies

(i) $\mathbf{aRb}$ if and only if $\mathbf{bRa}$;

(ii) $\mathbf{aRc}$ if and only if $\mathbf{bRc}$;

(iii) $\mathbf{cRa}$ if and only if $\mathbf{cRb}$; and,

(iv) $\mathbf{aRa}$ implies $\mathbf{aRb}$ (White and Reitz, 1983).

In principle, any social network can be reduced to a set of structurally non–equivalent positions that are each occupied by structurally equivalent actors. However, this intuition is of little value in practice as there are very few *exact* structural equivalences in social networks. The pragmatic response to this dilemma has been to develop methods that measure the extent to which each pair of nodes is equivalent, and then to mobilize some clustering algorithm. Each structural position (location) is now occupied not by actors that are exactly equivalent, but by actors that are sufficiently close to being equivalent. The use of a measure of equivalence and a clustering algorithm permits an analyst to establish a partition of the actors in a network.

For this idea to be mobilized in a network autocorrelation model, the underlying intuition is that equivalent actors are subject to equivalent processes that affect them by virtue of their occupancy of the same position in the network. Consider Figure 1, where the two nodes $\mathbf{P_1}$ and $\mathbf{P_2}$ send ties to non–overlapping sets of other actors.

This generic picture can be illustrated by the following examples. First, $\mathbf{P_1}$ and $\mathbf{P_2}$ are distinct parents linked to their respective children. A second situation could see $\mathbf{P_1}$ and $\mathbf{P_2}$ as former colonial powers linked to sets of their former colonies. For some variable of interest, it could be that $\mathbf{P_1}$ and $\mathbf{Q_1}$ are similar as a result of the dyadic link between them. Both could believe in "the empire" where the elite of $\mathbf{Q_1}$ have migrated from $\mathbf{P_1}$. An alternative view of a structural process would be one where $\mathbf{Q_1}$ through $\mathbf{Q_4}$ are similar, although there may be no direct link between them. Clearly, $\mathbf{Q_1}$ through $\mathbf{Q_4}$ are structurally equivalent and would be subject to the same process, for example, due to an unfavorable trading relations with the colonial power. Under the first model, $\mathbf{P_1}$ and $\mathbf{Q_1}$ through

Q_4, would all be similar, while under the second (structural equivalence) representation Q_1 through Q_4 would be similar by virtue of being structurally equivalent but, for some selected variable of interest, could be quite distinct from P_1. Empirically, if the variable of interest was measured and all five actors were close, then there would be support for the cohesion argument. Alternatively, if P_1 was quite different from Q_1 through Q_4, which in turn are similar to each other, then the suggestion would be that a structural equivalence mechanism rather than a cohesion mechanism was at work.

Figure 1.

Illustration of structural and regular equivalences.

A generalization of structural equivalence is regular equivalence where objects are regularly equivalent if they are equivalently connected to equivalent others. More formally, this can be expressed as

In a graph $< \mathbf{P}, \mathbf{R} >$ (defined above) an equivalence is a regular equivalence if and only if for all actors $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}\ \epsilon \mathbf{P}, \mathbf{aEb}$ implies

(i) aRc implies there exists d ϵ P such that bRd and dEc; and

(ii) cRa implies there exist d ϵ P such that dRb and dEc (White and Reitz, 1983).

Using Figure 1, Q_5 through Q_7 are structurally equivalent by virtue of being connected to P_2. However, Q_5 through Q_7 are not structurally equivalent to Q_1 through Q_4, since P_1 is distinct from P_2. But, it is clear that Q_1 through Q_4 are connected to P_1 in the same way that Q_5 through Q_7 are connected to P_2. Conversely, P_1 is connected to Q_1 through Q_4 in the same fashion that P_2 is connected to Q_5 through Q_7. In short, P_1 and P_2 are regularly equivalent while Q_1 through Q_7 are regularly equivalent. Although Q_5 through Q_7 are connected to a different colonial power, it could be argued that they are subject to the same process as it applies to all colonies regardless of the identity of, colonial powers. Similarly, parents occupy a role while, in relation to them, children occupy a complementary role. If a process is mediated by regular equivalence, then one would expect that Q_1 through Q_7 would be similar with regard to some variable while P_1 and P_2 would be similar to each other. It is worth noting that structural equivalence is a special case of regular equivalence in the sense that structurally equivalent actors are also regularly equivalent, but not vice versa.

Thus far, this discussion has sketched out a cohesion mechanism, a structural equivalence mechanism, and a regular equivalence mechanism. The implicit assumption is that if we know which mechanism is at work, then we can construct an appropriate weight matrix $\mathbf{W}$ that captures the interdependency among the actors. If e_{ij} denotes the extent of equivalence of $\mathbf{i}$ and $\mathbf{j}$ in some social structure, this may suffice for the interdependence measure. Alternatively, the set of $\{e_{ij}\}$ can be normalized in some fashion, for example:

$$w_{ij} = \frac{\max\{e_{ij}\} - e_{ij}}{\Sigma[\max\{e_{ij}\} - e_{ij}]}$$

Of course, there may be other ways in which the weight matrix can be constructed. At face value, the wry comment of Upton and Fingleton concerning the arbitrariness of $\mathbf{W}$ is pertinent. However, if the cohesion, or structural equivalence, or regular equivalence mechanisms can be specified in *advance*, it is possible to construct the appropriate $\mathbf{W}$ on *substantive* grounds. In principle, the way is then clear to mobilize all of the statistical machinery found within the rubric of spatial autocorrelation to formulate, estimate and test network autocorrelation models.[1]

Following Anselin (1988, pp. 34-5), a family of network autocorrelation models can be specified:[2]

$$\mathbf{y} = \rho_1 \mathbf{W}_1 \mathbf{y} + \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon} \tag{2.1a}$$

$$\boldsymbol{\epsilon} = \rho_2 \mathbf{W}_2 \boldsymbol{\epsilon} + \boldsymbol{\nu} \tag{2.1b}$$

where $\boldsymbol{\nu} \sim \mathbf{N}(\mathbf{0}, \sigma^2\mathbf{I})$. There are three special versions of the generic model specified in (2.1). When $\rho_1 = \rho_2 = \mathbf{0}$, equation (2.1a) reduces to the usual OLS population regression function while (2.1b) becomes the conventional specification of a normally distributed error term. For $\rho_1 = \mathbf{0}$ and $\rho_2 \neq \mathbf{0}$, equation (2.1) is the network disturbances model. Finally, when $\rho_1 \neq \mathbf{0}$ and $\rho_2 = \mathbf{0}$ we have the network effects model.

3. Issues stemming from network autocorrelation models

3.1. Multiple processes

If a cohesion model can be unequivocally specified it can be estimated and interpreted. Similarly, if an equivalence model can be specified, then it too can be estimated and interpreted. However, some network analysts posit a sharp distinction between cohesion models and equivalence models. If rendering a decision as to whether a cohesion process or an equivalence process is at work is necessary, it is a major disadvantage to use a model with a single regime of network effects. For example, Burt and Doreian (1982) estimated separately a cohesion model and a structural equivalence model in a study of the distribution of evaluations of major journals by scientists in a specific field. The relative performance of the two models were considered through an analysis of the residuals remaining when the separate analyses had been conducted. As the two mechanisms take the form of rival hypotheses it seems preferable to examine them competitively and directly. Rather than fit the two models separately and examine their residuals, it is preferable to have a model where both processes are explicitly included. Similarly, for a debate between structural equivalence mechanisms and regular equivalence mechanism, it would be desirable to build a model with both present. Theoretically speaking, this task can be carried out in the following way (Doreian, 1989a) depending upon the practical issues involved in estimating a model with two regimes of network effects.

A model with two regimes of effects [3] autocorrelation can be written as

$$\mathbf{y} = \rho_1 \mathbf{W_1 y} + \rho_2 \mathbf{W_2 y} + \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon} \tag{3.1}$$

with $\boldsymbol{\epsilon} \sim N(0, \sigma^2 \mathbf{I})$. With $\mathbf{A} = \mathbf{I} - \rho_1 \mathbf{W_1} - \rho_2 \mathbf{W_2}$ and $|\mathbf{A}|$ as the Jacobian of the transformation from $\boldsymbol{\epsilon}$ to $\mathbf{y}$, the log-likelihood function can be written as

$$\ln(\mathbf{L}) = \text{constant} - (N/2)\ln\sigma^2 - \frac{1}{2\sigma^2}[\mathbf{z}'\mathbf{z} - 2\boldsymbol{\beta}'\mathbf{X}'\mathbf{z} + \boldsymbol{\beta}'\mathbf{X}'\mathbf{X}\boldsymbol{\beta}] + \ln|\mathbf{A}| \tag{3.2}$$

Notationally, use of MLE leads to the same estimation equation for $\boldsymbol{\beta}$ as before [see equation (1.8)]:

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{z} \tag{3.3}$$

similarly for σ^2;

$$\hat{\sigma}^2 = (\mathbf{z}'\mathbf{z} - 2\boldsymbol{\beta}'\mathbf{X}'\mathbf{z} + \boldsymbol{\beta}'\mathbf{X}'\mathbf{X}\boldsymbol{\beta})/\mathbf{N} \tag{3.4}$$

with $\mathbf{z} = \mathbf{A}\mathbf{y}$. The iterative estimation for the ρ_i is more complex, as is the asymptotic variance-covariance matrix for obtaining approximate standard errors for the estimated parameters (with $\mathbf{B}_i = \mathbf{W}_i\mathbf{A}^{-1}$ for $i = 1, 2$):

$$V(\omega, \rho_1, \rho_2, \boldsymbol{\beta}) = \omega^2 \begin{pmatrix} N/2 & \omega\, tr(\mathbf{B}_1) & \omega\, tr(\mathbf{B}_2) & \mathbf{0} \\ \omega\, tr(\mathbf{B}_1) & \mathcal{B}_{11} & \mathcal{B}_{12} & \omega X'\mathbf{B}_1\mathbf{X}\boldsymbol{\beta} \\ \omega\, tr(\mathbf{B}_2) & \mathcal{B}_{21} & \mathcal{B}_{22} & \omega\mathbf{X}'\mathbf{B}_2\mathbf{X}\boldsymbol{\beta} \\ \mathbf{0}' & \omega\boldsymbol{\beta}'\mathbf{X}'\mathbf{B}_1'\mathbf{X} & \omega\boldsymbol{\beta}'\mathbf{X}'\mathbf{B}_2'\mathbf{X} & \omega\mathbf{X}'\mathbf{X} \end{pmatrix}^{-1} \tag{3.5}$$

where

$$\begin{aligned}
\mathcal{B}_{11} &= \omega^2[tr(\mathbf{B}_1'\mathbf{B}_1) + tr(\mathbf{B}_1)^2] + \omega\boldsymbol{\beta}'\mathbf{X}'\mathbf{B}_1'\mathbf{B}_1\mathbf{X}\boldsymbol{\beta} \\
\mathcal{B}_{12} &= \omega^2[tr(\mathbf{B}_1'\mathbf{B}_2) + tr(\mathbf{B}_1\mathbf{B}_2)] + \omega\boldsymbol{\beta}'\mathbf{X}'\mathbf{B}_2'\mathbf{B}_1\mathbf{X}\boldsymbol{\beta} \\
\mathcal{B}_{21} &= \omega^2[tr(\mathbf{B}_2'\mathbf{B}_1) + tr(\mathbf{B}_2\mathbf{B}_1)] + \omega\boldsymbol{\beta}'\mathbf{X}'\mathbf{B}_1'\mathbf{B}_2\mathbf{X}\boldsymbol{\beta} \\
\mathcal{B}_{22} &= \omega^2[tr(\mathbf{B}_2'\mathbf{B}_2) + tr(\mathbf{B}_2)^2] + \omega\boldsymbol{\beta}'\mathbf{X}'\mathbf{B}_2'\mathbf{B}_2\mathbf{X}\boldsymbol{\beta}.
\end{aligned}$$

3.2. Distinct types of actors

The two regime model of network effects is plausible for a community of scientists in an invisible college. Scientific leadership, and its corresponding material and psychic rewards, accumulate and evolve over a scientific career. Even if a scientific elite can be distinguished from the bulk of the members of an invisible college, the model retains plausibility. But for the motivating example of nation states bound into a global system, plausibility is stretched even for the two regime model. If colonies can be distinguished from colonial powers, or if core states, semi-peripheral states, and peripheral states are subject to distinct (but coupled) processes, the two regime model as stated loses its plausibility. The crux of the problem is that the qualitative distinctions on the actors may need to be incorporated into the model.

One rather simple, but effective, approach to this problem takes the form of incorporating the distinctions into the matrix $\mathbf{X}$. Snyder and Kick (1979) observe that world system theorists and dependency theorists have proposed no adequate operational criteria for classifying countries into the world system positions. Various *ad hoc* definitions have been proposed, and it is not surprising that the empirical status of some nations, for example Spain, is completely ambiguous as to which position contains them. Snyder and Kick's solution to this problem rests on the conceptualization of structural equivalence. The world system conceptualization (Wallerstein, 1974) is fundamentally structural and it seems reasonable to try and distill world system positions from structural data.

Snyder and Kick's (1979) proposal is straight forward: use structural data to generate structural positions. They used four tie types—trade relations, treaties, exchange of diplomats and military interventions—for the structural data. Each relation generates a nation–by–nation matrix. These matrices were stacked and analyzed jointly to obtain a partition in terms of structural equivalence by use of CONCOR (Breiger, Boorman, and Arabie, 1975).[4] The authors identified ten non–equivalent positions, across which 118 nations were distributed. Using world system terms, one position was clearly the core, another three positions can be viewed as belonging to the semi–periphery, with the remaining six blocks as parts of the periphery.

In 1979, network autocorrelation models were not an option.[5] Working within a regression framework, Snyder and Kick included the structural data by means of a set of dummy variables (omitting one to avoid an exact linear dependence among the regressors). Regression models, or rather, the parameter estimates and inferential decisions, are frequently challenged. Snyder and Kick's work was no exception—but the basis for the critics' objections did not include issues of network autocorrelation. Considerations of regression diagnostics and curvilinear relations suggested that Snyder and Kick's analysis (and theoretical arguments) were not supported. However, when Nolan (1983) reanalyzed the Snyder and Kick data using only three positions–core, semi–periphery, periphery—the initial findings were supported. Clearly, there are limitations to the number of dummy variables that can be included in a regression to capture structural positions when the data points are interdependent.

While using dummy variables to represent qualitative differences between nation states is direct and practical, it is not clear that the underlying mechanisms of the world system are adequately modeled. For models of income inequality or economic growth of nation states the incorporation of dummy variables amounts to little more than the fitting of mean values for the nations of different sectors (together with slope shifts if necessary). The central idea of the world system theorists is that nations of the various positions are locked into reciprocal mechanisms advantageous to one group and disadvantageous for another. It seems more appropriate to generate directly a model that focuses on the mechanisms themselves.

One simple way to incorporate two distinct types of actor is to partition $\mathbf{y}$, $\mathbf{X}$, $\mathbf{W}$, and $\boldsymbol{\epsilon}$ so that an effects model is written as:

$$\begin{pmatrix}\mathbf{y}_1\\ \mathbf{y}_2\end{pmatrix} = \begin{pmatrix}\rho_1\mathbf{I}_1 & \mathbf{0}\\ \mathbf{0} & \rho_2\mathbf{I}_2\end{pmatrix}\begin{pmatrix}\mathbf{W}_1 & \mathbf{0}\\ \mathbf{0} & \mathbf{W}_2\end{pmatrix}\begin{pmatrix}\mathbf{y}_1\\ \mathbf{y}_2\end{pmatrix} + \begin{pmatrix}\mathbf{X}_1 & \mathbf{0}\\ \mathbf{0} & \mathbf{X}_2\end{pmatrix}\begin{pmatrix}\boldsymbol{\beta}_1\\ \boldsymbol{\beta}_2\end{pmatrix} + \begin{pmatrix}\boldsymbol{\epsilon}_1\\ \boldsymbol{\epsilon}_2\end{pmatrix} \tag{3.6}$$

in obvious notation. Ignoring the network autocorrelation term, this is exactly the two population model of Zellner's (1962) seemingly unrelated regression model, treated at length in Theil (1971). The block diagonal form of $\mathbf{W}$ means that the eigenvalues of $\mathbf{W_1}$ and $\mathbf{W_2}$ can be determined separately and used to give the eigenvalues of $\mathbf{W}$. Although, the two types of actors have distinct equations:

$$\begin{aligned} \mathbf{y}_1 &= \rho_1 \mathbf{W}_1 \mathbf{y}_1 + \mathbf{X}_1 \boldsymbol{\beta}_1 + \boldsymbol{\epsilon}_1 \\ \mathbf{y}_2 &= \rho_2 \mathbf{W}_2 \mathbf{y}_2 + \mathbf{X}_2 \boldsymbol{\beta}_2 + \boldsymbol{\epsilon}_2 \end{aligned} \tag{3.7}$$

the maximum likelihood method can be used directly with the known eigenvalues of $\mathbf{W}$. In the interpretation of a partitioned $\mathbf{W}$, the two types of actors are kept distinct. For nations of Type $\mathbf{i}$, only the values of $\mathbf{y}$ for other Type $\mathbf{i}$ nations are used in the prediction equations. Using the nations example, with $\mathbf{y}$ a measure of economic growth, the core and non–core nations can have distinct $\mathbf{W_i}$ and $\rho_{\mathbf{i}}$. Within the two classes of actors the weights, and, by implication, the underlying processes, can be quite different and can be differentially important (depending upon the values of $\rho_{\mathbf{i}}$). At face value, a $\mathbf{W}$ constructed via regular equivalence could take the partitioned form given the core nations would be maximally like each other and maximally unlike non–core nations. Similarly non–core nations will be maximally like each other and unlike core nations with regard to (regular) position in the network.

However, dependency arguments go beyond saying that there are distinct processes for core and non–core nations. The claim is that First World nations benefit at the expense of Third and Fourth world nations. By repatriation of capital and extraction of profits, First World corporations and nations benefit while Third and Fourth world nations lose not only their resources, but also control over resources, and hence suffer from a distorted development. If this argument is correct (or indeed if the counter argument that all nations benefit is correct), then the model stated in equation (3.6) becomes inadequate. The weight matrix still can be partitioned but it would take a more complicated form, *i.e.*,

$$\mathbf{W} = \begin{pmatrix} \mathbf{W}_{11} & \mathbf{W}_{12} \\ \mathbf{W}_{21} & \mathbf{W}_{22} \end{pmatrix}$$

The role of $\mathbf{W_{11}}$ is that of $\mathbf{W_1}$ while $\mathbf{W_{22}}$ plays the same role as $\mathbf{W_2}$. However, the matrices of greater interest will be $\mathbf{W_{12}}$ and $\mathbf{W_{21}}$ as they represent the way in which the classes of nations have an impact on each other. In tandem with the partitioned form of $\mathbf{W}$, these are four network effects parameters. The within position parameters are ρ_{11} and ρ_{22}, while ρ_{12} and ρ_{21} are the between position parameters. Letting

$$\mathbf{R} = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}$$

the model can be stated as:

$$\begin{pmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{pmatrix} = \mathbf{R} \otimes \mathbf{W} \begin{pmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{pmatrix} + \begin{pmatrix} \mathbf{X}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{X}_2 \end{pmatrix} \begin{pmatrix} \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \end{pmatrix} + \begin{pmatrix} \boldsymbol{\epsilon}_1 \\ \boldsymbol{\epsilon}_2 \end{pmatrix} \tag{3.8}$$

where $\otimes$ is the Kronecker product. If only First world nations benefit then $\rho_{\mathbf{21}}$ would be positive and $\rho_{\mathbf{12}}$ would be zero or negative (or at least smaller than $\rho_{\mathbf{21}}$). If all nations benefit, then both $\rho_{\mathbf{12}}$ and $\rho_{\mathbf{21}}$ would be positive. Patterns inside the $\mathbf{W_{ij}}$ also could be of interest. The major problem with the model of (3.8) is that it may be intractable. The

MLE methods thus far, rest on the simple representation of $\ln|\mathbf{A}|$ as $\Sigma(1-\rho\lambda_i)$ where the $\{\lambda_i\}$ are the eigenvalues of $\mathbf{W}$.

It would appear that this simple decomposition is prohibited by the partitioned form of $\mathbf{W}$ and 4 network autocorrelation parameters unless either ρ_{21} or ρ_{12} is zero. If, from an estimation view, this model is intractable, then the straightforward use of dummy variables, obtained from the structure of the network, as proposed by Snyder and Kick (1979) has great appeal.

3.3. Boundary effects

As defined earlier, a social network is a set of actors, $\mathbf{P}$, over which one (or more) social relation(s) are defined. Although the existence of $\mathbf{P}$ is taken as given, the empirical problem of locating its boundaries remains a persistent and vexing problem. For virtually all $\mathbf{P}$, we know there are network ties that cross the boundary between $\mathbf{P}$ and all other actors. Consider Figure 2 where the upper panel shows a network from which a sub–network (second panel) has been extracted for a network analysis. If an influence process is at work, and if it is activated through the network ties, then actors g, h, i, j, and k are beyond the boundary of $\mathbf{P}$ (made up of a, b, c, d, and e). The actors on the boundary of $\mathbf{P}$ are affected by actors in the (selected) network as well as some actors outside $\mathbf{P}$. As an exact analogue of the spatial case, this network example confronts the same boundary problems as in spatial systems. At face value, then, network autocorrelation models can benefit from the experience generated through the use of spatial autocorrelation models. Unfortunately, this understates the problems found in network autocorrelation models.

For a cohesion based model, f is affected little by the boundary location as its entire ego–network is contained within the sampled network. Similarly for e. Of course, actors outside $\mathbf{P}$ can have an impact on f, but only through actors in the studied network. However, for positional models (for example, both structural and regular equivalence), mis–specification of the boundary is extremely consequential as the position of an actor is made up of all ties (present and absent) across the *whole* network. If the graph of Figure 2 (a) is accurate, then the positions of all actors in panel (b) are changed by the omission of network members. In particular, the construction of $\mathbf{W}$ will change dramatically and the estimated network autocorrelation model is likely to be misleading.

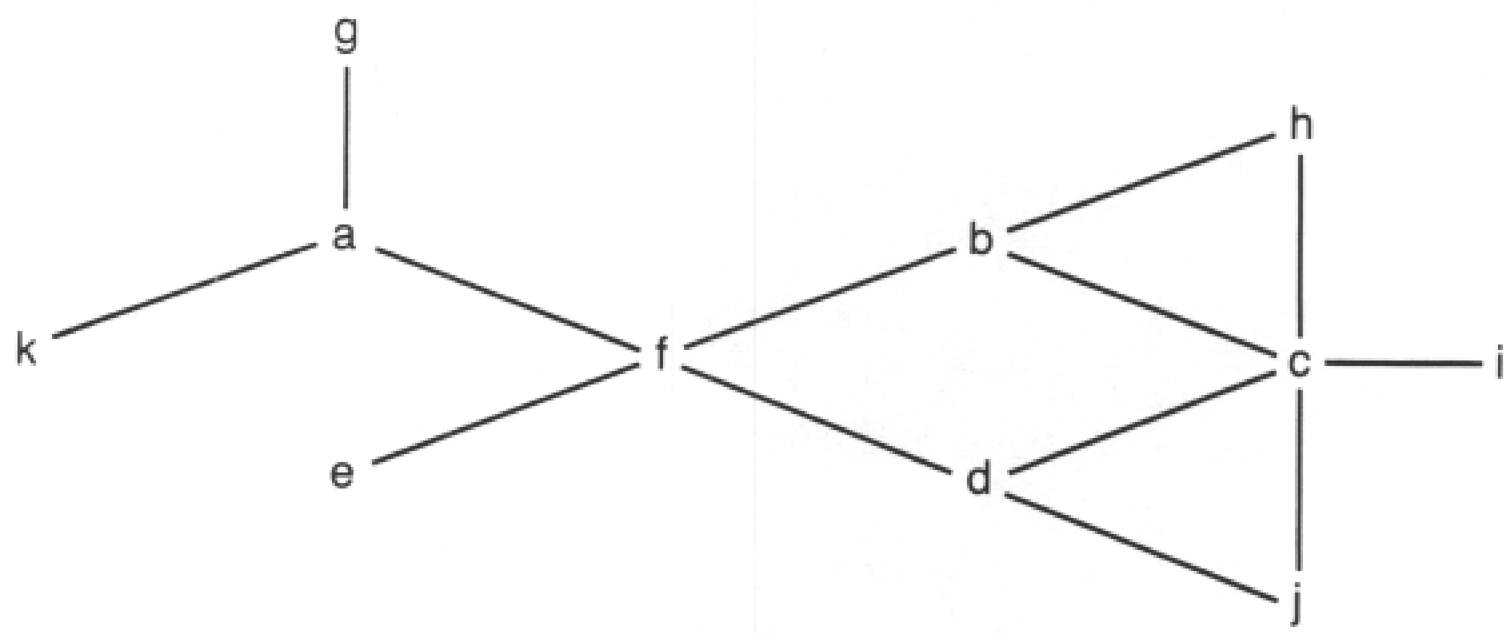
A special case of the boundary problem is the omission of an actor from $\mathbf{P}$. Taking Figure 2(b) as the true group, it is possible that data are not collected from an actor known to be in $\mathbf{P}$—say, by oversight, respondent refusal, *etc.* Imagine that data are not present for f in Figure 2(b). The structure that remains [Figure 2(c)] is radically changed. So much so, that any network analysis of the data is pointless. In contrast, omitting e is far less consequential for a subsequent network autocorrelation model. Anselin (1988) draws on work of Griffith (1983, 1985) to point to a way in which the consequences of boundary effects can be studied. [6]

Consider a network made up of two parts, the nodes of a particular group, denoted G, and nodes not in G, but in the wider network, denoted H. For a network autocorrelation model, as specified in (1.6), recognition of included and excluded actors leads to

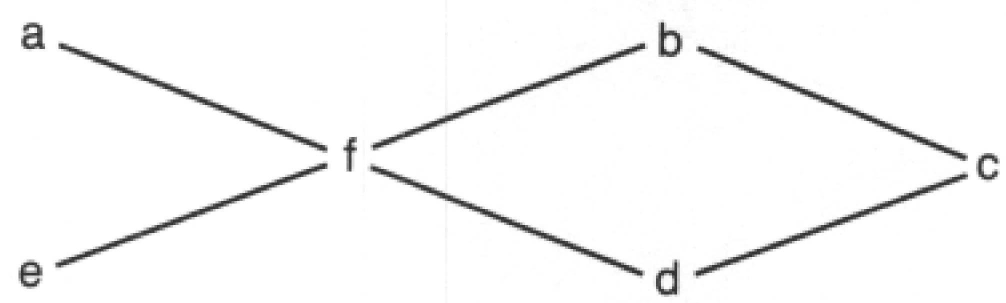
$$\begin{pmatrix}\mathbf{y}_G\\ \mathbf{y}_H\end{pmatrix} = \rho\begin{pmatrix}\mathbf{W}_{GG} & \mathbf{W}_{GH}\\ \mathbf{W}_{HG} & \mathbf{W}_{HH}\end{pmatrix}\begin{pmatrix}\mathbf{y}_G\\ \mathbf{y}_H\end{pmatrix} + \begin{pmatrix}\mathbf{X}_G\\ \mathbf{X}_H\end{pmatrix}\boldsymbol{\beta} + \begin{pmatrix}\epsilon_G\\ \epsilon_H\end{pmatrix} \tag{3.9}$$

Figure 2.

Examples of boundary problems in networks.



(a) An intact social network



(b) A subgroup embedded in the social network

(c) The subgroup with a missing node

with the obvious partitions of $\mathbf{W}$ and $\mathbf{y}$. The actual model estimated, with no recognition of H, would be

$$\mathbf{y} = \rho \mathbf{W}_{GG}\mathbf{Y}_G + \mathbf{X}_G\boldsymbol{\beta} + \boldsymbol{\epsilon}_G \tag{3.10}$$

while from (3.8) the corresponding equation is (Anselin, 1988, p. 175):

$$\mathbf{y}_G = \rho \mathbf{W}_{GG}\mathbf{Y}_G + \rho \mathbf{W}_{GH}\mathbf{y}_H + \mathbf{X}_G\boldsymbol{\beta} + \boldsymbol{\epsilon}_G \tag{3.11}$$

Re-writing, this becomes:

$$\mathbf{y} = \rho \mathbf{W}_{GG}\mathbf{Y}_G + \mathbf{X}_G\boldsymbol{\beta} + (\rho \mathbf{W}_{GH}\mathbf{y}_H + \boldsymbol{\epsilon}_G) \tag{3.12}$$

There is then an unknown network dependence term $(\rho \mathbf{W}_{GH}\mathbf{Y}_H + \boldsymbol{\epsilon}_G)$ with a distinct network autocorrelation structure. This error term is unlikely to have zero mean, nor will it be spherical (Anselin, 1988, p. 176). Further complicating matters is the fact that $\mathbf{Y}_G$ and $\mathbf{Y}_H$ are interdependent (equation 3.8) so that the error term in (3.11) is no longer independent of $\mathbf{Y}_G$.

Clearly, one line of attack is use (3.8)–(3.12) as the basis for simulation studies. Another is substantive and empirical. Consider the example of a network of social service agencies dealing with, in one way or another, the population of children and youth. This network is distributed across many sectors including mental health, health, criminal justice, social welfare, education, and employment sectors (Doreian, Woodard and Musa, 1989). Boundaries within and between these sectors are fuzzy, but cores and boundaries can be specified. A k-core is a set of nodes in a connected graph, such that considering only the nodes in the k-core, each node has in-degree and out-degree of at least k. As k defines a threshold, boundaries can be established experimentally, within and between sectors, with a view to examining the consequences of excluding sets of agencies.

This operationalization of boundaries rests on a particular data structure, obtained by a snow-ball sampling scheme. Starting with the central set of agencies (acknowledged by all as in the core), directors and staff are asked to list the other agencies they need to interact with in order to service their clientele. When another organization is cited enough times, it is added to the agency list, and data are obtained from it until no more organizations are added. With a low threshold, the network is expanded well into the peripheral agencies and beyond any reasonable boundary to the network. Data are then available on agencies outside $\mathbf{P}$, and boundary effects can be examined in the relevant context of the network.

3.4. Aggregation issues

Given a network, equivalence ideas are used to provide two complimentary reductions:

(i) to join nodes together in a single position, and generate a set of non-equivalent positions (blocks); and,

(ii) simultaneously, collapse relations between nodes to define relations between the constructed blocks.

The initial formulation of structural equivalence (Lorrain and White, 1971) was given in terms of category theory where the product of morphisms was crucial. If one morphism represents "mother of" and a second morphism represents "brother of," then their compound (product) will be a morphism (the product is closed) that ought to correspond exactly to "maternal uncle of." In its initial formalization, the construction of positions via collapsing of objects and morphisms jointly became impractical for all but small networks. The available algorithms for getting a structural equivalence partition are all rather crude attempts to create a practical partition which retain as much of the initial conception as possible. However, the partitioning of nodes and links is done in sequence—a partition of the nodes then collapsing ties. In fact, both the nodes and the ties are aggregated in terms of some equivalence conception. Consider the example in Figure 3 in terms of regular equivalence. The graph of 9 nodes can be reduced to 4 blocks and the ties between blocks are constructed from the ties between nodes, in each block, to nodes in the other blocks. The rows and columns of sociomatrix C, in Figure 3 have been permuted so that block members are together.

The image matrix can be constructed in a variety of ways depending on the criteria chosen. For example, one criterion could be that the presence of any link between blocks suffices to define a tie between blocks, or that the density of ties between blocks exceeds some threshold value (usually the overall density for the tie in the network). Under either criterion, the image matrix is the 4–by–4 matrix in Figure 3. Apart from providing a simpler and more easily understood network, the hope is that image matrices can form the building blocks for a structural theory of relations.

In terms of network autocorrelation, each block is fundamental and the nodes in that block provide indicators of it.[7] Rather than formulate an autocorrelation model in terms of individual nodes, such a model can be formulated in terms of blocks—if they are fundamental. This, however, leads directly to problems of aggregation as nodes are aggregated into positions. If an actor is placed incorrectly in a block, then the aggregation will have the same spill–over problem described by Anselin (1988, p. 12), generating network dependence. Serious as this is, there is yet another aggregation problem stemming from the aggregation of ties between blocks. The risk here is very serious as it leads to inaccurate [8] construction of $\mathbf{W}$.

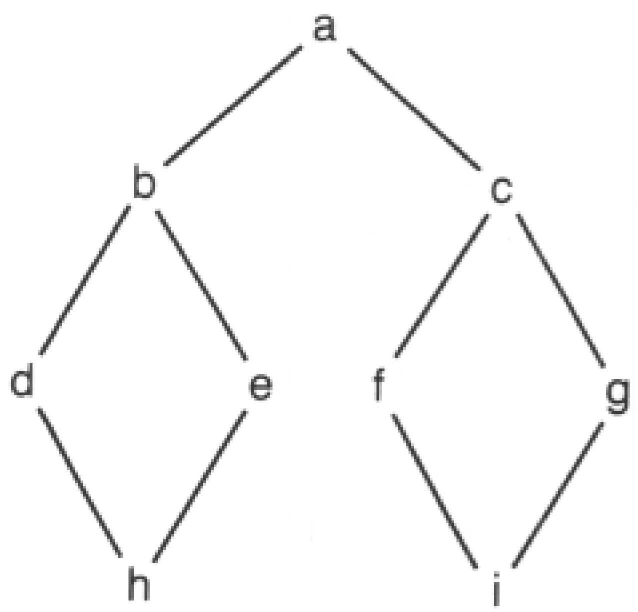
3.5. Mixtures of processes

The discussion of two regimes of network effects was couched in terms of rival structural mechanisms–cohesion versus structure equivalence, or structural equivalence versus regular equivalence–where the linear model provided an inferential framework. Two rival structural accounts, were competitively examined. Of course, one outcome could well be that ρ_1 and ρ_2 are both non–zero and that both mechanisms, via $\mathbf{W}_1$ and $\mathbf{W}_2$, are operative in generating the $\mathbf{y}$ as it is distributed over the network. However, it may be necessary to go beyond this formulation to one where the mechanisms are explicitly coupled.

Consider Figure 4, where the nodes represent political actors and the lines represent strong political ties in a hypothetical graph. It is a graph whose structure renders the issue of deciding *between* structural versus regular equivalence fruitless (cf. Doreian, 1988). When structural equivalence is considered, the partition yields two political alliances (Figure 4, upper panel). In addition, when regular equivalence is considered, the partition returned is a complementary one and provides additional insight into the structure of the group. Actors f and g have an integrative role between the alliances; actors a, c, d, k, and i all provide further integration within these alliances; and actors b, e, h, j, and l play no structural role beyond being peripheral and buried in a larger grouping. Given the structural equivalence partition, the regular partition also is coherent. The actors f and g are boundary spanners of

Figure 3.

Homomorphic reduction of a network to an image.



(a) A nine node network

		K	L		M				N	
		a	b	c	d	e	f	g	h	i
K	a		1							
L	b	1			1	1				
	c	1					1	1		
M	d		1						1	
	e		1						1	
	f			1						1
	g			1						1
N	h				1	1				
	i						1	1		

(b) Permuted sociomatrix

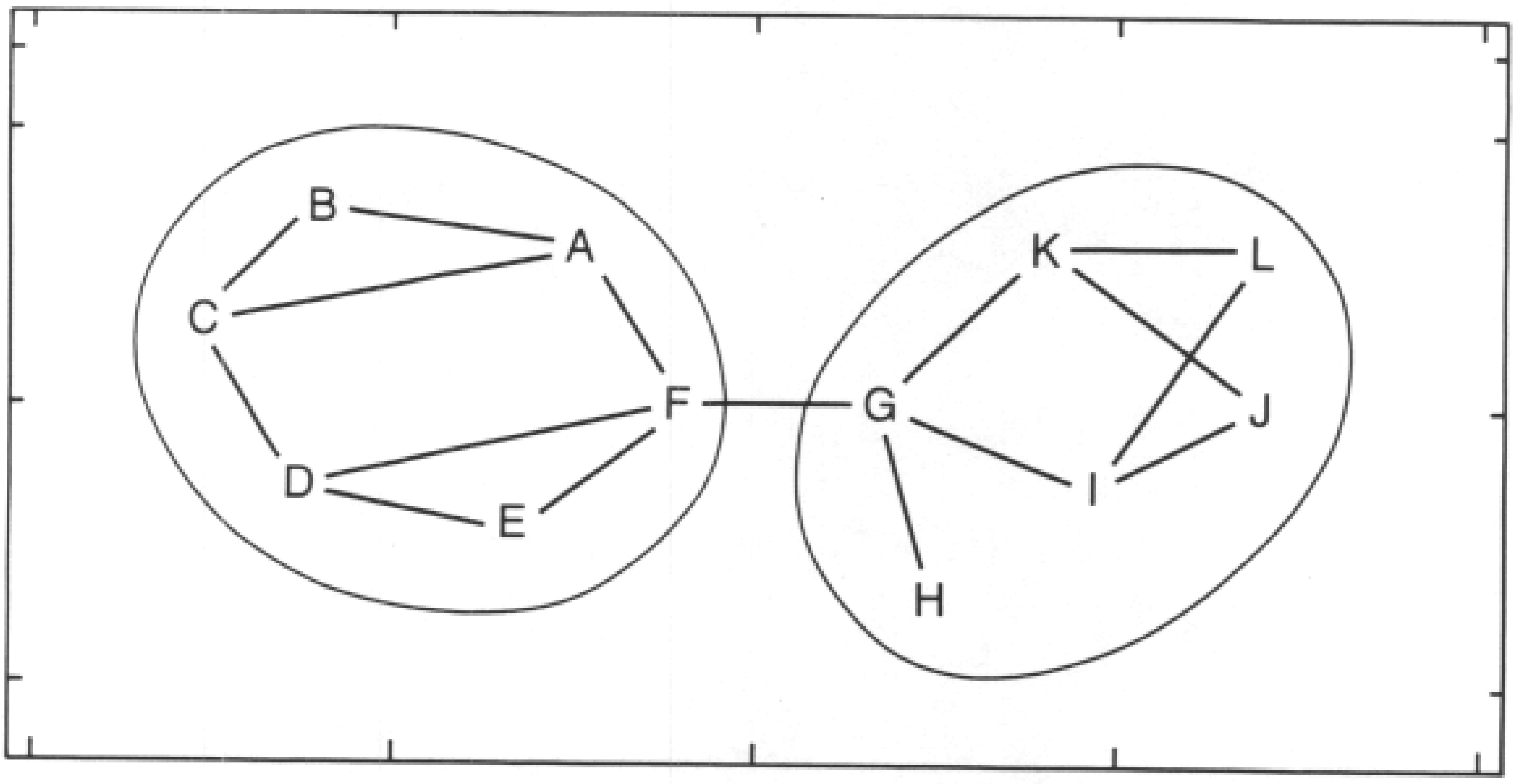
	K	L	M	N
K	0	1	0	0
L	1	0	1	0
M	0	1	0	1
N	0	0	1	0

K—L—M—N

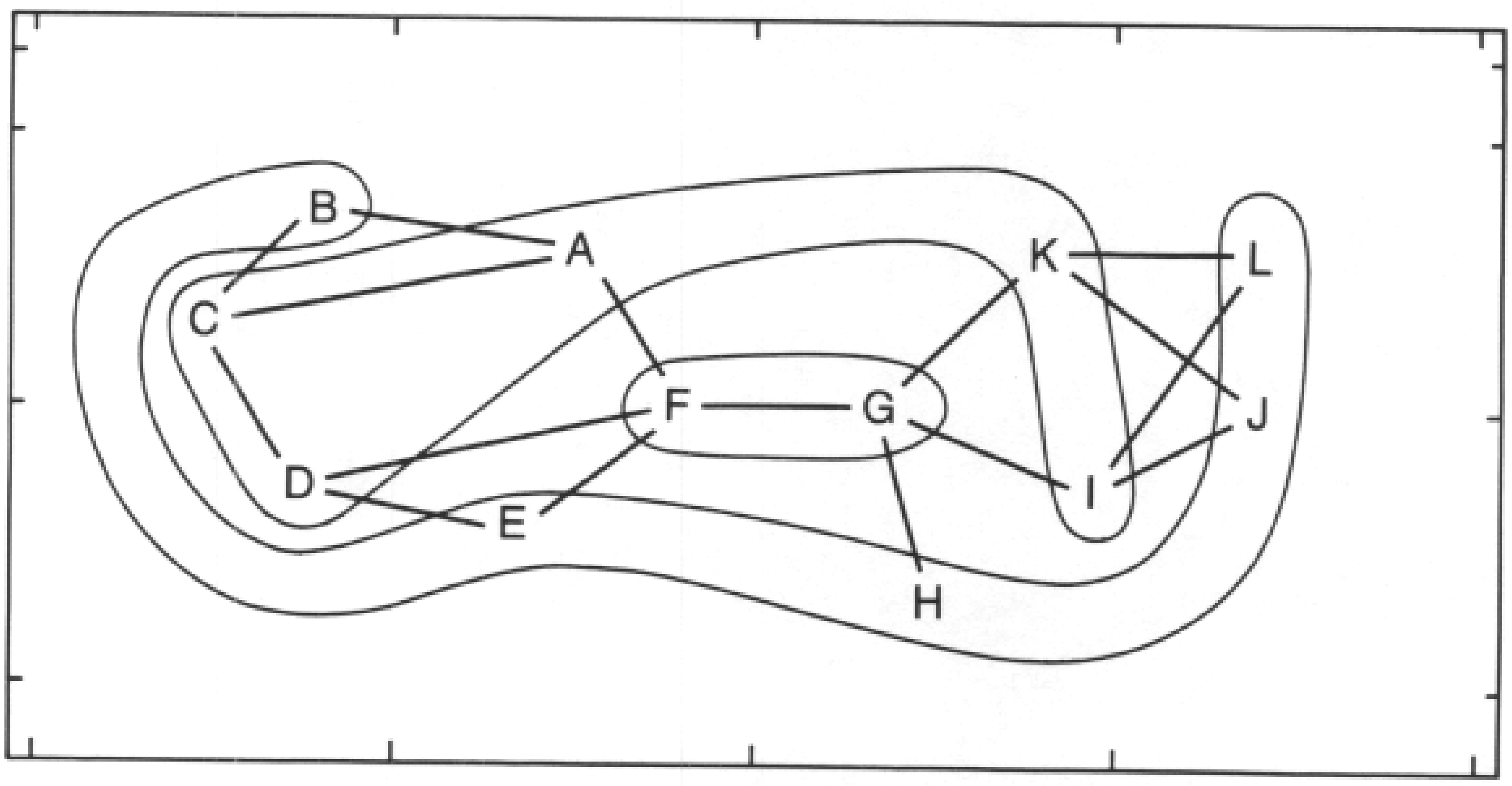
(c) Image matrix under regular equivalence and image graph

Figure 4.

Two complementary partitions of graph nodes.



(a) Partition based on structural equivalence



(b) Partition based on regular equivalence two systems. If the political dynamics involve taking a position with regard to some issue, it is clear that an account based on the cleavage between coalitions is important. But to mediate the conflict, the boundary spanners play a critical role and serve as conduits into their own coalitions. One would expect that boundary spanners are more moderate in their political views than those having no integrative role. There may be a cohesion mechanism inside the alliances and regular equivalence mechanism between them.

4. Dynamic models

Most analyses of networks are cross–sectional and avoid many issues raised by the inclusion of time. As noted by Barnes and Harary (1983), this is a serious omission. An empirical situation is likely to exhibit change in three possible ways:

(i) changes in the values of variables characterizing the nodes;

(ii) changes in the network ties; or,

(iii) changes in the nodal attributes together with changes in the network.

Only the first appears to be straightforward. Differential equations (or difference equations) can be used to model changes in the nodal attributes from within the perspective of structural control (Doreian and Hummon, 1976). In essence, equation (4.1) is derived as a model of an equilibriating mechanism:

$$\frac{dy(t)}{dt} = \gamma[\mathbf{y}^{\star}(t) - \mathbf{y}(t)] \tag{4.1}$$

where γ is sensitivity parameter. Equation (4.1) can be integrated and the solution system is used as a set of estimation equations. The connection to network autocorrelation (Doreian, 1989a) comes from specifying

$$\mathbf{y}^{\star}(t) = \rho\mathbf{W}\mathbf{y}(t) + \mathbf{X}(t)\boldsymbol{\beta} + \boldsymbol{\epsilon}(t) \tag{4.2}$$

the network effects model (with one or two regimes) is used to model the control values and the integrated process equation gives the estimation equation. There is literature on space–time models (see Upton and Fingleton, 1985; Anselin, 1988). Modeling the changes of nodal properties is difficult, but the problems inherent in modeling the changes of $\mathbf{W}_{ij}$ seem much harder. The problem is one of modelling $\mathbf{W}(t)$ and is both technical and substantive.

To model change in the network ties it may be best to use specific structural theories as a source. For example, structural balance (Cartwright and Harary, 1963) can be mobilized to study change from the premise that social actors prefer balance to imbalance. A sketch of doing this in a dynamics perspective is provided by Hunter (1979).

A second example can be taken from the literature on interorganizational networks where there are hypotheses concerning the formulation (and continuance and dissolution) of inter–organizational ties. Thus, occupational diversity, internal flexibility, professionalization, and large budget size all are seen as conducive to the formation of inter–agency ties. Common definitions of problem areas and agreement on issues of concern both lead to higher quality ties between agencies. Also, the greater the level of turbulence in the environment, the fewer the co–operative ties. Many hypotheses can be compiled to provide a theoretical basis for studying change in the composition of networks.

In terms of network autocorrelation models, network dynamics involve consideration of $\mathbf{y}(t)$, $\mathbf{X}(t)$ and $\mathbf{W}(t)$. Relative to the spatial case, it may be that the volatility of social networks and their inherently changing character will make it more difficult to build and estimate network autocorrelation models.

5. Conclusion

Network autocorrelation models draw their inspiration from the success of spatial autocorrelation modeling efforts as the connection between them is direct (via $\mathbf{W}$). The common focus for the two efforts is the recognition that data points are interdependent. The problem of modeling interdependent systems in geographic space is isomorphic to the problem of using network tools in 'social space'. It follows that each group can learn from the other. Thus far, geography, in the vanguard of autocorrelation modeling efforts, has forged the foundations for modeling interdependent systems. However, as more social scientists recognize the need to incorporate social structure into regression and other analyses, more people will be working on the common problem. In principle, when solutions are found outside geography they would be helpful for geographers.

Some of the problems inherent in network autocorrelation models seem much more severe than in the spatial case. Social networks change much more quickly than spatial configurations. The really tough problem is the specification of the process by which $\mathbf{W}(t)$ changes. If this problem is solved, it will benefit all social science—including geography. Perhaps, at some stage, we can envision coupling the two concerns of interdependence modeling and deal with the social processes in geographic space.

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NOTES

1. More precisely, a specific model is treated in this fashion. Given autocorrelated data, it is very difficult to distinguish an effects model from a disturbances model *in the data*. The choice between the models should be made prior to the data analysis (Doreian, 1980).
2. Anselin's specification is broader than (2.1) with $\boldsymbol{\nu} \sim N(0, \Omega)$ where Ω is diagonal and heteroskedastic.
3. Brandsma and Ketellapper (1979) contains the parallel treatment for two regimes of disturbance autocorrelation.
4. Attaining the goal of delineating world system positions on the sole basis of the ties among nations, is a major methodological advance. However, it does not remove the inherent ambiguity of the "verbal" classifications: the boundaries between positions retain fuzziness. CONCOR is a splitting clustering procedure where the analyst can stop splitting clusters at any point. The issue of fuzzy boundaries becomes more complicated when other structural equivalence algorithms are used. Another popular algorithm is STRUCTURE (Burt, 1989) where the distance between positions is measured as the Euclidean distance between vectors made up of a (sending) row and a (receiving) column. These distances are clustered to delineate positions. The two methods can, and frequently do, provide distinct partitions. It is an open problem as to what network properties either algorithm is responsive (Doreian, 1988).
5. Even now, a model with 4 regimes of network effects is very impractical, if not impossible.
6. Anselin (1988, pp. 174-5) shows that when spatial units are omitted from spatially autocorrelated models, the impact of the excluded areas is not confined to the boundary areas. For a cohesion model, however, this is less consequential.
7. Burt (1976) takes this one step further by defining types of positions–primary, broken, sycophant, *etc.*–and measuring the extent to which each node (not block) occupies a type of position. These variables are used in LISREL to build models of network phenomena. Forgotten in such analyses is the notion of network autocorrelation which may compromise the whole use of LISREL. However, something very important in Burt's approach is the use of confirmatory factor analysis to check that the nodes put into a common block/position are indicators of that position.
8. This is not necessarily a mis–specification of $\mathbf{W}$, as the analyst may correctly specify $\mathbf{W}$ in terms of, say, structural equivalence. When $\mathbf{W}$ is constructed, however, the mis–assignment of nodes corrupts it.

DISCUSSION

"Network autocorrelation models: problems and prospects"

by Patrick Doreian

Network autocorrelation models are a fusion of two methodologies from associated fields. Network models are sociological tools designed to categorize social interaction along prescribed pathways among social forces. As models of social process, they have given rise to much insightful analysis and interpretation. Spatial autocorrelation models are constructs designed to describe statistical interdependence among geographic neighbors. As geographical constructs, they have been extremely useful in describing and explaining spatial structural dependence (*e. g.*, Cliff and Ord, 1981). Patrick Doreian discusses the fusion of these two concepts into network autocorrelation models, tools that can be used to study interactions along social networks, accommodating the interdependence of network nodes. Modeling these interdependencies, he argues, will improve the accuracy and reliability of analytic network models. In essence, it will make the models more accurate and realistic.

Doreian's approach is innovative in that it goes beyond a simple application of geographic methodology to sociological problems. He adapts the method to the specifics of his problem, using the construct of geographic structure (or nodal links) to constrain the sociological models of interaction. This fusion should lead to models that are more representative of the true processes under study. Rather than being constrained by the geographic model, Doreian has modified the concept of network models to better fit current views of social interaction.

In biology and medicine, similar growth through cross-disciplinary fertilization with quantitative geography is possible. Not only can the concept and models of nonindependence be used, but the explicit use of geographic information adds new insight. I now will draw examples from evolutionary theory, ecology and epidemiology to illustrate this point.

Much of the theory of evolutionary biology is based upon the assessment of the genetic structure of populations. This structure is determined by the countervailing forces of natural selection, reproductive recombination (genetic drift) and mutation that are mediated by demographic and environmental influences. In essence, one looks to see which organisms are most similar and which are not, and then one tries to explain these differences as occurring through chance variation (recombination and mutation) or some selective force. Chance mutation does not lend itself well to geographic modeling (at the population scale) as it is thought to occur randomly through the genome (for a contrary view, one should see recent work by Cairns, Overbaugh, and Miller, 1988). Both recombination and selection, however, can have strong geographic components, and the study of the resulting geographic patterns has led to much insightful evolutionary analysis (*e. g.*, Endler, 1977). More recently, evolutionary biologists have begun to model the spatial pattern of the environment, and it is in this realm that I anticipate the most important advances will be gained by using geographic models in conjunction with evolutionary processes.

A second illustration stems from ecology, where succession is one of the principal paradigms of ecological thought. Plant communities vary over space and time in response to changing environmental conditions, and succession is the pathway over which these changes occur. For a long time succession was thought to be a unidirectional, temporal progression from well dispersed, rapid growth, short lived, ephemeral species (*e. g.*, weeds) to poorly dispersed, slow growth, long lived species (*e. g.*, oak, pine and redwood trees). Then ecologists noticed that succession was not unidirectional, but rather, depending upon local conditions and disturbance rates, could head in a variety of directions. More recently, ecologists have hypothesized that succession can occur spatially as well, and that it is simply the response of plants to a changing environment. And yet, few models have been built that adequately capture plant succession. Those that do succeed incorporate some component of geographic structure.

In trophic ecology, food webs have been used as schematic depictions for community structure. Dating back to the early days of ecology (*e. g.*, Lotka, 1925; Elton, 1927; Lindemann, 1942; Odum, 1969), food web diagrams have been used to show species–species interactions, predator–prey relationships, energy flow along trophic pathways, and other aspects of community structure and function. As qualitative tools, interactions or flows between nodes (*e. g.*, species) are shown as connected lines, while lack of direct interaction or flow is denoted by the absence of a path. Sometimes quantitative estimates of flows along links are provided to show the strength of flow, where these estimates represent broad–scale averages over time. Recently, quantitative interest in the size, structure and complexity of food webs has arisen (*e. g.*, Cohen, 1978; Cohen, Bariand and Newman, in press; DeAngelis, Post and Sugihara, 1983; Pimm, 1982). By comparing length and size (number of nodes) and structure of the foods across habitats, communities and biomes, researchers have drawn inferences about theoretical ecology regarding species interactions within these groupings.

To date, most studies of food webs have concentrated on the binary connection matrices describing species interactions that are similar to Doreian's social networks. While field research has documented the existence of these links, few quantitative evaluations have been undertaken. One intriguing approach for investigating the functioning of food webs would be to statistically model environmental and food web dynamics. For example, for a terrestrial system, one could monitor population densities, and light, temperature, moisture and nutrient levels over time. Using the food web model for this system (with 1s on the diagonal), one could fit Doreian's network autocorrelation model [his equation (2.1)] to the data. This would fit parameters to the food web links that would be useful for descriptive purposes, as well as allowing for perturbation analyses to be undertaken. At the current time only binary networks (*i. e.*, qualitative models) and purely theoretical, quantitative models have been explored. By adding nodal interdependence to derive quantitative, data–based models, it is likely that ecologists can achieve a more fine–scaled resolution to species relationships.

The final example comes from environmental epidemiology, which is a rapidly growing field of investigation. As the development of synthetic chemicals expanded after World War II, and the public's awareness of the ubiquity and potential hazard of these substances has grown, health scientists are assuming an increasing role in their characterization and study. Epidemiologists who study patterns of disease have been confronted with a new paradigm of disease causation, and slowly are adapting their methodology to address these issues.

The standard model of infectious disease causation is that an infectious agent (or vector) is a source of exposure and risk. Once inside the host organism, the infection is a biological entity that grows. The infectious agent (or vector) has a period and strength of infectivity and those coming into contact with a carrier of the agent may develop the disease. Their probability of illness is mediated by the length of exposure, the route of exposure, the strength of the infectious agent, the individual's own susceptibility, and other unknown risk factors. Generally, this model is simplified. The strength of the agent is a function of the disease that is chosen for study, and hence held constant within a study. Each route of exposure is considered important or not, and accordingly included or not included in the analysis. But, its importance is not scaled. And the unknown risk factors are considered to be distributed randomly through the population and of no predictive importance for the group under study. Therefore, disease incidence and prevalence models only are based upon the probability of contact via 'risky pathways' and the length of exposure. To make projections about disease spread and the probability of an epidemic under these models, one must develop only a history of contact among the individuals or population under consideration, and the agent. However, the strength, distribution and magnitude of the agent may be affected by exogenous factors, such as weather or food source. These factors often are omitted for infectious disease models because they are unknown, or their range of variations is thought to be sufficiently small as to not affect the model markedly. Further, the ability to characterize the variation of these factors is limited. Additionally, infectious diseases tend to be acute and have short latencies or induction periods (AIDS being a notable exception). This temporal compression facilitates their study.

One observation is that the study of infectious diseases would benefit from models analogous to Doreian's network models. Contact models are binary connection matrices that fail to accommodate other parameters of infectivity. By assessing infectivity, one could derive estimates for many of these parameters that would increase our knowledge of the disease process. While some such models exits, most ignore this approach (and few model the interaction).

In environmental epidemiology, the nature of exposure and disease is more complicated. As with infectious disease, probability of incidence also is based upon length of exposure, route of exposure, strength of agent, susceptibility, and other risk factors. However, in this case the strength of the agent is not constant, but varies by the type of agent, its exposure pathway and its concentration. Environmental epidemiologists often model exposure as a function of proximity to a source of pollution and the frequency with which one encounters the source, as well as the strength (or concentration) of the source. Most sources of exposure are geographically coherent in space. For example, they may be plumes downwind or downstream from a source, or parts of a community drawing water from the same source. We can study the disease process by comparing spatial patterns of the exposure agent to those of disease. Our recent work in disease cluster investigation has found that models consistent with environmental exposures may give different results than models consistent with infectious exposures (Wartenberg and Greenberg in press).

In summary, the utilization of geographic information in biology and medicine may lead to enhanced analysis of problematic situations. Most models fail to accommodate spatial (or dependency) structure, and thus obscure a certain level of resolution. By taking advantage of this information and using models of the sort Doreian proposes, investigators in these fields stand to gain substantially.

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