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"A SIMULATION STUDY OF BARRIER EFFECTS IN SPATIAL DIFFUSION PROBLEMS"

by

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FOREWORD

Every other Wednesday night the Michigan Community of Mathematical Geographers holds a joint seminar in Brighton, Michigan. Brighton is a small community closest to the point of minimum aggregate travel from the three universities involved.

Academic credit is given students who attend the seminar. Mr. Yuill is a student of the seminar and developed his interest in theoretical geographic barrier problems and in simulation techniques as an outgrowth of discussion of that topic presented to the seminar by Professor Nystuen in the Spring of 1964. The paper Mr. Yuill produced and which is herein presented was accepted as his Master's Thesis at the University of Michigan. It also received limited distribution as Technical Report No. 1, Spatial Diffusion Study, (O.N.R. Task No. 389-140, Contract No. 1228(33)).

We are grateful to Professors Duane F. Marble (Northwestern University) and Forrest R. Pitts (University of Pittsburgh) directors of that project for underwriting the cost of typing and preparation of the plates for reproduction.

The Editor, April 1965

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A SIMULATION STUDY OF BARRIER EFFECTS IN SPATIAL DIFFUSION PROBLEMS

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INTRODUCTION

Barriers play an integral role in restricting and shaping human activity, and in determining the distributional patterns of many phenomena. However in geography as well as in many other social sciences, the term "barrier" is frequently used and accepted at a very superficial level. An extremely general definition may be used such as that found in dictionaries: e.g., "Any obstruction; anything that hinders approach; any limit or boundary ..."[1] Although barriers in geography are usually considered with respect to questions of spatiality, this restriction still does not produce a more precise definition. There has been a strong tendency in geography to regard a barrier as an impediment to all activities regardless of their nature. Some of the devious paths into which this question may lead are apparent in the writings of the environmental determinists. For example, Semple in her American History and Its Geographic Conditions states: "At the end of the first century of permanent settlement they (the English settlers) found themselves in possession of a narrow strip of coast, shut off from the interior of the country by an almost unbroken mountain wall ..."[2] "The Appalachian system which,

1. ____, Webster's New Collegiate Dictionary, G. & C. Merriam Co., Springfield, Massachusetts, 1953.

2. Semple, Ellen Churchill, American History and Its Geographic Conditions, Houghton Mifflin Co., New York, 1903, p. 37.

together with the ice-worn highlands of New England, presented such an insuperable barrier to the early colonists, extends from the Green Mountains of Vermont to the pine-covered hills of Alabama."[3] This view however, is called into question by Brown and others: "Early maps of North America showed high mountain ranges occupying much of the interior. These fancied mountains gradually disappeared from the maps and were replaced by the various "chains" of the Appalachians. At first, the Appalachians were thought to be so formidable as to prohibit easy communication across them, but this belief too was abandoned in the light of exploratory accounts."[4] Brown's statement raises a serious question about regarding a barrier as something immutable. If the mountains were no longer regarded as complete barriers to the interaction of the people, did they then cease to be barriers at all? This dilemma again appears in Wolfe's recent work: "Precise definition of the terms 'barrier' and 'corridor' as they are used in the present context would be desirable, but it is not possible to provide them: barriers do not always serve as barriers nor corridors as corridors."[5]

One solution to the question of the nature of a barrier is indicated by Mackay: "The inhabitants separated by a boundary do not, except in very unusual circumstances, live in complete isolation from each other. On the contrary, a constant stream of human interaction flows back and

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3. Ibid, p. 38

4. Brown, Ralph H., Historical Geography of the United States, Harcourt, Brace & World Inc., New York, 1948, p. 96

5. Wolfe, Roy I., Transportation and Politics, D. Van Nostrand & Co., New Jersey, 1963, p. 15.

forth across a boundary . . . . If we can estimate, with reasonable precision, the effect of physical and cultural boundaries (e.g. a river or political boundary) upon each type of interaction, we will possess a powerful tool for regional analysis and boundary studies."[6]

A logical consequence of this is that if the effect of a barrier is to be understood, then this effect must be viewed within the context of the activity to which the barrier is supposedly an impediment. The barrier then may be actually defined in terms of the activity; a functional relationship. This concept of dynamic process appears to be crucial to the study of barriers in a spatial context. If barriers are studied only in relation to static distributions, then the whole character of the barrier may be easily misinterpreted or distorted. It is within the context of this paragraph that the study reported upon here is undertaken.

In geography, distributions of observed items in the real world are generally not manipulated at the level at which they are originally obtained. Normally, these distributions are represented in an abstract symbolic fashion on maps. The mapped (abstracted) representation is then manipulated rather than the raw data, and the extraneous facets of the real world are ignored. What has been done is that the raw data has been abstracted to form a simple model in which only the essential characteristic of the observations, their spatial distribution, has been considered.

It is, of course, possible to represent a barrier in this abstract

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6. Mackay, J. Ross, "The Interactance Hypothesis and Boundaries in Canada", The Canadian Geographer, No. 11, 1958, pp. 1-8.

sense. Further, it may then be possible to deal with the effects of barriers within an abstract two-dimensional space where the barriers also are represented by symbols. (See Figures I and II). It is easily seen that by abstracting only the functional aspect of the barrier from the observed data, two seeming dissimilar barriers may be represented symbolically in much the same fashion. This, of course, raises an important question for the researcher--are barriers, which are observed to be physically dissimilar, actually functionally similar? If so, then there may exist a class of common spatial effects of barriers despite severe differences in their original nature.

Origin of Barriers

Although barrier situations are encountered in many types of geographic research, there appear to be only two principal forms when their spatial representation is considered. The first, and most obvious, is what will be called an external barrier. The origin of these barriers is completely independent of the activity to which the barrier is functionally related. An example might be some natural physical feature, e.g., a swamp, and its impact upon interpersonal communications. (See Figure III).

Internal barriers on the other hand, are a direct result of the spatial arrangement of the activity under consideration. This is most common when the activity represents an interaction among a particular set of points which are in turn a subset of a much larger point set. An example of this might be interaction among members of some ethnic group which is unevenly mixed throughout a larger population. (See Figure IV).

It is important to recognize that both internal and external

FIGURE I

MOUNTAIN RANGE AS A PHYSICAL BARRIER

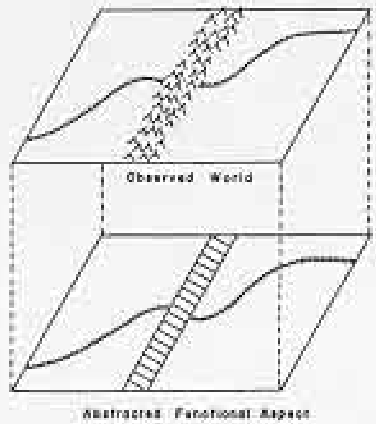


FIGURE II

POLITICAL BOUNDARY AS A CULTURAL BARRIER

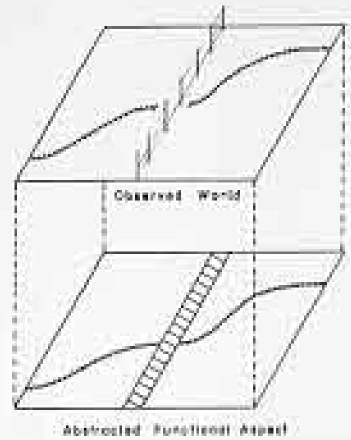


FIGURE III

EXTERNAL BARRIER

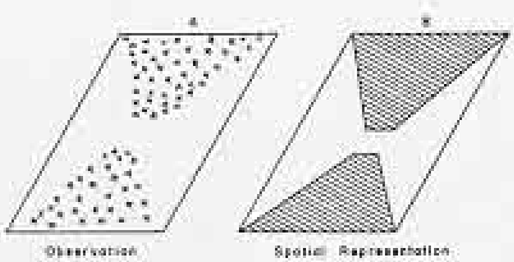
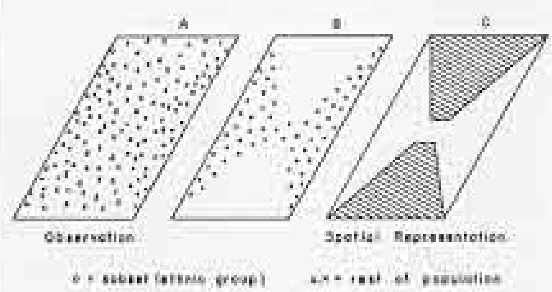


FIGURE IV

INTERNAL BARRIER



barriers may be spatially abstracted in an identical fashion (Figures III-b, IV-c). But, though these diverse barriers may be represented abstractly in the same symbolic fashion, the question of the similarity of their spatial effect upon their respective activities must be considered. While a certain similarity may seem evident at first glance, further consideration indicates that the case is somewhat more complex--that the effect of a barrier is really composed of two elements: functional effects and shape effects.

Functional Character of Barriers

One may readily imagine a multifold proliferation of the possible functions of a barrier. However if consideration is limited to the spatial aspects, it becomes possible to examine only a few essential characteristics.

Since a barrier is the creation of a dynamic spatial process, its functional effects must be also defined in terms of the process. These effects fall within three general categories: *absorbing*, *reflecting*, and *permeable* barriers.

Absorbing Barriers

This is defined as a barrier that absorbs energy from the activity which comes into contact with it. The action is turned neither away nor back, but is completely stopped in its original direction of movement. As a consequence, the total amount of energy in the interaction system is diminished by the amount absorbed by the barrier.

Reflecting Barriers

This barrier, unlike the absorbing, does not affect the total sum of energy of the activity. Instead, that portion of the energy which comes into contact with the barrier undergoes an abrupt change of direction without change in momentum.

Permeable Barriers

Permeability affects the spatial process or activity only in that some of the activity which contacts the barrier is able to pass through it. However Nystuen has pointed out an important dichotomy in the action of permeable barriers. They may serve as filters, allowing the penetration of only certain portions of an activity, e.g., selective tariffs. On the other hand the barrier may act as a screen in which case those portions of the activity which possess sufficient energy are able to penetrate the barrier.[7]

Combinations

It is of course highly unlikely that a barrier can be adequately represented in terms of only a single function. Almost any example which one might specifically choose to illustrate one of the particular functions mentioned above, would be found to contain at least one of the other functional effects. Further, although these three types may be discussed independently, they are all compatible within the same barrier. Figure V-a represents individuals (or unit elements) of some process encountering first single function barriers, then a multifunction barrier. If the total energy of the spatial activity is then considered, Figure V-a may be further symbolized as in Figure V-b.

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7. Personal communication from Dr. John D. Nystuen.

FIGURE V-a

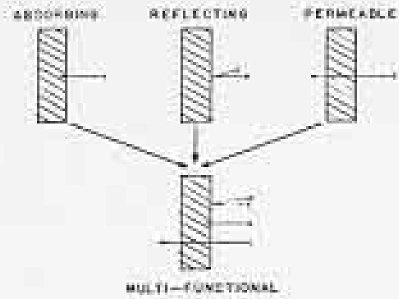


FIGURE V-b

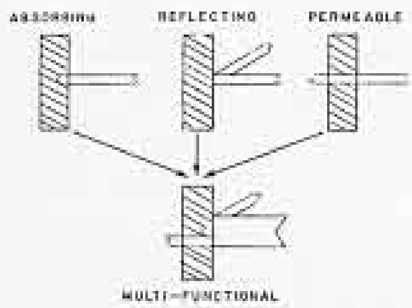


FIGURE VI

GRAPH OF BARRIER COMBINATIONS

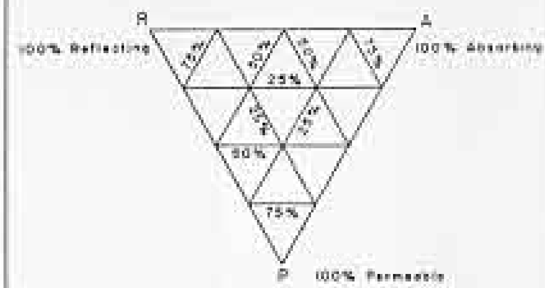


FIGURE VII

ABSORBING BARRIERS — TYPES

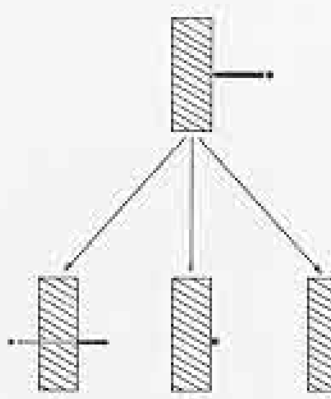
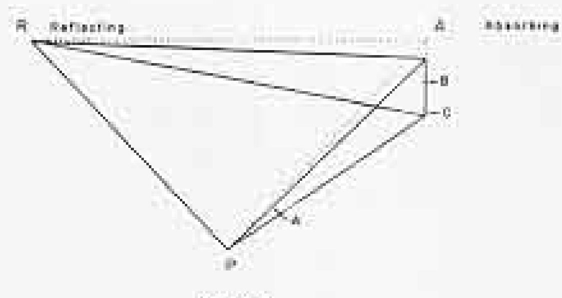


FIGURE VIII

COMBINATION UNIVERSE OF TOTAL ENERGY



If it is assumed that all portions of the activity which contact a barrier must be accounted for within the three functional categories, then the following relationship holds for the distribution of the available energy:

$$E_t = E_a + E_r + E_p$$

Where: E_t = the energy contained within the portion of the spatial activity which contacts the barrier.

E_a = the fraction of E_t which is absorbed by the barrier.

E_r = " " " " reflected by the barrier.

E_p = " " " " which penetrates the barrier.

This relationship is expressed graphically in Figure VI.

Variations within E_r and E_p have no effect upon E_t for by definition they only change the spatial distribution of the activity. With E_a however, the internal variations are significant for they directly control the defined decrease of energy. Figure VII illustrates three major variations of an absorbing barrier starting in each case with an individual or unit element encountering a barrier. In (a) the individual is able to continue through the barrier but his velocity is reduced. E_t is consequently diminished by that energy loss. In (b) the motion of the individual is completely halted resulting in a greater diminution of E_t than was true in (a). In (c) the individual itself is destroyed and E_t is reduced to zero. This range of possibilities may be expressed by adding another dimension to Figure VI to represent the change in E_t. This added dimension (see Figure VIII) is perpendicular to the plane ARP representing zero energy change in E_t. Since any combination which includes an absorbing barrier causes some decrease in E_t the resulting solid figure is entirely below the ARP plane except at the line RP. This solid figure represents the universe from thich the functional barriers of this study are drawn.

Shape of a Barrier

The second principal property of a barrier is connected solely with its spatial representation - <u>its shape</u>. This is in many cases the most important property of a barrier. However although shape is an integral part of almost every geographic distribution, little has been done to provide a systmeatic classification of shape. Many times a shape is compared to its nearest Euclidian counterpart or again, a classification may be more or less arbitrary as Bunge indicates: "Another common method of treating shapes is to devise a classification and subjectively assign shapes to it. Oxbow, circular, shoestring, and rectangular are typical classes. Often these classifications are so vague that it is difficult to have much confidence in the assignment of shapes of the various categories. Another difficulty is that the classes are so arbitrary."[8] However a barrier can, in many cases, be represented by one or a combination of simple Euclidian shapes without much loss of accuracy. It is beyond the scope of this investigation to attempt a solution to the problem of classification of shape. A few barriers with simple shapes will be considered, with the hope that the results will provide a basis upon which more complex analogies may subsequently be constructed.

As mentioned earlier a barrier may be abstractly represented in terms

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8. Bunge, William, <u>Theoretical Geography</u>, Lund Studies in Geography, Series C, General and Mathematical Geography, No. 1, Gleerup, Lund, Sweden, 1962

of its functional and spatial (shape) characteristics. This study is then to be conducted entirely upon this basis, considering only the elements of function and shape which are common to all barriers. Hopefully, knowledge gained from investigations in this area will be applicable to all spatial barrier situations in geography, whether they be physical, political, or cultural.

It is impossible to deal with all the ramifications of barriers in a paper of this limited scope. Both the range of material investigated and the investigative techniques themselves must be severely limited. Hence the types of barriers to be considered here will be very simple ones.

A variety of methods exist for dealing with barrier situations. Certain approaches may utilize analogies drawn from the physical sciences such as those from hydraulics or electrical networks. Many of these, although very useful in specific cases, suffer from limited flexibility.

The technique chosen here is a Monte Carlo simulation of a simple diffusion situation. Monte Carlo methods comprise that branch of experimental mathematics which is concerned with experiments on random numbers....In the case of a probabilistic problem, the simplest Monte Carlo approach is to observe random numbers, chosen in such a way that they directly simulate the physical random processes of the original problem, and to infer the desired solution from the behavior of these random numbers.[9]

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9. Hammersley, J.M., and D.C. Handscomb, Monte Carlo Methods, London Methuen and Co., 1964, p. 2.

Since the diffusion model used in this paper is derived principally from the work of the Swedish geographer Hägerstrand, a brief resume of that important work is in order. In the following condensation all quotations are taken from one of Hägerstrand's papers.[10]

Hägerstrand's Spatial Diffusion Models

A highly significant process which may explain many nebula distributions is the diffusion of techniques and ideas through the network of social contacts. This process of diffusion was studied in a rural area in southern Sweden with two innovations, a government subsidy to improve pasture and control of bovine tuberculosis.

Since the vehicle for the diffusion process was the network of social contacts, these were closely investigated. "A good many observations suggest that this network has a definite spatial structure which probably is rather stable, that is the links connect different places with probabilities which presumably change only slowly and thus to some extent are predictable."

Interpersonal communications were judged to be most important for information transmission at the local level. The interaction level was measured via such surrogates as telephone traffic and local migration, which were felt to be measures independent of the diffusion data. It was concluded that "the communication links of the average individual on the local plane must very rapidly decrease in number with increasing distance or in the sample region roughly with the square of the distance."

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10. Hägerstrand, Torsten, "On Monte Carlo Simulation of Diffusion", in Quantitative Geography (William Garrison, ed.) forthcoming.

Using this information the diffusion of an innovation was simulated by Monte Carlo methods. For the first model an even population distribution and an ideal transportation surface were assumed. The rules governing the operation of the system were the following:

1. Only one person carries the item at the start.
2. The item is adopted at once when heard of.
3. Information is spread only by telling at pairwise meetings.
4. The telling takes place only at certain times with constant intervals (generation intervals) when every carrier tells one other person, carrier or non-carrier.
5. The probability of being paired with a carrier depends upon the geographic distance between teller and receiver in a way determined by empirical estimate.

Based upon the network of social contacts a mean information field (contact field) was constructed for the area, and it was assumed that this field represented the probability of contact, as a function of spatial separation, for any two persons. For simplicity the MIF was modified to the form of a rectangular grid of 25 cells (5 cells on a side) with the innovation carrier located in the center cell. (This grid coincides exactly in orientation and dimension with the grid division of the study area). The probability of each cell being contacted by the innovation carrier (center cell) was approximately $1/\text{distance}^2$.

Following the rules of the model the diffusion was simulated over a number of generations then compared to the actual diffusion in the study area. Although a comparison was made only in very general terms, there were striking visual similarities in the spatial patterns.

In later variations of this basic model, Hägerstrand changed some of the parameters and operating procedures in order to obtain a more realistic representation of the diffusion process. Two important problems with which he had to deal were those of study area boundaries and barriers. The boundary problem revolved around the treatment of cells near the edge of the study area so that they would have the same opportunity of being contacted by an innovation carrier as cells in the interior. Hägerstrand's solution was to add a border two cells in width to the study area (Figure IX). Individuals in the border could be contacted and could contact their counterparts in the study area. This permitted the innovation to move outside the main study area and then come back in again. In this manner the probability of contact was the same for each cell within the study area.

The barriers were topographic restrictions (long lakes and forests) which distorted the previously assumed even transportation surface. Hägerstrand recognized two categories of barrier here: weak in which communication was reduced; and absolute in which communication was completely cut off. Figure X illustrates the representation of this situation in the Hägerstrand simulation. The barriers were placed between the cells, a one-dimensional representation. There is no possible interaction over an absolute boundary. Across a weak boundary the communication was cancelled about half the time by a random process.

Although Hägerstrand was not primarily concerned with barriers in his study, he recognized their importance and devised a method for investigating their effects. It is his work which has given impetus to the present study of barrier effects.

STRUCTURE OF THE SIMULATION MODEL

The diffusion model used in this study is based upon and is somewhat similar to the one proposed by Hägerstrand. However in order to meet the requirements of this particular study certain structural changes were necessary. These arose from the limited objectives of the present study, as well as from technical limitations of the specific computer program which was employed.

Since the primary emphasis in the present instance is upon the barrier rather than on the diffusion itself, the model may be simplified by omitting certain details relevant only to specific diffusion situations. Several important changes are made in terminology. Many terms were borrowed from Hägerstrand for they are equally applicable to an abstract diffusion model. However it must be noted that there is no generic carry-over of terms. They apply as used here, only to the spatial functions.

Cell: This is the areal unit into which the study area is subdivided. The cell either contains a number of items or members, or it is empty, or it contains some suppresion device. In the latter case, the cell is designated as a barrier and reacts in a specified manner upon the spatial course of the diffusion. Therefore a barrier is considered as an area-based function rather than as a line-based function as in Hägerstrand's work.

Transmitter: A member of the population (in a cell) which in interacting according to the rules of Monte Carlo simulation, attempts to contact members in the same or another cell.

Acceptor: A member of the population of a cell which has been contacted by a transmitter and which becomes a transmitter itself in all succeeding generations.

One further change involves the ommission of Hägerstrand's operational rule number one since it will not be relevant in the present context.

Since each cell represents a unit of interaction or a barrier, the floating grid (MIF) which determines the areal probability of contact was modified. The five-square grid or probability field of Hägerstrand was changed to a smaller three-square grid of nine cells, and the probabilities were adjusted so that their sum in the nine cells totaled one. This contraction of the grid made possible the use of barriers and borders which were only one cell wide. The attendant ease in programming is obvious.

Further, the potential adopters within a cell were not considered individually but statistically. That is, a certain proportion of the population is a cell is considered to be acceptors, rather than singling out specific individuals.

Operational Procedure

The resultant computer simulation model works in the following manner:

For a transmitter in any cell, there is a given probability of contact with the surrounding cells in addition to contact within its own cell. Figure XI shows the symmetric mean information field which surrounds the transmitter. This field (or grid as it is represented here) is centered upon each cell which has transmitters in any given generation. The Monte Carlo process is applied as many times to a cell as there are transmitters, then the system moves on to the next cell containing transmitters.

When a cell has been contacted by a transmitter, a further probability function is called into play. The probability of contacting a member of that cell's population not previously contacted is equal to $(1 - N_a/N_p)$, where N_a is the number of acceptors in that cell and N_p is the total population of the cell.

Once a valid contact is made, a member becomes an acceptor and transmitter beginning with the next generation. If (statistically) an already contacted member of the population of a cell is again contacted, that transmission is regarded as lost.

Functional Barriers

Since it is impossible to incorporate infinite variability into the program, some limitations of the barriers tested had to be imposed. Operationally, this consisted of limiting the functions of the barriers to five, which are described as they function within the computer program. Given the transmission situation in Figure XII, the effect of each functional barrier is easily demonstrated.

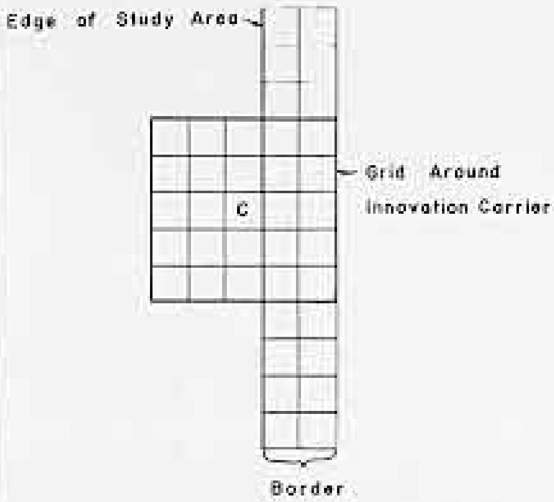
Super Absorbing Barriers (Figure XIII)

With this class of barrier (represented approximately by (c) in the universe of Figure VIII), the transmitters are temporarily destroyed but may be replaced by subsequent contact from other transmitters.

Absorbing Barriers (Figure XIV)

When an absorbing barrier is contacted the transmission is lost for that generation, but the transmitter itself is unchanged by the operation. Its position in the functional universe (Figure VIII) is

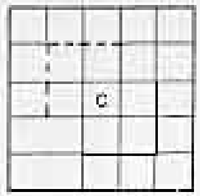
FIGURE IX



C = Innovation Carrier

FIGURE X

HÄGERSTRAND'S BARRIER REPRESENTATION



Solid Line — Absolute Barrier

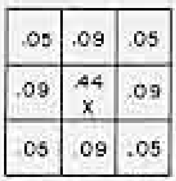
Broken Line — Weak Barrier

C= Innovation Carrier

FIGURE XI

PROBABILITY OF CONTACT

BY TRANSMITTER



X = Transmitter

FIGURE XII

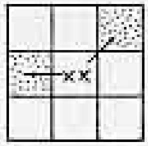
Initial Conditions:

Two Transmitters in the Center Cell

Arrows show the Cells the Transmitters Attempt to Contact

x = transmitter

o = acceptor



▒ — Barrier Cells

FIGURE XIII

SUPER ABSORBING BARRIERS

Generation N

Generation N+1

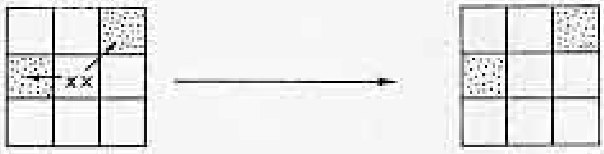


FIGURE XIV

ABSORBING BARRIERS

Generation N

Generation N+1

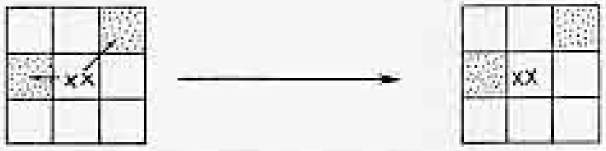


FIGURE XV

REFLECTING BARRIERS

Generation N

Generation N+1

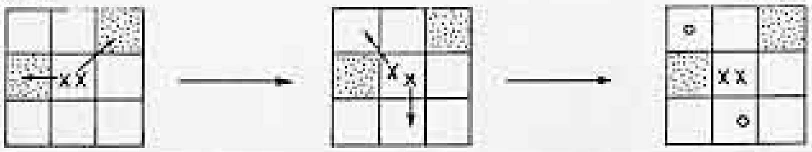


FIGURE XVI

DIRECT REFLECTING BARRIERS

Generation N

Generation N+1

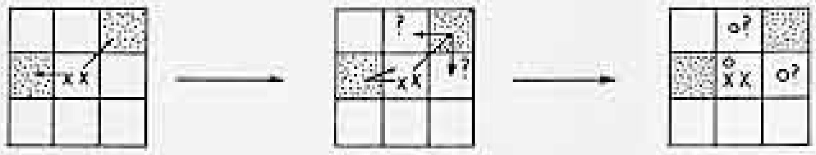


FIGURE XVII

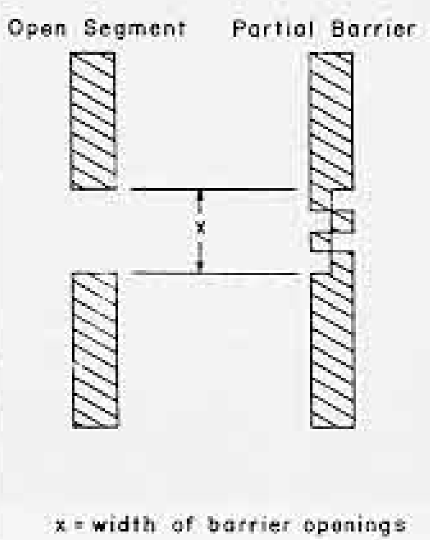
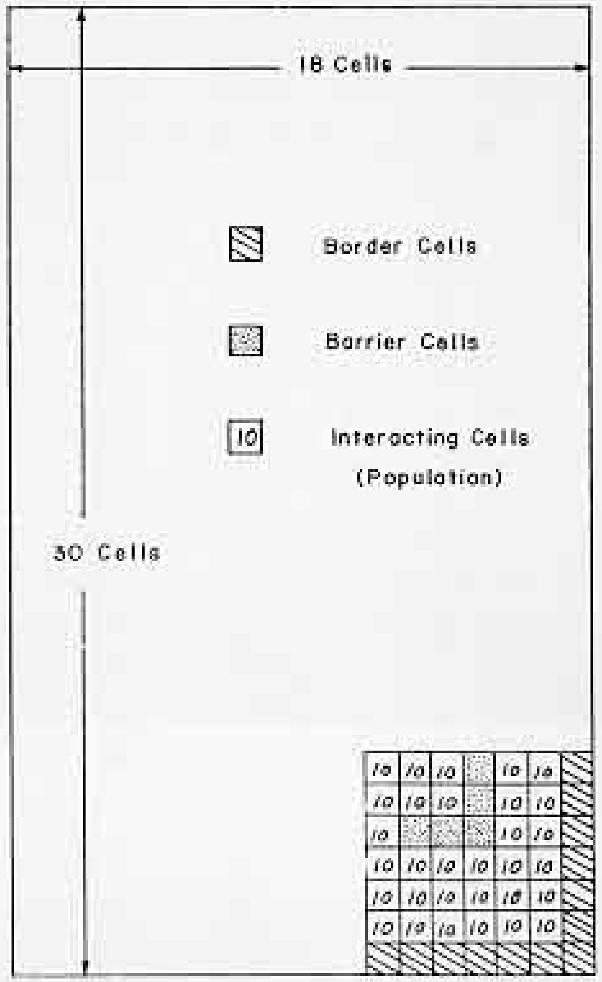


FIGURE XVIII

SIMULATION AREA



about that of (b).

Reflecting Barriers (Figure XV)

In this barrier (R in Figure VIII), the transmitter is given an opportunity to transmit again within the same generation.

Direct Reflecting Barriers (Figure XVI)

In this barrier (R in Figure VIII), the contact is deflected to the nearest cell toward the transmitter. (Distance is measured from cell center to cell center). In the case of a corner cell, which has two equally near neighbors, the choice between them is a random one. Since the mean information field is symmetrical, the deflection by all the corner cells will be alike and those of all side cells alike.

Permeable Barriers

Permeability, in comparison with other barrier functions, is a negative quality since the more permeable a barrier, the more it loses its function as a barrier. In the theoretical universe depicted by Figure VIII the permeability of a barrier can vary from zero to 100 percent. However in the computer program developed for this study, a permeable barrier is somewhat difficult to represent since each cell is assumed to be either a barrier of a specified nature, or a cell with interacting members. This obviously precludes intermediate positions in the permeable spectrum. To partially obviate this difficulty an attempt was made to approximate an intermediate permeable barrier through a structural spatial distribution of barrier cells. Figure XVII shows such a distribution, a checkerboard arrangement, together with a completely open segment of a barrier for comparison.

Size of the Study Area

Since the cell is the basic spatial unit in this computer simulation, the dimensions of the simulation area are likewise expressed in these units. For this investigation the process was confined to an area of 30 by 18 cells or a total of 540 cells. Of this however, the 92 cells which form the outside edge are excluded since their function is solely to confine the simulation to the specified area and to provide "border bounce" capability. This size of area was considered the best compromise between an area of sufficient extent and efficient use of computer time. Figure XVIII shows the simulation area. The computer program utilized in this study is detailed in Appendix A.

RESULTS OF THE SIMULATION STUDY

The simulation model was applied to a variety of barriers in order to test the effects of each characteristic in insolation - or more precisely, to hold the others constant. The characteristics examined here were the three functional effect series (absorbing, reflecting and permeable) plus some simple elements of shape.

In attempting to analyze the results of the simulations it was found that identical criteria could not be used in all situations. The disparity of shape among the barriers investigated made it impossible to find one type of measurement which would satisfy all requirements. Therefore the problem of comparing different measures arose. This is not as grave as would appear since each barrier situation was intended to supplement the information obtained from the others. Further, all the measures were based directly upon the spatial response of the diffusion simulation so they are genetically closely related.

The immediate results and deductions from each group of tests are first given separately (following), then in combination.

Separate Group Results

Group I: Parallel Barriers Perpendicular to a Linear Diffusion Wave Front

The purpose of this group was to test the effect on the diffusion wave of 1) varying the width of the channel between two parallel barriers, and 2) differences arising out of absorbing and reflecting functional effects; permeability was held constant at zero. (See Figure XIX).

The measure used here of the influence or effect of these barriers, was the rate of advance of the diffusion wave front between the barriers. The distance between the two barriers (x) took on the successive values of 1, 2, 3, 4, 6, & 9 cells, and the functional nature of the barriers varied from absorbing to reflecting.

Approximately 20 generations of the simulation were required for the diffusion front to pass the length of the barriers. For each generation the rate of advance was calculated and the means of the various combinations were found. Figure XX shows these means plotted against the width between the barriers for each of the four functional forms tested.

From a general knowledge of spatial and functional restrictions certain results can be reasonably predicted. The most obvious of these are:

a) As the distance (x) between the barriers increases, there is a decreasing effect of the barriers upon the rate of advance of the diffusion wave front.

b) In narrower channels, the absorbing and super absorbing barriers retard the diffusion wave the most.

Although these predictions are generally substantiated, there appear unexpected variations:

a) The reflecting and direct reflecting barriers apparently have little effect upon the rate of advance of the diffusion wave. Figure XX shows little variation with distance between barriers, and in fact, the variations may be entirely due to the nature of the Monte Carlo process.

b) In contrast, the absorbing and super absorbing barriers do show a response to the width of channel, but the response is non-linear. In Figure XX, there appears to be a definite break in the trend at a channel width of four cells, with the rate of advance remaining approximately constant for greater widths. This aspect will be discussed further in a subsequent section.

Group II: Passage Around A Bar Barrier Parallel to a Linear Diffusion Wave.

The spatial configuration encountered here is probably one of the most common representations of a barrier. The purpose of tests on this group is to examine the spatial effect of the barrier in terms of its length and function; as in Group I the permeability is held constant at zero. Since the effect of the barrier is particularly spatial, rate of advance as a measurement is not applicable because it would tend to obscure some of the more important aspects of the situation. For this reason the measurement criteria used here are the differences in time (Δt) between the generation in which the linear diffusion wave

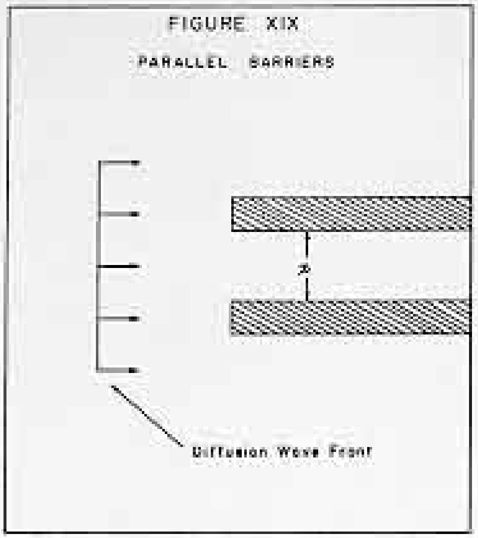


FIGURE XIX
PARALLEL BARRIERS

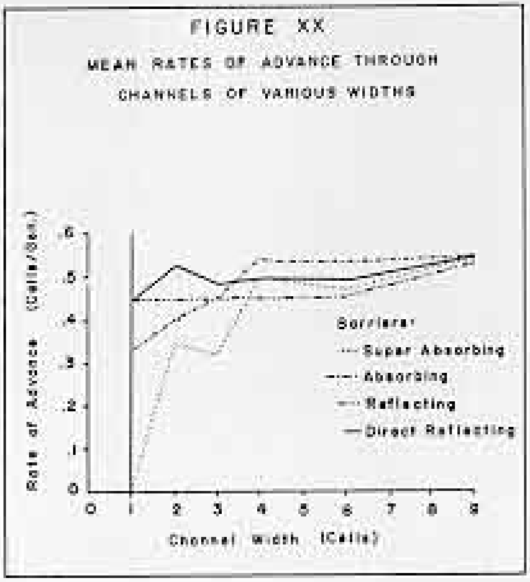


FIGURE XX
MEAN RATES OF ADVANCE THROUGH CHANNELS OF VARIOUS WIDTHS

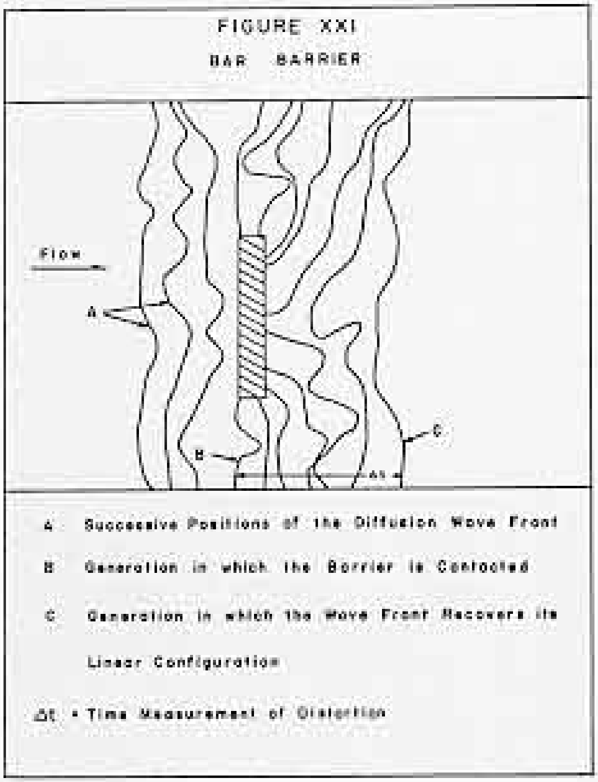


FIGURE XXI
BAR BARRIER

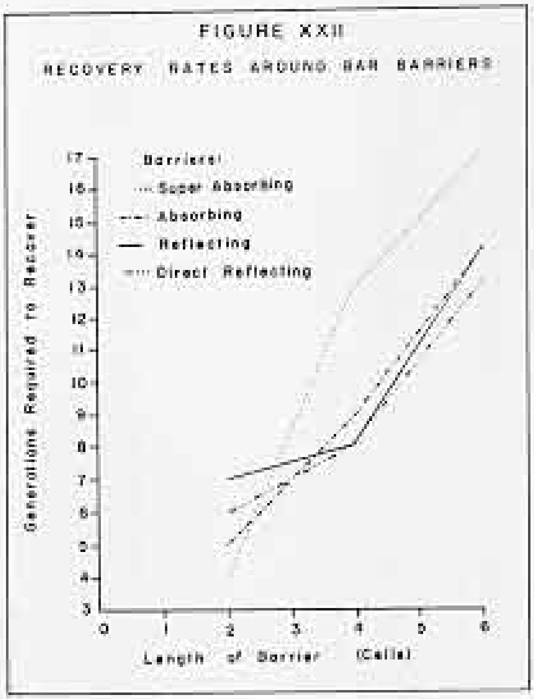


FIGURE XXII
RECOVERY RATES AROUND BAR BARRIERS

first encounters the barrier (Figure XXI - b), and the generation in which the wave recovers its linear form after passing the barrier (Figure XXI - c). Figure XXI shows an actual pattern of diffusion around a barrier.

The spatial distortions caused by all members of this group are alike - varying only in a matter of degree-- and as a consequence, the various functions and lengths of barriers may be readily compared. Since the recovery time (Δt) of the plane diffusion wave is also dependent upon the amount of its distortion, Δt may be used as a measure of this distortion. (The units of Δt are computer generations--equal are arbitrary.)

The effects of four absorbing and reflecting barriers were tested on barriers of 2, 4, and 6 cells in length. Again as in Group I, there are certain expected effects. Figure XXII indicates that in general the longer the barrier the greater the recovery time (Δt) required by the diffusion wave (due to greater distortion). Also absorbing barriers introduce more distortion than the reflecting barriers.

For each length of barrier the functional types were ranked in order of decreasing recovery time (Δt) as shown in Table I. These ranks were summed for each type of barrier giving the results shown in Table II.

This result conforms to expectations as the absorbing barriers have a higher rank (lower totals) and a longer recovery time. However an examination of the data generated for barriers two cells in length shows the rank order completely reversed, producing a singular exception to the general pattern. This fact, although by itself of no great significance, will be discussed further in connection with other unexpected deviations.

TABLE I

Barrier Type	Length: 2 cells R.T.	2 cells R.O.	4 cells R.T.	4 cells R.O.	6 cells R.T.	6 cells R.O.
Super Absorbing	4	4	13	1	17	1
Absorbing	5	3	9	2	14	2.5
Reflecting	7	1	8	3.5	14	2.5
Direct Reflecting	6	2	8	3.5	13	4

R.T. -- Recovery Time in Generations R.O. -- Rank Order

TABLE II

Barrier Type	Sum of Ranks*
Super Absorbing	6.0
Absorbing	7.5
Reflecting	7.0
Direct Reflecting	10.5

* From Table I

Group III: Passage of a Linear Diffusion Wave Through an Opening in a Barrier Parallel to the Wave Front

This group represents a barrier situation of common occurrence, particularly in the physical sciences, and its common occurrence has led to the accumulation of a great deal of empirical information on this particular spatial restriction. Hence the general results here can be predicted with moderate precision. This is fortunate, since no good measure of the diffusion wave shape was found.

For this study the barriers were two cells in width, impermeable and the size of opening (x) varied from two to six cells. Figure XXIII illustrates the effect of a five cell wide barrier opening upon a linear diffusion wave.

From Figure XXIII it is seen that the resulting curvilinear form in the stages after passage is about as expected. It was further found that as the opening in the barrier became narrower, the shape of the wave front approached that created by a point source. Definite observed differences in the spatial pattern of the diffusion did appear when absorbing and reflecting barriers were contrasted, as shown in Figure XXIV.

Since the rate of advance in the original direction of the plane wave (as shown by the arrow) remained approximately the same, the differences in shape are due to the portions of the wave near the barrier (a and a'). There are no other factors to consider; the functional effects of the barriers must be responsible for the difference. To test this hypothesis, the distances traveled by the diffusion wave for reflecting and super absorbing barriers were compared in the cells immediately adjacent to the barriers. Figure XXV is a plot of distance against time (expressed in generations) for the two cases.

Root mean square calculations of regression lines (rate of advance of the diffusion wave along the barrier in the region of a and a') gave slopes of 0.56 for the reflecting barrier and 0.39 for the super absorb-

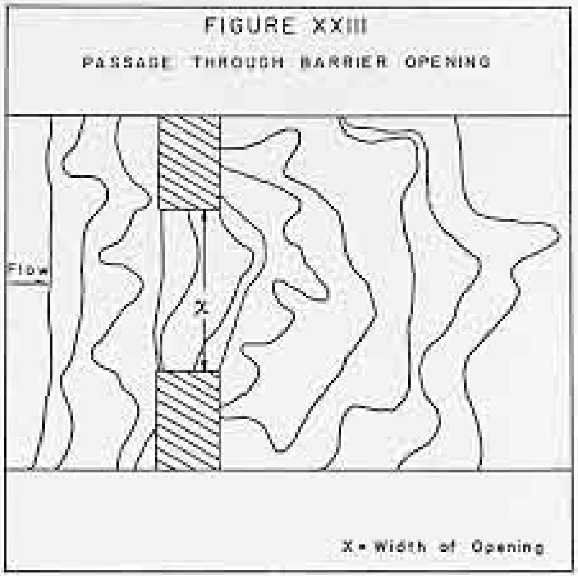


FIGURE XXIII
PASSAGE THROUGH BARRIER OPENING

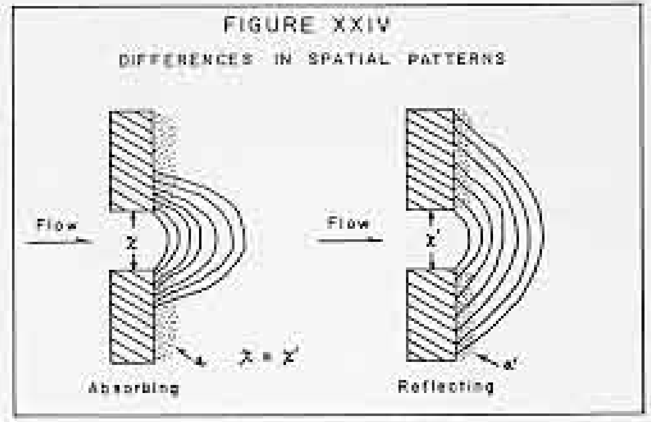


FIGURE XXIV
DIFFERENCES IN SPATIAL PATTERNS

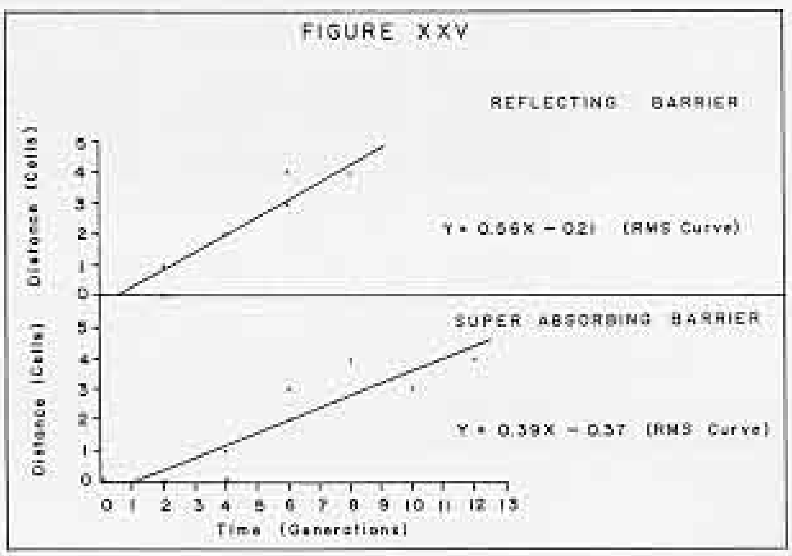


FIGURE XXV

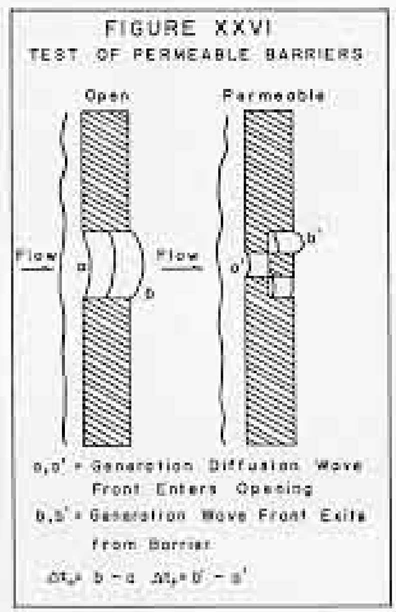


FIGURE XXVI
TEST OF PERMEABLE BARRIERS

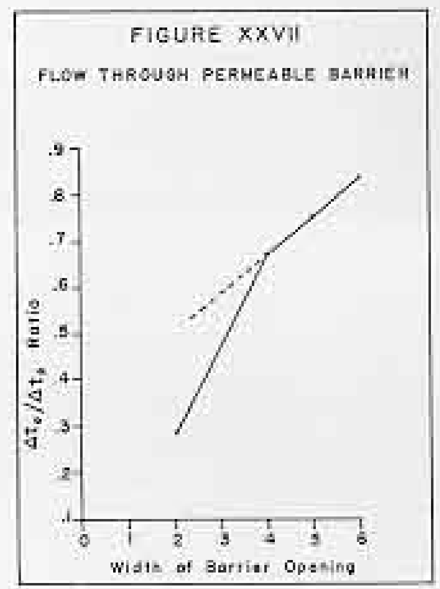


FIGURE XXVII
FLOW THROUGH PERMEABLE BARRIER

ing barrier. When these rates of advance are compared with Figure XX it is seen that the rate of advance for the reflecting barrier is approximately that of an unobstructed diffusion wave, while that for the super absorbing barrier is significantly smaller. This substantiates by measurement a conclusion which is indicated by the functions of the barriers - that there is friction between the absorbing barrier and the passing wave of diffusion which acts as a drag upon the wave and distorts its shape.

Group IV: Permeable Barriers

This group was measured in conjunction with the barrier configurations of Group III. Openings of 2, 4, 5, and 6 cells were tested using a series of super absorbing barriers. The permeable barriers were approximated by the checkerboard configuration of Figure XVII.

There was some difficulty in finding a suitable measure for this group. Although not entirely satisfactory, the best measure was to compare the time required for the diffusion wave front to pass from one side of the barrier to the other in permeable and open barriers. The time difference was expressed in generations (Figure XXVI).

The time difference for each size of barrier opening was expressed as a ratio $\Delta t_o/\Delta t_p$ (See Figure XXVI), which was then plotted against the width of barrier opening in Figure XXVII.

If the points are connected with straight line segments there appears to be a definite regularity among the upper three. This would seem to indicate that the longer the permeable barrier the greater the rate of advance of the diffusion wave through it. In actuality, Figure XXVII shows only that the longer the barrier the greater is the probability that the diffusion wave will pass through it at some point with the same rate of advance as if there were no barrier there. But this conclusion is even more unsatisfactory in terms of the objective, for little has been shown about the real effect of a permeable barrier. Furthermore the shape of the diffusion wave was too often determined by only the point at which the wave first passed through the barrier. Therefore this particular spatial approximation of the barrier function of permeability does not seem to be a particularly fruitful line of inquiry.

Figure XXVII does indicate something of importance, however. The ratio $\Delta t_o / \Delta t_p$ of the barrier opening of two cells does not follow the general trend of the other points. If there is a relationship between the ratio and the width of barrier opening, then it may be a non-linear function. There even may be more than one relationship involved.

Results and Implications

Results

The most obvious feature of the combined results is the dominant effect of shape in the diffusion pattern. This response to shape is not surprising, and it is one that is readily observable in the physical sciences - particularly in the flow of fluids. However there are significant variations in this general pattern which are generated by the functional effect of the different barriers. These are as follows:

Reaction to Function. On the basis of functional effects, the absorbing and reflecting barriers are naturally expected to display certain differences. Figure XXV illustrates such a difference. However, referring to Figure XX it is seen that the difference is due to only the absorbing barriers performing as expected. The absence of any noticeable reflecting barrier effect upon the wave front advance is at first surprising, but is actually due to the operation of the computer program. Since only probabilities within the floating grid are changed and not the boundaries, there is little spatial effect by these reflecting barrier functions.

Persistent Irregularity in Data. In considering three of the investigated groups there is found a curious pattern of irregularity. (See Figures XX, XXII, and XXVII). Between the values of two and four cells, there appears either a definite change in slope or in the case of a multi-curve graph (Figure XXII), a reversed order. This is especially significant in that this irregularity is found among widely different barrier configurations measured by different methods. It was concluded that the anomaly was most likely related to the floating grid since the dimensions of the grid (three cells square) fall exactly in the center of the critical area of change (two - four cells).

If indeed, the anomaly can be related only to the grid, then there must be a causal relationship. Such a relationship might be structured as follows:

Let the floating grid be represented by a circle with a diameter of three arbitrary units. When the circle encounters parallel barriers of Group I as shown in Figure XXVIII the spatial relationship is more apparent. Part 'a' shows that though the circle may touch a barrier it is not necessarily affected by it since its movement is really not restricted. When two sides of the circle come into contact with the barriers ('b') restriction begins, for movement is then curtailed in the direction perpendicular to the flow vector. Part 'c' represents an even more drastic curtailment of movement. The absorbing and super absorbing barriers of Figure XX show exactly this effect. (The reflecting barriers do not show the effect for the reasons mentioned earlier).

In a related manner, the expected functional order of Group II (See Figure XXII) breaks down completely at barrier lengths smaller than the floating grid diameter. Figure XXIX illustrates how the ratio of grid diameter to barrier length is a critical factor in determining the effect of that barrier.

Figure XXVII of Group IV further indicates a much greater spatial interference for barrier openings of less width than the grid diameter.

By implication and logical extension of the above results, the following conclusions were reached:

1. A circle (representing the probability field of an individual) is really only affected by the barriers with which it comes into contact. Further, the degree of influence or effect is proportional to the amount of contact between the circle and the barrier.

2. When contact is made with a barrier there is a spatial effect on the circle depending upon the function of the barrier. Figure XXX shows these different spatial effects of the absorbing and reflecting barriers.

If the area of contact is 'x' in part 'a' then for an absorbing barrier ('b') the area of the circle is reduced by that amount. The reduction in area however, is generalized as a reduction in the radius of the circle.

For the reflecting barrier ('c') the effect is not a reduction

FIGURE XXVIII

UNIT CIRCLE ENCOUNTERING PARALLEL BARRIERS

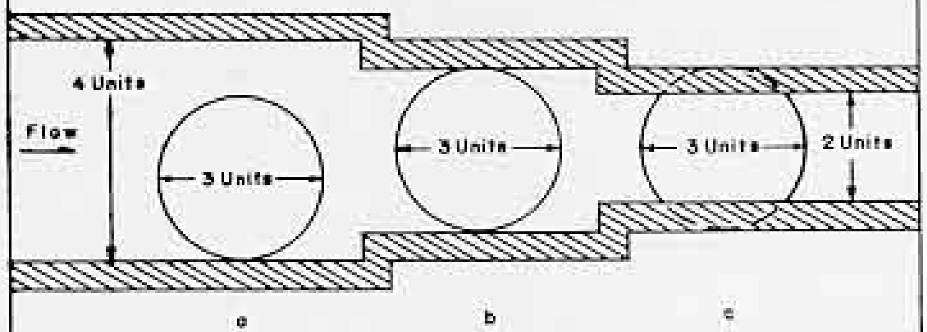


FIGURE XXIX

UNIT CIRCLE ENCOUNTERING BAR BARRIER

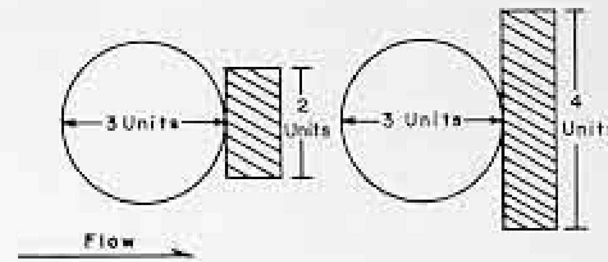
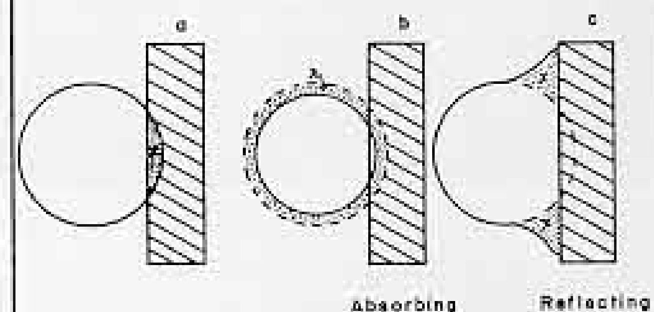


FIGURE XXX

SPATIAL EFFECT OF BARRIERS ON UNIT CIRCLE



but a distortion of area with the center of the circle remaining stationary.

3. As far as permeability is concerned, the results here offer few definitive guidelines. However since retardation of the diffusion wave front was noted, there are several logical spatial representations:

a/ The floating grid diameter is reduced while passing through the barrier.

b/ The grid size is untouched but the barrier is enlarged to the proper effective distance via a map transformation.

APPLICATION OF RESULTS

There is almost an infinite variety of barrier situations to which these results may be applied. A few will be given here for the purpose of illustration. First however, the abstract processes must be put into a more concrete form.

The floating grid, it must be remembered, is based upon the mean information field of an individual. It is further related to real situations by means of the concept of uncertainty of location. Rarely in the dynamics of human events is the individual stationary. It is not reasonable, then, to consider him as a point location in a dynamic simulation. A more accurate representation is a closed curve which delineates the area of the individual's most probable occurrence. Within this areal domain however, there are important distinctions in the probability distribution. For a traveler or nomad there is almost an equal probability of his being at any particular point within the area. Bur for an individual with a fixed place of residence there is a much greater probability of his being near that residence. For convenience the circle is adopted as the closed curve representing the individual.

The first illustration is the use of the results to simulate travel rates by land (in the United States in the early 19th. century) by using only the gross physical features. Figure XXXI shows various time distances from New York City using very few and very simple assumptions on rates of travel and permeability. For the model these assumptions consist of dividing the area under consideration into three regions, each of which is considered homogeneous with respect to rate of travel. The regions are: 1) An area of unrestricted movement, corresponding roughly to the Atlantic coastal plain, the piedmont, and associated valleys. 2) An absorbing but permeable barrier which reduces the energy (rate of travel) to one-third that of the unrestricted region. This corresponds to the Appalachian System. 3) An impermeable, reflecting barrier corresponding to the Adirondack Mountains.

Figure XXXII is a map of actual recorded rates of travel in 1800. The shape of the theoretical model agrees well with the actual although there are some distinct differences in the time rates to reach the same point. The importance of this illustration however, is that this very rudimentary simulation of a functional barrier can produce reasonable spatial results.

Barriers are also very important in the spatial configuration of cities. Probably the most efficient area for a city is a circle, but functional barriers often transform this shape during the process of growth. A common case is a river of sufficient width so as to hinder easy passage across it. In the settlement and growth of a city, a river of this magnitude may be regarded as a reflecting barrier (See Figure XXX-c).

and for interchange across it, an absorbing but permeable barrier. Neglecting changes in modes of transportation the generalized resulting pattern of a city is shown by Figure XXXIII.

For comparison some diagrams of actual cities on rivers are shown in Figure XXXIV. The prevalence of cities similar to this all over the world (Bay City, Michigan; Düsseldorf, Germany; Sault Ste. Marie, Canada- to mention just a few) affords an excellent starting place for a solid study of city shapes and the response of the growth mechanisms to barriers.

CONCLUSIONS

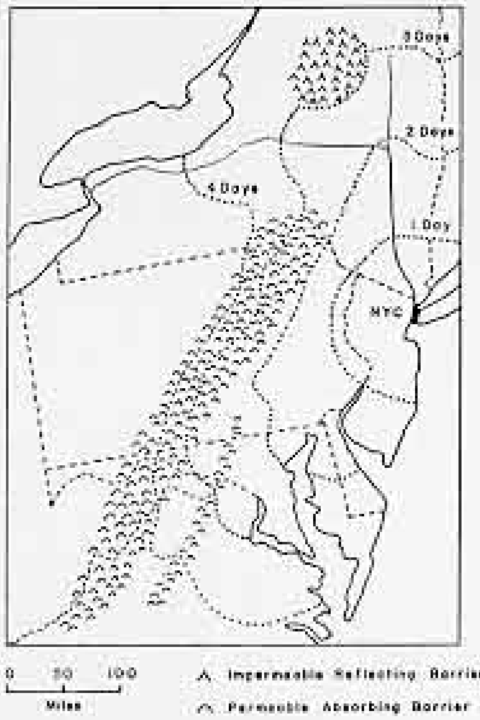
The avowed aim of this study has not been completely fulfilled. There are perhaps more unanswered questions about the spatial nature of barriers than before. However several significant points have emerged. The first is the outgrowth of considering a barrier as a function of a process or activity, and revolves about the idea that the range of a barrier may be limited by the range of an individual interactor or unit of the activity involved. That is, the barrier is only effective when it is in spatial conflict with the closed curve describing the dynamic location of an individual unit. Although this may scarcely be regarded as established or proven, the results in this study give strong evidence in support of it.

The second point is the significance of shape in the measurement of barrier effects. No common measure could be found which was satisfactory for all the barrier shapes tested. Instead each shape necessitated a different form of measurement ranging from rate of travel - a linear measure, to distortion of a plane wave - an areal

FIGURE XXXI

THEORETICAL TRAVEL TIME FROM NEW YORK CITY ABOUT 1800

Mode of Travel — Stagecoach

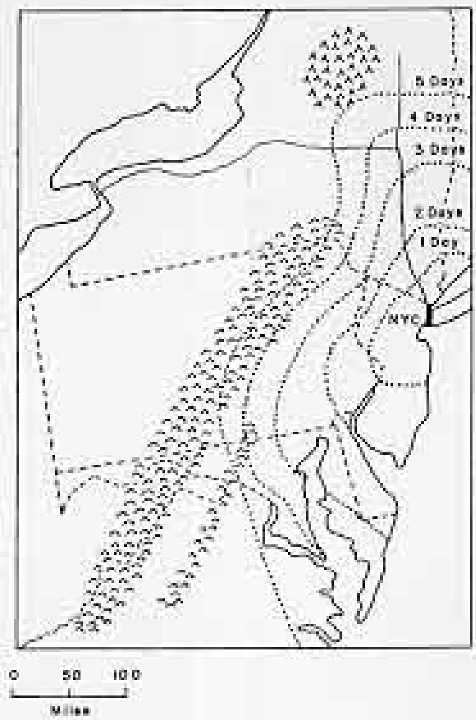


Barrier Absorbs 2/3 the Velocity of the Vehicles

FIGURE XXXII

RATES OF TRAVEL — 1800

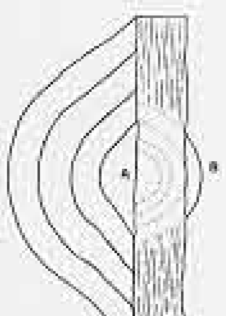
(Rate from New York City by Stage)



after Brown, Op. Cit., p101

FIGURE XXXIII

GENERALIZED CITY GROWTH PATTERN ON RIVER

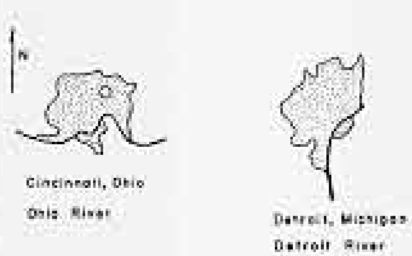


A — Start of City

B — Related Settlement

FIGURE XXXIV

DIAGRAMS OF RIVER CITIES



or two dimensional function. This indicates that for direct comparison of barriers, shape is a most important property. A further implication is of a hierarchy of barrier properties with shape being primary and functional effects being secondary.

The third point is that though different measures were required by different barrier configurations, the quality actually measured in each case was the spatial distortion of the advance of the diffusion wave front. In each case time was the principal element (rate of advance, recovery time, etc.) required to measure the diffusion. The implication is that for geographic problems of movement in space, time is an essential element. This is especially true for diffusion models for which time is both a regulating and a limiting factor. Time is therefore inextricably connected with space in all diffusion processes. The question now arises about the place of time in a steady state flow around barriers such as quotas in international trade. Since time is not an essential part of this flow, are the results gained from this diffusion model valid for such flow? It is suggested that this would be an interesting line of inquiry for further investigation.

A few primitive rules or ideas have been advanced in the study for dealing with spatial effects of barriers. For more exact and better definition of these concepts, work is needed on many facets: the effect of size changes in the floating grid, spatial distortion of non-linear diffusion wave fronts, specific combinations of barriers, interference caused by multiple diffusion, and many others. With refinements introduced by further investigation along the lines mentioned here, these notions will become useful tools in geographic research.

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APPENDIX A: Program Listing

The Monte Carlo model used in this study was programmed for the IBM 7090 computer at the University of Michigan. The program is written in MAD (Michigan Algorithmic Decoder); the reader who is unfamiliar with this language is referred to the MAD manual which is available from the University of Michigan Computing Center. The principal parts of the program are as follows:

Data Input

The population distribution, barrier structure, values of the floating grid (MIF), and the point of termination are read in and stored. In this process certain sequences must be preserved: The population must be read in before the barriers, and the storage locations for acceptors must be cleared to zero before the initial acceptors are read into the memory. Note that use is made of the simplified input-output option of MAD.

Simulation Initiation

This is the starting point for the two basic loops which make up the main portion of the program. The major loop (JUMP1 - LOOP1) centers the MIF sequentially on each cell in the simulation field. The inside loop (JUMP2 - LOOP2) allows each cell to transmit once per generation for each acceptor in that cell. The destination of each transmission is also determined here.

Reaction of the Individual Cell to a Transmission

Depending upon the population of the cell, the transmission encounters either a barrier or a cell with interacting members. (Cell Populations $\leqq$ zero indicate barriers).

Termination

At the end of each generation, a current status report is printed which shows the spatial distribution of acceptors by cells. The simulation terminates when the number of acceptors in a specified cell passes a predetermined level.

The subroutine RANDOM provides a means of generating random numbers which are uniformly distributed over the interval $0 \leqq x \leqq 1$. RANDOM employs the power residue method of random number generation, and the periodicity is $2^{35} - 1$.

```
        R SYSTEMATIC BARRIER STUDY. MONTE-CARLO SIMULATION.
        R TOTAL FIELD 18 BY 30 CELLS.  THE MEAN INFORMATION FIELD IS
        R A SQUARE GRID OF 9 CELLS.  THE GRID CENTER IS ON CELL(N) AS
        R (N) VARIES FROM 1 TO 540.  VALUES OF THE GRID ARE BASED
        R UPON HAGERSTRAND MODELS.

         PRINT COMMENT $1$
         INTEGER I,N,X,QTS,QTA,QTR,QTD,Z,IAX,SAB,ABSORB,REF,DREF
         DIMENSION POP(540),ACCEPT(540),RECEPT(540),Z(1),AT(1),QTS(1),
        1QTA(1),QTR(1),QTD(1),IAX(1),SAB(100),ABSORB(200),REF(100),
        2DREF(100),FOLK(1),FIELD(10),CELL(50)

        R DATA INPUT

        R FIELD SPECIFIES VALUES OF FLOATING GRID (OR MEAN INFORMATION
        R FIELD).

         THROUGH ALPHA, FOR I = 2, 1, I .G. 9
ALPHA    READ DATA FIELD(I)

        FOLK SPECIFIES POPULATION OF INTERACTING CELLS.

         READ DATA FOLK
         THROUGH SET1, FOR N = 1, 1, N .G. 540
         POP(N) = FOLK
SET1     ACCEPT(N) = 0.

        R THE FOLLOWING READS IN SUBSCRIPTS OF THE CELLS WHICH ARE TO
        R BE BARRIERS.  CARD FORMAT IS 16 INTEGER FIELDS OF 5 COLUMNS
        R EACH.

        R BARRIER              PROGRAM DESIGNATION     CELL POP VALUE
        RSUPER ABSORBING             SAB                      -3.
        RABSORBING                   ABSORB                   -2.
        RREFLECTING                  REF                      -1.
        RDIRECT REFLECTING           DREF                      0.

         READ DATA QTS
         READ FORMAT CARD, SAB(1)...SAB(QTS)
         VECTOR VALUES CARD = $16I5*$
         THROUGH BETA, FOR I = 1, 1, I .G. QTS
BETA     POP(SAB(I)) = -3.
         READ DATA QTA
         READ FORMAT CARD, ABSORB(1)...ABSORB(QTA)
         THROUGH GAMMA, FOR I = 1, 1, I .G.QTA
GAMMA    POP(ABSORB(I)) = -2.
         READ DATA QTR
         READ FORMAT CARD, REF(1)...REF(QTR)
         THROUGH DELTA, FOR I = 1, 1, I .G. QTR
DELTA    POP(REF(I)) = -1.
         READ DATA QTD
         READ FORMAT CARD, DREF(1)...DREF(QTD)
         THROUGH KAPPA, FOR I = 1, 1, I .G. QTD
KAPPA    POP(DREF(I)) = 0.

        R THE FOLLOWING READS IN THE INITIAL ACCEPTORS .

         READ DATA IAX
         THROUGH SET2, FOR I = 1, 1, I .G. IAX
```

```
SET2      READ DATA CELL(1), ACCEPT(N)

          R Z = SUBSCRIPT OF CELL TERMINATING THE PROGRAM.
          R AT = VALUE OF ACCEPTORS IN CELL(Z) AT WHICH PROGRAM ENDS.

           READ DATA Z, AT

          R INPUT DATA VERIFICATION.  PRINTS OUT 18 BY 30 CELL ARRAYS
          R SHOWING INITIAL ACCEPTORS AND BARRIERS.

           PRINT COMMENT $1  INITIAL ACCEPTORS$
           PRINT FORMAT OUTPUT, ACCEPT(1)...ACCEPT(540)
           V'S OUTPUT =$1H4,1F3.0,29F4.0/(1H0,1F3.0,29F4.0)*$
           PRINT COMMENT $1 POPULATION AND BARRIER DISTRIBUTION $
           PRINT COMMENT $0 POP OF -3 IS SUPER ABSORBING BARRIER $
           PRINT COMMENT $  POP OF -2 IS ABSORBING BARRIER $
           PRINT COMMENT $  POP OF -1 IS REFLECTING BARRIER $
           PRINT COMMENT $  POP OF 0 IS DIRECT REFLECTING BARRIER $
           PRINT FORMAT OUTPUT, POP(1)...POP(540)

          R MAIN PROGRAM

          R GEN = NUMBER OF GENERATIONS.
          R RECEPT = ACCEPTORS OF EACH CELL IN ANY ONE GENERATION.
          R ACCEPT = CUMULATIVE NO. OF ACCEPTORS IN EACH CELL.
          R Q = TRANSMITTERS IN CELL(N)

           RNO = 0.
           GEN = 0.
JUMP1      THROUGH NULL, FOR N = 1, 1, N .G. 540
NULL       RECEPT(N) = 0.
           THROUGH LOOP1, FOR N = 1, 1, N .G. 540
           WHENEVER POP(N).E.0..OR.POP(N).E.-3..OR.POP(N).E.-2..OR.
          1POP(N).E.-1..OR.ACCEPT(N).LE.0.
           TRANSFER TO LOOP1
           END OF CONDITIONAL
           Q = 0.

          R THE FOLLOWING SPECIFIES THE CELL OF THE MEAN INFORMATION
          R FIELD WHICH A TRANSMITTER ATTEMPTS TO CONTACT.

JUMP2      Y = RANDOM.(RNO)
           WHENEVER Y .GE. FIELD(9)
           TRANSFER TO GRID9
           OR WHENEVER Y .GE. FIELD(8)
           TRANSFER TO GRID8
           OR WHENEVER Y .GE. FIELD(7)
           TRANSFER TO GRID7
           OR WHENEVER Y .GE. FIELD(6)
           TRANSFER TO GRID6
           OR WHENEVER Y .GE. FIELD(5)
           TRANSFER TO GRID5
           OR WHENEVER Y .GE. FIELD(4)
           TRANSFER TO GRID4
           OR WHENEVER Y .GE. FIELD(3)
           TRANSFER TO GRID3
           OR WHENEVER Y .GE. FIELD(2)
           TRANSFER TO GRID2
           OTHERWISE
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          TRANSFER TO GRID1
          END OF CONDITIONAL

         R GRIDS(1-9) SPECIFY REACTION OF THE CELL CONTACTED BY THE
         R TRANSMITTER.  WITH A POP OF ZERO OR LESS. THE CELL ACTS AS
         R A BARRIER.  A POPULATION GREATER THAN ZERO INDICATES A CELL
         R WITH INTERACTORS WHICH MAY BE CONTACTED TO BECOME ACCEPTORS.
         R THE PROBABILITY OF TRANSMITTING TO AN UNCONTACTED INTER-
         R ACTOR OF A CELL DECREASES AS THE RATIO ACCEPTORS/CELL POP
         R OF THAT CELL INCREASES.

GRID1     WHENEVER POP(N-31) .E. -3.
          ACCEPT(N) = ACCEPT(N) - 1.
          TRANSFER TO LOOP2
          OR WHENEVER POP(N-31) .E. -2.
          TRANSFER TO LOOP2
          OR WHENEVER POP(N-31) .E. -1.
          TRANSFER TO JUMP2
          OR WHENEVER POP(N-31) .E. 0.
          Y = RANDOM.(RNO)
          WHENEVER Y .GE. 0.500 .AND. POP(N-30) .G. 0.
          TRANSFER TO GRID2
          OTHERWISE
          TRANSFER TO GRID4
          END OF CONDITIONAL
          OTHERWISE
          CONTINUE
          END OF CONDITIONAL
          QUOT = ACCEPT(N-31) / POP(N-31)
          Y = RANDOM.(RNO)
          WHENEVER Y.GE.QUOT.AND.RECEPT(N-31)+ACCEPT(N-31).L.FOLK
          RECEPT(N-31) = RECEPT(N-31) + 1.
          END OF CONDITIONAL
          TRANSFER TO LOOP2
GRID2     WHENEVER POP(N-30) .E. -3.
          ACCEPT(N) = ACCEPT(N) - 1.
          TRANSFER TO LOOP2
          OR WHENEVER POP(N-30) .E. -2.
          TRANSFER TO LOOP2
          OR WHENEVER POP(N-30) .E. -1.
          TRANSFER TO JUMP2
          OR WHENEVER POP(N-30) .E. 0.
          TRANSFER TO GRID5
          OTHERWISE
          CONTINUE
          END OF CONDITIONAL
          QUOT = ACCEPT(N-30) / POP(N-30)
          Y = RANDOM.(RNO)
          WHENEVER Y.GE.QUOT.AND.RECEPT(N-30)+ACCEPT(N-30).L.FOLK
          RECEPT(N-30) = RECEPT(N-30) + 1.
          END OF CONDITIONAL
          TRANSFER TO LOOP2
GRID3     WHENEVER POP(N-29) .E. -3.
          ACCEPT(N) = ACCEPT(N) - 1.
          TRANSFER TO LOOP2
          OR WHENEVER POP(N-29) .E. -2.
          TRANSFER TO LOOP2
          OR WHENEVER POP(N-29) .E. -1.
          TRANSFER TO JUMP2
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           OR WHENEVER POP(N-29) .E. 0.
           Y = RANDOM.(RNO)
           WHENEVER Y.GE. 0.500 .AND.POP(N-30) .G. 0.
           TRANSFER TO GRID2
           OTHERWISE
           TRANSFER TO GRID6
           END OF CONDITIONAL
           OTHERWISE
           CONTINUE
           END OF CONDITIONAL
           Y = RANDOM.(RNO)
           QUOT = ACCEPT(N-29) / POP(N-29)
           WHENEVER Y.GE.QUOT.AND.RECEPT(N-29)+ACCEPT(N-29).L.FOLK
           RECEPT(N-29) = RECEPT(N-29) + 1.
           END OF CONDITIONAL
           TRANSFER TO LOOP2
GRID4      WHENEVER POP(N-1) .E. -3.
           ACCEPT(N) = ACCEPT(N) - 1.
           TRANSFER TO LOOP2
           OR WHENEVER POP(N-1) .E. -2.
           TRANSFER TO LOOP2
           OR WHENEVER POP(N-1) .E. -1.
           TRANSFER TO JUMP2
           OR WHENEVER POP(N-1) .E. 0.
           TRANSFER TO GRID5
           OTHERWISE
           CONTINUE
           END OF CONDITIONAL
           Y = RANDOM.(RNO)
           QUOT = ACCEPT(N-1) / POP(N-1)
           WHENEVER Y.GE.0.500.AND.RECEPT(N-1)+ACCEPT(N-1).L.FOLK
           RECEPT(N-1) = RECEPT(N-1) + 1.
           END OF CONDITIONAL
           TRANSFER TO LOOP2
GRID5      Y = RANDOM.(RNO)
           QUOT = ACCEPT(N) / POP(N)
           WHENEVER Y.GE.QUOT.AND.RECEPT(N)+ACCEPT(N).L.FOLK
           RECEPT(N) = RECEPT(N) + 1.
           END OF CONDITIONAL
           TRANSFER TO LOOP2
GRID6      WHENEVER POP(N+1).E. -3.
           ACCEPT(N) = ACCEPT(N) - 1.
           TRANSFER TO LOOP2
           OR WHENEVER POP(N+1) .E. -2.
           TRANSFER TO LOOP2
           OR WHENEVER POP(N+1) .E. -1.
           TRANSFER TO JUMP2
           OR WHENEVER POP(N+1) .E. 0.
           TRANSFER TO GRID5
           OTHERWISE
           CONTINUE
           END OF CONDITIONAL
           Y = RANDOM.(RNO)
           QUOT = ACCEPT(N+1) / POP(N+1)
           WHENEVER Y.GE.QUOT.AND.RECEPT(N+1)+ACCEPT(N+1).L.FOLK
           RECEPT(N+1) = RECEPT(N+1) + 1.
           END OF CONDITIONAL
           TRANSFER TO LOOP2
GRID7      WHENEVER POP(N+29) .E. -3.
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          ACCEPT(N) = ACCEPT(N) - 1.
          TRANSFER TO LOOP2
          OR WHENEVER POP(N+29) .E. -2.
          TRANSFER TO LOOP2
          OR WHENEVER POP(N+29) .E. -1.
          TRANSFER TO JUMP2
          OR WHENEVER POP(N+29) .E. 0.
          Y = RANDOM.(RNO)
          WHENEVER Y.GE.0.500 .AND.POP(N-1) .G. 0.
          TRANSFER TO GRID4
          OTHERWISE
          TRANSFER TO GRID8
          END OF CONDITIONAL
          OTHERWISE
          CONTINUE
          END OF CONDITIONAL
          Y = RANDOM.(RNO)
          QUOT = ACCEPT(N+29) / POP(N+29)
          WHENEVER Y.GE.QUOT.AND.RECEPT(N+29)+ACCEPT(N+29).L.FOLK
          RECEPT(N+29) = RECEPT(N+29) +1.
          END OF CONDITIONAL
          TRANSFER TO LOOP2
GRID8     WHENEVER POP(N+30) .E. -3.
          ACCEPT(N) = ACCEPT(N) - 1.
          TRANSFER TO LOOP2
          OR WHENEVER    POP(N+30) .E. -2.
          TRANSFER TO LOOP2
          OR WHENEVER POP(N+30) .E. -1.
          TRANSFER TO JUMP2
          OR WHENEVER POP(N+30) .E. 0.
          TRANSFER TO GRID5
          OTHERWISE
          CONTINUE
          END OF CONDITIONAL
          Y = RANDOM.(RNO)
          QUOT = ACCEPT(N+30) / POP(N+30)
          WHENEVER Y.GE.QUOT.AND.RECEPT(N+30)+ACCEPT(N+30).L.FOLK
          RECEPT(N+30) = RECEPT(N+30) + 1.
          END OF CONDITIONAL
          TRANSFER TO LOOP2
GRID9     WHENEVER POP(N+31) .E. -3.
          ACCEPT(N) = ACCEPT(N) - 1.
          TRANSFER TO LOOP2
          OR WHENEVER POP(N+31) .E. -2.
          TRANSFER TO LOOP2
          OR WHENEVER POP(N+31) .E. -1.
          TRANSFER TO JUMP2
          OR WHENEVER POP(N+31) .E. 0.
          Y = RANDOM.(RNO)
          WHENEVER Y.GE. 0.500 .AND.POP(N+30) .G. 0.
          TRANSFER TO GRID8
          OTHERWISE
          TRANSFER TO GRID6
          END OF CONDITIONAL
          OTHERWISE
          CONTINUE
          END OF CONDITIONAL
          Y = RANDOM.(RNO)
          QUOT = ACCEPT(N+31) / POP(N+31)
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          WHENEVER Y.GE.QUOT.AND.RECEPT(N+31)+ACCEPT(N+31).L.FOLK
          RECEPT(N+31) = RECEPT(N+31) + 1.
          END OF CONDITIONAL

         R TERMINATION AND OUTPUT

         R THE FOLLOWING ALLOWS EACH CELL TO TRANSMIT ONLY ONCE FOR
         R EACH ACCEPTOR IN EACH GENERATION.

LOOP2     Q = Q + 1.
          WHENEVER Q .L. ACCEPT(N)
          TRANSFER TO JUMP2
          END OF CONDITIONAL
LOOP1     CONTINUE
          GEN = GEN + 1.
          THROUGH ADD, FOR N = 1, 1, N .G. 540
ADD       ACCEPT(N) = ACCEPT(N) + RECEPT(N)

         R THE FOLLOWING SPECIFIES FORM OF THE OUTPUT.  THE ACCEPTORS
         R ( OF EACH OF THE 540 CELLS) ARE PRINTED OUT IN AN 18 BY 30
         R CELL ARRAY WITH THE GENERATION NUMBER PRINTED ABOVE THE
         R ARRAY.

          PRINT FORMAT COUNT, GEN
          V'S COUNT = $1H1, H*  GENERATION NUMBER =  *, F3.0 *$
          PRINT FORMAT OUTPUT, ACCEPT(1)...ACCEPT(540)

         R THE FOLLOWING CARDS TEST FOR THE CONDITION WHICH TERMINATES
         R THE PROGRAM.

          WHENEVER ACCEPT(Z) .L. AT
          TRANSFER TO JUMP1
          END OF CONDITIONAL
          END OF PROGRAM
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