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THE PHILOSOPHY OF MAPS

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2. Marc Anderson, "A Working Bibliography of Mathematical Geography," September, 1963.

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THE PHILOSOPHY OF MAPS

INTRODUCTION

The tone of the papers presented is extremely disjoint. The only commonality is the subject - philosophical questions arising from maps - and in some cases the authors did not see their comments as especially philosophical. The editor had to underscore the philosophical unity that impressed him and led to the selection of material. The editor did not feel that a review article would do since much of the material is original, and perhaps more deeply, who is to say that the various points of view might not all be fruitful and should be pursued simultaneously? Commalities cannot be forced by the editor dishonestly, that is, before he sees them.

This is a collection of papers written by seven geographers and a philosopher in which from the study of maps philosophical questions arise. Previous philosophical questions of geographers were those of the theory of knowledge and the philosophers were read from a great distance. Kurt Schaefer's long conversations with Gustav Bergmann, which questioned the epistomological exceptionalism of the science of geography, was the initial and the best representation of this dialogue.

But the tables have turned for now the geographers are raising questions to the philosophers. We no longer feel ourselves to be humble students before sages but slightly annoyed critics of philosophy's neglect of not just geography but all the visible, literal fields of knowledge including graphics and geometry. Now short of pugnacity, but past persistance, we return to philosophy in a new wave of interest with questions that we suspect may provide a challenge for philosophy today. We have changed the subject, too, from ground where we were the uninitiate--philosophy of science (and how we used to pour over Cohen and Nagel) to ground that is clearly our home territory--The Map. The older geographers, those that were horrified at our initial furious attack on maps as inferior to mathematical functionals, had substantial position on their side. But they were and are so religious about their commitment to the map--complete with religious persecutions for those that did not genuflect before the fundamentalist map thumpers--that they practically compelled our revolt. We were provoked. Why did Hartshorne's excellent universal methodology ignore maps? Why did the cartographers ignore methodology? All those Leroy Pens and Zipatone and never a philosophical question. What could this mean? Certainly nothing flattering.

The first paper is a short one by Waldo Tobler tying the geographer's traditional concern with "projections" into the main stream of "transformations." William Pattison is next. Pattison brought his notion of the literalness of geographic concepts to the Dedham Conference some years back which William Warntz and I also attended. This started both Warntz and myself to thinking and Warntz came up with the Venn diagram application at the conference itself, which he has developed over the years into the third paper. The fourth contribution is the editor's thinking about spatial prediction. Stephen Toulmin's excerpted article follows. Toulmin is the only philosopher who is quoted. Few philosophers have initiated an interest in geography, Kant and a handful, and no one other than Toulmin has shown an interest in maps. After Toulmin, comes Roland Martin, the interesting geographic mystic and then still another report by the editor. The final two papers are by Andrew Karlin and Richard Guyot, students. All bibliographic references are collected at the end. This is the complete collection of philosophical material on maps that has come to my attention.

(W. Bunge)

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What, if anything, is ultimately invarient in science? The impact of Waldo Tobler's paper is two-fold. To help geographers used to thinking in our ancient terms of "map projections" to broaden their scope, and to imply a question, "What portions of geography are not transformations?" Such a question leaves a classically trained geographer shaken. What a wierd world for geographers raised in a "factual" geography suffocated in parameters such as the highest point in the State of Wisconsin. What remains invariant that is not trivial? Perhaps philosophers will find this question dull after their earlier experiences with physics but if even geography is transforming itself into transformations, is this the ultimate fate of all scientific knowledge? Everything a mapping? The breadth of mapping might then expand all the philosophical discussion to follow, such as spatial prediction of mapable temporal phenomena.

Transformations

Waldo Tobler

A map projection can be considered a transformation applied to spatial point coordinates. The emphasis in the present work is on this class of geometrical transformations. It would be misleading to imply, by omission, that there are no additional types of transformations of interest to geographers. Only two elementary examples are presented here, but these suffice to introduce briefly some additional transformations.

As a first example consider the entire cartographic process. Geographical information is supplied to the cartographer and he then transforms this into a geographical map. Clearly this can be considered a signal processing operation, much like radio, and can be represented diagrammatically as

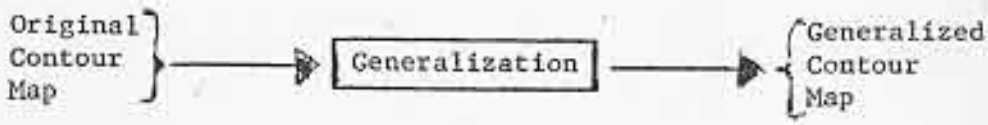
$$\text{input} \longrightarrow \boxed{\text{Transformation}} \longrightarrow \text{output}$$

The basic assumption is that the system is of high fidelity; that is, the map user expects that he can use the geographical map as an adequate representation of the input description of the environment. Alternately stated, the inverse of the cartographic transformation is map reading. In practice a map is never a complete representation of the environment. The electrical engineer would inquire about the transfer function characteristics of the signal processing system (the cartographer). Many of the detailed steps in the cartographic process can also be considered within this general framework. Map projection conversions are a particular case. Another step occurs in map generalization. This is worth examination in some detail.

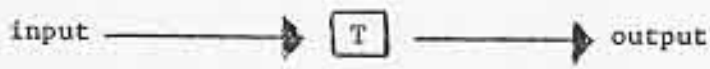
Let G denote an n by n matrix of topographical elevations taken at equally spaced geographical intervals. This matrix can be contoured using bivariate interpolation to produce a topographical map. Let S be an n by n smoothing matrix with entries as follows:

$$S = \begin{matrix} 3/4 & 1/4 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 1/4 & 1/2 & 1/4 & 0 & 0 & 0 & & 0 & 0 & 0 \\ 0 & 1/4 & 1/2 & 1/4 & 0 & 0 & & 0 & 0 & 0 \\ 0 & 0 & 1/4 & 1/2 & 1/4 & 0 & & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 1/2 & 1/4 & & 0 & 0 & 0 \\ \vdots & & & & & & & & & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & & 1/2 & 1/4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & & 1/4 & 1/2 & 1/4 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1/4 & 3/4 \end{matrix}$$

A generalization of the topographical map can be obtained by contouring the result of G pre- and post-multiplied by S, i.e., $G^* = S\,G\,S$. Diagrammatically:



which is equivalent to



The points of interest are that the inverse ($G = S^{-1}\ G^*\ S^{-1}$) exists, and that the transformation can be put in series ($G^{**} = S^2GS^2$, etc.). In other words, generalized maps can be further generalized, and can be ungeneralized.

Hagerstrand's theoretical model of the geographical spread of phenomena provides a second example. Here the transformation consists of transistion probabilities which map one state matrix, to use systems terminology, onto another state matrix. The transformation in this instance is stochastic and generally does not have an inverse. The point is that this example, like that preceeding it, does not disturb the geometrical relationships, as do map projections.

X X X X X XXX X X X X X X X X

If reality is not real, what is? Geographers have a sturdy tradition of climbing mountains and exploring distant places. William Pattison somehow

strikes that mood. By George, when we geographers say something we really mean it! The emotional impression is the exact opposite of Tobler's lead article. What could be realer than geography?

The Literalness of Spatial Thought

William Pattison

It is usual for practically everyone to organize his experiences spatially, by thinking of events according to their "whereness," or position. By structuring our experiences in this fashion we create for ourselves what may be called a spatial frame of reference. In contemplating the framework, one notices first the remarkable fact that all of its component concepts, starting with position, are to be taken literally. According to strictly spatial rules, a position is a location in space, not a way of designating a job assignment. Similarly, left and right are truly physical orientations, not political tendencies; and up and down are directions in which one can physically move or point, rather than figures of speech for use in describing pretended movement on "the ladder of success," for example. Likewise, distances are to be understood in terms of such units as inches and miles; which is to say that the concept of social distance, for example--valuable though it may be in appropriate frames of reference--has no validity in the context now under discussion.

The literalism of spatial thinking is difficult for many people, especially those in academic life, to acknowledge as legitimate, habituated as they are to using spatial terms almost exclusively in a figurative or metaphorical manner. Academic discourse, perhaps most notably in the humanities and the social sciences, relies heavily upon references to such motions as rising and falling, arriving and departing, emerging and penetrating, without "really meaning" what is said.

The discoursers speak of approaching a brink, of crossing a boundary, of fixing an aim, of reserving an area, or of locating an interface, while they

remain entirely in a world of metaphorical usage. They often require almost forcible reminding that there is a prototype world of sense perception in which all of these expressions have a literal signification--and to which the figurative usages themselves must be ultimately referred, for clarification of meanings.

Among essayists and other representatives of belles lettres, acceptance of spatial thinking is not uncommon. As keen an appreciation of its distinctiveness as could be wished for has appeared in the writings of the British author Freya Stark:

> Geologists may see their years in terms of time; musicians in some echo that dies in the air; the poet strains language beyond the bounds of telling;...but to us, who delight in maps, the idea of life inclines to be spatial--we see it moving point to point, like a road if we are inclined to attribute its shaping to men, like a river, if we have more feeling for the unexpectedness of nature.

To be sure, even here a strong suggestion of metaphorical intent is apparent, but happily the author is unambiguous in her allusion to maps, for the map is the expression par excellance of the kind of literal thought that we are seeking to understand.

X X X X X X X X X X X X X X X

If even point-set has literal meaning, how abstract are abstractions? Warntz, as usual, is dramatic. As Warntz is more than aware, if distances are measured in cost terms or some other unit, the earth is far from spherical, and therefore the variance of locations is considerable. The antipodal point of England for Mackinder in the proper geodesic coordinate system was not New Zealand, hardly anywhere near "half way around the world" but the heartland of Eurasia. Nordbeck on his visit at Wayne mentioned some work on locational variance that he and Hagerstrand were working on. The globe is as lumpy as a potato. It is not anywhere close to being round nor did Magellan go around it, even conceptually.

William Pattison's literalness bears fruit. What is considered more abstract by logicians-mathematicians or more fundamental to their thinking than point-set? But Warntz makes his own case.

Some Elementary and Literal Notions About Geographical Regionalization and Extended Venn Diagrams

William Warntz

Part I - Geographical Regionalization

All that we have to do, apparently, is to be willing to admit the feasibility of recognizing places on the earth's surface as though they could be elements in sets capable of assignment to variously defined classes. We wish to consider how this is related to regionalization as the geographer recognizes it.

Having done this, we are then in a position to employ graphical methods (cartographical for geographers) to show the results of our regionalizations as we perform the various operations possible such as union, intersection, and so on upon the sets involved. Maps showing regional classification can thus be regarded as logic diagrams. Mapping of sets is a general mathematical concept. Geographical mapping is merely a special case of this.

Consider the sets of places on the earth's surface as necessarily having spatial properties (location--absolute and relative) and capable of having non-spatial properties (climatic, geomorphologic, economic, political, linguistic, *ad infinitum*). How do we transform the former into the latter in intelligent, systematic, meaningful and useful ways?

First let us note (following Gardner, 1958) that a logic diagram may be "a two-dimensional geometric figure with spatial relations that are isomorphic with the structure of a logical statement."

Gardner adds that,

> "These spatial relations are usually (italics mine) of a topologic character, which is not surprising in view of the fact that logic relations are primitive relations underlying all deductive reasoning and topological properties are, in a sense, the most fundamental properties of spatial structures. Logic diagrams stand in the same relation to logical algebras as the graphs of curves (maps of areas) stand in relation to their algebraic formulas; they are simply other ways of symbolizing the same basic structure."

What happens if we say that sets consist of places on the earth's surface as elements and that all geometric properties and locational relationships, however transformed, are to be recognized in our manipulations? We wish to go explicitly from diagrams for non-spatial sets to geographical maps for spatial sets. The inverse of this, i.e., maps to logic diagrams, was recognized very early by the Harvard-educated (Class of 1859) logician and philosopher, Charles Sanders Peirce (1839-1914). At a time when specialization was taking command, Peirce remained universal. In an empirical age and place, he was a rationalist and a theorist. He stressed the role of theory building and how data are to be defined by where they fit into a theoretical structure. Peirce has been called the unexpected, exotic culmination of a physical geography tradition in the United States. I suggest that he be regarded as the inevitable culmination of a physical geography tradition which he not only maintained but also extended beyond his fellows.

At a time when the Venn graphical method of logic diagrams was being perfected for non-spatial sets, Peirce was developing graphical methods for analyzing in detail the structure of all deductive reasoning; breaking structures into their elements and giving each of the elements its simplest, most iconic geometrical representation possible. He considered that in this way the mind could "see" the logical structure in "a fashion analogous to seeing a geographical area when you look at a map."

He wrote that maps put before us pictures of thought. He inferred that maps could be experimented upon in a manner similar to the way a scientist experiments with a structure in nature. By altering mappings in various ways we could discover new properties of the structure not previously suspected. A map may be a device for invention and discovery of new truths as well as an instrument for proving, preserving, and recording old ones.

Alas! Peirce, the philosopher, was not influential among succeeding

generations of geographers, or philosophers for that matter, and we find ourselves, at present, having to argue deductively from mathematical-logical mapping to geographical mapping rather than inductively the other way around. Much time has been wasted. The Schaefer-Bergmann (1953) cooperation has marked the rebirth.

Dividing the earth's surface, or parts of it, into meaningful regions of various kinds is a valid and useful endeavor. Many geographers have turned their attention to this problem and as Bunge (1962) has shown, geographers independently have rediscovered the entire logic of classification. This, as Bunge remarks, has been "no mean intellectual feat." Bunge has provided a table of vocabulary equivalences for the terminology in regionalization and in general classification.

It is true that every square inch on the earth's surface differs in its non-spatial properties in some way from every other square inch. No two places, however small, are exactly alike. To know about the phenomena in such minute detail is virtually impossible. Moreover, such unorganized detail would have limited use, and, as in science generally, a method of grouping similar elements, in this case places, is, for many purposes, essential. The classification of the earth's surface lies at the heart of regional geography. The number and characteristics of the geographical classes and regions, so defined, obviously, and of necessity, depend upon the nature and the purpose of the classificatory scheme employed, and thus may be expected to differ from purpose to purpose. In this respect the essence of regional classification is like the problem of classification in general based on the principle of dichotomy in all academic disciplines. There is, however, one factor always present in regional classifications and that is earth location. It is this factor and the concepts related to it, such as concentrated, dispersed, clustered, evenly distributed, contiguous, etc., that are essentially geographic. It can be shown that the

terminology and method developed in science generally for classification are relatable precisely by vocabulary equivalences to those developed independently in geography through the years when that factor that is peculiarly geographical is added, namely location of the earth's surface. This view as noted above has been stated explicitly by Bunge and has been reviewed by Greeg who also acknowledged the interests of Gilmour, Cline, Simpson, Hettner, Hartshorne, et al. Golledge, Amadeo, and Haggett have commented meaningfully upon the problem.

Whereas regionalization is akin to classification, it is a mistake to think that anyone versed in classification procedures could thereby construct useful geographical regions. In addition to handling the important geographical factor of location successfully, the regionalizer must make appropriate decisions as to what significant non-spatial features or differentiating characteristics are to be included as criteria for the classification.

An understanding of non-spatial processes is imperative to establish superior classifications for regionalization purposes, but the study of non-spatial processes for their own sake "rather than for the ultimate classification (uniform regions) would appear to be outside the province of geography." Others are better equipped to handle these non-spatial processes than geographers.

All regionalizations like all classifications are arbitrary. No perfectly rational classification of phenomena exists independent of the use to which the classes are to be put. Similarly, there can be no perfect regionalization independent of purpose and a valuable regional division for one purpose may be quite inadequate for another. Geographers now generally agree that a region is an intellectual construct and that the concept of a "region as a concrete unit object" is indefensible. Bunge puts it well when he states that in order to produce "areal classifications of identical sorts no matter what differentiating characteristics are considered, it is necessary that there exist a perfect areal

correlation among all phenomena. This condition is not met on the earth's surface. As an alternative it is possible, but absurd, to insist on some one arbitrary areal classification as sacred and immutable."

A technical problem of any good classification (regionalization) is to establish the class limits (regional boundaries) in such a way that the differences among classes (regions) and the differences within classes (regions) are discernible and can be manipulated to desired ends.

From just these few considerations alone from among the number that Bunge has presented we find it easy to accept the idea that it is efficient and indeed necessary to consider "uniform" geographical regionalization as one kind of classification problem.

It seems possible now to go beyond this important beginning to additional concepts that may prove useful in our attempts to understand the essence of regionalization. In particular, it might be of benefit to make explicit the implicit link of geographical regionalization to Boolean algebra and Venn diagrams.

Boolean algebra is the algebra of logic, an abstract mathematical structure appearing in three different forms. It is the algebra of sets and to this we should specifically turn attention in our research.

To the mathematician or philosopher sets are considered to be just collections of objects. The objects which are the elements of the set may be material objects or purely conceptual intangible "objects" (or ideas). In fact, the basic theory does not need to specify. It is enough that a set is a collection of elements. The patterns of relations among sets is the concern. The most important idea involved is that any element in the "universe" is either a member of a given set or it is not. Now, this idea does not seem at first to hold much interest, but it turns out to be fruitful material for the human mind which generates an astonishing array of concepts and techniques in the presence of

this basic idea.

Thus, we see that set manipulation, like classification is a dichotomous procedure. And since all information can be coded and transmitted by employing a binary system, and since electrical circuits can be described in these terms, and since electronic computers operate on this basis, the possibilities for elaborate and complex data manipulation systems are great.

Scientists employing various set theory concepts in their various disciplines are interested in the patterns of relations among sets, but they also are inescapably concerned with the actual composition of sets, that is, with the classification of the "objects" of investigation into operational categories based on useful properties for the problem at hand. Thus, classification problem occurs at some stage in the development of every science and the recognition of classes is, in fact, crucial to its completeness.

Logic is not concerned with specific examples or unique individuals because generalization about individuality is per se a contradiction. Logic deals with members of a class. Scientists in their various disciplines may extend logical reasoning to individuals as members of a class, however.

As noted at the beginning of this paper, classification in geography is intimately linked with the concept of regions. Places classified according to their properties, when location is always included as one of the properties produce geographical regions varying according to the kinds of criteria and the limits employed including location.

To supplement the symbolic statements of the patterns of relations among sets when various operations are performed on them, graphical procedures are employed. The special diagrams used are called Venn diagrams after John Venn, a British mathematician of the nineteenth century who refined an earlier procedure of Euler. It is part of our purpose here to extend the use of such diagrams to the mapping of geographical regions by making use of properties already inherent

in Venn diagrams but as yet unutilized. We preface the following demonstration by noting again that in conventional set theory the graphical portrayal recognized the topological orderings among sets and portrays these. We shall attempt to utilize not only topological properties but various other geometric properties as well regardless of the statement by C. I. Lewis (1918) that for diagrammatic purposes only, the elements of the algebra of sets can be applied "to spatial entities such as continuous or discontinuous segments of a line, or to continuous or discontinuous regions in a plane" (italics mine). "The applications to regions in a plane gives the more workable diagrams, for obvious reasons. And since it is only (italics mine) for diagrammatic purposes that the application of the algebra of sets to spatial entities has any importance, we shall confine our attention to regions in a plane."

Not wishing to belabor a point, we nevertheless explicitly reject the notion that the spatial entities must be regarded, as above, only as analogies. We intend to apply spatial properties literally to real spatial distributions on the earth's surface when this may be done appropriately.

The conventional interpretation of the algebra of sets applies to classes taken "in extension." That is to say, a given letter symbol, say A, signifies not a class-concept, but rather the aggregate of all the objects, i.e., the entire membership, denoted by the class-concept. To say that A = B means not that the concept of class A may be regarded as a synonym for the concept of the class B, but that the classes A and B must consist of exactly the same members. They, therefore, are regarded as having the same extension.

Conventionally, it is also noted that classes may be divided into logical but not physical parts. We shall consider extension in a physical sense as well. Both number and areal extent are our objectives and while actual physical division on the earth's surface cannot be accomplished, we proceed as though this division can be portrayed graphically by maps. In other words, our consideration

of objects *in extenso* includes not only number but space.

We turn now to several purely existential assumptions:

(1) The existence of a universe class;

(2) The existence of a null class;

(3) The existence of more than one element. (Though this is not essential, it is usually assumed).

By a universe class, or simply universe, we mean a complete collection of all those, and only those, elements which belong within the realm of discourse, in a formal way. The universe is the "set of all sets" in which we are interested. Figure 1 gives us very simply the conventional Venn diagram of the universe of discourse.

Figure 1

The rectangular form is a mere convention. Other shapes, regular or irregular, could have been used. The important thing is that an intended universe is bounded and all of the points or unit areas within the bounded area of the plane may be regarded as elements in the universe. Not only does the shape of this universe have no meaning apart from convenience but also neither does the size of the diagram nor do distances, directions, and specific geometric locations within it have specific meanings. Neither does the location on the paper carry logical significance.

When various elements are arranged into various sets, nearness of elements to each other in the topological sense are important and "neighborhoods" must remain invariant though the geometrical locations of the various neighborhoods

is of no consequence. To repeat, the diagrams need only be regarded as having topological properties. These are what are important in the conventional Venn diagrams of the algebra of sets and we may do whatever we find convenient to do with the other geometrical properties of our diagrams to preserve the required topological properties in the patterns of relations among sets.

Now let us turn our attention again to geography. Here the universe is the whole surface of the earth. The earth's surface is the set of all sets and the places on it are the elements of the universe and its sets. Note that a place may be a point or an area on the earth's surface as necessary. For example, a given latitude-longitude designation would specify a point as place. On the other hand, we can regard the area of New Jersey as the place where the inhabitants of New Jersey live. For continuous distributions over the earth's surface, geographical places may be all or certain points and the elements in sets are infinite or finite in number as required. For other distributions geographical places may be all lines or selected lines, being thus infinite or finite. If areas (whether all or some) are the elements on the earth's surface, the sets, of necessity, are finite in number. Place, therefore, is to be defined operationally as required for the use to which the concept is to be put.

Essex County, Massachusetts, is a place in one context, the area of the United States in another, and the whole earth in yet another context. Whether places are points and thus there may be an infinity of them on the whole earth's surface or whether the whole earth's surface constitutes the one and only place when the appropriate non-spatial properties we wish our places to have are defined, is purely an operational consideration. The only property that all places have in common is the locative.

If areas are places then a region contains a finite number of elements with each element containing in itself an infinite number of point members. Additional considerations of denumberable and non-denumerable infinities lies beyond

the scope of this introductory presentation.

Now we want to show a diagram of the earth's surface as our universe. Moreover, regardless of the other non-spatial characteristics which the elements subsequently may be recognized as possessing, we insist at the outset that the locations be represented "correctly." We start, thus, with the irreducible geographical requirement, location in its full sense.

Figure 2 is to be regarded as a mapping of the earth's surface in both the mathematical and the cartographical sense. On this map:

Figure 2

there can be, and we assume there is, a one to one agreement between the points on the earth's surface and the points within the ellipse. Immediately, we run into problems in that the earth's surface, being that of a very nearly perfect sphere, is at once finite but unbounded. Our mapped earth diagram represents this finiteness and it is bounded. This is a consequence of the particular projection we have chosen to use to transform the spherical surface to the plane.

On the map in Figure 2 the kind of projection used can not be readily recognized because there are no outward and visible signs such as the conventional latitude and longitude grid or some well-known and equally recognized distributions.

The significant thing is that the full range of geometrical properties such as size (to scale), distance, shape, direction, and so on are now ordered by mathematical functions and we may later begin further consideration of properties of phenomena associated with places.

Let us assume that the earth's surface is portrayed in Figure 2 according to the Aitoff projection which shows the entire earth's surface.

Moreover, the Aitoff projection maps the entire earth according to a projection without singular points. We might be more comfortable with the idea that the area of the plane bounded by the ellipse in Figure 2 is really a representation of the earth's surface though as yet you have only the author's word for it, let us add a reassuring latitude-longitude grid. This, too, is a classification procedure involving defined sets. We continue our elaboration now with the mapped points of the entire earth's surface and a representation of the conventional coordinate system for defining and fixing locations. See Figure 3. We are now able to define places that concern us in terms of the geographic irreducibility--location.

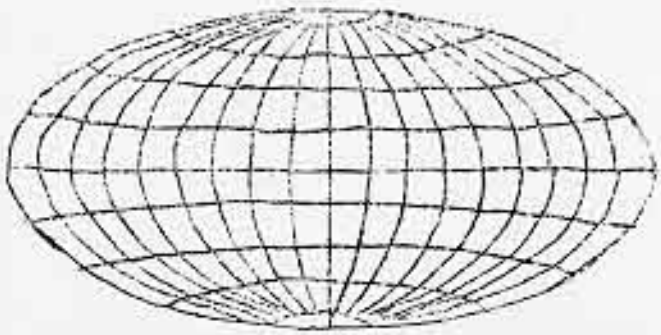


Figure 3

Let us now turn our attention to the important, existential assumption, the existence of a null class. This means simply the class in which no member has a certain given property. The null class contains all of the non-existing elements in the universe of discourse. It is the only class which contains members with incompatible properties. It is, in fact, made up only of such members. Graphically, the null class cannot be portrayed, nor can we map it as a geographical reality, as will be indicated subsequently.

Although it may be difficult to think of a universe of discourse of only one element, but the existence of more than one element is not essential because

the algebra would be just as acceptable. The algebra would still be pertinent, however trivial, if we operated on null classes alone.

In addition to the foregoing existential assumptions, Hilton notes certain operational assumptions, namely:

(1) The existence of a complement for every term;

(2) The existence of a sum for any two, or more terms; and

(3) The existence of a product for any two, or more, terms.

When we say that we assume the existence of a complement for every term, we mean that for every class of a given type, there is also a class of elements not of that type. Together, these two classes constitute the universe. The second class may be thought of also as the negation of the first class. This is, of course, exactly the same thing as the principle of dichotomy discussed above.

In Venn diagrams for conventional logic, the relation of a set and its complement are usually shown as in Figure 4. L is the specified property. All elements not in the L set are shown as L', that is, not L.

Figure 4

From an empirical earth-surface point of view, let L be the class of all elements (places) on the earth's surface that are regarded as dry land, i.e., solid ground, and L' be its complement, i.e., the places that are regarded as not dry land. And, let us insist upon a graphical portrayal in which the locations on the earth's surface of the elements in set L be regarded as inviolate and that these locations be ordered according to the map projection illustrated in Figure 3. The result of this is certainly familiar to all. It is the map of land and water surfaces shown in Figure 5. It is "accurate," of course, only to the extent that the scale permits.

Of course, such a map as in Figure 5 represents to most people more than "a mere classification" of the earth's surface. But, it is just that--a classification. So incomparably useful has this classification been throughout our history and so great have been the expenditures of time and other resources to secure all possible detail about elements so classified, that maps showing less than this particular classification of the earth's surface are regarded by most people--geographers included--as not being really maps at all.

Such a classification of the earth's surface is an example of an arbitrary classification that is so useful for so very many and such varied considerations that it is not ordinarily thought of in a classifacatory sense but rather regarded as a purely "natural" thing. Land and water as such are, we must admit, examples of non-spatial categories. The locations of these places have thus defined the areas shown, however.

Now when we introduce the full range of geometrical properties to Venn diagram methods of graphical portrayal of sets in which location is made explicit we find that <u>one type</u> may have many regional examples. Thus, the lack of compactness, i.e., the variance, in the distribution of dry land as a category has occasioned the development of different names for different parts of the fragmented land mass. We have given names to certain large pieces and have regarded them as "continents" and have similarly given names to smaller pieces and have regarded them as "islands," either as belonging to some continent or not as the case may be. Oceans, too, may be so regarded. We will investigate the "continent" and "oceans" questions in a bit more detail below. Here, however, let us consider additional aspects of non-spatial properties of places and classification in these terms.

After our first great classification of the places on the earth's surface into that which is <u>terra firma</u> and that which is not <u>terra firma</u>, we see that the number of "geographical regions" representing each of these classes seems to have

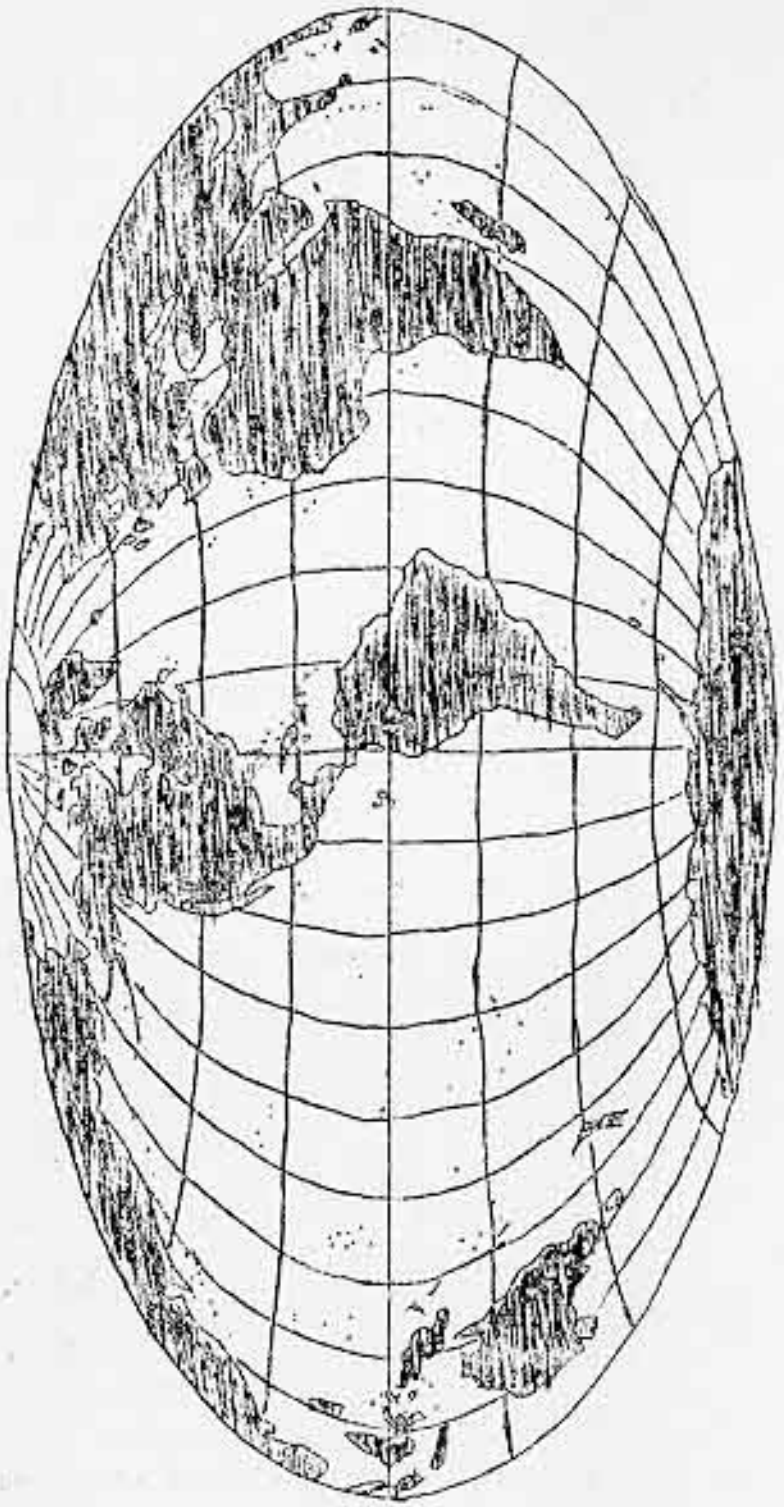


Figure 5

been more or less determined by sheer locational considerations.

Although the mapped distributions based on the land-water classification have proved to be of inordinate value through the ages one can readily think of a "base map" showing neither of these features. The above distinction, of course, becomes less and less significant as the atmosphere adjacent to the earth becomes the preferred space for travel. And, certainly, very long range guided missles for military purposes render the land-water "boundaries" more nearly inconsequential. Horrible thought!

As an example of another kind of classification of the earth's surface, let us consider certain temperature phenomena. Take, for example, as a class, the elements (places) having the property such that the average temperature of the coldest month is above 32 degrees Fahrenheit. The typical Venn diagram portrayal of this in a non-locational frame would be similar to that of figure 4 with whatever letter symbols are chosen to stand for this particular class and its complement. Again let L indicate dry land and H the property associated with elements for which the temperatures are as defined above. Note again that L and H are both non-spatial properties. These circumstances are shown in Figure 6 below.

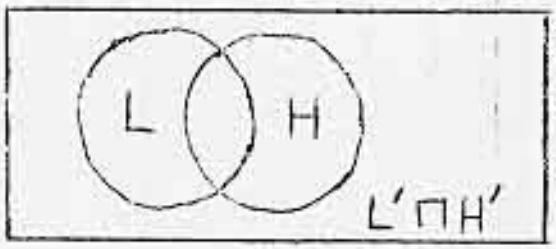


Figure 6

This figure now may be interpreted, for example, as follows:

If L and H are two sets we may derive a new set by considering the aggregate of all elements which are members of set L or set H or both. This new set is called the union of L and H or simply the sum of those terms. In conventional symbols we write $L \cup H$. Portrayed graphically (Figure 7) for elements (places) independent of their geographical location, the following emerges. The shaded

area denotes the union.

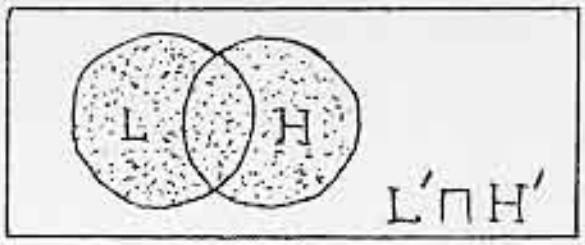


Figure 7

Of course many other concepts and operations exist such as inclusion, subset and proper subset, equality, disjunction, intersection, inverse functions, partitions, steps, miniterms, dualities, circuits, switchings, transformations and so on in an "astonishing array." Virtually every general concept or operation now clearly defined within set theory has its important special case within geography. Geography is in no way exceptional if we regard it as the science of location and spatial relations, i.e., from descriptive through classificatory through theoretical-predictive levels.

We give one more simple example. Figure 8, in its shaded area, shows the

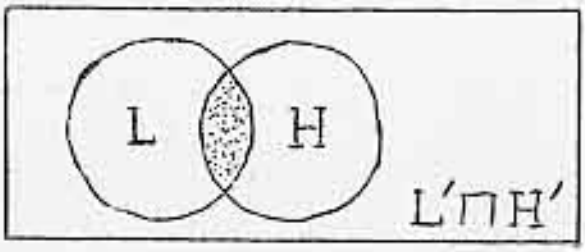


Figure 8

intersection of sets L and H, the product of those terms. In symbols we may write $L \cap H$ and thus specify the set containing elements (places, independent of actual earth location) which are members of both L and H.

Let us now preserve geographical locations of places and portray the above intersection, again according the Aitoff equal-area transformation from the earth sphere to the map plane.

The result is given in Figure 9. The great lesson here is that a single compact intersection non-geographically may map geographically into many "regions"

on the earth's surface.

Figure 9

Figure 10 may be more comforting because it shows the same information and has the remaining land areas indicated.

Obviously we could increase the number of sets we wish to consider and arrive at an intersection of sets such that only one point (or even none) on the earth's surface satisfies the non-spatial conditions imposed. In spatial terms such definition is possible and frequently highly desirable. The intersection determined by a specific latitude value and a specific longitude value is positionally uniquely determined, of course. For non-spatial properties with the location of the elements preserved, we may adjust our classifications to get the most meaningful results.

Figure 11 portrays the geographical mapping of that set of non-spatial properties whose elements are sometimes referred to as constituting the "Temperate Marine Type" of climate. These elements are at least a proper subset of those shown in figures 9 and 10. They also meet additional requirements concerning amounts and seasonality of precipitation and maximums imposed on the warmest month average temperatures.

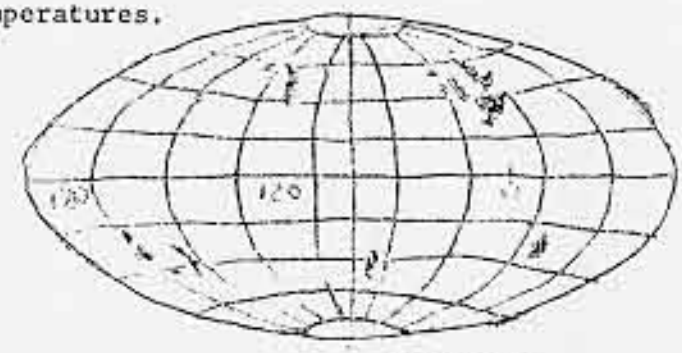


Figure 11

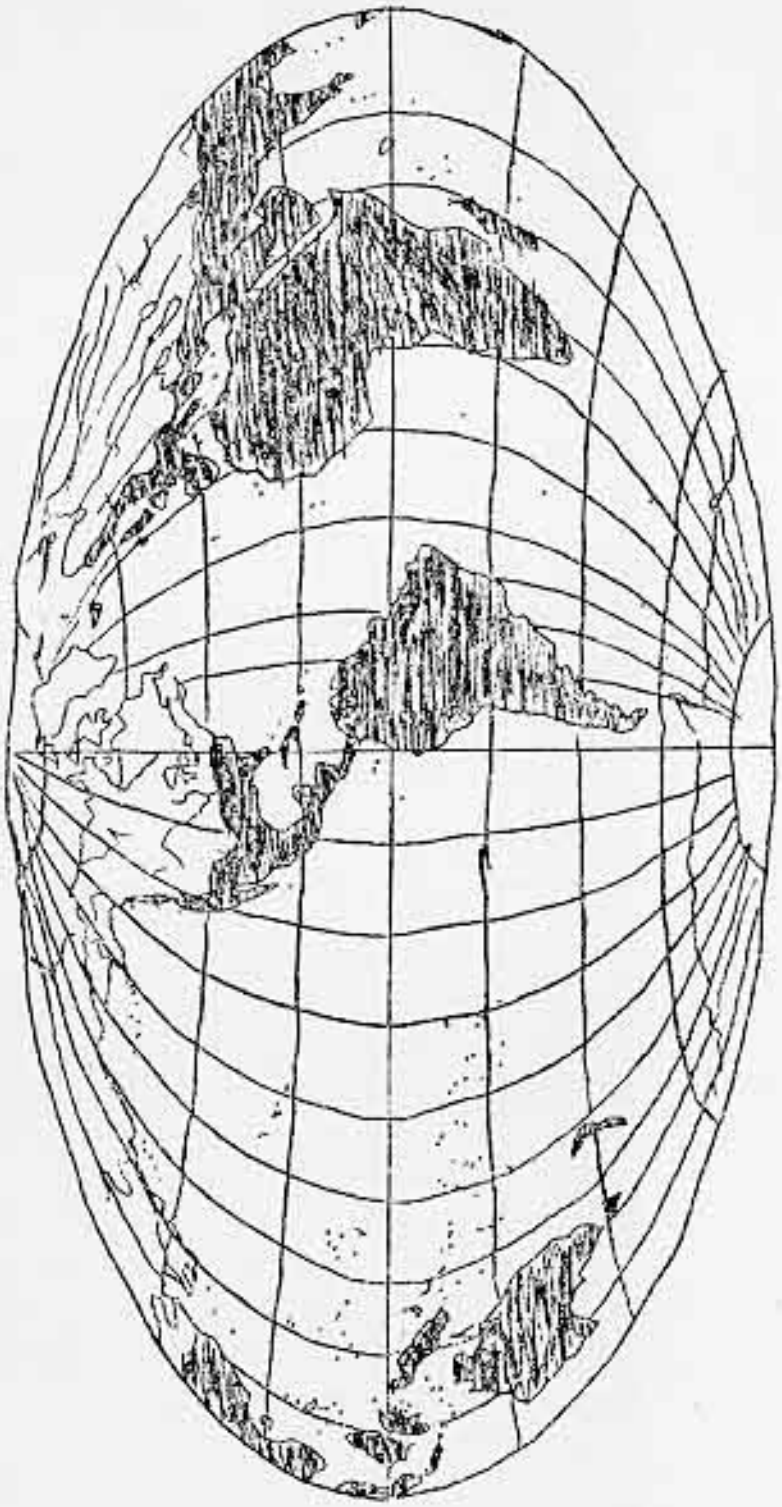


Figure 10

We will not follow this line of reasoning any farther here except to point out again that non-spatial property compactness may lead to geographical fragmentation. What is desirable non-spatially may be undesirable or awkward geographically.

For the degrees of freedom obtaining, we wish to maximize the ratio of variance among sets to the combined variance within sets. If we consider climatic types, income types, etc. for places (non-locationally) the conventional techniques of analysis of variance, of course, are pertinent and have meaning. We can arrive at a "Temperate Marine Type" in such a fashion. The geographical mapping that results, however, as noted, may be satisfactory. How should we group the fragmented "types of areas" (using type in the non-spatial sense) to give geographically useful and significant regions?).

Part II - Extended Venn Diagrams

Let us now consider variance in geographical terms alone. That is to say, we are concerned only with location of elements (places) and, at first approximation, we are willing to forego any consideration of non-spatial properties that places might be considered to have. In simple geographical terms averages are positions, deviations are distance (physical, time, cost, etc.) and variance is spatial. Degrees of freedom and probabilities retain their conventional roles.

If the regions into which we wish to divide the earth's surface are to be meaningful and convenient spatially for, say, planning or administrative purposes, spatial compactness may be desirable. Generally speaking the most efficient regionalization achievable for a given set of places in terms of both their non-spatial and also their geographical variances can be obtained only as a deliberate compromise of these two kinds of variance. To the degree that for the criteria use, the differences among places within regions are insignificant compared to the differences among regions we may regard our regions as individually homogeneous and collectively heterogeneous and our regionalization as a technical

success for the criteria used. Again we note that location is the sine qua non of geography. If this factor's variance ratios alone are to be regarded, then the "ideal" regionalization based on location and the metric of distance results in a mosaic of hexagonally-shaped regions on the plane. Another tesselation is required for a closed spherical surface. Such a distribution alone simultaneously maximizes the distance among the centers of regions and minimizes the distances of all points within a region from that region's center. If other criteria of places than just their locations are included in the classification, other spatial patterns emerge depending upon the relative "weights" assigned to the various criteria. Whether we have chosen the appropriate other criteria on which to base a particular regionalization, i.e., whether we have regions of practical value for their intended purposes is, as noted, of crucial importance. We shall not venture here to discuss the various rationales and strategies for effecting the required compromises between the two types of variance and interpreting the results. Rather, we chose to conclude this paper with additional comments about spatial variance alone.

No two places coexist locationally by definition. Places are unique in the limit as to location, but this introduces no new idea or consternation into classification theory. Individuals are unique but grouping is at once necessary and efficient. Minimum distance variation between places occurs for adjacent, i.e., contiguous places. Maximum distance variation for places on the earth's surface obtains, of course, when the places are antipodal. Thus, in locational terms alone, any grouping procedure putting places into sets might regard contiguous places as most likely candidates for mutual inclusion in the same set and antipodal places as least likely candidates for the same set.

If spatial sets are regions then the Temperate Marine type of climate has many geographical regions of the same non-spatial type. Perhaps we would wish to include England and France within the same region. But, this region perhaps should not include its non-spatially identical New Zealand. New Zealand is

virtually antipodal spatially to England. It perhaps should be included in another geographical region but of the same non-spatial type.

Can we think of some pertinent problem such that pure spatial variance of places alone is the significant phenomenon independent of their non-spatial properties? Perhaps! But, on a perfectly spherical earth with places differentiated by location but not differentiated in terms of non-spatial properties, every place is an equally efficient average (central) location because the total variance is a constant regardless about which place it is measured.

It seems as though spatial variance of places on a regular closed geometrical surface like that of the assumed spherical earth becomes significant only when we select out certain places (arbitrarily, or in terms of certain non-spatial properties, hypothetically, theoretically, or empirically) such as the places occupied by persons, the places deemed to be within a certain class of vegetation forms, the places regarded as exhibiting certain climatic conditions, and so on.

The most nearly pure spatial example involving the least contrived and perhaps most generally significant non-spatial classification (at least in this and the most recent epoch in human history) is the one based on the land-not land (i.e. water) distinction noted earlier. We return to the "number of continents" problem. This is clearly not an idle question when viewed in terms of spatial variance and the number of sources yielding the most efficient description.

At work through the ages, though never fully consciously announced, has been the understanding by mankind that the specific recognition of the separate continents includes the considerations of the centers and boundaries of continents such that the present classification of the earth with seven continents might approximate closely the "ideal" ratio of the distances among all points within these continents and distances among the continents. This is, of course, subject to appropriate examination and verification. We should test to see

whether or not this is so. And, too, should Europe and Asia so-called be regarded separately or as Eurasia? Recognition of Antarctica as a separate continent (or is it a large island?) seems easy enough, but what about North America and South America? Here the isthmus connecting them (artificially pierced by the Panama Canal) renders them contiguous. But, this may not be enough to cause us to regard North America and South America so-called as one continent as the actual spatial distributional shapes of these two land masses perhaps causes the average distances between the points of the two continents to be too great for the narrow land link to overcome. On the other hand, actually non-contiguous land masses such as "adjacent" enough islands may be included with a given continent as for example Japan as part of Asia.

In connection with all of the above it is interesting to note that the meaning of the word "continent" as an adjective is "held together, contained, or restricted."

"How many continents are there?" is thus seen to be not just a rhetorical question. It is valid and geographers classifying the earth's surface should attempt to answer it and support their answers with results obtained by careful methodology based on the analysis of the variance of locations within as compared to the analysis of the variance of locations among continents based on the various assumptions as to what are continents. Appropriate statistical tests would provide pertinent measures of significance based on varying degrees of freedom. The fragmented spatial distribution of our non-spatial class L has made it relatively easy for mankind acting collectively to reach a certain tentative concensus through means not actually explicitly agreed upon for the class L and the associated number of and names for continents.

We are currently designing a small research effort to attack directly this problem by way of an "iterative-weighted functions of locational values" computer program. Research into the literature of the history of continent recognition and naming is also contemplated.

Did Europeans divide Eurasia differently when they knew that Australia existed than they did when they did not? Why? How much larger would Greenland have to be to be recognized as a continent assuming its approximate present location? Or, how much more isolated would Greenland have to be to be recognized as a continent given its present size? It may be that there is no position on the earth that a "Greenland" might assume that would occasion continenthood given its present size. Clearly, continenthood is a function of size, shape, and position as the concepts of the analysis of spatial variance reveal. At optimum location for continenthood how much larger might a "Greenland" have to be?

Suppose all of the dry land of the earth to be gathered together in one maximally compact, and thus circular, continent. (Before continental drift?). Would there be an advantage to recognizing sub-division into continents based on locational factors alone? Clearly not! The idea of continents in a physical sense (apart from political, cultural, economic, etc. considerations) takes on meaning in the real world only in the presence of fragmented land masses and/or the not altogether convex shapes of these land masses. In fact it is the obvious matching of concavities with convexities that makes the continental drift hypothesis such a pleasing notion.

The basic ideas of the theory of convex sets are natuarally and easily appreciated when examined in a geographical context, rather than in a non-spatial one. In geography the meaning is direct, obvious, and literal but nonetheless powerful. This is true in general of a large part of set theory as developed by Georg Cantor (1845-1918) who provides the foundation of the theory of point-sets, of real functions, and of topology. In this respect geography is truly elegant. In fact it may be the least exceptional science. What about the complement of the class L, that is L' and its regions of location? We may regard this at the outset as being not only the negation of L but as constituting in its entirety another class, say W, or the water surface of the earth. This, too, can be

mapped. In fact we have done it in figure 5 for the very reason that W and L gave no members in common and together they constitute our universe of discourse, all the places on the surface of earth.

One final comment. In general in statistical analysis of variances, squared values for deviations are customarily used because of their obvious significance in terms of certain model distributions such as the Gaussian normal. Then, too, squared values have lovely algebraic properties--no negatives, for example. In the geography of continenthood, all deviations are themselves positive and squaring to avoid negative values is not necessary although it may still be desirable for other reasons. The power of the distance to be involved is, of course, a matter of considerable importance. We can define a center as a place where the appropriate extreme value for a spatial moment occurs and we define any spatial moment about any point (following Neft, 1966) as:

$$M'_n = \int r^n DdA,$$

where r is distance, n is an exponent and DdA is the density of elements over the infinitesimal bit of area. The assumption of different values for n occasions different centers. What is most reasonable for given circumstances is a difficult problem and is often more clearly dramatized by geographical problems than non-spatial ones.

Some one should write a book on regionalization. It might well have the following title:

> "The Algebra of Sets, Geographical Regions, and the Use of Some Obvious But Hitherto Ignored Properties of Venn Diagrams--Or The Logic and Method of Geographical Regionalization as an Extension of Boolean Algebra Examining the Relationship to General Classification Theory, Symbolic Logic, the Algebra of Sets, Computers, Electronic Circuitry, and Logic Diagrams with Special Consideration of Automated Logic Machines and the Representation of Systems of Geographical Regions Based on a Rational Interpretation of Non-Spatial Differences and Geographical Distances, that is to say the Theory and Application of the Geographical Analysis of Variance."

We sometimes make our greatest gains in geography when it is our basic concepts that are simple and natural and our techniques that are sophisticated.

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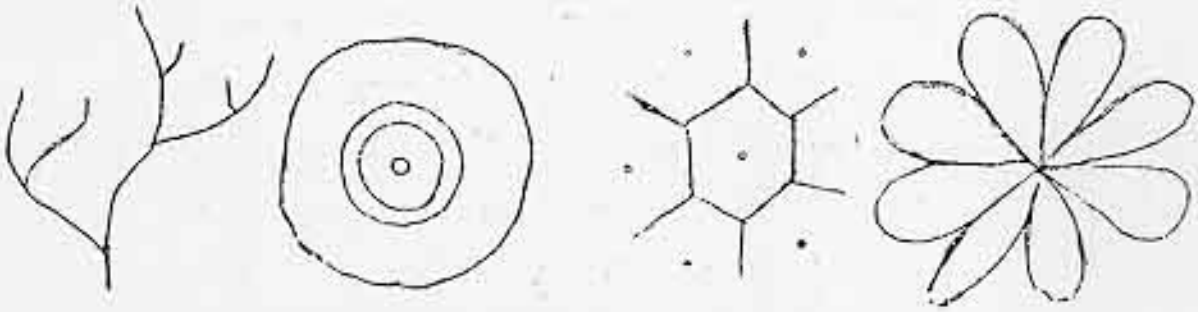
What is implied if the concept of pure spatial prediction has validity? Geography, like other mathematized sciences before, is searching for the correct coordinate system and point of origin. Tobler is our Copernicus. The Geographic Projection, the one we still seek, the one much more important than the infinite projections mastered, is the Uniform Plain, which is the geographic equivalent of "other things equal" assumptions in other sciences. Once the space is properly projected, the patterns (our primatives are the dimensions) both probalistic and extremum (with the function to be minimized some concept of nearness) should emerge and be more testable. Somehow the patterns and the coordinate system should be related functionally. This is a Grand Design of Theoretical Geography. In none of this work is it necessary to refer to time. Movements can often be eliminated as well: Space and perhaps movements, but never time. This work led the editor to the work on pure spatial prediction. Time is so damnably invisible, but space we can see. Kurt Schaefer's great sentence, "Patterns are morphological laws." implies what philosophically? A visible law? The law itself seeable? The law of a line and an area as near to each other as possible (with certain other minor restraints) is a dendrite (river, sewer system, oil pipeline)?



The following comments were excerpted from Theoretical Geography (1966).

Spatial Prediction

William Bunge

Abstraction rules supreme these days. With the overthrow of Kant, the rise of non-Euclidean geometry and the scientific success of the new physics, anything that is non-abstract is definitely out of fashion. As students are introduced to the notions of dimensions and hyperspaces, they are simultaneously admonished not to take them seriously but immediately gain the sophistication of considering dimensions as a little boy's version of variables. Real terrain surfaces are demeaned as organically inferior to mathematical ones--pedagogical crutches for the weak minded.... In mathematics, it is considered the most flagrant gauchery to use a diagram. "Graphics" is thought to be an inflated title for "mechanical drawing." In fact, all the intrinsically visible subjects;

geography, graphics, and geometry, are suspected of being really grade school subjects, fit only for brains that are still undergoing biological maturation and whose harmfully misleading approach will have to be undone later. No one is arguing for a return to Greek Naiveté. Certainly invisible abstractions can be real and are necessary even in geography, but why is this in contradiction to the thought that perhaps concrete visability might also be real? A truth is no truer for being obtuse or sophisticated. In an age of mental Schaum tortes can not nature's fresh peaches also be worth savoring? Whitehead, you have gone too far!

Abstraction is so much the rage that even geographers have difficulty imagining a pure spatial prediction. Asking students for examples is always a disappointment. Their examples typically include the eventual draining of the Great Lakes or the creation of vast cities in Southern California. Always time is present as a variable even if space is present in their examples. But time need not be present at all to make a prediction. Consider the profile, or side map, of a wave pattern. (Figure 1).

Figure 1

Wave Pattern

This could be the march of temperature over the seasons, a time prediction. It could also be a profile of the Ridge and Valley Region of Appalachia. Standing on the crest of the last wave to the right, what can be predicted? --that summer will be followed by fall and so forth in time or that the peak of the ridge will be followed by the side of the valley and so forth in space. Consider an idealized topographic sheet. (Figure 2). What will the adjacent sheet be like?

Predicted Map:

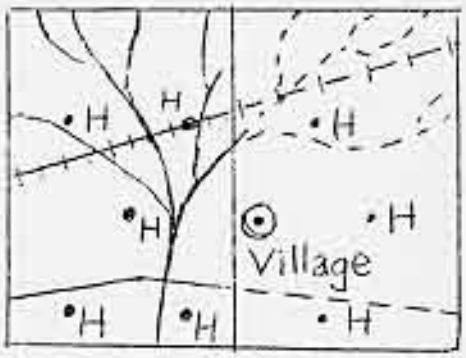


Figure 2

GIVEN

PREDICTED

The railroad and highway might be continued as straight lines and the river pattern idealized. The farther in space away from the known location the less confident the predictions, just as confidence is lost in the Weather Bureau the farther in the future the prediction is made. Some might object that such a prediction is merely an extrapolation. But most clearly with the spatial prediction of the village, an object that did not appear on the known sheet, this is not true. With improvement of our skills it is conceivable that Koppen's Hypothetical Continent be extended to a Hypothetical Globe. No extrapolation is involved. Geographers predict a village because geographic theory demands it. If geographic theoriticians provide geographic experimenters, including explorers, with the most likely sites of the missing Mayan cities, compute their k's, their rank-size rule, estimate the number of hamlets, compute their average spacing and so forth are they not predicting? To place our finger on the map of Yucatan and say, "There," is no less impressive a prediction than an astronomer pointing in the Heavens to a missing planet or Babe Ruth at the right field wall.

The most compelling case for pure spatial prediction is the seeming isomorphism of concepts with the concepts of temporal prediction. The temporal scientist's past and future corresponds to geography's behind and ahead; their event, our place; their moment, our location. It is the visability of spatial prediction which makes geography so instrinsically visible. The case should not be overstated. For example, it is most likely that the solution to the mapping of the uniform plain will not be a visible solution. Limits exist to literalness just as to abstraction...

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In what sense is the map a theory? One effect of reading Toulmin is to put mathematics into a proper perspective. Toulmin (1953) writes elsewhere in his excerpted chapter, "...the mathematician remains the servant of the man who knows when and how the results of his computations can be applied. Jeans and Eddington were both primarily mathematicians, and in their popularizations of physics gave prominence to the mathematical side of the subject, but the results were in certain respects misleading: the physics is not in the formula, as they suggested and as we are often inclined to suppose, any more than being able to find your way about is part of a map. The problem of <u>applying</u> the theoretical calculus remains in physics the central problem, for a science is nothing if its

laws are never used to explain or predict anything." Science is the queen of mathematics, but the sciences seem so weak in preparing their students in philosophy that it all occurs in the mathematics departments where young minds are pumped full of the Christian Science view of the world that afflicts mathematicians. More power to them if such a Ptolemaic view of the intellectual universe spurs on starving mathematicians, but the sciences should not allow their students to be so mapped by intellectual default. Philosophy of mathematics is what the brighter students must somehow overcome to arrive at philosophy of science. As Toulmin comments, "The physics is not in the formulae..." A question plagued me for years, "Though Newton could predict the motions of the moon, how did the moon know where to go? Can the moon integrate?"

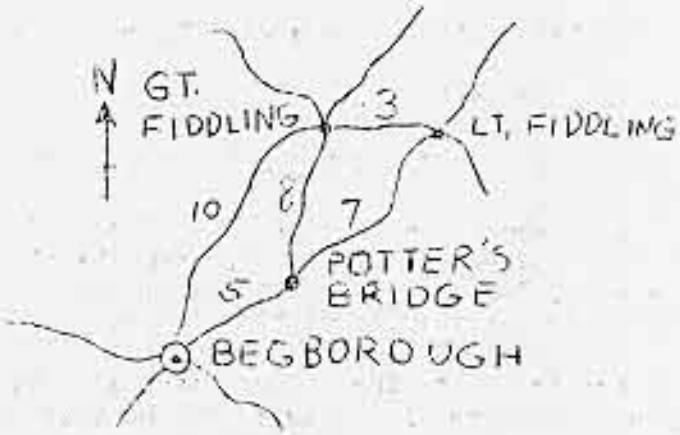
As to possible extensions of Toulmin, he goes a rye when he mis -answers his superb question "...what exactly corresponds in cartography to laws of nature in physics?" He assumes it must be map projections which he justly criticizes but it is patterns, geographic laws, like Christaller's or Thünen's, or the host of new ones, that corresponds to laws of nature in physics. He also makes, the minor-to-the-discussion error, of assuming that the task of map projections is to "preserve certain chosen features" of the surface of the Earth, when, of course, the major task is to create new features that Nature missed, such as Mercator's straight loxodromes accomplish. We need a terrible "crooked" projection, the Uniform Plain maker, that will make the simple "distortions" of physics appear as child's play.

Though for purposes of his discourse there is no error in comparing the fundamental map with complete theory in physics, geography does not view a complete map as its complete theory. The map is merely the experimental data controlled through the laboratory instrument of the projection; the spatially manipulated locations of the earth's surface. Geography as the science of location, in its most sophisticated labor, seeks to predict locations. The most fundamental theory is hueristically defined as placing objects of different dimensions as near to each other as possible, with some feeling for probabilistic convergences with the extremum statement. The following comments were excerpted from Toulmin's book, The Philosophy of Science, 1953.

Theories and Maps

Stephen Toulmin

...Consider, for instance, the imaginary motoring map opposite, showing the town of Begborough and its environs.



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We can ask about this section of map a question similar to Mach's question: namely, what relation it bears to the set of geographical statements that can be read off it, such as "Potter's Bridge is 5 m. NE of Begborough on the road to Little Fiddling", and "Great Fiddling is 3 m. due West of Little Fiddling."

How are we to answer this question? Certainly the map cannot be said to be deduced from the set of geographical statements nor, in a logic-book sense of the phrase as opposed to a Sherlock-Holmesian one, are the statements deduced from the map. For in a deductive inference, such as "Fish are vertebrates, mullet are fish, so mullet are vertebrates", the same terms appear both in the premises and in the conclusion; whereas here the 'conclusions' read off may be statements, but the 'premise' is a map and contains no 'terms' at all. Only where premises and conclusion are comparable in the way that "Fish are vertebrates" and "Mullet are vertebrates" are comparable, is there room for a deductive connexion, so the relation between the map and the geographical statements must be of a different, non-deductive kind. At the same time, the map need not be said, in Mach's sense, to 'contain' anything which cannot be expressed as a geographical statement of the kind included in our set: everything which one could read off from the map of this sort. Though the map and the geographical statements are not deductively related, one need not conclude that the map goes beyond the surveyor's readings; since it does not present us with additional information of a novel kind, but represents the same information as the statements in a different manner. This example shows that, when we are presented with two logically incomparable forms of expression, the question whether or no one form of expression contains more than the other is quite independent of the question whether or no the one can be deduced from the other. In fact, unless the expressions are of logically similar kinds, there can be no question of such deduction...

The aggregate of discrete observations is transformed into a simple and connected picture, much as the collection of readings in a surveyor's notebook

is transformed into a clear and orderly map.

The consequences of this analogy are worth noticing. For if someone asks, "Doesn't the map tells us that Potter's Bridge is 5 m. NE of Begborough, and a whole lot of similar things?", we can only answer "Yes and No." Certainly, if you know how, you can read off from the map a great range of geographical information; but the map on the one hand, and the geographical statements on the other, tell us things in very different ways. A man might own Ordance Survey maps of the whole country, and yet, for lack of a training in map-reading, be quite unable to tell us anything of a geographical kind; likewise, a man might have memorized all the currently accepted laws of nature and even know a vast amount about the calculative side of mathematical physics, and yet not be equipped to explain or predict any of the phenomena observed in the laboratory...

In the traditional logical account of the sciences, one encounters certain difficulties when explaining how it is that experiments are used to establish theories. In the first place, physicists seem to be satisfied with far fewer observations than logicians would expect them to make: one finds in practice none of that relentless accumulation of confirming instances which one would expect from reading books on logic. This divergence is partly to be accounted for by the logicians' confusion between laws and generalization--one would hesitate to assert, say, that all ravens were black if one had seen only half a dozen of the species, whereas to establish the form of a regularity in physics only a few careful observations are needed--but this is not the whole story. There is also a second, related difficulty to be overcome: that of explaining how subsequent applications of a theory are related to the observations by which the theory was originally established.

To take the two difficulties together: it is worth noticing that they arise for theories as much as, and no more than, for maps. Not all the applications to which a theory is put need have been specifically made in the course of the experimental investigation by which it was established. But nor need all

the things which can be read off from a map have been specifically put in. A child might wonder how it was possible ever to produce a map at all, since to tread every inch even of a small area, and to measure all the distances and directions that one can read off from a map, would take an unlimited length of time. This, of course, is the marvel of cartography: the fact that, from a limited number of highly precise and well-chosen measurements and observations, one can produce a map from which can be read off an unlimited number of geographical facts of almost as great a precision. But it is not a marvel calling for a general explanation, for only in some regions can the techniques be implicitly relied on. In irregular country it is always possible to be misled, and the number of observations which have to be made per square mile will be much greater in some areas than others--just how many are needed being something the practising cartographer must be able to judge.

Correspondingly, it is a fact that many physical systems have been found whose behavior can be similarly 'mapped.' Having made a limited number of highly accurate observations on these systems, one is in a position to formulate a theory with the help of which one can draw, in appropriate circumstances, an unlimited number of inferences of comparable accuracy. Thus it is always possible that the next time Boyle's Law is applied, the particular combination of pressure and volume concerned will be being observed for the first time. But again, though this fact is in its way a marvel, it is not one requiring a general explanation, any more than is the possibility of mapping. For here, too, how far the behavior of a given system consists of phenomena which can be mapped in a simple way, and just how many observations will need to be made before we can be confident that our theory is a trust-worthy one, are things which will vary very much from system to system and which it is part of a physicist's training to learn to judge...

The imaginary road map of the region between Begborough and the Fiddlings which we discussed a few pages back, need not be the only map of the region.

There will also be some more elaborate physical maps drawn to a larger scale and showing a great deal more detail. In such maps as these, roads will perhaps be drawn to scale, not represented by lines of purely conventional widths, while towns and villages will be marked, not as mere dots and blobs of standard sizes, but as having definite shapes and made up of individual streets and blocks of houses.

Now a number of things should be noticed about the relation between the road map and a physical map of the same region. In the first place, many things can be mapped on the physical map which there is no way of putting into the road map: this is a consequence of the ways in which the two maps are produced, and of the comparative poverty of the system of signs used on the road map. On the other hand, given the physical map, one could produce a satisfactory road map: all that appears on the road map has its counterpart on the more elaborate map, even though in a different form. But this does not mean that the road map is not, of its kind, an unexceptionable map of the region. Providing that it is not thought of as having irrelevant pretensions, there is nothing wrong with it: indeed, for some applications one will be able to discover the things one wants to know, e.g., distances by car, more easily from the road map than from the physical one. Finally, it is worth noticing what happens if we mix up the systems of signs used on two different kinds of map. There are some motoring maps in which one finds town-outlines and other features sketched in on top of the simple road pattern: but since only distances along roads can be given a satisfactory interpretation on such maps, the result is usually confusing, and the simply blob for a town is more consistent with the general scheme of the map.

The relation between geometrical optics and the wave-theory is not unlike that between a road map and a detailed physical map. Thus the fact that one can explain on the wave-theory, not only all the phenomena that can be accounted for on the geometrical theory, but also why the geometrical account holds and fails to hold where it does, is like the fact that one can construct a road map from a

physical map; but again it is not a sign that the geometrical theory need be superseded for all purposes. Road maps did not go out of use when detailed physical maps were produced. It shows only that, as one can produce a road map from a physical one but not vice versa, so one could produce a ray-diagram from the wave-theory picture of an optical system, but not vice versa. The conceptual equipment of the geometrical theory, like the system of signs on a road map, is too poor for one to do with it all that can be done with the wave-theory. Indeed, the notion of a light-ray is an artificial one in very much the way that the conventional-width road is, and has to be abandoned in the wave-theory because the accuracy with which one wants to answer questions about optical phenomena is too great for the conventional picture to be retained. No more can one, from a simple motoring map, answer questions about the distance from the northern verge of one road to the middle of another--these are things that a map of that type does not pretend to show. Again, since there is no room within geometrical optics for representing the phenomena of diffraction, a physicist would hardly think it worth while to give any indication on a ray-diagram of the shapes of any diffraction-fringes he observed: they would be just as out of place there as town shapes are on a bare motoring map.

If we look at the relation between different theories from this angle, we can notice some points of importance about the notion of a 'fundamental' or 'basic' theory. One finds that, at a given stage in the history of physics, there is commonly one theory, at any rate in a particular field, which is regarded as the basic theory: this theory is thought of as capable of accommodating all the phenomena to be observed in that field. Now two questions need to be asked. Since it will never be the case that all the phenomena have in fact been explained, all that need be claimed is that the basic theory can in principle explain them all: the first question is, what are we to understand by this claim? Secondly, when physicists talk about explaining everything, what are the criteria by which they would judge that everything had in fact been

explained?

It is helpful to compare the basic theory with the fundamental map on which the Ordance Survey might record all the things which it is their ambition to record. This would, of course, be a map drawn on the very largest scale, but it would not be the only true map of the country: rather it would be the one which most fully and precisely represented the region mapped, and the one from which by appropriate selection and simplification all others could be produced. For many purposes it will be too elaborate to be of practical use, but for some purposes none else will do, and the lover of cartography for its own sake must have a special place for it in his heart.

The value of the comparison lies in this: it suggests that the standards of what constitutes a complete theory in physics may change. For we could say that the fundamental map was complete only if it showed all the things which in that region it was the cartographer's ambition to record. Now it is always possible for cartographers to develop fresh ambitions: the criteria of the completeness of a map are, accordingly, at the mercy of history. So are they with the theories of physics. One is at first inclined to suppose that the physical sciences have a definite goal, the same for Aristotle, Newton, Laplace, Maxwell, and Einstein, but a closer look at the history of the subject will show the mistakenness of this idea. Rather there is at any given stage a standard of what sorts of things require explaining: This is something with which scientists grow familiar in the course of their training, but which is hardly ever stated. The standard accepted at any time determines the horizon of physicists' ambitions at that time, the goal which for them would have been reached if 'everything'--i.e., everything thought of as requiring explanation--had been found a place in the theories of physics...

It is, then, still in cases where our interest is in how one might 'get somewhere,' i.e., produce or counteract some spotlighted development, that we talk about causes--though the destination need not be one that we care about

either way. From this we can see why the term 'cause' is at home in the diagnostic and applied sciences, such as medicine and engineering, rather than in the physical sciences. For the theories of the physical sciences differ from those of the diagnostic and applied sciences much as maps differ from itineraries. If the term 'cause' is absent from the physical sciences, so also a map of South Lancashire does not specifically tell us how to get to Liverpool. To a man making a map, all routes are as good as each other. The users of the map will not all be going the same way, so a satisfactory map is route-neutral: it represents the region mapped in a way which is indifferent as between starting-points, destinations and the like. An itinerary, however, is specifically concerned with particular routes, starting-points and destinations, and the form it takes is correspondingly unlike that of a map. Often enough, of course, a map be used to work out the itinerary for a particular journey, and from one map an indefinite number of routes may be read off, as occasion requires. But, from its form, there is nothing about a map to show that it is to be used for this, rather than any other of a wide range of purposes...

This analogy shows us something about the relation between the fundamental and applied sciences, and about such phrases as 'applied physics.' For in many fields of science practical skills preceded theoretical understanding, and even provided the first data for systematic study. Sundials were in use for centuries before their operation was properly understood, and there are still plenty of familiar processes, in cooking for instance, about whose physico-chemical nature we have only the sketchiest of ideas. There is therefore only a part of engineering which can be called 'applied physics', even though this part may be continually growing and may in some divisions, such as atomic energy, be all but exhaustive. This state of affairs also has its natural counterpart in cartography. For a long time, travellers relied on itineraries rather than on maps; Greek seamen and Roman legionaries as often as not followed set routes for which itineraries had been written out; there must still be today a few more remote

parts of the world which are totally unmapped, but around which a guide could take one; and even in our own well-mapped country we all know some short cuts and refinements that are shown on no map. So though the preparation of itineraries may in fact often be applied cartography, it need not be. Itineraries preceded maps. The development of cartography has given us a *way* of understanding the relations between different routes, and at the same time a source of new itineraries whose possibility had not previously been recognized. And there may be some parts of the world so remote, so mountainous, that one could hardly hope to work out itineraries for them except by first mapping them from the air...

In cartography, too, there is a good deal which has to be contributed by us before there can be a map at all, and this contribution is again of an unmysterious kind. Cartographers and surveyors have to choose a base-line, orientation, scale, method of projection and system of signs, before they can even begin to map an area. They may make these choices in a variety of ways, and so produce maps of different types. But the fact that they make a choice of some kind does not imply in any way that they falsify their results. For the alternative to a map of which the method of projection, scale and so on were chosen in this way, is not a truer map--a map undistorted by abstraction: the only alternative is no map at all. To draw an analogy between a cartographer's method of projection and the ichthyologist's fish-net would accordingly be misleading. There is no question of falsification here. Quite the reverse: it is only after all these decisions have been taken and a map has been produced, that the question can even be raised, how far the product of the cartographer's work is true to the facts, for only then will there by anything which can be true to <u>or</u> falsify them...

The existence of the Absolute Zero can be compared with the existence of the boundary in a map of the World drawn to a stereographic or orthographic projection. On these projections, the surface of the Earth does not cover the whole of any sheet of paper you use, as a Mercator's map is capable of doing,

but fills only two circles. If there is blank space round the circles, that is not because the cartographer has chosen to cut off the map half-way up Greenland, say, but because, the nature of the projection being what it is, no point on the Earth can be mapped outside the circles. One can, of course, decide to make the circles as large as one chooses; but, however large one decides to have them, there will still be a boundary, whereas a map drawn to Mercator's projection is capable of going on indefinitely.

If we prefer, it is open to us to stop using a map of one kind and start using one of the other kind; and to abolish the boundary in this way shows nothing about the area we are mapping. The presence or absence of such a boundary tells us nothing about the surface of the Earth. The same is true in physics. One can, if one chooses, change over from the ordinary ideal gas scale to a logarithmic scale, which extends without limit in both directions; and to make this change implies nothing about actual thermal phenomena. In neither case does one, by changing the method of representation, burke any facts about the World. . .

What, then, of the question, "Do electrons exist?" How is this to be understood? A more revealing analogy than dodos or Ruritania is to be found in the question, "Do contours exist?" A child who had read that the equator was 'an imaginary line drawn round the center of the earth' might be struck by the contours, parallels of latitude and the rest, which appear on maps along with the towns, mountains and rivers, and ask of them whether <u>they</u> existed. How should we reply? If he asked his question in the bare words, "Do contours exist?", one could hardly answer him immediately: clearly the only answer one can give to this question is "Yes and No." They 'exist' all right, but do they <u>exist</u>? It all depends on your manner of speaking. So he might be persuaded to restate his question, asking now, "Is there really a line on the ground whose height is constant?"; and again the answer would have to be "Yes and No", for there is (so to say) a 'line', but then again not what you might call a <u>line</u>. . .

And so the cross-pruposes would continue until it was made clear that the real question was: "Is there anything to show for contours--anything visible on the terrain, like the white lines on a tennis court? Or are they only cartographical devices, having no geographical counterparts?" Only then would the question be posed in anything like an unambiguous manner. The sense of 'exists' in which a child might naturally ask whether contours existed is accordingly one in which 'exists' is opposed not to 'does not exist any more' or to 'is non-existent', but to 'is only a (cartographical) fiction'. . .

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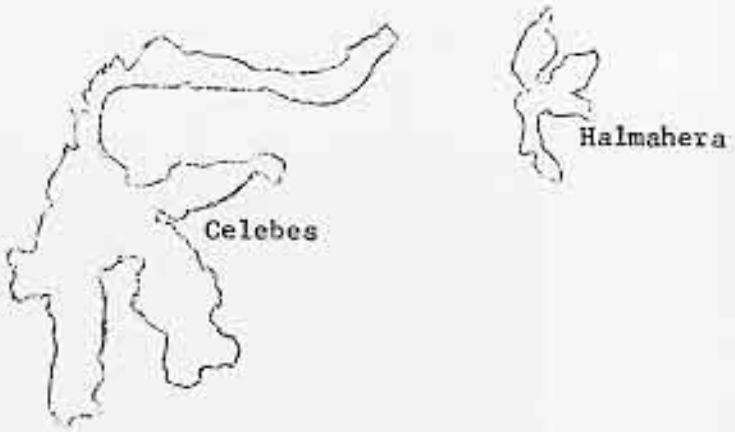
What, if anything, besides language can we read? Martin's contribution is maddening. It is original and sweeping, a man who sees "letters arranged into sentences on the face of the map." What of Wegner whose continents literally fit like a jog saw puzzle and who died trying to prove a "mere" jig saw puzzle idea himself? At the Brighton meeting of MICMOG where Martin presented his material again in the spring of 1967, we all were messmerized by his concepts and confessed the little secret mysteries we had snatched from Mother Map over the years. For instance, since a child I have personally been struck by the enormous similarity between Celebes and Halmahera, both in the same part of the world, both with impossibly similar shapes and orientations. The similarity is so striking as to call for a small map.



Martin refers to his work as a "new concept" in geography. He looks at the map fresh, like a child might year after year and he sees things. "The Quebec shore of the Ottawa River expresses the opposite E. Ontario Shore, as can be expected of any river. The corresponding Africa and America shores, though across oceans do the same."

Philosophically, must all languages be written by men? Do scientists literally try and "read" nature? What does a geographer mean when he says he is "reading" a map? Is a map a language? Is the landscape? The following comments were excerpted from Martin's thesis.

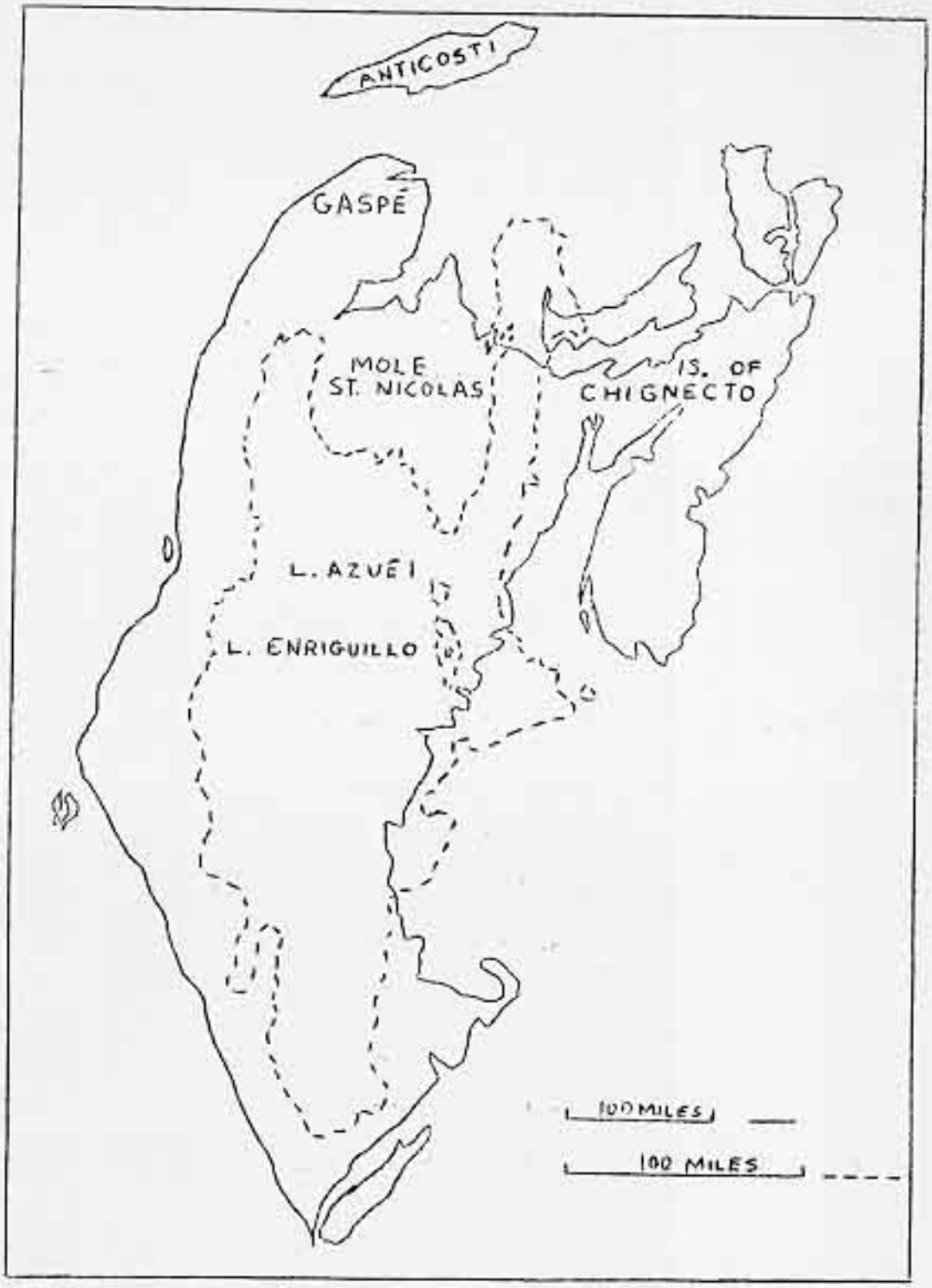
The Earth as a Living Body

Roland Martin

The first difficulty confronting us was in finding a starting point in the physical actuality of the world represented on the map. Ask any geographer if he knows of any two lands in the world having the same features of same relative proportions. Not only will he deny such knowledge but he is likely to go on to deny the very possibility of the existence of such two similar lands, or at best he will only state that he had never given the matter any thought or consideration, let alone the benefit of a tentative investigation.

If the relationships are uncomplicated, why did not someone perceive them merely by close observation? First of all, in order to find something one has to look for it. In this case, what one would be looking for is not the features of a particular land entity--such as geographers normally look into--but a relationship. The idea of a relationship is entirely abstract and exists only as a concept of the mind until it rests between two objects that enter in relation.

In addition, it still takes more than an abstraction implanted in the mind to prime the process. There are several other impediments to be overcome before gaining access to this first conjugated observation. The Rockies and the Appalachians are oriented primarily from north to south, while the Notre Dame Mountains and Ridge of Nova Scotia are oriented generally to the northeast, and the Cordillera Central and La Hotte--La Selle Mountains of Haiti are oriented from west to east. With such divergent orientations as are the case between Haiti and North America, the similarity between the two land units hardly jumps to the eye. The observer seeking to investigate the relationship between the two units would have to confront a map of the island with a map of the continent and wheel them freely. He might then orient the two western peninsulas of Haiti in line with Alaska and the northern end of the Appalachians so that the mountain ranges fall parallel to one another.



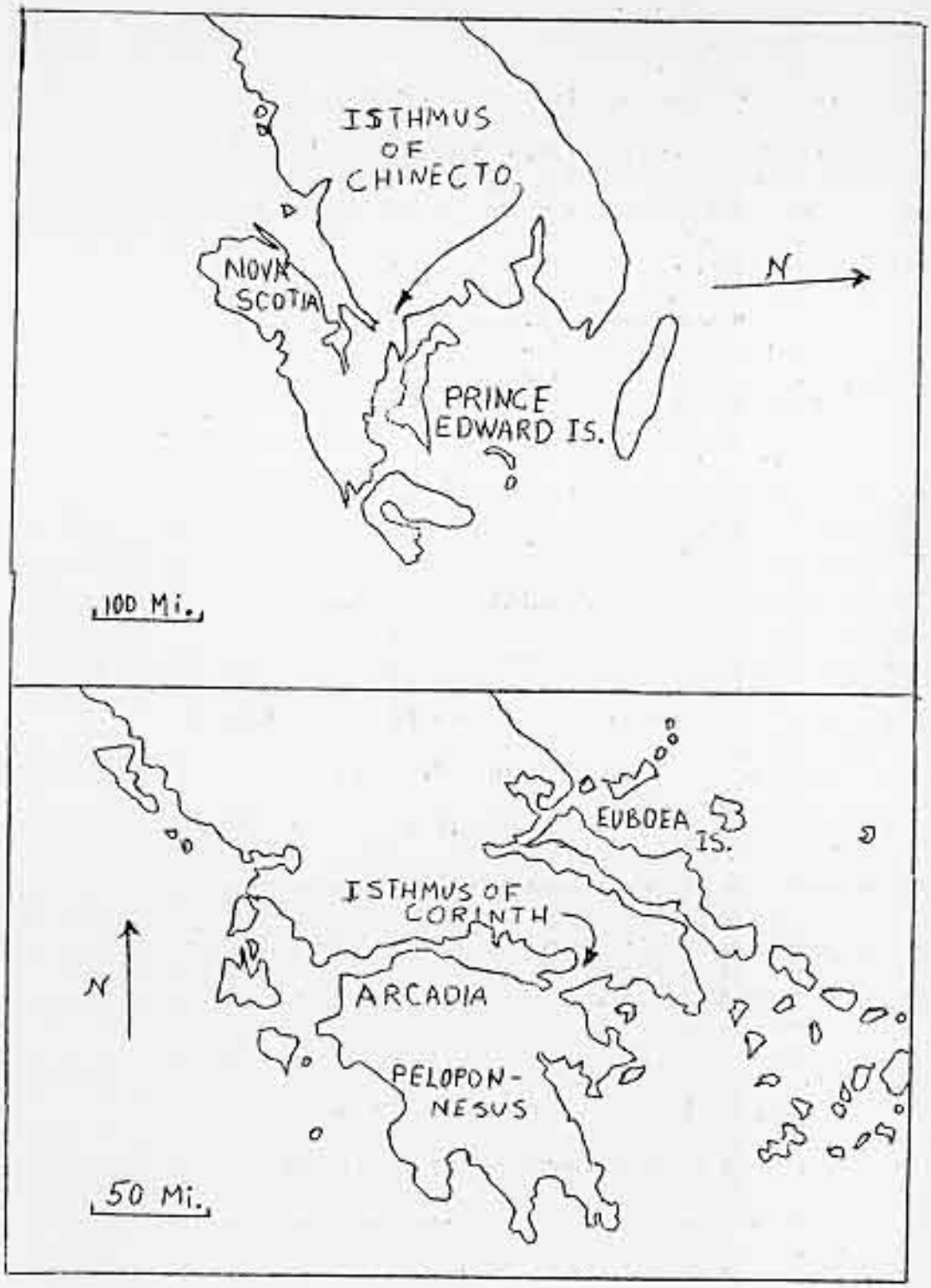
Haiti and Acadia

What is the task of the investigating geographer? To learn to separate the systematic aspect of each land from the physiognomic. The physiognomy throws a thousand veils over that self-same ever-recurrent system. The distinction is essential, fundamental. How will the geographer deal with it?

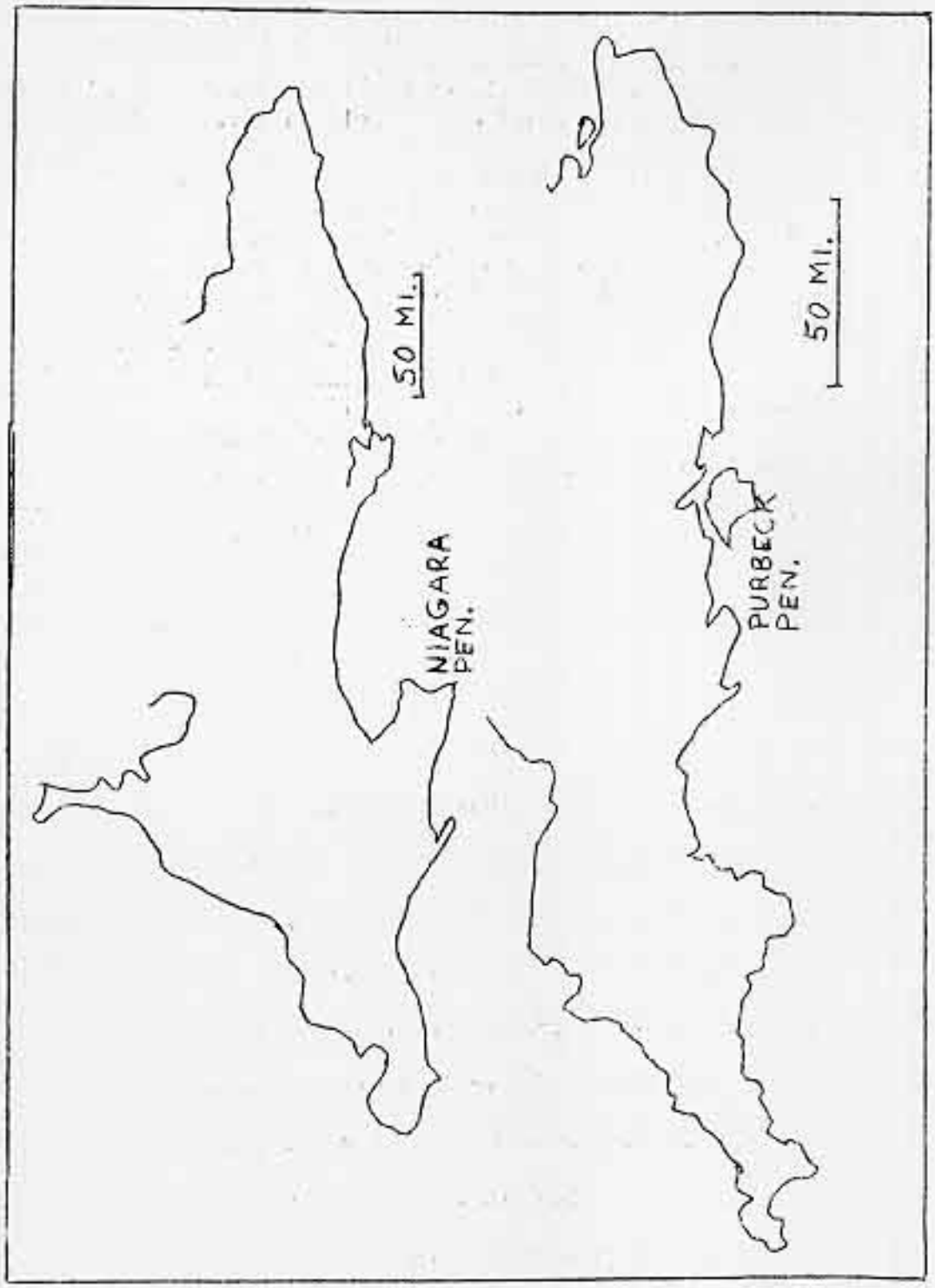
Our own way offered no short-cuts. It consisted of spending thousands of hours spread through consecutive days and weeks and months and years amounting to two decades in the perusing of maps: these of varying scales, symbolisms, contour lines; hydrographic or orographic; physical, political, economic or historical, etc. Once one becomes thoroughly familiar with the individual features of the world, and their numerous images are stored in the mind, the close observation of any area on a map brings back to the consciousness related images stored in the memory. Should I mention Brittany, will the reader be tempted quickly to think of Alaska or Turkey, to name only two others, will he be tempted to go a step further and think of the county of Retz, across the Loire, appended to and belonging to Brittany? Will he think of the Alaska Panhandle as another appendage? Will he think of the Sandyak of Alexandrette returned to Turkey two decades ago? These countless items of reference constitute a fund of experience indispensable to build on further and more intricate experiences. The investigator needs to be involved with the more obvious experience that areas such as Alaska, Brittany and Turkey afford before he can move on to less obvious ones such as Wisconsin or the German coast of the North Sea. Between this last area and Alaska, the structure is not the same, the physiognomy is not the same, but the profile of both coastlines is the same.

Thus the process is successively compounded from the more obvious to the less obvious, and thence to the introduction of concepts. Does the Alaskan type of morphological occurrence constitute a natural appanage of the west? Along the same line, but as an opposite, do such peninsulas as Kamchatka or Gallipoli constitute a natural appanage of the east?

The process of identification of parts is followed by one of distribution



Greece and Acadia



South Channel Coast and South Coast of Great Lakes

and organization. A cluster of features two or three or four times repeated is a clue to what to expect (or what yardstick to bring) in an additional area. Thus are identified what we have called land isolates, or self-contained areas presenting a recognizable complex of large-scale features.

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How do we prove truth? In an age of philosophical abstraction a rather strong materialistic rebound is not unexpected. Besides, as graphic a subject as geography, with its field boots, landscapes and feel of the earth's surface, this earthy subject, could not be held in Whitehead's hypnotic trance for long.

While the discussion of maps as related to proof is somewhat buried in the middle of the effort, it was the triggering notion of the article and opens the possibility of proving certain geometries by direct geometric methods again. Yes, we geographers know what happened to hidden theorems in the straight edge and compass. We also know just how impossibly immature the state of the mathematical arts is relative to some of our basic problems.

One great advantage of formal mathematical proofs not mentioned is the possibility of those mysterious simplifiers popping out such as i's and e's, the strange parameters. But maps might have their own parameters, strangely reoccurring patterns.

The material is part of a larger effort in preparation.

Truth

William Bunge

The human mind, like the other organs of the body, evolved for reasons of straight Darwinian biological survival. There is no reason to believe that the brain contains any more adornment than the human stomach... It exists no more for its own sake, for self-gratification, than does the liver. The separation of mind and matter is a false separation. No such dichotomy exists. Only matter went into the creation of the brain. The mind need not consist of magical nonsensical stuff for religion to be meaningful. Whispy, imaginary vapor in the cranium is too crude a foundation for religion. The mind is obviously not in perfect balance with its environment or it would never have produced the radioactive poisons that so threaten it with extinction, but then neither is the rest of the human organism so well adjusted. The feet too seem half way from hands to hoofs, from prehuman to human. Still, "thoughts" with possibly some

efficient low level random noise, and certainly all "systems of thought" are either a direct response to current survival or past, that is, thoughts are as much evolutionary features as fingernails.

What system of thoughts exist in the human mind? It is not necessary that the brain divide its survival work as neatly and with as much even-handed balance as a college catalogue. A mixture of computers of varying practical-survival importance might be the most efficient system with considerable cross communication between the organ's subparts.

The table shows examples of systems of thought. What is the history of these forms? Many appear in prehuman evolutionary stages. Chess and go, the greatest games, came into being considerably after man came down out of the tree. The first maps, drawn in the dust of some cave, have been vastly improved. Certain truth systems, such as science and mathematics, came into conscious division of labor only most recently. It is to be expected that other aspects of the human mind will be discovered and new truth systems emerge into consciousness.

On the surface, the truth systems seem totally unrelated with the exception of the already stressed fact of their Darwinian common origin, but other commonalities emerge. Each truth system has a "pure" form. Many practitioners of the special circuits in the brain feel themselves to be totally motivated by "impractical" impulses. "Art for art's sake," "Mathematics for its own sake," and so forth. But the ultimate survival purpose of the function being performed does not have to be clear to the practitioner. A farmer might take pride in his ability to plow a straight center furrow and might even enter a plowing contest "for its own sake," but we can plainly see this activity is related to growing food. The farmer hardly thinks to himself every second he is farming, "I'm growing food so the species can survive." Such a constant thought signal would not be survival efficient. Idealism, no matter how strong the subjective pull of it, can be explained as materially efficient, but materialism cannot be explained, with any conviction, on the basis of idealism. It is true our minds might be

Thought System	Science	Humor	Religion	Games	Justice	Art	Senses
"Pure" Form	theory	contra-diction	faith	play	laws	rules	sensations
Proofs	experiment	jokes	worshipping	contests	trials	works of art	feeling
Truth death life	disproven proven	obscene humorous	blasphemous spiritual	losing winning	illegal legal	ugly beautiful	painful pleasurable

Thought System	Logic	Instinct	Space	Emotion	Wisdom
"Pure" Form	relation-ships	reactions	patterns	feelings	judgments
Proofs	"mathematical" proofs	living	maps	getting to know someone or something	experience
Truth death life	false true	a don't a do	lost oriented	hated loved	foolish wise

sitting in a warm saline solution in the year 8,000,000 and our sense nerves being fed artificial perceptual stimuli from an enormous computer, but this is too tortured. Rod Serling does not have the believability of Charles Darwin.

The "for its own sake" feeling has been extremely seductive and has led to all sorts of nonsense including the essentially unflattering assertion that much mental work is "useless." But this is so much in contradiction to the facts, such as the tremendous utility of "useless" mathematics, that a mysterious explanation had to be offered that the conscious pursuit of useless curiosity was useful. How metaphysical! Yet this "rationale" has much currency even among scientists...

Some calculations have been made as to the amount of digital computer apparatus that would be necessary to recapitulate the mental processes of the human mind. The calculation indicates an enormous computer requiring great energies and producing serious quantities of heat. But why not a series of computers specially designed for special survival problems; a mix of computers pragmatically evolving with some abilities to intercommunicate. Such a strategy would explain several mysteries, including the tiny size of the brain relative to its ability to compute.

Scientists and mathematicians often achieve success by direct reliance on a sense of symmetry or even more deep feeling of general beauty. How could this be? Is this magic? But assuming that the mind can afford no more ornamentation than the rest of the organs, perhaps the aesthetic sense is survival prone, a direct analogue computer cutting through the "normal" logic of "formal proof."

Mach writes:

> ...In every symmetrical system every deformation that tends to destroy the symmetry is complemented by an equal and opposite deformation that tends to restore it.... One condition, therefore, though not an absolutely sufficient one, that a maximum or minimum of work corresponds to the form of equilibrium is thus supplied by symmetry. Regularity is successive symmetry. There is no reason, therefore, to be astonished that the forms of equilibrium are often symmetrical and regular.

Toth writes:

> Besides...classical theory, regular figures may be approached in another way, starting from the observation that extremum postulates often involve regularity. Classic theory starts with a more or less arbitrary definition of regularity. Here, in turn, regular arrangements are generated from unarranged, chaotic sets by the ordering effect of an economy principle, in the widest sense of the word. This theory may be called the genetics of regular figures.

When Maxwell "mysteriously" balanced the partial differential equations he was being efficient. Tobler and I and countless others have had similar experiences in dealing with mathematical solutions and I have had a sense of symmetry and similitude especially in discovering why rivers and other lines are dendritic. It was originally on purely aesthetic grounds that I favored Christaller's "fixed k" assumption over Losch's. This removes the mystery. The human mind developed a set of circuitry that short cuts the logic of extremum and simply "instinctively" selected that which was "beautiful." But the deeper question is not the question as to why aesthetics helps scientists and mathematicians but is artistic expression a direct path to truth? If symmetries and perhaps much deeper instincts toward beauty help "formal" logicians, what do they do to artists themselves used directly in their hands....

Much special computing capacity is devoted to spatial problems. The seeking of game, the hiding from enemies, the searching for food, the problem of being lost, all are deadly biological survival situations. A cab driver can find a close approximation by "instinct" of the least time path down a polar geodisic coordinate system that is constantly changing shape. He goes extremely "straight" in an extremely complicated crooked world without even the benefit of high school calculus....

The fundamental reason for proof is that the human mind-survival computer needs help in directly perceiving the truth when making certain survival judgments. Proof helps reduce human error and therefore helps survival. Error is the key; the total error in judging the truth so that the organism can properly

respond to its environment and not make so many deadly mistakes that the species perishes. Notice perfection is not the goal. All organisms make mistakes, even individual deadly ones. Squirrels fall out of trees to their deaths, birds fly into mountains, fish drown in air and so forth. If a "perfect" solution to any one problem were computed, the brain would have to be too large in the circuitry to afford the "correct" answer. "Perfect" answers are too expensive. This is the Error of Mathematicians in spite of their modern acceptance of probability theory. It makes no survival sense to have a perfect computer if other errors are going to nullify the "logical" perfection as we know from the theory of errors. Consider the errors. One, the errors of perception of the real world, data input. This error is usually much greater than the form in which the data is presented suggests. For instance, maps of rainfall or anything else are almost totally believed but a great deal is not known about the data: The individual error in each rain gauge; the great distance between rain gauges: since no one has ever done a detailed study of the falling rain, say every meter over 100,000 square kilometers, who knows if the shape of the approximating function so blithely drawn by the TV weatherman bears any resemblance to the actual surface whatsoever? The behavior of the second derivatives of such a map look suspiciously like the contours of the earth's terrain and the actual rainfall might be more closely approximated by a step function for all we know. Two, the errors of approximation in order to fit the system of logic to be used. In practice, we always abstract from the data, already at best dimly perceived, in order to fit our system of proof. In applied mathematics even continuous functions of continuously conceived real life variables are only approximately fit. Not only do the least square errors enter, but the selection of the function to be fit can give great error, for instance, the least squares fit of the downtown skyline to a sine function! Once the imperfectly recorded data is squeezed into some form of proof, we encounter the third error, the imperfection of the logic. The practical problems of survival that mankind faces prove intractable to the

proof systems developed. Certain cheating emerges. For instance, in the proof system of mathematics, the mathematicians insist on stupendous errors in cramming the data into the existing logical machinery. If a problem from the real world of survival arises as to the location of a highway between city A and city B, all known factors involved affecting the highway location, pollution, travel time, cost to builder and so forth can be placed in some common utile measure and this continuous distribution mapped. If the surface is not topologically simple, and it will not be, analysis will not be possible to apply to trace out the total least cost line. So, blithely, the problem is "simplified" into a series of squares with some average utile cost assigned to each square. Now the mathematics might prove manageable and assuming not too big a matrix and a muscular computer, the "exact" solution might be forthcoming. But look what the mathematician has done! In order to secure a "perfect" mathematical solution in the algebras, he has forced an increase in the error of the second type. He has forced the data into a discontinuous algebra. But if the error of the second type is increased to the point where it is larger than the "logical" errors, what is the survival-gain? The "practical" solution is in greater error though the mathematical part is perfect. The <u>fourth error</u> is that mankind has no complete theory. Even if perception were perfect, data fitting perfect, the proof mechanism perfect, the theory would be incomplete. With the highway problem perhaps future switching to electric batteries in automobiles will eliminate the pollution consideration and thus this future effect must be partially discounted from the present location problem and on and on. Since the death of the illusion that Newton had determined the Universe to be a Great Clock, science is increasingly pessimistic about its ability to have a universal theory of Truth. Science's work will never be done. How much Truth can the human mind be expected to understand? It has been a great human error to assume that the function of the brain was to discover The Ultimate Truth. Blessed are the humble, baby. The function of the brain is to enable the human species to survive, a modest goal,

just like the function of the nervous system of the ant is to enable that creature to survive. We therefore must stop looking for the Ultimate Truth in theory. The Mathematician's Error is his refusal to accept that the best the human mind can do is to distribute the error in such a way as to arrive at the nearest thing to Truth that its limited capacity will ever allow so that we can continue to exist as a species. All the mathematician's insistence on "perfection" has done is up the errors among the other three sources of error, thus enfeebling the Truth of the applied product.

Having faced a Total Theory of Errors, we can sensibly raise the question of what is proof. The mind has found Darwinian virtue in what we called "formal proof," mathematics-logic. Crows can count up to a certain number of hunters entering and leaving the woods and avoid a shooting up to a small number. We do not have to recapitulate the evolutionary history of this advantage. The proof of the pudding lies in the mind's ability to think in this direction just as the best proof of the utility of the human eyelash lies in its existence. But the mind needs help. The help is called "proof." For example,

Prove: $8a + 16b - 3c = -10$
Given: $16a + 32b = 6c - 20$

1. $16a + 32b = 6c - 20$
2. $8a + 16b = 3c - 10$
3. $8a + 16b - 3c = -10$ Q.E.D.

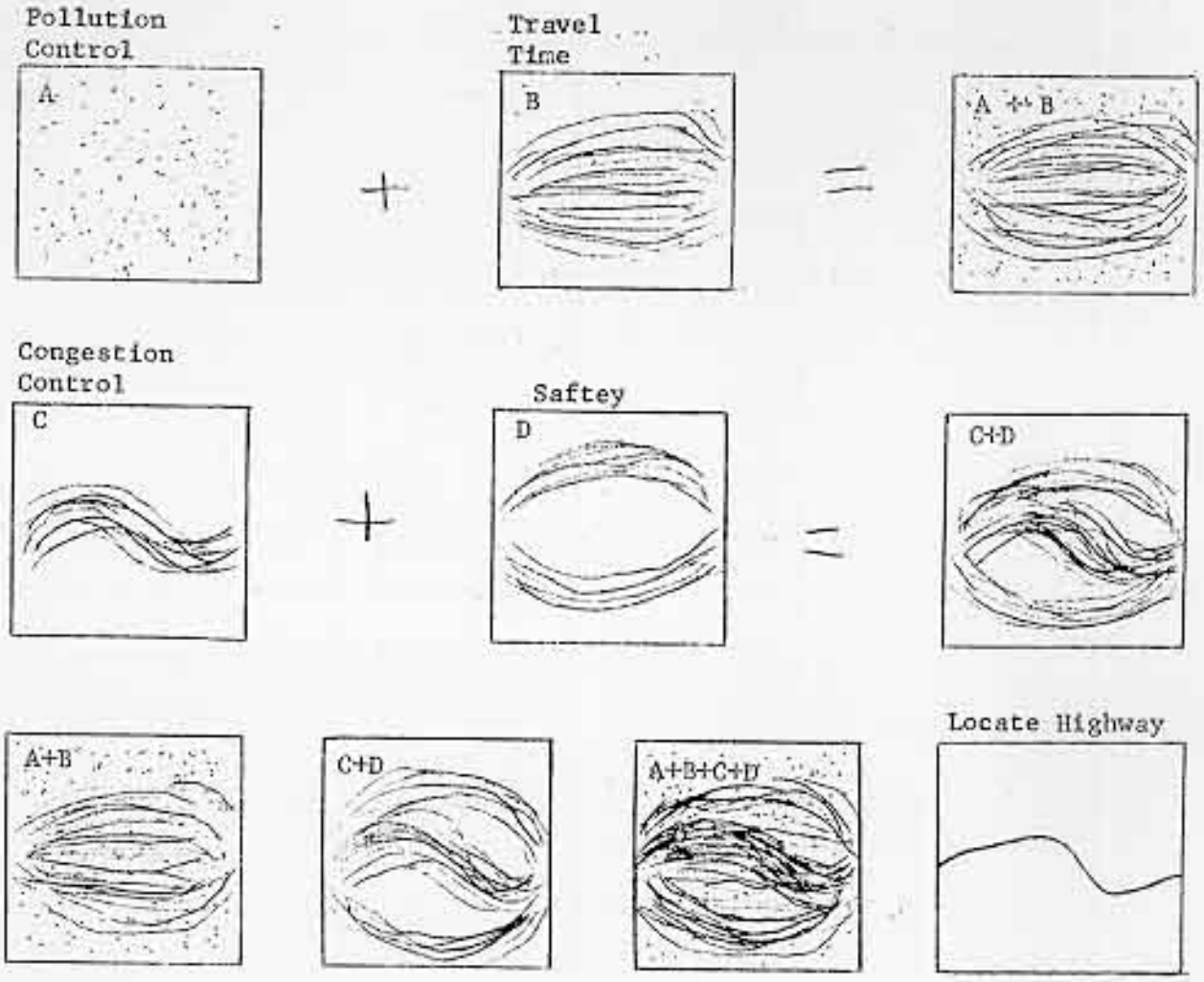
You might object that the proof was trivial and unnecessary. But you might argue that all proofs are trivial and unnecessary, as R.A. Fisher did. Proofs are a waste of time since mathematical truth can be directly perceived. But most logicians feel better with the proofs, the step by step visual aid so that the human computer, which did not develop this memory in many of us, can be aided. Not all "formal" proofs are so full of certitude. In the problem of finding the best location of a highway, the problem might be even too much for the best modern computer and use of the most modern algorithm in finding the single best solution. Then an approximately good solution might be found. The

approximate solution might contain an estimate of the error, the approximation of the approximation as in probability theory, and it might not. Simple simulation techniques might be used to establish a tolerable error. Scientists are rather well aware that all is fair in love and science. Any dirty method will do.

The proof lies on the sheet of paper but the truth of the proof lies in the human mind. The proof does not give birth to itself. Placing Step One on the paper and leaving in a warm saline solution does not give rise by birth to the other steps. The "proof" is a visual aid and the judgment of the Truth is always in the human mind. All proof systems, even "abstract" mathematics, are perceptual. Mathematicians should examine what they really do. Put little black marks on white pieces of paper.

But logic-mathematics is not the only Darwinian brain circuitry that exist. Men have different kinds of thoughts, different brain processes for arriving at survival-truth. Toulmin points out that the map does not tell the viewer anything that the viewer does not compute in his mind. To "prove" that some town is so many miles from another one can submit a map and measure the distance on the map. Assuming the map is free of error (preposterous) one can exactly (ridiculous) prove the distance. But to come closer in illustration to mathematical proof, return to the problem of locating the highway between two towns. A map of every factor that seems to enter can be made as Alexander has done. Each factor can be weighed by its relative merit as Alexander has not done but Roberts has. The weighing can be done perceptually so that a factor three times more important is three times blacker. One map can be placed on another and the two combined. If the darkness of the imposed maps begins to make the map solid black in appearance in any part, the value can be lightened on the combined maps. After all are combined, the best looking, darkest appearing route can be traced by a pencil. The proof is complete. Man starting with the same approximate value systems, the given set of original maps, will draw approximately the same

ultimate highway system.



Q.E.D.

But think of the error! But proof has nothing whatsoever to do with error elimination, only error reduction! Just listen to the outcry of the mathematician. To be "correct" why not find a numerical utile value for each point on the map (really, each point!), that is, an approximate utile value for approximately each value on the map and then find the line integral (intractable!), well, all right an algebraic approximation, and assuming the computer will not smoke too badly, you will be given an "exact" and "provable" best single solution, provided your dirty old theory was true. But how can the error of the map proof be estimated? Well, it obviously is not an infinite error, in fact, it seems to be pretty close to the Truth, besides we do not know of a better way to estimate highway location, and on top of that a better way may never be a "for-

mal" say, it might simply be an improved visual way. Look what is going for the maps as systems of proofs--all the evolutionary apparatus of space computing. Error might be estimable by psychological testing, but remember error in the proof might be desirable: why have a proof that is better than data input or incomplete theory. Maybe the human mind also evolved its own practical theory of error. It can be argued that replication is difficult. What Total Problem in real life is not, but approximate replication should be achieved. It can be argued that novices would not do as well as the practiced. This is different with mathematical proofs? It can be argued that none of the steps are necessary. Well good, if you happen to be an R. A. Fisher of maps, but most of us find perceptual errors reduced enough to make the map steps worthwhile. It can be argued that we have no sound theory at all for drawing in the line of the highway from the darkened area of the map. It looks pretty good to my evolved eye-mind and that is good enough for most highway locations. After all, how do we locate highways in real life?

Artists also use proofs. The beauty-truth is perceived in the eye and mind of the beholder and if we all were R. A. Fisher type artists we would not have to have the proof of the "work of art" to see the artistic truth. We could all be direct artists and in part we all are. What do artists say about their work? They say that they start with a certain situation, a set of "givens" and the worl more or less forces itself if they, the artist, have the courage to be honest. The novel writes itself, the play itself, the painting draws itself, step by step and it just has to come out the way it does. There are more random artistic works, more error in the proof and perhaps less in the viewer, for errors must be balanced and cannot this interpretation be given to the debate against the perfection of classical realistic art? The truth may be perfect on the canvass but this merely shifts the error to the perceiver, or the original data, or the total theory of truth, which is incomplete in artists as well as scientists.

How does one know, when does the mind tell us, that the proof is complete?

Again, coming back to evolutionary essentials greatly simplifies the problem. Proof is a step by step transformation that cuts down perceptual error. It is complete when the problem, given by life, is transformed to the point where the person or group puzzled by the problem knows what to do. All thought systems--science, humor, religion, games, justice, art, senses, logic, instincts, space, emotion and wisdom tell us that war in this age is disproven, obscene, blasphemous, losing, illegal, ugly, painful, false, a don't,lost, hated and foolish. All systems of truth signal Death.

But not all truths require proof. It is foolish to prove what is known to be true. Now immediately people are going to come running out of the woods displaying every paradox in the book. Yes, intuition can be terribly wrong. But more impressively, it can be terribly right. How many decisions are made by "formal" proof? What is the percentage of error from intuitive failures? And even what about Goedel and truths that lie outside proof even in formal logic? It is wasteful and ultimately deadly dangerous, to spend time proving the obvious.

X X X X X X X X X X X X X X X X

What is a "natural" language? The last two articles are the works of students. Young minds, like young atheletes, are the best ones. Karlin is going to peal off the map patterns and read the language of the maps like a latter day Rosetta Stone. This paper, the most formal, somehow pulls much together.

Shapes as a Group

Andrew Karlin

Geographers have always studied shapes, but we have rarely worked directly with them in a rigorous way. There are probably several reasons for this neglect, but perhaps the two most important are, first, our tendency to regard a region's shape as the spatial limits to some phenomena under study, and, second, a lack of systematic and rigorous methods. Currently, however, we are discovering the importance of shape in its own right. Two good examples of this are Bunge's Theoretical Geography and Alexander and Manheim's The Use of Diagrams in Highway

Route Location.

If, however, we find it useful to manipulate shapes, or what is the same thing here, figures, how are we to do it? But this is really two questions. First, if we manipulate shapes are our answers meaningful? For example, if we impose a circle over a triangle does the result have any significance? Moreover, is any curlicue a figure in the same sense as a circle, with its tidy geometry and simple equation? Second, if we attribute some meaning to the sum, what are the mechanics of the addition? In practice, of course, these questions of meaning and technique are bound together.

Bunge has suggested that we give formal answers to these questions through group theory. A group, according to Keyser, is a special type of system. That is, a group is a class, or collection of things, with some definite rule, or way, in accordance with which any member of the collection can be combined with either itself or any other member. More precisely:

> Let S denote a system consisting of a class C (whose members we will denote by $\underline{a}$, $\underline{b}$, $\underline{c}$ and so on) and of a rule of combination (which rule we shall denote by the symbol o, so that by writing, for example, $\underline{a}$ o $\underline{b}$, we shall mean the result of combining $\underline{b}$ with $\underline{a}$). The system S is called a group if and only if it satisfies the following four conditions:
>
> (a) If $\underline{a}$ and $\underline{b}$ are members of C, then $\underline{a}o\underline{b}$ is a member of C: that is, $\underline{a}o\underline{b} = \underline{c}$, where $\underline{c}$ is some member of C.
>
> (b) If $\underline{a}$, $\underline{b}$, $\underline{c}$, are members of C, then $(\underline{a}o\underline{b})o\underline{c} = \underline{a}o(\underline{b}o\underline{c})$... that is, the rule of combination is associative.
>
> (c) The class C contains a member i (called the identical member or element) such that...$\underline{a}oi = io\underline{a} = \underline{a}$...
>
> (d) If $\underline{a}$ be a member of C, there is a member $\underline{a}'$ (called the reciprocal of $\underline{a}$) such that $\underline{a}o\underline{a}' = \underline{a}'o\underline{a} = i$...
>
> Other definitions of the term "group" have been proposed and sometimes used. The definitions are not all of them equivalent but they all agree that to be a group a system must satisfy condition (a).

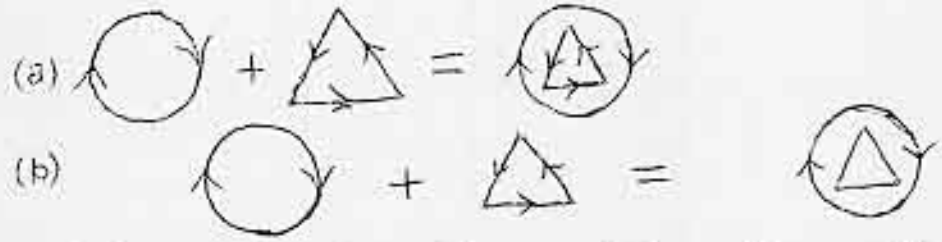
Let us assume that any figure or shape has an equation, though perhaps unknown. This resolves the semantic problem of figure and shape raised earlier. Further, let us assume that we can assign vectors to figures, an assumption first suggested by Nystuen. For convenience' sake, a "counter-clockwise" figure is positive and a "clockwise" figure negative, as the figures below illustrate.

positive

negative

Our rule of combination, then, is addition. To add two figures, we impose one upon the other. For example, put each pattern on a separate sheet of paper, and then lay one sheet upon the other. Place the two sheets on a light table. The pattern showing through is the sum, and may be traced directly. Adding a positive to a negative figure which is otherwise identical results in a blank or an undirected figure, the counterpart of zero in the number system. (Although we normally think of two equal but opposite vectors as canceling each other and would expect the result to be no figure, it is perhaps useful to let an undirected figure act as a place-holder equal to blank paper.)

Two questions remain, however. First, what is the algebra? Alexander and Manheim look for the number of times shaded areas coincide, measuring coincidence by the intensity of the shading. They use the simple additive algebra we are accustomed to, $\underline{a} + \underline{a} = 2\underline{a}$. But a Boolean algebra is also possible: $\underline{a} + \underline{a} = \underline{a}$. That is, when we lay a pattern over an identical pattern the result is one pattern, not two. The lines are not twice as black as before. This first question of algebras suggests the second. When we talk about figures we may mean either the outline alone or the area within the outline. Their different additions are shown below.

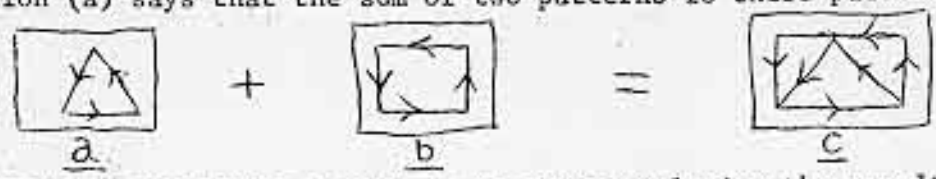


In case (a) we add just the outlines, and the sum is a positive triangle "inside" a negative circle. In case (b), the addition of areas, the sum is a negative circle with a triangular "hole" or zero-area within itself. The intersection of a positive and a negative figure is an undirected zero vector; that

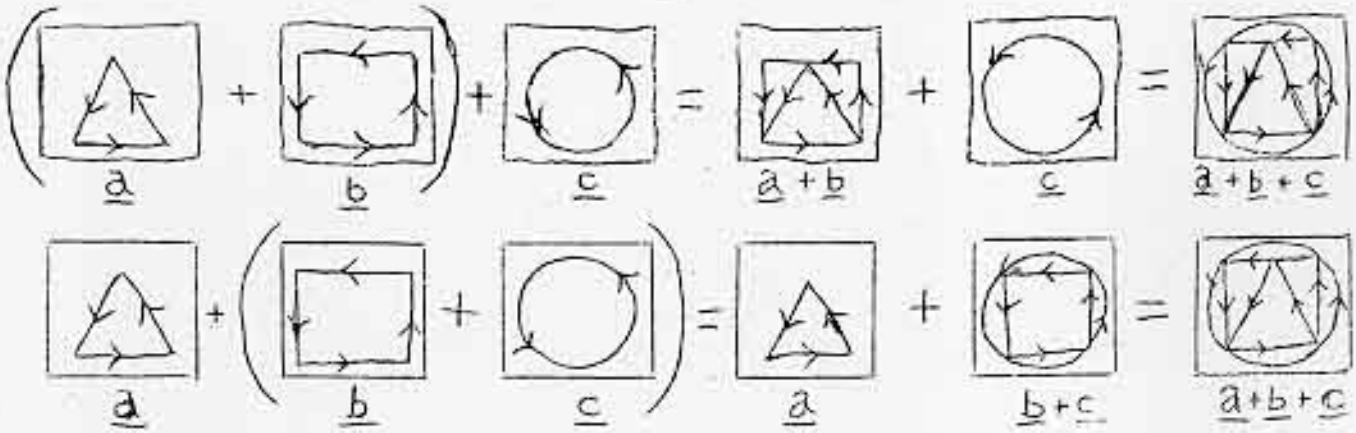
is, an undirected vector whose length is zero.

Returning to Keyser's criteria with all this in mind, let us take the simplest case--adding outlines by Boolean algebra. The other cases are all analogous to this.

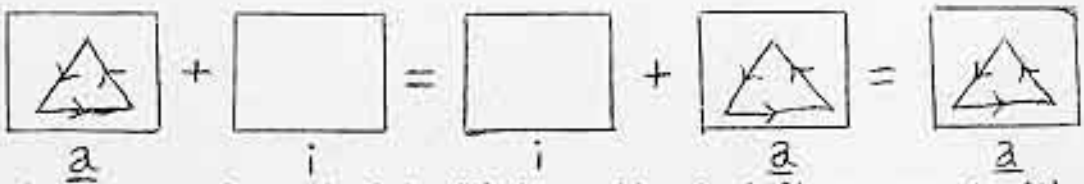
Condition (a) says that the sum of two patterns is third pattern.



(B) says that overlaying a with b and then overlaying the result with c yields the same pattern as overlaying b first with c, then a.



(C) is fulfilled by using a blank paper as the identical member.



The reciprocal required in (d) is an identical figure except with a different direction.

There is an alternative way of working with figures developed by John Pfaltz and Azriel Rosenfeld for computer applications. It is conceptually simple, although not conveniently workable with a pencil and paper. "Any region can be regarded as a union of maximal neighborhoods of its points, and can be

specified by the centers and radii of these neighborhoods." This set of centers of maximal neighborhoods often forms a centrally located stick figure and so is often called a "skeleton." There are algorithms both for converting regions (or figures) to skeletons and for regenerating regions from skeletons. Pfaltz and Rosenfeld describe the set-theoretic operations for determining the union, the difference, and the intersection of two regions. The identical member here is a skeleton of centers with zero radius. Thus, these skeleton figures are also a Group.

The immediate point of this paper has been that figures are a group. But more importantly, what I have hoped to show is this: we geographers can handle shapes in an exact, simple, and direct way. When we talk about patterns on the map we mean just that, but we have almost always either spoken of shapes in a vague and abstract way, or, attempting to be more precise, converted shapes to abstruse mathematical functions. But there is no point, no gain, in moving away from our data before it is collected.

X X X X X X X X X X X X X

The last paper is filled with the traditional cry of pain over the basic disarray that prevades the logic of geography. But to the editor it raised an interesting question. Is the four color problem "really" geographic?

Two Theorems for Geography

Richard Guyot

Geography has forsaken its core for its adjectivally applied fields: physical, economic, urban, cultural, perceptual, ad. nauseum. The literal meaning of geography is: earth + graphe (description). According to Cassirer, the philosopher, the core of description is some generic concept (1923). The description and study of form is morphology, therefore the morphology of things on the earth is geography. Science is a way of knowing and the science of geography is the study of patterns as morphological laws (Schaefer, 1953, p. 226-49).

Geometry is nominally the "measure" of, or a calculus for figures "on the earth". There are actually many kinds of geometries, some are metric, others, such as topology, are relational. If patterns can be conceptualized by metrics or relations then there is a need to accumulate useful morphology theorems under a geographic geometry.

Geographic geometry can be discerned in the stacking of airplanes over airports and pistons in the Ford Rouge Plant. Few geographers do field work let alone see the pure spatial implications in sub-assembly and assembly lines. The dendrite can be discerned in sewers, trees, commuters (Warntz and Bunge, Geography, The Innocent Science, 196?) and in crystal growth in metals (Bell Telephone Laboratories' ad in Scientific American, Sept., 1967, p. 33). Time and motion studies could be called applied geography. For some reason geographers rarely consider anything shown at a scale larger than 1:24,000. They seldom consider anything with an R.F. greater than one or anything that occurs indoors. Since geography can't logically be a jack-of-all-trades it cannot be a study of everything on the surface of the earth. If geography is the study of processes on the surface of the earth then it must contain practically all knowledge. If it is interactions then it should logically be all interactions. It should not be the study of everything no one else wants to study (e.g., climates, superficial sociology, and left-over landforms from geology). Thus geography can fall in less academic disfavor by proceeding logically with the use of patterns and other spatial constructs for description. The solution to geography's lack of academic legitimacy is to develop a core field of theoretical or general geography. This geography must be made up of patterns of elements.

In reality this has been done through terms as Mackinder's Heartland, Colby's Centrifugal and Centripetal (which have been renamed gravity models), and time honored site and situation to cite a few examples not used by "mathematical geographers". The new generation of mathematical geographers neither recognizes its predecessors' accomplishments nor passes much beyond neanderthal mathematics

as compared to the level used by freshman engineering students. A quantitative flash-in-the-pan resulted from Sputnik in 1957 when all the humanities lost out to psuedo-science. The mathematical geographers have failed because they discovered well-known statistical techniques (to the social sciences) and borrowed pieces of theory without building generic concepts.

It is wrong to require a "new" science to provide all the answers instantaneously. However, it is also wrong to allow people to feel they must mathematize course titles without providing them the elements of a logical system. Mathematics is but one logic system. One can't "prove anything with statistics". Statistics and mathematics are abstract tools. Any fault or gain lies in the intellectual work done in applying abstractions.

Such criticism of quantitative geography has already been made by many. "The changing of the key concept is in itself more likely to be a recombination of older knowledge rather than something completely new" (Bunge, "Simplicity," 1968). The "old geography" will work provided there is at least a framework. Geographic knowledge must be set down in the form of definitions and axioms from which geographic theorems can be built.

As a start, what follows is an attempt to define mapping and map features. In the spirit of the Micmog Discussion Papers two working theorems are presented without formally determining proper axioms. Only through further work can the definitions used be worked up into good axioms.

Definitions

A dimension is normally defined in dimension theory as "...$\leq$ n if an arbitrarily small piece of the space surrounding each point may be delimited by subsets of dimension $\leq$ n-1." The dimension 0 is bounded by the empty set $\{-1\}$ (Hurewicz and Wallman, 1941, p. 10-24). This definition is not rigorous enough for mathematicians but will provide a guide for an intuitive geographic definition. In geography, the definition of an object (herein called a figure) of n

dimensions is that it is bounded by a figure of dimension n-1. For the sake of visual conceptualization only, a figure may be considered an object. For example, a line is bounded by points. Areas are bounded by lines. Volumes are bounded by surfaces (Warntz and Bunge, Geography the Innocent Science, The dimension $n < 0$ will be defined later. All dimensions are integers.

Each n-figure may be considered as an uniform region according to set theory or Bunge (1966, p. 14-26). For example if an agricultural area ($n = 2$) is divided by a line (n-1) it is implied that a difference exists on either side of this line. This line is a common boundary between two different types of agriculture such as wheat and corn. Therefore each common boundary of dimension n-1 separates distinguishable figures (regions). If two figures are not distinguishable then their separation is not distinguishable. That is, a partition that bounds nothing does not exist. Conversely if two figures are not distinguished by a partition then they are uniform throughout and exist as one figure only. This concept has already been established in Hudson's Unit Area method. It only has to be expanded to n dimensions.

The Meeting Theorem

This theorem only states the upper limit to the number of dimensions that figures might meet in. Meeting is defined as the contact in common in the form of a mutually inclusive figure of dimension n_c (called a meeting figure) between figures of all the same dimension n. That is, a figure of dimension n_c exists such that every part of it will be simultaneously common to m figures, all of the same n.

Where: n_c is the dimension of any meeting figure.
m is the number of figures, all of the same n, that are meeting.

n_{c_m} is the maximum dimension of a meeting figure for m, n dimensional figures. This maximum dimension is the highest absolute value of n_c. Absolute value is the highest integer regardless of sign.

Everywhere in the meeting figure is common to all m of the n dimensional figures.

For example: two lines ($n = 1$) meet in a point ($n_{c_2} = 0$), two surfaces ($n = 2$) meet in a line ($n_{c_2} = 1$), and two volumes ($n = 3$) meet in a surface ($n_{c_2} = 2$). Therefore, when $m = 2$ the maximum dimensioned meeting figure between <u>two</u> n-figures is n-1. This can be assumed since the highest dimension that is common to both n-figures is their n-1 boundary. Further, <u>one</u> figure of dimension n meets itself in itself ($n_{c_1} = n$).

Parallel lines, surfaces and other similar cases in higher dimensions obviously don't meet. However two lines lying together, having no "width" (n-1), must meet in their length. If they are both the same length then they can be classified as the same identical line. If a line is laid along the middle of a longer line the meeting theorem delimits three different segments: the left end of the long line, separated by a point (n-1 boundary) from the end segment is the short line segment (which is identical for both lines), and after another $n-1 = 0$ point the right end of the long line. This is simply Hudson's unit "area" method in one dimension.

Since two figures of n dimensions meet in $n_{c_2} = n-1$, then <u>this</u> n-1 meeting figure will meet the n-1 boundary of <u>a third</u> figure of the same n. Such that:

$(n-1)'$ meets $(n-1)''$ in $(n-1) -1 = n-2$. Thus $n_{c_3} = n-2$.

Since:

$$n_{c_1} = n$$
$$n_{c_2} = n-1$$
$$n_{c_3} = n-2.$$

By induction:

$$n_{c_m} = n - a, \quad \text{where } a = m-1.$$
$$n_{c_m} = n - (m-1)$$
$$n_{c_m} = n - m + 1.$$

It is useful to construct a table of the values of n-m+1 for ordinary values of m and n:

Where: n is the dimension of the figures.
m is the number of figures of the same n that are meeting.

m \ n	1	2	3	4	5	6 ..... m
0	0	-1	-2	-3	-4	-5
1	1	0	-1	-2	-3	-4
2	2	1	0	-1	-2	-3
3	3	2	1	0	-1	-2
4	4	3	2	1	0	-1
. . .						
n	n	n-1	n-2	n-3	n-4	n-5 ..... n-m+1

After n_{c_m} approaches zero the dimension of meeting becomes negative. A negative dimensioned figure is defined as an imaginary figure, much like the imaginary number i. For simple combinations it is possible to show the need for imaginary figures. Given three points, $n_{c_3} = -2$. Three points determine the shaded surface (Fig. 1a) which will "connect" or could "meet" these three points. Given three lines, $n_{c_3} = -1$. This implies (fig. 1b) that there "exists"

Figure 1

at least one imaginary meeting figure of -1 dimension which will be common to ("connect") the three lines (one of the three possible ones is shaded).

The imaginary dimensions of n_{c_m} provide the highest absolute valued dimension which could connect any given number of figures of the same dimension n. For example four points could be co-planar and would only need a -2 surface to connect them. At the worst, the fourth point could be not co-planar and a -3 dimensional imaginary figure would be common to the four points. This agrees

with the theorem which states $n_{c_4} = -3$ is the highest dimensioned imaginary figure needed for <u>any</u> four points. The same can be done intuitively for lines and surfaces.

The range for real meeting figures ($n_c > 0$) is: $n \geq n_c \geq 0$. The range for imaginary meeting figures is: $0 \leq n_c \leq n_{c_m}$ (assuming the absolute value of the imaginary dimensions is taken). For example, eighteen volumes might meet in a line (the axis of a cake cut in eighteen pieces). However, no eighteen volumes can meet in a surface (n-1). They can meet in a point or "less" (imaginary figures). Thus any number of figures can meet in the <u>boundary of a boundary</u> or n_{c_m} for $m \geq 3$. Only one n-figure can meet in itself $n_{c_1} = n$.

The dimension of meeting for figures of different n is the n_{c_m} for the lowest common n. Thus if a line, a surface, and a volume all meet they will meet according to n_{c_m} for three lines.

A mathematical group must be both associative and commutative with respect to operations. Meeting figures are commutative but not associative. For example:

Where: o indicates the operator of "meeting".

$n' \circ n'' \Rightarrow (n-1)'$, and $n''' \circ n'''' \Rightarrow (n-1)$.

Combining these two n-1 figures associatively:

$(n-1)' \circ (n-1)'' \Rightarrow (n-1) -1 = n-2$.

But, $n_{c_4} = n-m+1 = n-3$.

Further, each time one sub-set of m, n-figures is combined with another sub-set, a degree of freedom is lost. n_{c_m} holds for any m regardless of configuration or constraints. As can be seen above, only with certain configurations can four volumes meet in n-2 (a line). For example:

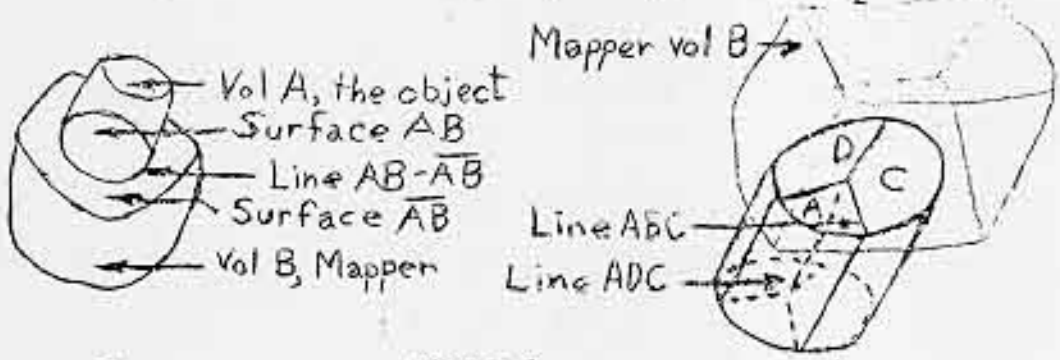
$$\begin{array}{l} n^1 \circ n^2 \Rightarrow (n-1)' \searrow \\ \qquad\qquad\qquad\qquad (n-1)' \circ (n-1)'' \Rightarrow (n-2)^1 \circ (n-2)^2 \Rightarrow (n-2)-1 = n-3 \\ n^3 \circ n^4 \Rightarrow (n-1)'' \nearrow \qquad\qquad\qquad\qquad \text{(lowest common dimension)} \\ n^5 \circ n^6 \Rightarrow (n-1)''' \longrightarrow (n-1)''' \end{array}$$

But, $n_{c_6} = n-m+1 = n-5$. Therefore <u>any</u> six volumes may be connected by an imaginary figure of dimension no larger than n-5 = -2. However only a certain

six volumes can meet in n-3 = 0.

The Mapping Theorem

Mapping is the recording of distinguishable objects such as occurs in the "unit area method". A map is defined as all or part of the n-1 figures which bound any n-figure that has been designated as a mapper. Any n-figure can be a mapper. A mapper of dimension n will map or record in the form of meeting figures all n-figures with which it meets. Therefore, again in the identity case, a map of a map is a map, since the meeting figure for n_{c_1} is n. A map is the common figure or interface between figures of the same n that meet. For example two volumes (n=3) A and B meet in a common surface (n_{c_2} = 2). If volume B is the mapper its n-1 surface will be the map on which m-1 other three-dimensional "objects" will touch. The map will show the surface of B composed of two regions: the common surface AB and the surface "not AB" (Fig. 2a). The regions AB and $\overline{AB}$ (read not AB) being surfaces have their own meeting figure, (n-1-1) = n-2, line AB-$\overline{AB}$. This line is common to both surfaces and also common to both volumes. This illustrates the distinction of the meeting figure n_{c_m} as the maximum dimensioned figure which is mutually inclusive. Two volumes could meet in a line or a point but they can not meet in any figure higher than n_{c_2}.



a Figure 2 b

If three volumes A, B, and C, with B the mapper meet, then the n_{c_3} meeting figure will be line ABC (Fig. 5b). Since the surface of mapper B not touching A or C is feature-less, it may be disregarded. Thus a map feature is defined as the meeting figure which defines (recognizes) any change in the mixture making

up a common mapper surface. Where surface feature AB changes to surface feature CB the feature-bounding line ABC exists. Figure 2b can be expanded to include four volumes, three "objects" plus the mapper B. The mapper will be excluded from the description but will be implied in calculating m for n_{c_m}. The three volume-objects A, C, and D all meet in line ADC. Since $n_{c_4} = 0$, three volumes can meet the mapper, the fourth volume, in a point. These three volumes form three different surface features or regions. Therefore these three regions meet in a maximum dimension of $n_c = 0$. Any three or two surfaces may meet in a line but these regions are bound by the constraint that they are the boundary of four volumes meeting at a point. The n-1 features must follow the constraints placed on them by the set of m, n-figures.

Implications of the Mapping Theorem

Most usual maps are two dimensional maps of the interface between the air volume (mapper) and the earth volume. Corn and wheat are actually three dimensional crops. A map of water, corn and wheat with air the mapper is shown in Figure 3.

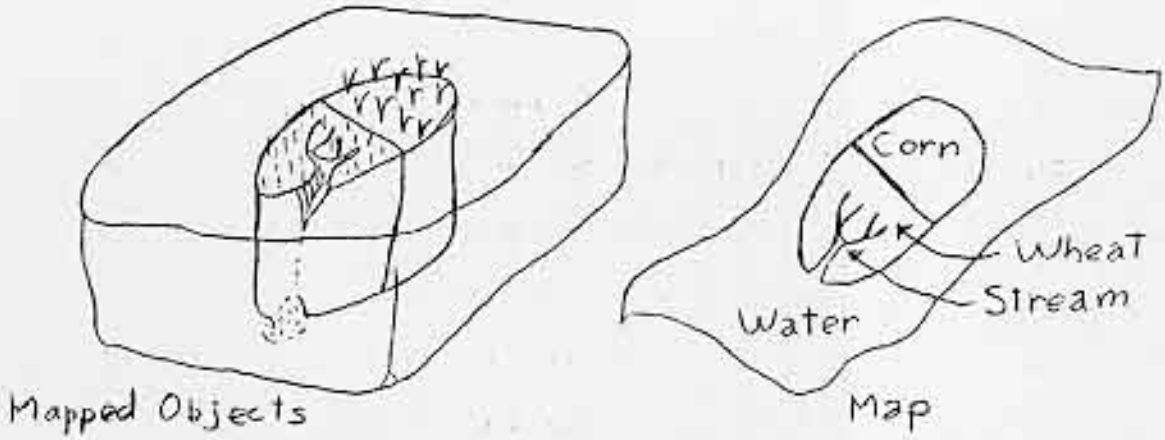


Figure 3

A contour "map" is simply the air volume mapping its interfaces with interval elevations volumes. From sea level to, say, 10 ft. is one volume. Where this volume and the water and air volumes meet is a line called the shoreline (Warntz and Bunge, Geography, The Innocent Science, 196?). The next line is

where the 0-10 ft. volume meets the 10-20 ft. volume, and so forth. The resulting two dimensional surface is a bit lumpy but it <u>is</u> a surface and can be flattened by transforming it (projecting on) to a flat surface.

A geologic cross section (n=2) and a planimetric map (n=2) meet in a line (n_{c_2} = 1) (Fig. 4).

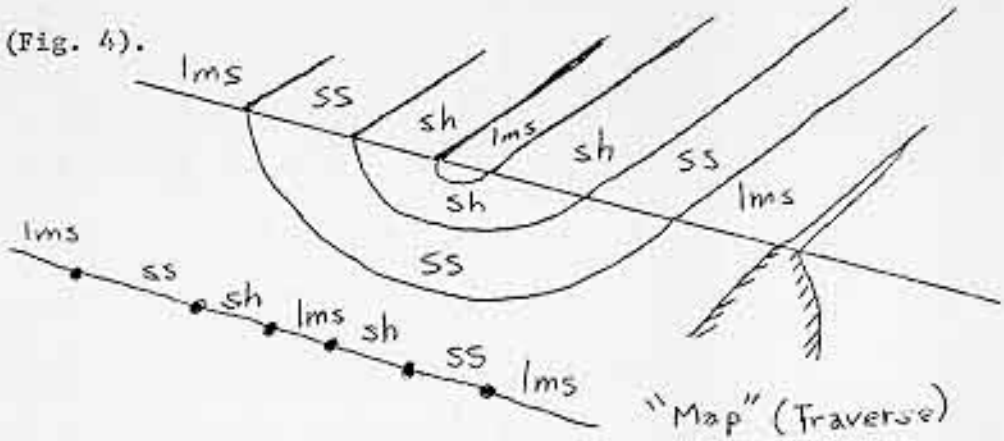


Figure 4

Imaginary figures (n<0) are maps. Given a group of points a planimetric map can be made. If the points have data associated with them a contour map (n=2) or even a three dimensional model can be made. The mapper volume for an aerial photo is the cone of reflected light from the objects that "meet" it. Oil and coal seams can be mapped using limestone for the mapping volume. Mariners sometimes still use an "armed lead" (sounding lead with a tallow filled depression in the base) to record the sea bottom. The lead is the mapper, the tallow is the meeting figure and it picks up sand, mud, or nothing if the bottom is bare rock.

Assume Figure 4 is a block diagram and an intrusive dike (which is a volume) touches the air volume in a <u>line</u>. The surrounding mantle rock intervenes so that the maximum meeting figure n_{c_3} = 2 is not realized. This line begins at a point and ends at a point but does not partition the earth's surface. However it is, in fact, a boundary of a boundary (n-2). The mapper only maps n-1 interfaces. Thus lines on maps may be from volumes or from surfaces intersecting. The stream in figure 3 changes from lines to an estuary. The stream is still everywhere an earth (wheat), air, and water interface.

If lines and areas can be explained as meeting figures, then patterns are

meeting figures. Patterns in two dimensions will have counterparts in n dimensions. Therefore all objects may be mapped in some way. Table 1 shows that two 3 dimensional figures and three four dimensional figures both meet in two dimensions. In fact, sixteen 17 dimensional figures also may meet in two dimensions. This implies that the same 2 dimensional map may represent an infinite number of objects of the appropriate dimensions. The same map patterns and morphological laws might run through a wide number of interacting objects.

Coloration and the Four Color Problem

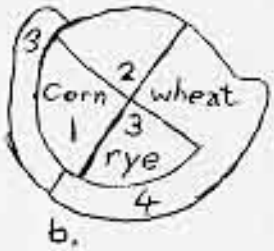
The four color problem suffers in part because of a lack of definition for coloring. The following definition is offered in the context of distinguishable objects.

Although a line distinguishes two regions, both regions cannot be colored the same color because it would imply they are identical (Cayley, 1879). Different colors would improve the distinguishing characters of lines. The same color could be used again if regions could be particularly well distinguished. A line bounds regions, a point bounds lines, thus a point bounds a boundary. A boundary of a boundary should provide the necessary distinguishing power for using the same color.

If four regions, corn, oats wheat, and rye meet at a point (Fig. 5a) then the corn-oats boundary changes to a corn-rye boundary at a n-2 bounding point.

a.

Figure 5



b.

If the n-1 boundary of the corn does not share any n-1 boundary with the wheat then both may use the same color. It can be seen that the oats and rye intervene when the meeting figure is n-2 (the center point). Thus only three colors are required provided the m+1th region does not share a n-1 boundary with whichever

color is available.

It is possible, as in Figure 5b, that a 4th region may share some of the n-1 boundary at some place removed from the n-2 meeting figure. This contingency may be handled by allowing four colors to be used. Since the last region does join up due to its configuration (Fig. 5b) the set of m figures is meeting associatively. Each time the m+1th region, due to configuration, meets associatively with a "distant" region one degree of freedom is lost. If no foresight is used, an infinite number of colors could be used.

On a line map, made from intersecting surfaces, the number of colors needed may only be two. Such a map could be made from a string with alternate black and white yarn segments. If the number of segments is even then the ends of the string may be tied together. If the number of segments is odd then a black will be tied to a black unless one black is changed to a third color. This is because meeting cannot be done associatively and maintain a maximum number of choices.

Four colors may be necessary on a finite surface. Since the surface of a cylinder joins itself four colors are also needed at most. The typical solution on a sphere is to cut a hole in the sphere or expand a point. However, this hole or point must be colored since a solid sphere (or hollow ball, like a geoid) is the mapper. A torus requires six colors since the inside of the hole permits an additional joining confrontation. Needing more than 6 on a torus, 5 on a sphere, 4 on a finite surface, or 3 on a line is due merely to careless choosing of colors. Two or more starting points for the coloring will require additional colors as does starting at one point and returning to that point a multiple number of times. Each return to the starting point may require an additional color.

Conclusion

Arguments of primacy of labor seem fruitless. Who is the queen to whom, mathematics to science, science to philosophy? It is like asking which part of the watch is more vital? Which parts of a watch which if they were removed would still allow the watch to function? Geographers seem to clearly need the skills of philosophers. Hopefully we have raised interesting philosophical questions for why else would they bother with us? The commerce should be mutually helpful.

The simplest review is a listing of the questions raised by the authors. Tobler asks, "What, if anything, is ultimately invarient in science?" Pattison's question is "If reality is not real, what is?" Warntz asks, "If even point-set has literal meaning, how abstract are abstractions?" Bunge's first question is, "What is implied if the concept of pure spatial prediction has validity?" Toulmin partially answers his own question, "In what sense is the map a theory?" Martin wants to know, "What, if anything, besides language can we read?" Bunge's second question reads, "How do we prove a truth?" Karlin implies the question, "What is a 'natural' language?" And Guyot asks, "Is the four color problem really geographic?" It is not likely that geographers will do as well solving philosophical problems as philosophers, helpful as the answers might prove to geography.

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