Foreword

In these two short papers Professor Tobler demonstrates both the originality and analytical ability so typical of his work. Some of the ideas he expresses have been developed from conversations held by the members of the Michigan Community of Mathematical Geographers over the past year. The advantages of extending analytical techniques of one dimension to other dimensions is one example.

His analysis of map generalization by numerical means has produced surprises. I did not expect that some smoothing operators would prove to have inverses. A class of such operators is demonstrated here. Some of our readers may be surprised at the usefulness of computer produced maps. Professor Tobler has assembled programs which produce such maps with ease and speed and at costs under the costs of manually producing the same.

The two ideas I find most exciting in these papers are the emergence of an explicit operational definition of site or neighborhood conditions by use of local operators and the suggestion that by use of frequency filters and similar techniques periodicities in space may be revealed.

The Editor

January 1966
Numerical Map Generalization

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ABSTRACT

Mathematical procedures for the generalization of isarithmic maps are derived. It is shown that the level of generalization for different purposes may be specified in advance, and that under certain conditions the original map can be recreated from the generalized map.
ACKNOWLEDGEMENTS

The author is indebted to the computing center of the University of Michigan for support of this study. J. Harbaugh made available the two-dimensional Fourier analysis computer program. Colleagues and students in the Michigan Inter-University Community of Mathematical Geographers contributed numerous comments. The data employed for the examples came from a digitalization by R. Yuill of a contour map given on page 92 of Garnier (1963).
The process of map generalization bears a certain resemblance to the process of abstracting textual materials, such as books and articles. As a condensation one expects that only the most relevant and important items will be retained in the simplified version. It is of some theoretical interest to speculate on the extent to which this process can be formalized. As a form of "picture abstracting" the process of map generalization is probably a simple case of a more general problem of pattern analysis, which in turn may be considered to be the basis for the important inductive approach to any knowledge. In particular, it might be anticipated that a study of map generalization might also illuminate the process of scientific generalization, especially as practiced in fields which traditionally make frequent use of maps.

Mapped data may be considered to consist of point (or zero-dimensional) phenomena, and line, area, intensity, and flow symbols. Generalization consists of the application of a transformation which modifies the map data. Pillewizer and Topfer (1964) have considered generalization of point symbols, and Perkal (1958) has contributed to the problem of generalizing lines and areas. The present discussion examines the generalization of "three-dimensional" phenomena, which are assumed to be continuous. Information of this type is most often shown on maps by isarithms of various types; contours, isopleths, isobars, and so on.

As a priori criteria one would expect that generalization of isarithmic maps should somehow eliminate "small scale" features. If generalization is considered as a transformation it is natural to inquire whether the process can be reversed. If topographical maps are generalized the result should conform to appropriately modified accuracy standards. Finally, certain statistical parameters (mean, standard deviation, and so on) should be preserved by the transformation.
Trend analysis provides one form of map generalization. The geographical
trend of population density (persons per square mile) in Michigan is (Tobler,
1964a)

\[ D = 6314 - 68.7 \times \text{latitude N} - 36.9 \times \text{longitude W}, \]
or, in words, the population declines towards the northwest. The "plane" de-
 fined by this equation is in some sense a geographical generalization of the
distribution of population in the state of Michigan. The average population
density is preserved and the generalization represents about 8% of the observa-
tions. A quadratic generalization of the same data,

\[ D = 207529 - 4547 \times \text{lat. N} - 2389 \times \text{long. W} + 52 \times \text{lat. N} \times \text{long. W}, \]
accounts for some 15% of the data. In both of these instances the generaliza-
tion is extreme, though cases can be cited (Krumbein and Graybill, 1965) for
which this technique of trend analysis is most useful.

Another possible method of contour generalization is to smooth the values
by a moving average, as in the treatment of time series data. For expository
convenience consider first only a profile. This can be represented by a single
valued function \( Z = f(x) \). From this profile select a finite sample of values
\( Z_1 \) at a set of discrete locations \( x_1, x_2, x_3, \ldots, x_i, \ldots x_n \). For notational
convenience let \( \Delta x = x_i - x_{i-1} = \) a constant for all \( i \). Smoothed values \( Z^* \) can
be obtained from the original values from the following linear equation

\[ Z_i^* = \frac{\sum_{m=-k}^{m=k} w_m Z_{m+i}}{\sum_{m=-k}^{m=k} w_m} \]
except for obvious modifications at the ends of the interval. The \( W \)'s are
weights, so that \( Z^* \) is a weighted moving average. This can be written, for
\( k = 1 \) and with the sum of the weights equal to unity, as the matrix equation
\[ Z^* = Z \cdot S \]

where \( Z \) is the \((1 \times n)\) vector of \( Z_i \) and \( S \) is an \((n \times n)\) smoothing matrix of the form

\[
S = \begin{bmatrix}
 3/4 & 1/4 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
 1/4 & 1/2 & 1/4 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
 0 & 1/4 & 1/2 & 1/4 & 0 & 0 & \ldots & 0 & 0 & 0 \\
 0 & 0 & 1/4 & 1/2 & 1/4 & 0 & \ldots & 0 & 0 & 0 \\
 & & & & & & & & & \\
 & & & & & & & & & \\
 0 & 0 & 0 & 0 & 0 & 1/2 & 1/4 & 0 & \ldots & 0 \\
 0 & 0 & 0 & 0 & 0 & 1/4 & 1/2 & 1/4 & \ldots & 0 \\
 0 & 0 & 0 & 0 & 0 & 1/4 & 1/2 & 1/4 & \ldots & 0 \\
\end{bmatrix}
\]

which represents a three point binomially weighted moving average. For \( k > 1 \) the matrix is simply less sparse. Note that a second smoothing can be obtained as

\[ Z^{**} = Z^* \cdot S \]

or, more generally

\[ Z^M = Z \cdot S^M \]

Repeated application of the smoothing operation of course results in increased generalization.

Under certain conditions the original values can be restored from the smoothed values by solving the above matrix equation to obtain

\[ Z = Z^* \cdot S^{-1} \]

In other words, a generalized profile can be ungeneralized! The condition is the existence of the inverse matrix \( S^{-1} \). The necessary theorems demonstrating the existence of this inverse are in the appendix. In general,
the exact inverse requires an inverse weighting using all \( n \) of the smoothed values. An approximate, three weight, inverse for the matrix shown can be constructed from the weights \(-0.5, +2.0, -0.5\), however (Holloway, 1958). This is less exact but is more practical for large \( n \).

If the sum of the weights in the smoothing operation is equal to unity, then the mean of the series is invariant. The extremes are usually reduced, however. In some cartographical situations it seems important that the extrema be retained. Several simple operators can be devised which restore the extremes. For example, let

\[
Z'_i = A + BZ_i
\]

where:

\[
B = \frac{Z_{\text{max}} - Z_{\text{min}}}{Z_{\text{max}} - BZ_{\text{min}}}
\]

This normalization restores the maximum and minimum of the series.

A numerical example is given in Table I and is illustrated in the accompanying figures.

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<th>( Z )</th>
<th>( Z^* )</th>
<th>( Z'^* )</th>
<th>( Z''^* )</th>
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<td>622</td>
<td>641</td>
<td>614</td>
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</tbody>
</table>

**TABLE I**

PROFILE GENERALIZATION
PROFILE GENERALIZATION

Original Profile

Smoothed Profile

Twice Smoothed Profile

Profile Smoothed Twenty Times

Figure 1a
PROFILE GENERALIZATION

\[ Z' = Z - Z^* + \overline{Z} \]

The Small Scale Features

\[ Z'' = Z^* - Z^** + \overline{Z} \]

The Medium Scale Features

\[ \hat{Z} = Z^* s^{-1} \]

Restored Values obtained from \( Z^* \) by use of the 3 weight approximate inverse \( s^{-1} \)

\[ \hat{\tilde{Z}} = Z^* s^{-1} \]

Restored values obtained from \( Z^* \) by use of the exact inverse \( s^{-1} \)

Figure 1b
PROFILE GENERALIZATION

Smoothed, then Normalized

Fourier expansion of 12 terms

Fourier expansion of 8 terms

Fourier expansion of 4 terms

Figure 1c
The smoothing operation can be applied to generalize contour maps, not only profiles. Let the value of a surface at any point \( i,j \) be defined by \( Z_{ij} = f(x_{ij}, y_{ij}) \). In most cartographical situations this is a single valued function. For convenience take \( \Delta x = \Delta y = \text{unity} \), and a \( k \)th two-dimensional weighted moving average can then be defined by

\[
Z_{ij}^* = \frac{\sum_{p=-k}^{p=k} \sum_{q=-k}^{q=k} W_{pq} Z_{i+p, j+q}}{\sum_{p=-k}^{p=k} \sum_{q=-k}^{q=k} W_{pq}}
\]

This is easily implemented on a digital computer. In matrix form, the process can be written as

\[
Z^* = S \cdot Z \cdot S^T
\]

where \( Z \) is now a matrix of "elevational" data, and \( S \) is a conformable smoothing matrix as in the previous case. Repeated smoothings can be obtained by repeated matrix multiplications. The inverse operation ("ungeneralizing, or roughening") is given by

\[
Z = S^{-1} \cdot Z^* \cdot S^{-1}
\]

The data matrix \( Z \), of course, need not be square, but the smoothing matrices are then of different orders.

A somewhat more practical procedure than the matrix formulation, and perhaps easier to visualize, is to consider the contour generalization to be the resultant of two operations taken in sequence, one applied to all the east-west profiles, then the other applied to all of the north-south profiles. The amount of smoothing in these two directions is usually the same, but could also differ. Construction of a nine-point two dimensional weighting function from the one dimensional case then follows by cross multiplication from a table such as given here:
Construction of an approximate inverse weighting function is similar, e.g.:

\[
\begin{array}{c|ccc}
1/4 & 1/2 & 1/4 \\
1/4 & 1/16 & 1/8 & 1/16 \\
1/2 & 1/8 & 1/4 & 1/16 \\
1/4 & 1/16 & 1/8 & 1/16 \\
\end{array}
\]

The common cartographic rational for map generalization is retention of legibility under reductions of scale. It is suggested here, however, that it may also be appropriate to generalize maps, not only because of scale changes, but also simply for different map purposes. In other words, the map information is to be filtered in accordance with the use of the map. A brief discussion of spatial frequency filters is therefore appropriate.

It is known that a single valued profile can adequately be represented as the Fourier sum of a large number of sine and cosine curves of varying frequency, amplitude, and phase. Similar results hold for the two dimensional case. The moving average operates as a transformation to modify the amplitudes and phases of the individual trigonometric terms (which are usually referred to as frequencies), and this modification is not the same for all terms. The amount (which can be calculated) by which the amplitudes and phases of individual frequencies are modified depends on the weights employed in the moving average. The binomial weighting employed in the previous examples has certain advantages (e.g., large scale features are retained and small scale features are filtered out, and phases are unchanged) but is only one of many possible weighting schemes. The simple (or equally weighted)
moving average might, at first glance, appear more advantageous, but it has certain undesirable properties; for example, some peaks may become troughs. Since the frequency response of particular weighting schemes may be calculated, it might be expected that one could apply the procedure in reverse. To some extent filters (weights) can be devised to eliminate, or emphasize, features of any size (frequency), and approximate inverse filters can also be obtained. This suggests that it is possible to specify the level of generalization in an a priori fashion for different map scales. For example, at a map scale of 1:500,000 features having a wavelength of less than one half kilometer can probably not be shown. Holloway (1958) demonstrates explicitly the amount of generalization (that is, the weighting function) required (in his notation set $\sigma = 0.167$ kilometers). This example is elaborated in the appendix.

Also implicit in the construction of frequency filters is the notion that, if one knows the geographical scale to which some process is sensitive, one can mathematically generalize a map specifically for this purpose, and this map will differ from that for some other process. For example, topographic features of wavelengths greater than 25 kilometers and less than 0.10 meter are probably not important for the construction of airports. An appropriate filter would eliminate these large and small scale features but would leave medium sized features. Unfortunately we do not have much information concerning the wavelengths to which geographical phenomena are sensitive. It is also suggested that Perkal's (1958) $\tau$-generalization procedure consists of a form of frequency cut-off (truncation of the Fourier series).

An example of contour map generalization by the procedures described is given in Table II and the accompanying illustrations.
<table>
<thead>
<tr>
<th>ELEVATIONS EMPLOYED IN CONTOUR GENERALIZATION</th>
</tr>
</thead>
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<tr>
<td>495  540  575  580  800  930  650  610  510  500  460  475  510  480  480  530  480  455</td>
</tr>
</tbody>
</table>

1) Elevations in feet, 500 grid interval of 0.5 mile.
MAP GENERALIZATION

Original Map Z

Smoothed Map Z*

Twice Smoothed Map Z**

Thrice Smoothed Map Z***
MAP GENERALIZATION

Smoothed - Then Normalized

Original Map - Doubled Contour Interval

Small Scale Features
Removed During 1st Smoothing
MAP GENERALIZATION
Using Trigonometric Polynomials

285 Terms

143 Terms

81 Terms

49 Terms
MAP GENERALIZATION

Map Restored Using Approximate Inverse

Map Restored Using Exact Inverse
It is also possible to consider any individual contour as a line, and attempt line-generalization, as of say, a coastline. As the map scale becomes smaller and smaller, less and less of the coastline detail can be shown. This situation is discussed at some length in the available literature. Alternately, as Perkal suggests, different generalizations of the coastline may be appropriate for different purposes. One computer technique for coastline generalization consists of thinning. Given a string of positional coordinates for sequential points along the coast, points \( j \) are eliminated if

\[
D_{ij} < \epsilon
\]

where \( j = i + k, \ k = 1, 2, \ldots, n, \)

and in the cartesian case,

\[
D_{ij} = (x_i - x_j)^2 + (y_i - y_j)^2 \]

and \( \epsilon \) is some minimal distance, for example, twice the minimal plotting size of the automatic drawing instrument. This approach has already been employed with considerable success and is more fully discussed elsewhere (Hershey, 1963; Tobler, 1964b). Thinned data cannot be restored, however.

If the coastline were a single valued function it could be generalized as a topographic profile. The coastline, however, is not single valued. It might be decomposed into single valued intervals but a parametric representation in terms of arc length \( s \) is more convenient. Let the coastline be represented by the complex function

\[
Z = f(x + iy), \quad i^2 = -1,
\]

where \( s = s(s) \) and \( y = y(s) \). For equal intervals \( \Delta s \) of \( s \) the \( j^{th} \) point of the coastline is

\[
Z_j = x(s_j) + iy(s_j).
\]

A smoothing can then be accomplished by taking

\[
Z^* = Z \cdot S
\]

where \( S \) is a smoothing matrix as before. Since the coastline closes, \( k \) values
Generalization of Coastline by Thinning. After Tobler (1964b).

Machine drawn from data compiled at 1/50,000

Tracing from 1/5,000,000
Jet Navigation Chart showing area involved
of $Z_j$ could be added to the beginning and end of the complex vector $Z$, and $S$ becomes an $n + 2k$ by $n + 2k$ matrix. Different smoothings could in fact be applied to the real and imaginary parts of $Z$, though it is difficult to imagine why this might be done.

Examination of a number of published contour maps at different scales but of the same area suggests that the practical cartographer generalizes by thinning after enlarging the contour interval in approximately the inverse ratio of the map scales. This process is easily formalized, e.g., on a digital computer, and appears suitable for the navigational usage of maps. The filtering procedure, on the other hand, seems to have advantages for more advanced theoretical investigations. For example, property values for individual parcels of real estate within a city (as might be obtained from the assessor's office) typically show considerable variation from one parcel to the next. The space smoothing technique can be employed to investigate the existence of geographical patterns of property values. The generalization facilitates the recognition of patterns because it appears to be true, as Holloway (1958, p. 386) suggests, that we do "high-pass filtering in our mind's eye."

Several additional smoothing techniques can be found in works on numerical methods, and these can generally be extended to the two-dimensional case fairly easily. Virtually all of these techniques make the assumption that "there is a strong antecedent probability that if the observations had been more accurate the curve would have been smooth" (Whittaker and Robinson, 1944), where smooth means that higher order differences vanish. In the topographic situation, at least for purposes of generalization, it can be assumed that the errors are negligible and the concept of a filter seems more appropriate. In terms of geographical theory one might argue that the pattern-generating processes are subject to random disturbances, which we wish to
eliminate by the generalization procedure.
APPENDIX I

The Inverse of the Smoothing Matrix

Definitions

(1) A matrix \( A = [a_{ij}] \) is diagonally dominant if
\[
\left| a_{ii} \right| \geq \sum_{j=1 \atop j \neq i}^{n} \left| a_{ij} \right|, \quad 1 \leq i \leq n
\]

(2) A matrix \( A \) is reducible, \( n \geq 2 \), if there exists an \( n \) by \( n \) permutation matrix \( P \) such that
\[
P^{-1} A P = \begin{bmatrix}
A_{11} & A_{12} \\
0 & A_{22}
\end{bmatrix}
\]
where \( A_{11} \) is an \( r \) by \( r \) submatrix, \( A_{22} \) is an \( (n-r) \) by \( (n-r) \) submatrix, \( 1 \leq r \leq n \).
If such a \( P \) does not exist then \( A \) is irreducible.

(3) A directed graph is strongly connected if for any ordered pair of nodes \( N_j \) and \( N_k \) there exists a directed path \( N_j N_{j1}, N_{j1} N_{j2}, \ldots, N_{jr} N_k \) connecting \( N_j \) and \( N_k \).

(4) A matrix \( A \) is irreducibly diagonally dominant if \( A \) is irreducible and diagonally dominant with the strict inequality valid for at least one \( i \).

Theorems

(1) A matrix is irreducible if, and only if, its directed graph is strongly connected.

(II) A matrix which is irreducibly diagonally dominant is non-singular.

Remarks

The 18 by 18 smoothing matrix \( S \) used in the generalization is irreducibly diagonally dominant. The inverse matrix \( S^{-1} \) can in fact be exhibited explicitly, but is omitted here (interested parties can obtain copies from the
An "inverse" $s^{(-1)}$ constructed from the approximate inverse weighting function $(-.5, +2, -.5)$ has the property that

$$s^{(-1)} \cdot s \neq I$$

as was expected.
APPENDIX II

Determinination of a Binomial Weighting Function

Procedure

(1) Given (in appropriate units, e.g. kilometers):
   (a) The observation interval $\Delta x$
   (b) The "cut-off" wavelength

(2) Set the cut-off wavelength = 3$\sigma$

(3) The interval between observations then corresponds to
   \[
   \frac{\Delta x}{\sigma}
   \]
   on the abscissa of the normal curve.

(4) Record (e.g., CRC Standard Math Tables, 12th ed., pp. 245-249) the ordinates of the normal curve from zero to at least 16 in increments of $\frac{\Delta x}{\sigma}$.

(5) Adjust the values obtained to yield a weighting function which sums to unity by dividing by $\frac{\sigma}{\Delta x}$.

Example

(1) a) Suppose data are available at every 0.1 km. (e.g. $\Delta x = 0.1$).
   b) On a 1/500,000 scale map features having a wavelength of less than one half km. cannot be shown. The cut-off frequency is 0.5 km.

(2) $3\sigma = 0.5 \Rightarrow \sigma = 0.167$ km.

(3) $\frac{\Delta x}{\sigma} = \frac{0.1}{0.167} + \frac{0.1}{0.2} = 0.5$

(4) From the table of the normal curve:

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<tr>
<td>2.5</td>
<td>0.0175</td>
<td>0.0088</td>
<td>W+5</td>
</tr>
<tr>
<td>3.0</td>
<td>0.0044</td>
<td>0.0022</td>
<td>W+6</td>
</tr>
</tbody>
</table>

The 13 weights obtained can be used to smooth the data. Note that by taking $\Delta x = 0.5$, the map is over generalized, since $3\sigma = 0.6$ km not 0.5 km as specified in the initial conditions. This is hardly of consequence since perfect cut-off filters cannot be realized in practice (cf. Holloway).
References


A. V. Hershey, 1963, The plotting of Maps on a CRT Printer, NWL Report No. 1844


