A METHOD FOR THE STEPWISE SEPARATION OF SPATIAL TRENDS

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The problem of separating trends occurs in very similar terms in the analysis of time series[1] and of space series. Trends are general tendencies persisting through time or through space that cause time or space series to increase or decrease "smoothly." In order to isolate them, the original series are split into components associated respectively with the trend and with the residual, such that the summation of their corresponding terms reproduces the original series.

Techniques of time series analysis can be and have been extended to space series. The interpolation of planes and surfaces to geologic series[2] closely parallels the fitting of lines and curves to time series. The only difference is that functions of two spatial coordinates (functions of two variables) are used in the first case, and functions of time (of one variable) in the second. Also, the mobile averaging can be easily extended to two dimensional cases, for instance, by

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[1]See, for instance, H. T. Davis, The Analysis of Economic Time Series, Detan Printing Company, Colorado Springs, 1941.

[2]A Discussion and bibliography is contained in R. L. Miller and J. S. Kahn, Statistical Analysis in the Geological Sciences, Wiley, 1962, p. 394-.

replacing the value of each term of a space series by some average of the values of the terms within a given distance from it.[3]

The Method

The methods proposed in this paper are expecially suited to extract spatial trends,[4] but may be applied to time series as well. Basically, they are designed to give a quantitative expression to smooth changes functionally related to distance from points or lines of reference. In particular, these methods may be used in order to identify the smooth decline of the "influence" of an urban center, or of a cluster of urban centers with distance from a point of reference that may or may not coincide with downtown areas. The methods identify trends by means of optimal points of reference or origins. A point of reference is considered optimal when the correlation between some predetermined transformation of the distance of the series terms from it, and the values of the terms are not smaller than for any other admissible point of reference. In order to clarify the concepts involved, a simple time series example is now discussed. Suppose, for instance, in the time series below

Value	5	4	3	2	1	2	3	4	5
Time	t	t+1	t+2	t+3	t+4	t+5	t+6	t+7	t+8

that each term of the series has a value and a time coordinate. If the

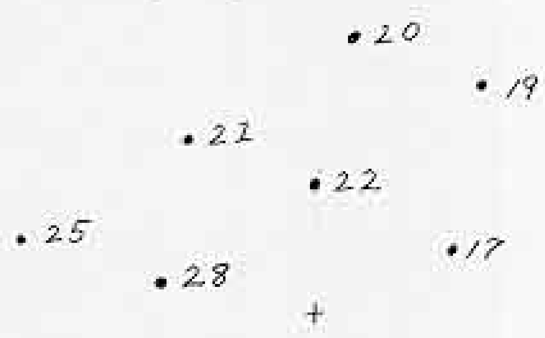
[3]See P. E. Potter, "The Petrology and Origin of the Lafayette Gravel," Part I: Minerology and Petrology, Journal of Geology, 63, (1955): pp. 1-35.

[4]The first application of the methods described in this paper can be found in R. K. Semple, A Quantitative Separation and Analysis of Spatial Trends in the Viability of Small Urban Centres in Southern Ontario. Unpublished M.A. Thesis, University of Toronto, 1966.

time coordinate t+4 is taken as origin, distances of the terms from it can be easily obtained. The correlation between these distances and the respective term values is 1. Clearly the correlation between term values and distance from any other origin would be less than 1, so that in this case, t+4 is the optimal origin. A regression, in which time distances from the optimal origin (or some transformation of them) is the independent variable and the values of the time series are the dependent variable, could be calculated and residuals obtained. In the example above the residuals are all zero. But suppose that this was not the case. Then the procedure could be repeated on the residual time series, and a new optimal origin determined, new residuals obtained, and so on, iteratively. These iterations would be interrupted when the residuals are random or smaller than a threshold. The difference of the initial time series minus the last residual time series constitutes a trend component.

For extracting spatial trends, a two-dimensional generalization of the procedure above can be applied. The concept of optimal origin is easily extended to space series. For example, suppose the series identified by the following diagram:

Figure 1



The circles indicate the location of the terms of the space series and the numbers adjacent to them are their values. The cross is an arbitrary "origin." The distance between the cross and each of the circles (or some function of this distance) can be calculated for each "term" and correlated with the values of the terms. An origin is optimal if the correlation coefficient associated with it is not smaller than the one associated with any other admissible origin.

The procedure for extracting spatial trends involves the following steps:

1. An optimal origin is obtained.
2. The values of the space series terms are regressed against the distance of the locations of the terms from the optimal origin. The values predicted by the regression identify a trend associated with the optimal origin.
3. The residuals, that is the difference between the original values and the predicted values, form a new space series that can be again separated into a trend and a new residual. To this effect, a new optimal origin is located, such that the correlation between the distances from it to the location of the terms of the space series, and the residuals from the first regression is maximal.
4. A new regression can be calculated and new residuals obtained.

The procedure can be repeated again and again. Each time it yields an optimal origin, a space series incorporating the portion of the trend associated with the optimal origin used, and a space series consisting of residuals. The iterations can be stopped when the residual space series is random.

The variance of the original space series can be decomposed into the variance of the trend component associated with the successive optimal origins plus the variance of the residual. A rule of thumb for deciding when to stop the extraction of new trend components from the residual may be based on the proportion of the total variance in the last (nth) trend component. A threshold is set and trend components are extracted only when their variance is greater than the threshold.

The Scope of the Technique

This procedure for the extraction of spatial trends is eminently applicable when empirical observations or theoretical considerations suggest that a given spatial variable tends to have values the higher (or lower), the closer they are to given locations. However, it can be shown that the procedure is also effective for trends related to straight or curved lines. Suppose, for instance, the space series in the following figure:

Figure 2

27 0	25 0	26 0	27 0	25 0	26 0
30 0	29 0	31 0	30 0	30 0	29 0
26 0	27 0	25 0	25 0	26 0	27 0

High values occur along the central "ridge," and decline on both sides of it. The values of the variable decline with distance from a line rather than from a point, and therefore the trend can be associated with a line. Suppose the procedure discussed above is applied. One of the points along the central ridge would be chosen as first optimal origin, and the linear regression of the values of the series against a transformation of the distances from the optimal origin would be calculated. Clearly, the residuals will tend to be larger on the portions of the central ridge away from the first optimal origin. Therefore, the second optimal origin would be located on the ridge, away from the first origin. A similar reasoning can show that the procedure will tend to locate, along the central ridge, a sequence of points that will capture the trend.

The same will happen for other kinds of trends. Each optimal origin will specify further portions of them until purely random residuals remain.

When the value of a variable in a space series tends to decline (or to increase) with distance from certain "poles" the procedure will yield optimal origins that will tend to coincide with these poles. In this case the optimal origins have a substantive meaning. If instead the spatial trend is related to lines rather than points, the optimal origins taken separately do not have meaning other than a formal one.

Similar considerations apply to the trend components associated with individual optimal origins. Whenever these origins correspond to poles their trend components identify the areas where the influence of the poles is felt, and it can be illuminating to map them.

Instead, when line trends are extracted, only the sum of the trend components associated with all the optimal origins has a substantive meaning.

The method proposed can be useful to geographers in different respects. It decomposes space series into two components containing respectively smooth variations of variables over space, and residuals. These two layers can be investigated, for instance, by multiple regressions, in order to analyse separately the causative factors related to trends and those related to residuals from trends.

Furthermore, within some substantive contexts, the optimal origins and the trend components associated with them may identify cores of regions, and areas in which the influence of these cores is felt.

The spatial trends to which this procedure is suited involve a non-linear decline with distance from a point or line of reference. The trends, therefore, can be visualized as mounds, ridges, or hills rather than as traingular cones. Suitable transformations of the variables can be used so that the influence of each optimal origin declines more than proportionally with distance.

The transformations suggested are 1) the logarithmic transformation of the values of the series terms, and 2) the reciprocal of the distance from the optimal origins--increased by one. If the first transformation is used, the relationship between trend and distance from optimal origins is obtained in the following form:

$$V_{ij} = \exp (a_1 - b_1 d_{1ij} + a_2 - b2ij + \cdots + a_n - b_{nij})$$

Where V_{ij} is the value of the space series term with coordinates i and j, d_{kij} is the distance of the term with coordinates i and j from the kth optimal origin, and a_k and b_k are regression coefficients.

With the second transformation instead, the relationship between trend and distances is:

$$V_{ij} = a_1 + \frac{b_1}{d_{1ij} + 1} + a_2 + \frac{b_2}{d_{2ij} + 1} + \cdots + a_n + \frac{b_n}{d_{nij} + 1}$$

<u>An Empirical Application</u>

In the example that follows it was attempted to separate spatial trends from data on population growth in small urban centers in Southern

Ontario.[5] The computer program used is discussed in detail in Appendix A and only its basic structure is described here. The input of this program consists of coordinates and the values of the terms of a space series.

Optimal origins are searched for by superimposing over the study area successively finer grids. The differences of the largest and the smallest ordinate and abscissa values of the space series are calculated. The largest of the two differences is chosen as the length of a square study area. The sides of this square area are then divided into fifteen equal parts which identify a 15 by 15 grid. From each intersection of the grid the distance to every term of the space series is calculated, and the (transformed) distance is correlated with the values of the terms so that a 15 x 15 matrix of correlation coefficients is obtained. The largest correlation coefficient and its coordinates are identified. In order to determine more precisely the actual coordinates of the optimal origin, the four diagonal correlation coefficients adjacent to the one identified are used to define the limits of a second finer grid from which a 10 x 10 matrix of correlation coefficients is calculated. Again the largest correlation coefficient and its coordinates are identified. One final 8 x 8 matrix of very fine coefficients is then obtained, and the largest coefficient in it is assumed to indicate the actual point of highest correlation on the study area. This point is taken to be the optimal origin and the values of the space series are regressed against the transformed

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[5]For the locations and identification of the towns in the study area see Table 1 and Map 1.

distances from the optimal origin to the locations of the space series terms. The variance explained by the distances is calculated and its ratio to the variance of the space series obtained.

The regression residuals are then calculated. They provide the new space series on which the steps above are repeated. The procedure is iterated until an optimal origin is obtained, such that the distances from it explain less then five per cent of the variance of the residual. The program is then terminated. The program outputs: 1) the coordinates of the optimal origins, 2) the residuals from the trends associated with each optimal origin and the final residuals, 3) the proportion of the variance of each residual accounted for by each component.

The program was used to analyse the population growth of incorporated towns of Southern Ontario with a 1951 population between one and five thousand. Different areas of the province appear to be characterized by different growth rates. Thus, the purpose of the experiment was to identify and separate these trends. The input consisted of geographical coordinates and of the percentage population change between the 1951-1961 census period (See Table 2). Five optimal origins were obtained which account for over 90 per cent of the population change's variance (See Table 3).

Growth rates tend to decline with distance from three optimal origins clustered in a "horseshoe" fashion around the western end of Lake Ontario and increase with distance from a two optimal origin located at Parry Sound and Woodbridge. (See Map 2).

A test of the effectiveness of the procedure was carried out by regressing the original space series values, the spatial trend values and the residuals from the trend against variables selected to represent the most likely "causes" of spatial trends. (See Map 3 for trend component). If these variables really express causes of regional trends of viability, and if the procedure proposed is effective in separating a trend component from a residual, it necessarily follows that:

$$R_T > R_V > R_R$$

where R_T, R_V and R_R are the multiple correlation coefficients of the regional variables and, respectively, of the trend component (R_T), the original space series value (R_V), and the residuals (R_R).

In other words, the variables will be more related to the trend measures than to the original space series which includes a residual in addition to the trend. Also the original variables will be more related to the original space series values than to the residuals that presumably do not contain trend elements.

In order to calculate the multiple correlation coefficients, a BIMED 29 stepwise multiple regression program was used. This program regresses independent variables, one at a time against a dependent variable until their added explained variance falls below a specified level. Correlation and regression coefficients are output.

The following variables were used: (See Table 4).

- X_1 township population density
- X_2 value of farm sales per farm
- X_3 township agricultural assessment in dollars per acre of land
- X_4 market potential measures
- X_5 improved acreage as a percentage of occupied

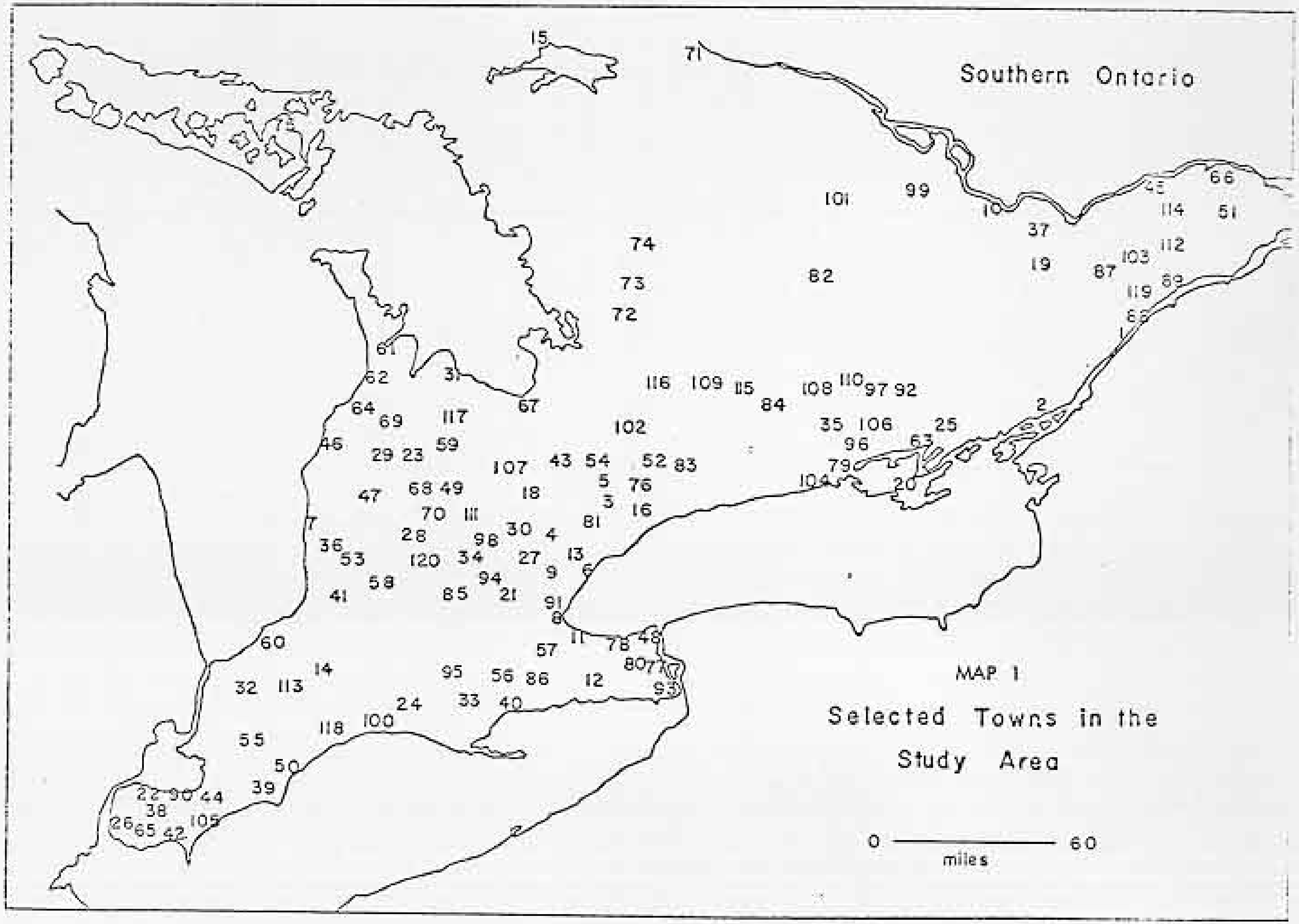
The market potential measures were obtained from the following,

$$MP_i = \sum_{j \neq i} a_i b_i \; / \; d_{ij} \qquad i,j = 1,2 \ldots n$$

Table 1

Selected Towns in the Study Area

1. Prescott
2. Gananoque
3. Richmond Hill
4. Georgetown
5. Aurora
6. Port Credit
7. Goderich
8. Stoney Creek
9. Milton
10. Arnprior
11. Grimsby
12. Dunnville
13. Streetsville
14. Strathroy
15. Sturgeon Falls
16. Markham
17. Penetanguishene
18. Orangeville
19. Carleton Place
20. Picton
21. Hespeler
22. Tecumseh
23. Hanover
24. Aylmer
25. Napanee
26. Amherstburg
27. Acton
28. Listowel
29. Walkerton
30. Fergus
31. Meaford
32. Petrolia
33. Delhi
34. Elmira
35. Campbellford
36. Clinton
37. Almonte
38. Essex
39. Blenheim
40. Port Dover
41. Exeter
42. Kingsville
43. Alliston
44. Tilbury
45. Rockland
46. Kincardine
47. Wingham
48. Niagara
49. Mount Forest
50. Ridgetown
51. Alexandria
52. Usbridge
53. Seaforth
54. Bradford
55. Dresden
56. Waterford
57. Caledonia
58. Mitchell
59. Durham
60. Forest
61. Wiarton
62. Southampton
63. Deseronto
64. Port Elgin
65. Harrow
66. Vankleek Hill
67. Stayner
68. Harriston
69. Chesley
70. Palmerston
71. Mattawa
72. Gravenhurst
73. Bracebridge
74. Huntsville
75. Little Current
76. Stouffville
77. Chippawa
78. Beamsville
79. Brighton
80. Fonthill
81. Woodbridge
82. Bancroft
83. Port Perry
84. Lakefield
85. New Hamburg
86. Hagersville
87. Kemptville
88. Cardinal
89. Morrisburg
90. Belle River
91. Waterdown
92. Tweed
93. Crystal Beach
94. Bridgeport
95. Norwich
96. Frankford
97. Madoc
98. Elora
99. Eganville
100. Port Stanley
101. Barry's Bay
102. Sutton
103. Winchester
104. Colborne
105. Wheatley
106. Stirling
107. Shelburne
108. Havelock
109. Fenlon Falls
110. Marmora
111. Arthur
112. Chesterville
113. Watford
114. Casselman
115. Bobcaygeon
116. Beaverton
117. Markdale
118. West Lorne
119. Iroquois
120. Milverton



MAP 1

Selected Towns in the Study Area

Table 2

LOCATION AND GROWTH OF SMALL TOWNS
IN SOUTHERN ONTARIO (1951-1961)a

Town	1	2	3
RICHMOND HILL	320	260	6600
STREETSVILLE	300	290	3437
STONEY CREEK	303	325	2144
GEORGETOWN	285	285	1983
MARKHAM	332	262	1674
AURORA	320	250	1618
MILTON	290	300	1297
PORT CREDIT	310	295	977
BANCROFT	428	248	960
STOUFFVILLE	335	253	881
GRIMSBY	312	328	856
CHIPPAWA	348	343	848
FONTHILL	328	338	646
BRADFORD	313	266	579
CRYSTAL BEACH	347	360	566
PRESCOTT	588	172	525
BEAMSVILLE	319	329	483
BRIDGEPORT	240	298	471
ALLISTON	292	236	451
ACTON	280	285	439
ORANGEVILLE	275	258	414
CLINTON	175	285	371
WOODBRIDGE	310	260	369
WATERDOWN	290	314	367
DELHI	245	361	362
AYLMER	211	366	351
WHEATLEY	106	427	334
KEMPTVILLE	579	145	317
STRATHROY	167	348	314
PORT PERRY	350	240	314
CALEDONIA	285	338	308
STAYNER	230	251	305
GODERICH	163	273	299
UXBRIDGE	342	240	297
BELLE RIVER	88	410	296
ROCKLAND	598	93	293
ELMIRA	243	288	289
NIAGARA	344	324	287
BLENHEIM	139	407	281

Table 2 (Page 2)

	1	2	3
WATERFORD	260	353	273
STURGEON FALLS	287	22	267
LAKEFIELD	401	208	267
TECUMSEH	075	405	263
NEW HAMBURG	232	310	262
SUTTON	327	220	259
PORT DOVER	266	366	256
ESSEX	080	390	251
ARNPRIOR	527	108	249
HANOVER	211	234	246
MARMORA	441	202	236
ALMONTE	540	128	223
AMHERSTBURG	059	427	223
BRIGHTON	439	244	222
FOREST	140	340	222
COLBORNE	429	249	206
MEAFORD	241	191	206
EXETER	178	311	196
STIRLING	451	219	195
PETROLIA	130	354	194
WINCHESTER	598	137	190
HAGERSVILLE	277	350	188
DURHAM	225	233	185
NORWICH	239	348	183
BARRY'S BAY	439	105	181
DESERONTO	486	228	181
WALKERTON	203	237	180
FRANKFORD	448	230	179
ALEXANDRIA	644	104	178
HARROW	072	434	176
VANKLEEK HILL	640	092	172
HESPELER	260	304	170
EGANVILLE	477	100	168
BEAVERTON	341	208	161
DUNNVILLE	309	355	157
LISTOWEL	215	275	154
NAPANEE	492	223	154
KINGSVILLE	085	435	150
TWEED	467	207	147
MOUNT FOREST	249	292	145
DRESDEN	127	382	143
CHESTERVILLE	606	135	141
MITCHELL	199	301	135
PICTON	480	155	132
FERGUS	254	278	131
TILBURY	108	412	130
ST. MARY'S	202	319	121
GANANOQUE	545	213	114
HAVELOCK	427	207	113
WINGHAM	192	259	105

Table 2 (Page 3)

	1	2	3
CASSELMAN	616	117	103
ARTHUR	245	265	102
ELORA	276	210	102
RIDGETOWN	147	396	101
WIARTON	205	178	94
LITTLE CURRENT	155	057	93
HARRISTON	221	259	92
BRACEBRIDGE	330	150	91
CARDINAL	597	165	91
MADOC	455	200	86
MARKDALE	236	220	82
PENETANGUISHENE	286	176	79
MATTAWA	371	027	78
WATFORD	153	350	77
CAMPVELLFORD	434	219	75
SOUTHAMPTON	189	201	69
SEAFORTH	184	292	65
KINCARDINE	169	231	63
MILVERTON	218	291	53
PORT ELGIN	188	207	47
SHELBURNE	267	242	46
IROQUOIS	601	160	46
FENELON FALLS	370	198	42
WEST LORNE	163	385	38
GRAVENHURST	326	162	24
CHESLEY	215	220	15
CARLETON PLACE	543	135	15
BOBCAYGEON	382	199	2
PALMERSTON	223	266	-12
MORRISBURG	610	152	-21
PORT STANLEY	195	377	-21
HUNTSVILLE	335	122	-30

(a) Note: -30 means that a town's population declined by -3.0 percent between 1951 and 1961.

1,2 Map coordinates
3 Percent population change 1951-1961

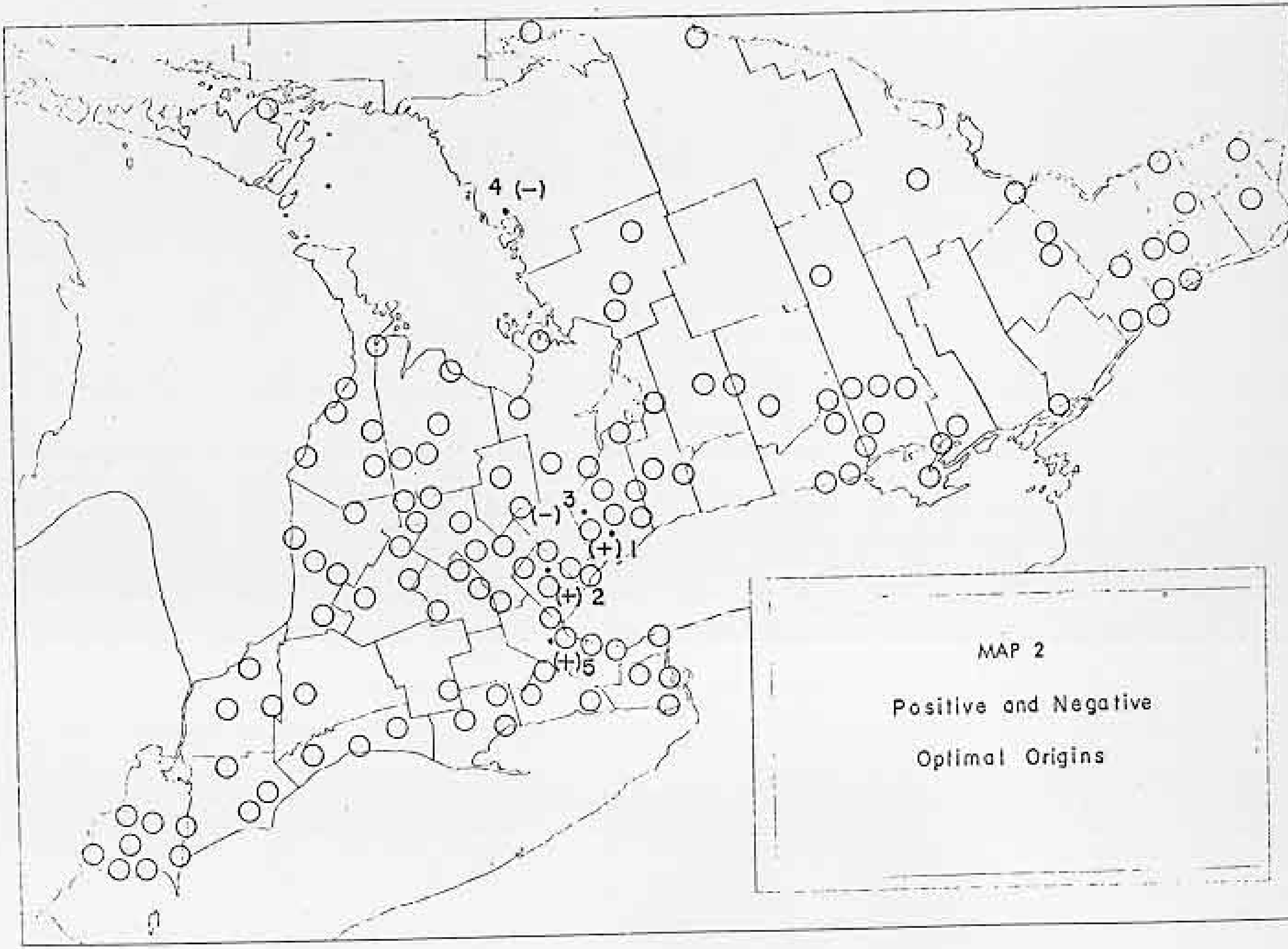

4 (–)
(–) 3
(+) 1
(+) 2
(+) 5
MAP 2
Positive and Negative
Optimal Origins

Table 3

Town		Location of Optimal Origins Coordinates		Cumulative Explained Variance
		X	Y	
Richmond Hill	(+)	322	260	68.8%
Milton	(+)	296	291	85.1%
Woodbridge	(-)	311	263	88.5%
Parry Sound	(-)	257	123	89.5%
Stoney Creek	(+)	303	326	93.7%

(+) Decline in growth with distance from optimal origin
(-) Increase in growth with distance from optimal origin

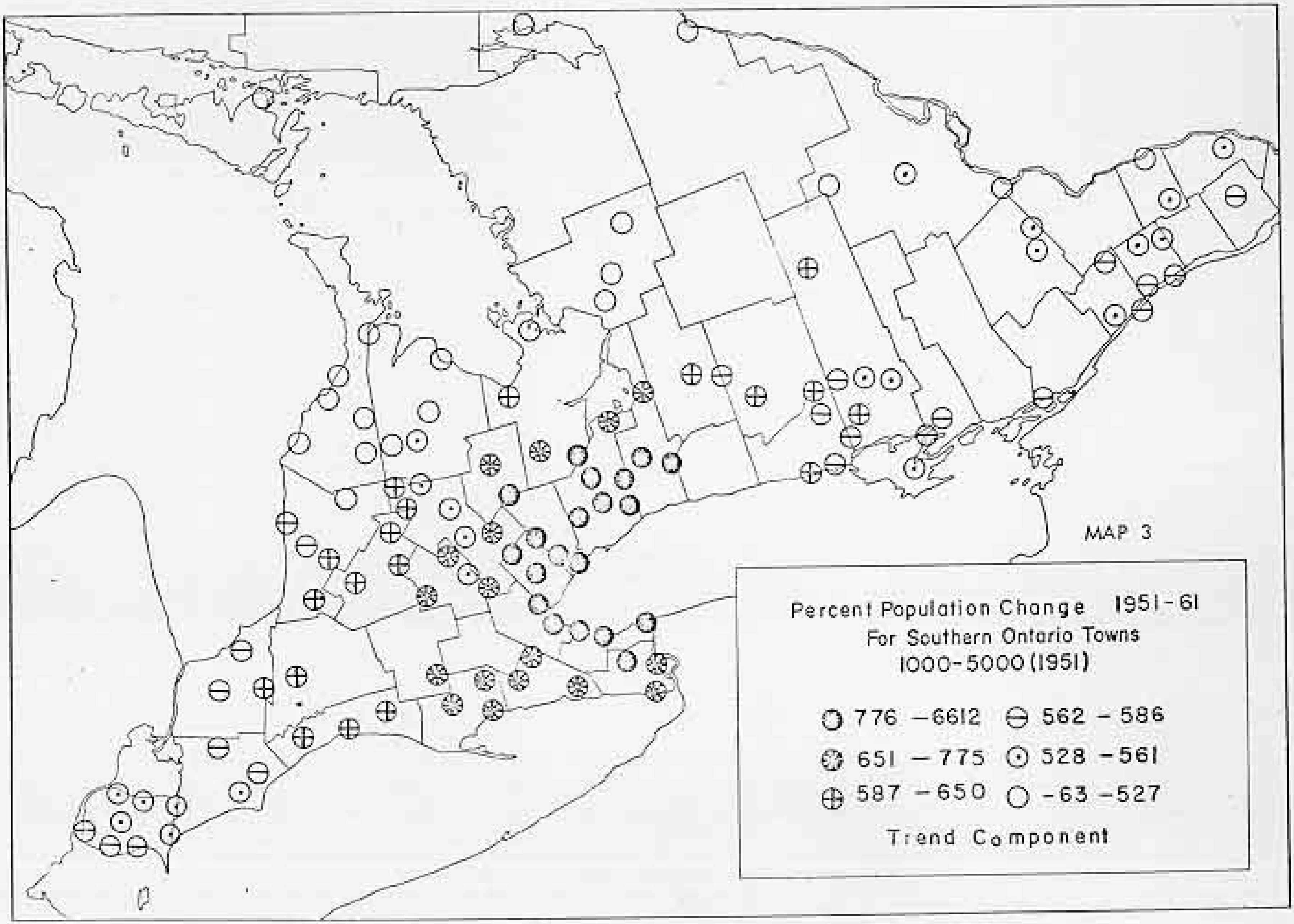

MAP 3
Percent Population Change 1951-61
For Southern Ontario Towns
1000-5000 (1951)
776 - 6612
562 - 586
651 - 775
528 - 561
587 - 650
-63 - 527
Trend Component

Table 4

Town	1	2	3	4	5	6
RICHMOND HILL	6600	2198	34192	272	6242	358
STREETSVILLE	3437	6770	7157	1455	3339	98
STONEY CREEK	2144	5997	21233	905	2138	6
GEORGETOWN	1938	1878	4148	528	999	984
MARKHAM	1674	2198	34192	275	1502	172
AURORA	1618	2809	4192	161	1397	221
MILTON	1297	4728	4148	1483	1160	137
PORT CREDIT	977	6770	7157	1455	1074	-97
BANCROFT	960	458	1224	8	230	730
STOUFFVILLE	881	2809	34192	215	1621	-175
GRIMSBY	856	4348	3160	527	657	199
CHIPPAWA	848	9270	3691	557	356	492
FONTHILL	646	1588	3691	210	433	213
BRADFORD	579	648	2366	68	515	64
CRYSTAL BEACH	566	1653	3691	412	319	247
PRESCOTT	525	672	272	98	165	360
BEAMSVILLE	483	1994	3160	400	457	26
BRIDGEPORT	471	1038	4685	47	176	-71
ALLISTON	451	722	2366	68	401	50
ACTON	439	1878	4148	128	791	-352
ORANGEVILLE	414	325	292	60	456	-42
CLINTON	371	600	656	55	208	163
WOODBRIDGE	369	2997	34192	391	504	-135
WATERDOWN	367	2455	21233	258	640	-273
DELHI	362	1032	1148	135	288	74
AYLMER	351	1027	1142	112	235	116
WHEATLEY	334	1000	927	108	180	154
KEMPTVILLE	317	201	272	33	192	125
STRATHROY	314	682	3793	63	223	91
PORT PERRY	314	533	5128	78	497	-183
CALEDONIA	308	599	663	179	405	-97
STAYNER	305	1008	2366	35	236	69
GODERICH	299	232	656	54	192	107
UXBRIDGE	297	346	5128	100	579	-282
BELLE RIVER	296	877	2541	101	176	120
ROCKLAND	293	710	223	42	157	136
ELMIRA	289	1038	4685	135	345	-56
NIAGARA	287	2323	3160	425	413	-126
BLENHEIM	281	723	927	110	188	93
WATERFORD	273	713	1148	91	322	-49

Table 4 (Page 2)

Town	1	2	3	4	5	6
STURGEON FALLS	267	1160	621	13	32	235
LAKEFIELD	267	1218	1438	136	235	32
TECUMSEH	263	1242	2541	130	202	61
NEW HAMBURG	262	808	4658	104	299	-37
SUTTON	259	1127	5128	166	366	-107
PORT DOVER	256	2655	1148	117	304	-48
ESSEX	251	1138	2541	71	174	77
ARNPRIOR	249	883	919	36	152	97
HANOVER	246	642	810	30	145	78
MARMORA	236	131	1224	5	212	24
ALMONTE	223	713	464	60	187	36
AMHERSTBURG	223	1329	2541	75	202	21
BRIGHTON	222	643	538	38	218	4
FOREST	222	871	1371	69	209	13
COLBORNE	206	462	539	50	253	-47
MEAFORD	206	525	810	22	-19	225
EXETER	196	551	656	63	224	-28
STIRLING	195	334	1224	28	220	-25
PETROLIA	194	513	1371	48	208	-14
WINCHESTER	190	601	201	52	161	29
HAGERSVILLE	188	450	663	83	349	-161
DURHAM	185	201	810	16	179	6
NORWICH	183	735	1339	24	293	-110
BARRY'S BAY	181	173	919	5	156	25
DESERONTO	181	368	1224	28	214	-33
WALKERTON	180	602	403	54	147	33
FRANKFORD	179	2287	1224	71	203	-24
ALEXANDRIA	178	301	205	31	192	-14
HARROW	176	1206	2541	133	175	1
VANKLEEK HILL	172	2457	346	47	191	-19
HESPELER	170	2533	4685	176	341	-271
EGANVILLE	168	138	919	17	170	-2
BEAVERTON	161	461	5128	80	285	-124
DUNNVILLE	157	490	663	120	353	-196
LISTOWEL	154	508	995	58	248	-94
NAPANEE	154	579	289	48	211	-57
KINGSVILLE	150	1636	2541	170	205	-55
TWEED	147	274	1224	24	183	-36
MOUNT FOREST	145	475	1986	37	378	-233
DRESDEN	143	871	927	85	210	-67
CHESTERVILLE	141	601	201	52	161	-20
MITCHELL	135	389	995	58	224	-89
PICTON	132	1384	230	61	187	-55
FERGUS	131	766	1986	55	392	-261
TILBURY	130	750	2541	84	180	-50

Table 4 (Page 3)

Town	1	2	3	4	5	6
ST. MARY'S	121	828	995	68	250	-129
GANANOQUE	114	761	566	78	202	-88
HAVELOCK	113	132	1438	24	221	-108
WINGHAM	105	723	652	111	-35	142
CASSELMAN	103	402	223	26	191	-88
ARTHUR	102	233	1986	31	316	-214
ELORA	102	766	1986	55	172	-70
RIDGETOWN	101	269	927	58	216	-115
WIARTON	94	155	403	44	-31	126
LITTLE CURRENT	93	206	60	4	73	30
HARRISTON	92	518	1986	44	235	-143
BRACEBRIDGE	91	573	282	63	22	69
CARDINAL	91	989	272	78	195	-104
MADOC	86	231	1224	16	182	-96
MARKDALE	82	209	810	24	148	-66
PENETANGUISHENE	79	2144	2366	111	-63	142
MATTAWA	78	568	621	5	76	2
WATFORD	77	535	1371	52	216	-139
CAMPBELLFORD	75	562	538	38	203	-128
SOUTHAMPTON	69	744	403	53	88	-19
SEAFORTH	65	553	656	53	220	-155
KINCARDINE	63	474	403	45	125	-62
MILVERTON	53	458	995	62	266	-216
PORT ELGIN	47	744	403	53	79	-32
SHELBURNE	46	237	292	32	322	-276
IROQUOIS	46	421	201	49	194	-173
FENELON FALLS	42	413	464	54	231	-189
WEST LORNE	38	418	1142	55	221	-183
GRAVENHURST	24	331	282	58	47	-23
CHESLEY	15	225	403	35	133	-118
CARLETON PLACE	15	394	464	29	188	-173
BOBCAYGEON	2	277	464	46	211	-209
PALMERSTON	-12	518	1986	44	280	-247
MORRISBURG	-21	495	201	50	194	-215
PORT STANLEY	-21	1033	1142	60	237	-258
HUNTSVILLE	-30	350	282	32	72	-22

1 Percent population change 1951-1961
2 Township population density
3 Market potential
4 Agricultural assessment
5 Regional component of population change
6 Residuals

where MP_i is the market potential of the ith county, d_{ij} is the distance from the ith county to the jth county, and a_i and b_i are, respectively, disposable income and the population of the ith county.

The results of the analysis are given in Table 5.

Clearly R_T is greater than R_V and R_V is greater than R_R as predicted. Also, the variables selected by the stepwise program for the prediction of the spatial trend (X_4 and X_3) are different from the ones selected for predicting the residuals from the trend (X_1). This implies that the components into which the original space series is split have altogether different characteristics, as it should be expected if the procedure proposed is effective in separating spatial trends and residuals.

Conclusion

The technique discussed in this paper is particularly suited to separating trends that can be expressed as functions of distances from points and lines of reference, and it can be used for the quantitative identification of cores of regions, growth poles and depressed areas. It may provide additional tools for the geographic research that aims at results in the form of precise and testable statements.

Table 5

RESULTS OF THE MULTIPLE REGRESSION AND CORRELATION ANALYSIS

$y_1 = .053x_4 + .892x_3 + 85.6 \qquad R_T = .726$

$y_2 = .051x_4 + 1.05x_3 + 56.1 \qquad R_V = .693$

$y_3 = .032x_1 - 46.04 \qquad R_R = .248$

y_1 = trend component

y_2 = percentage population change

y_3 = residuals from the spatial trend

x_1 = township population density

x_2 = township agricultural assessment

x_3 = market potential

APPENDIX

i) A Program for studying the relationships of Spatially Distributed Variables with Distances from points

ii) General Flowchart

iii) Program

A PROGRAM FOR STUDYING THE RELATIONSHIPS OF
SPATIALLY DISTRIBUTED VARIABLES
WITH DISTANCES FROM POINTS

LANGUAGE: Fortran IV

OUTPUT:

I. Initial Data
   a. The 'X' and 'Y' co-ordinates and the magnitude of the spatially distributed variable

II. Three correlation matrices
   a. An initial 15x15 matrix of correlation coefficients
      i. The location and magnitude of the largest correlation coefficient (absolute value)
      ii. The location and magnitude of the four diagonal correlation coefficients.
   b. A second 10x10 matrix of fine correlation coefficients
      i. The location and magnitude of the largest fine correlation coefficient
      ii. The location and magnitude of the four adjacent diagonal coefficients
   c. A third 8x8 matrix of very fine correlation coefficients
      i. The location and magnitude of the largest very fine correlation coefficient

III. The actual 'X' and 'Y' map co-ordinates of the point of highest correlation

IV. The residuals from the correlation analysis
   a. The residuals are printed eight to a row with the first residual corresponding to the first item of the initial data. The residual corresponding to the second item of data is found in row one, column two; and so on...

V. The explained variance due to the regression of the distance from the initial point of highest correlation to each item and the magnitude of each item.

VI. A second set of three correlation matrices
   a. The output is of the same form as sections two through five. This time, however, the explained variance is due to the regression of the distances from the second point of highest correlation to each item and the magnitude of the residuals for each item from the initial correlation.

VII. A third set of three correlation matrices
   a. Output continues until explained variance of a set of residuals on the previous set of residuals falls below five per cent.

RESTRICTIONS: This program handles up to 500 items and their co-ordinates.

CARD PREPARATION:

First Control Card

cols 1-5 1 if residual print-out is desired, 0 otherwise.
6-10 1 if reciprocal transformation plus one of distance required.
11-15 1 if log transformation of distance required.

Second Control Card

cols 1-3 number of items (variables)
4-40 variable format statement defining the format of the card input (The first three fields should be in A-conversion and are used for the variables' names). The variable format should begin with 1X. The subsequent fields can be numeric or blank.

For example:

a statement (1X, 3A6, 3F9.0) reserves column 1 for machine control, cols 2-19 for the variable name, and cols 20-28 for the 'X' co-ordinate, cols 29-37 for the 'Y' co-ordinate, and cols 38-46 for the magnitude of the variable.

Data cards: The data cards are punched in accordance with the variable format statement contained in the second control card.

DECK-MAKE-UP:

i. $JOB Monitor Control Card
ii. Fortran IV Source Program Cards
iii. $DATA Control Card
iv. First Control Card
v. Second Control Card
vi. Data Cards

<u>Note</u>: This program is designed to be read on input-tape 5 and write on output-tape 6. It can be adjusted to suit the local computer installation be changing the 5 and 6 on source program cards SPADIO12 and SPADIO13 respectively.

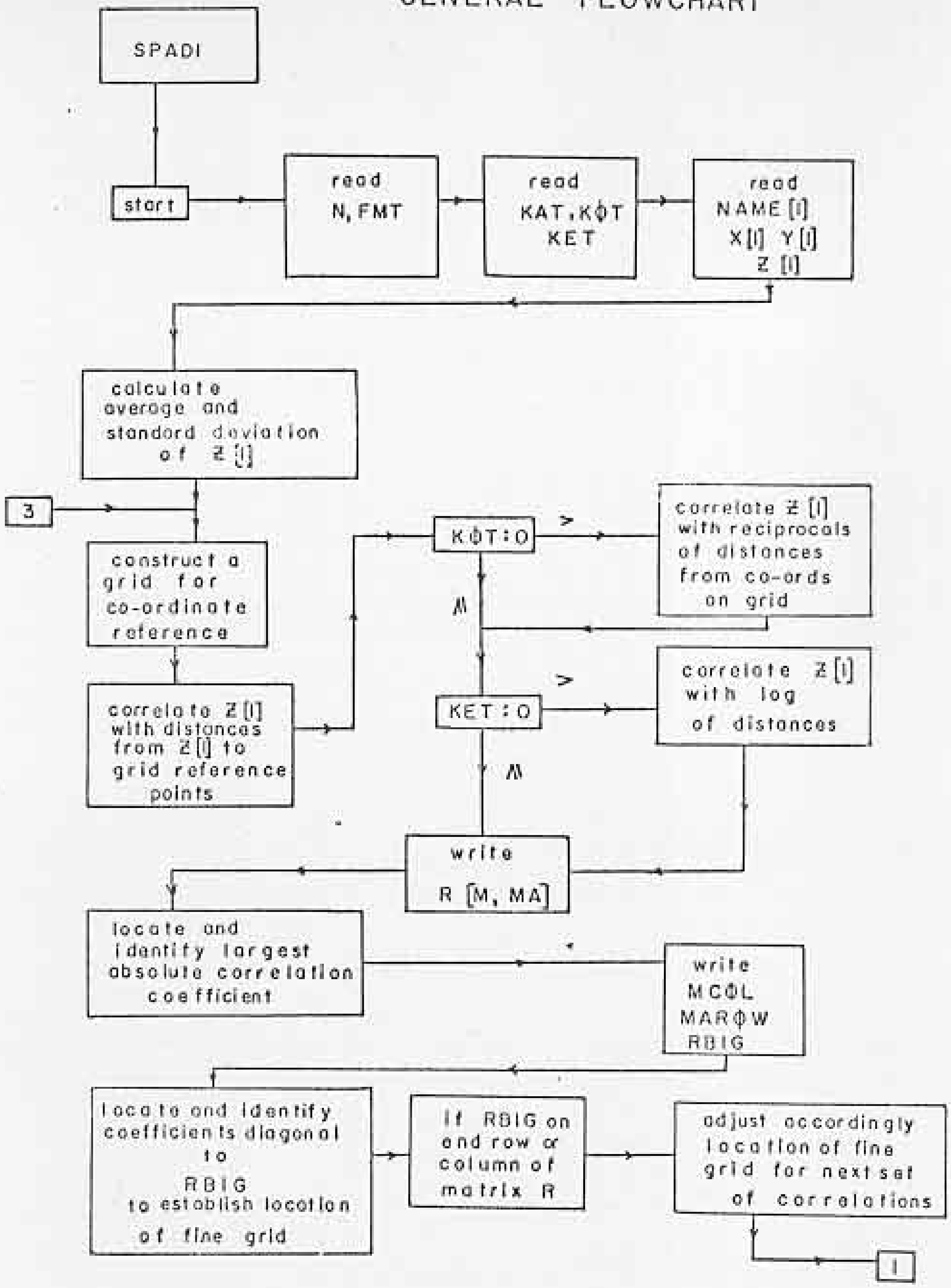
LIST OF SYMBOLS USED IN THE PROGRAM

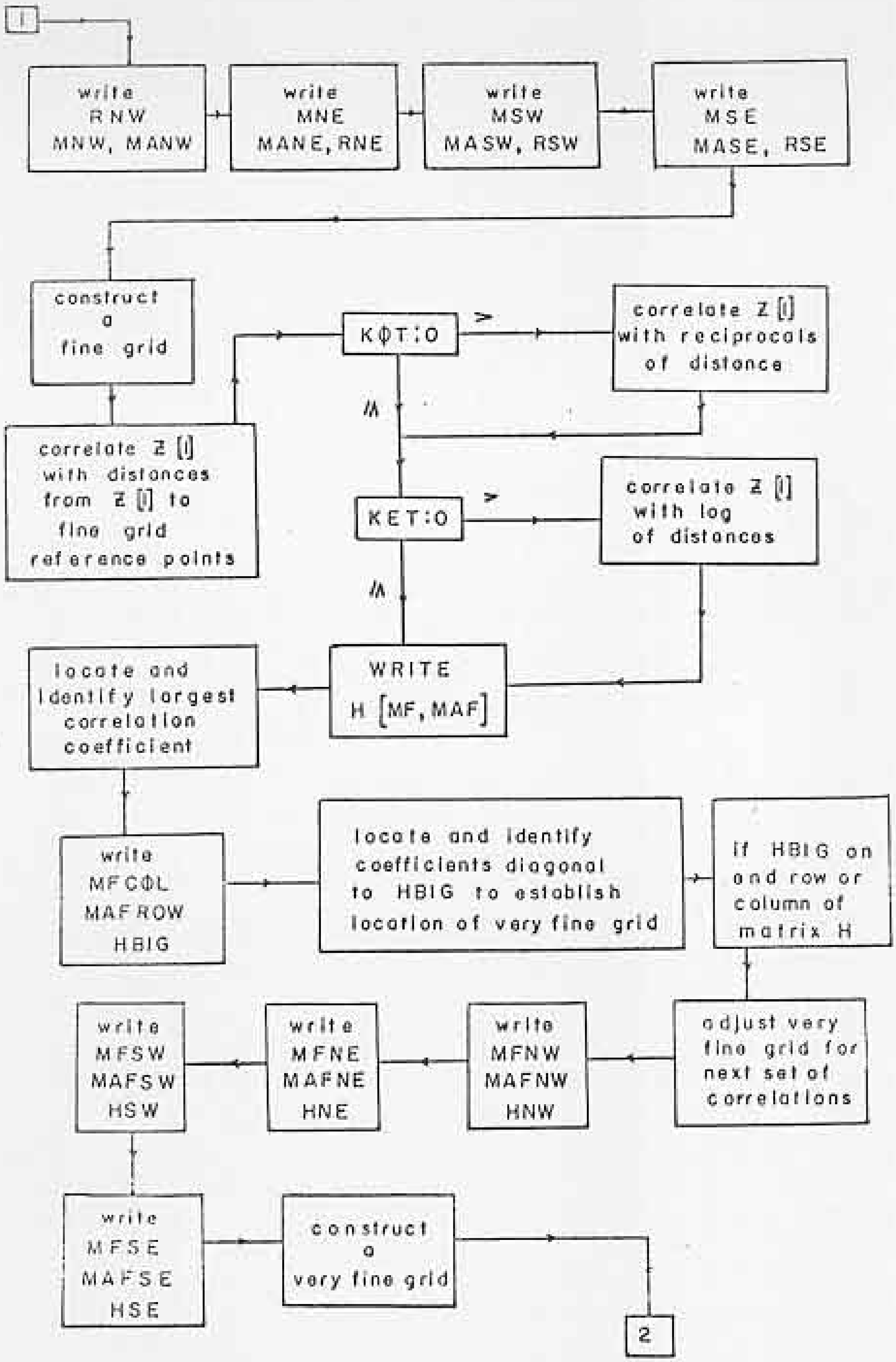
FLOW-CHART

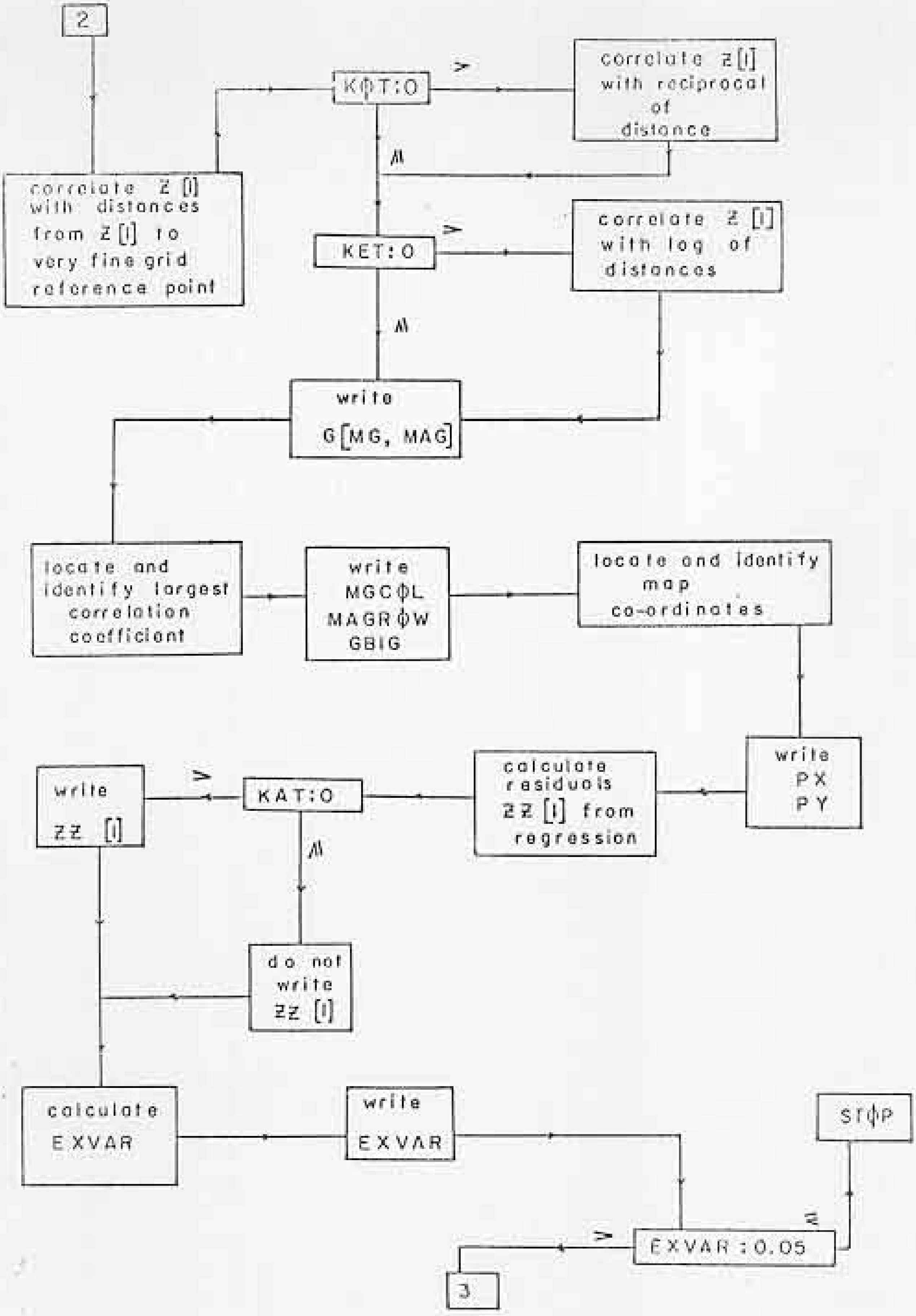
N	Number of spatially distributed variables
FMT	Variable format
KAT	Option for residual print-out
KOT	Option for reciprocal transformation
KET	Option for log transformations
NAME(I)	Name of variables
X(I)	X co-ordinate of each variable
Y(I)	Y co-ordinate of each variable
Z(I)	Value assigned to each variable
R(M,MA)	Initial matrix of correlation, M and MA equal 15
MCOL	The column of the large correlation coefficient
MAROW	The row of the large correlation coefficient
RBIG	The largest correlation coefficient (Absolute value)
MNW	Column to the left of RBIG
MANW	Row above RBIG
RNW	Upper left diagonal correlation coefficient to RBIG
MNE	Column to the right of RBIG
MANE	Row above RBIG
RNE	Upper right diagonal correlation coefficient to RBIG
MSW	Column to the left of RBIG
MASW	Row below RBIG
RSW	Lower left diagonal correlation coefficient to RBIG
MSE	Column to the right of RBIG
MASE	Row below RBIG

RSE	Lower right diagonal correlation coefficient to RBIG
H(MF,MAF)	Matrix of fine correlation coefficients, MF and MAF = 10
MFCOL	The column of the largest fine correlation coefficient
MAFROW	The row of the largest fine correlation coefficient
HBIG	The largest fine correlation coefficient
MFNW	Column to the left of HBIG
MAFNW	Row above HBIG
HNW	Upper left diagonal correlation coefficient to HBIG
MFNE	Column to the right of HBIG
HNE	Upper right diagonal correlation coefficient to HBIG
MFSW	Column to the left of HBIG
MAFSW	Row below HBIG
HSW	Lower left diagonal correlation coefficient to HBIG
MFSE	Column to the right of HBIG
MAFSE	Row below HBIG
HSE	Lower right diagonal correlation coefficient to HBIG
G(MG,MAG)	Matrix of very fine correlation coefficients MG equals MAG equals 8
MGCOL	Column of the largest very fine correlation coefficient
MAGROW	row of the largest very fine correlation coefficient
GBIG	The largest very fine correlation coefficient
PX	The 'X' co-ordinate of the point of highest correlation
PY	The 'Y' co-ordiante of the point of highest correlation
ZZ(I)	The residuals
EXVAR	Explained variance.

GENERAL FLOWCHART




1
write RNW MNW, MANW
write MNE MANE, RNE
write MSW MASW, RSW
write MSE MASE, RSE
construct a fine grid
correlate Z [I] with distances from Z [I] to fine grid reference points
KØT:0
>
correlate Z [I] with reciprocals of distance
≥
KET:0
>
correlate Z [I] with log of distances
≥
WRITE H [MF, MAF]
locate and identify largest correlation coefficient
write MFCØL MAFROW HBIG
locate and identify coefficients diagonal to HBIG to establish location of very fine grid
if HBIG on end row or column of matrix H
adjust very fine grid for next set of correlations
write MFNW MAFNW HNW
write MFNE MAFNE HNE
write MFSW MAFSW HSW
write MFSE MAFSE HSE
construct a very fine grid
2



2
KØT:0
>
correlate Z[I] with reciprocal of distance
≥
correlate Z [I] with distances from Z[I] to very fine grid reference point
KET:0
>
correlate Z [I] with log of distances
≥
write G[MG, MAG]
locate and identify largest correlation coefficient
write MGCØL MAGRØW GBIG
locate and identify map co-ordinates
write PX PY
calculate residuals ZZ [I] from regression
KAT:0
>
≥
write ZZ [I]
do not write ZZ [I]
calculate EXVAR
write EXVAR
EXVAR : 0.05
>
≤
STØP
3

```
   95 CONTINUE
      WRITE(NOUT,3)
    3 FORMAT (1H1,5X,11H REGRESSION,5X,7H NUMBER,I3)
      WRITE (NOUT,6)
    6 FORMAT(16X,25H CORRELATION COEFFICIENTS)
      WRITE (NOUT,7) ((R(M,MA),M=1,15),MA=1,15)
    7 FORMAT(3X,15F8.4)
C
C     SECTION TO LOCATE AND IDENTIFY THE LARGEST CORRELATION COEFFICIENT
C
      RBIG=R(1,1)
      MCOL=1
      MAROW=1
      M=1
      MA=1
  100 IF(ABS(R(M,MA))-ABS(RBIG)) 102,102,101
  101 RBIG=R(M,MA)
      MCOL=M
      MAROW=MA
  102 IF(MA-15) 103,105,105
  103 MA=MA+1
      GO TO 100
  105 IF(M-15) 106,120,120
  106 M=M+1
  107 MA=1
      GO TO 100
  120 CONTINUE
      WRITE (NOUT,8)
    8 FORMAT(10X,7H COLUMN,10X,4H ROW,10X,6H VALUE)
      WRITE (NOUT,9) MCOL,MAROW,RBIG
    9 FORMAT(14X,I3,11X,I3,8X,F8.4)
C
C     SECTION TO LOCATE AND IDENTIFY THE CORRELATION COEFFICIENTS
C     DIAGONAL TO  RBIG
C
      MCEL=MCOL
      MAREW=MAROW
      IF (MCEL-1) 160,160,170
  160 MCEL=2
  170 CONTINUE
      IF(MAREW-1)180,180,190
  180 MAREW=2
  190 CONTINUE
      IF(MCEL-15) 210,200,200
  200 MCEL=14
  210 CONTINUE
      IF(MAREW-15) 230,220,220
  220 MAREW=14
  230 CONTINUE
      RNW=R(MCEL-1,MAREW-1)
      MNW=MCEL-1
      MANW=MAREW-1
      RNE=R(MCEL+1,MAREW-1)
      MNE=MCEL+1
      MANE=MAREW-1
      RSW=R(MCEL-1,MAREW+1)
      MSW=MCEL-1
      MASW=MAREW+1
      RSE=R(MCEL+1,MAREW+1)
      MSE=MCEL+1
      MASE=MAREW+1
  235 CONTINUE
      WRITE(NOUT,10)
   10 FORMAT(7X,4H MNW,7X,5H MANW,10X,4H RNW,7X,4H MNE,7X,5H MANE,10X,
     14H RNE)
      WRITE (NOUT,21) MNW,MANW,RNW,MNE,MANE,RNE
      WRITE (NOUT,12)
   12 FORMAT (7X,4H MSW,7X,5H MASW,10X,4H RSW,7X,4H MSE,7X,5H MASE,10X,
     1 4H RSE)
      WRITE(NOUT,21) MSW,MASW,RSW,MSE,MASE,RSE
C
C     SECTION TO OBTAIN A FINE GRID
C
      VA=(2.0*VA)/10.0
  240 XX=SMAX
      AAA=MNW
      XXX=XX+((AAA-1.0)*VA)
      YY=SMAY
      BBB=MANW
      YYY=YY+((BBB-1.0)*VA)
      MF=1
      MAF=1
C
C     SECTION TO CALCULATE THE CORRELATION OF Z AND THE DISTANCES
C     FROM XXXYYY AND LOCATE IT IN THE MATRIX H
C
  241 DO 242 I=1,N
  242 D(I)=SQRT(((X(I)-XXX)**2)+((Y(I)-YYY)**2))
C
C     RECIPROCAL TRANSFORMATION     KOT
C
      IF (KOT) 249,249,248
  248 DO 981 I=1,N
  981 D(I)=1.0/(D(I)+1.0)
```

```
C
C     PROGRAMME BY E.CASETTI AND R. K. SEMPLE
C     DEPARTMENT OF GEOGRAPHY UNIVERSITY OF TORONTO OCTOBER  1968
C
C     PROGRAMME FOR STUDYING THE RELATIONSHIPS OF SPATIALLY DISTRIBUTED
C     VARIABLES WITH DISTANCES FROM POINTS
      DIMENSION X(500),Y(500),Z(500),R(15,15),NAME(500,3),FMT(6),A(500)
      DIMENSION H(10,10),B(500),G(8,8),C(500),D(500)
      DIMENSION ZZ(500),BAR(500),SLOPE(500)
C
C     SECTION TO READ AND PRINT DATA
      NIN=5
      NOUT=6
      READ (NIN,999) KAT,KOT,KET
  999 FORMAT (3I5)
      READ (NIN,1) N,FMT
    1 FORMAT (I3,6A6)
      WRITE (NOUT,4)
    4 FORMAT (/20X,4HDATA,/)
      READ (NIN,FMT)(((NAME(I,J),J=1,3),X(I),Y(I),Z(I)),I=1,N)
    5 WRITE (NOUT,FMT)(((NAME(I,J),J=1,3),X(I),Y(I),Z(I)),I=1,N)
C
C     SECTION TO CALCULATE THE AVERAGE AND STANDARD DEVIATION OF Z
C
      RA=N
      ZA=0.0
      DO 50 I=1,N
   50 ZA=ZA+Z(I)
      ZA=ZA/RA
      ZVA=0.0
      DO 55 I=1,N
      Z(I)=Z(I)-ZA
   55 ZVA=ZVA+(Z(I)**2)
      ZVA=ZVA/RA
      ZVA=SQRT(ZVA)
C     SECTION TO OBTAIN A GRID
   56 SMAX=AMIN1(X,1,1,N,1)
      BIGX=AMAX1(X,1,1,N,1)
      SMAY=AMIN1(Y,1,1,N,1)
      BIGY=AMAX1(Y,1,1,N,1)
      DX=BIGX-SMAX
      DY=BIGY-SMAY
      IF(DX-DY) 51,51,53
   51 VA=DY/15.0
      GO TO 54
   53 VA=DX/15.0
   54 XX=SMAX
      YY=SMAY
      M=1
      MA=1
C
C     CALCULATE THE CORRELATION OF Z AND THE DISTANCES FROM XXYY AND
C     LOCATE IT IN THE MATRIX R
C
   59 DO 60 I=1,N
   60 A(I)=SQRT(((X(I)-XX)**2)+((Y(I)-YY)**2))
C
C     RECIPROCAL TRANSFORMATION        KOT
C
      IF(KOT) 69,69,68
   68 DO 980 I=1,N
  980 A(I)=1.0/(A(I)+1.0)
   69 CONTINUE
C
C     LOG TRANSFORMATION               KET
C
      IF (KET) 64,64,991
  991 DO 62 I=1,N
   62 A(I)=ALOG(A(I))
   64 CONTINUE
      AA=0.0
      DO 65 I=1,N
   65 AA=AA+A(I)
      AA=AA/RA
      AVA=0.0
      DO 70 I=1,N
      A(I)=A(I)-AA
   70 AVA=AVA+(A(I)**2)
      AVA=AVA/RA
      AVA=SQRT(AVA)
      DO 75 I=1,N
   75 R(M,MA)=R(M,MA)+(A(I)*Z(I))
      R(M,MA)=R(M,MA)/(AVA*ZVA*RA)
      IF (MA-15) 80,85,85
   80 MA=MA+1
      YY=YY+VA
      GO TO 59
   85 IF (M-15) 90,95,95
   90 M=M+1
      MA=1
      XX=XX+VA
      YY=SMAY
      GO TO 59
```

```
   20 FORMAT(7X,5H MFSW,7X,5HMAFSW,10X,4H HSW,7X,5H MFSE,7X,6H MAFSE,
     1 10X,4H HSE)
      WRITE(NOUT,21) MFSW,MAFSW,HSW,MFSE,MAFSE,HSE
   21 FORMAT(7X,I3,7X,I3,7X,F8.4,7X,I3,7X,I3,7X,F8.4)
  505 CONTINUE
C
C     SECTION TO OBTAIN A VERY FINE GRID
C
  509 GA=(2.0*HA)/8.0
      XX=SMAX
      YY=SMAY
      XXX=XX+((AAA-1.0)*VA)
      CCC=MFSW
      XXXX=XXX+((CCC-1.0)*HA)
      YYY=YY+((BBB-1.0)*VA)
      DDD=MAFSW
      YYYY=YYY+((DDD-1.0)*HA)
      MG=1
      MAG=1
C
C     SECTION TO CALCULATE THE CORRELATION OF Z AND THE DISTANCES
C     FROM XXXXYYYY AND LOCATE IT IN THE MATRIX G
C
  541 DO 542 I=1,N
  542 C(I)=SQRT(((X(I)-XXXX)**2)+((Y(I)-YYYY)**2))
C
C     RECIPROCAL TRANSFORMATION       KOT
C
      IF(KOT) 549,549,548
  548 DO 982 I=1,N
  982 C(I)=1.0/(C(I)+1.0)
  549 CONTINUE
C
C     LOG TRANSFORMATION     KET
C
      IF(KET) 547,547,993
  993 DO 546 I=1,N
  546 C(I)=ALOG(C(I))
  547 CONTINUE
      CC=0.0
      DO 543 I=1,N
  543 CC=CC+C(I)
      CC=CC/DA
      CVA=0.0
      DO 545 I=1,N
      C(I)=C(I)-CC
  545 CVA=CVA+(C(I)**2)
      CVA=CVA/DA
      CVA=SQRT(CVA)
      DO 550 I=1,N
  550 G(MG,MAG)=G(MG,MAG)+(C(I)*Z(I))
      G(MG,MAG)=G(MG,MAG)/(CVA*ZVA*DA)
      IF(MAG-8) 555,560,560
  555 MAG=MAG+1
      YYYY=YYYY+GA
      GO TO 541
  560 IF(MG-8) 565,570,570
  565 MG=MG+1
      MAG=1
      XXXX=XXXX+GA
      YYYY=YYY+((DDD-1.0)*HA)
      GO TO 541
  570 CONTINUE
      WRITE (NOUT,22)
   22 FORMAT (56X,35H VERY FINE CORRELATION COEFFICIENTS)
      WRITE(NOUT,23) ((G(MG,MAG),MG=1,8), MAG=1,8)
   23 FORMAT(25X,8F8.4)
C
C     SECTION TO LOCATE AND IDENTIFY THE LARGEST CORRELATION
C     COEFFICIENT IN THE VERY FINE GRID
C
      GRIG=G(1,1)
      MGCOL=1
      MAGROW=1
      MG=1
      MAG=1
  600 IF(ABS(G(MG,MAG))-ABS(GRIG)) 602,602,601
  601 GRIG=G(MG,MAG)
      MGCOL=MG
      MAGROW=MAG
  602 IF(MAG-8) 603,605,605
  603 MAG=MAG+1
      GO TO 600
  605 IF(MG-8) 606,610,610
  606 MG=MG+1
  607 MAG=1
      GO TO 600
  610 CONTINUE
      WRITE(NOUT,24)
   24 FORMAT(10X,7H COLUMN,10X,4H ROW,10X,6H VALUE)
      WRITE(NOUT,25) MGCOL,MAGROW,GRIG
   25 FORMAT (14X,I3,11X,I3,8X,F8.4)
C
C     SECTION TO CALCULATE ACTUAL MAP CO-ORDINATES OF HIGHEST
```

```
  248 CONTINUE
C
C     LOG TRANSFORMATION          KET
C
      IF(KET) 247,247,999
  999 DO 246 I=1,N
  246 B(I)=ALOG(B(I))
  247 CONTINUE
      BB=0.0
      DO 243 I=1,N
  243 BB=BB+B(I)
      BB=BB/DA
      BVA=0.0
      DO 245 I=1,N
      B(I)=B(I)-BB
  245 BVA=BVA+(B(I)**2)
      BVA=BVA/DA
      BVA=SQRT(BVA)
      DO 250 I=1,N
  250 H(MF,MAF)=H(MF,MAF)+(B(I)*Z(I))
      H(MF,MAF)=H(MF,MAF)/(BVA*ZVA*DA)
      IF(MAF-10)255,260,260
  255 MAF=MAF+1
      YYY=YYY+HA
      GO TO 241
  260 IF(MF-10) 265,270,270
  265 MF=MF+1
      MAF=1
      XXX=XXX+HA
      YYY=YYY-((BMF-1.0)*VA)
      GO TO 241
  270 CONTINUE
      WRITE(NOUT,14)
   14 FORMAT (56X,30H FINE CORRELATION COEFFICIENTS)
      WRITE (NOUT,15) ((H(MF,MAF), MF=1,10),MAF=1,10)
   15 FORMAT (10X,10F8.4)
C
C     SECTION TO LOCATE AND IDENTIFY THE LARGEST CORRELATION
C     COEFFICIENT IN THE FINE GRID
      HBIG=H(1,1)
      MFCOL=1
      MAFROW=1
      MF=1
      MAF=1
  300 IF(ABS(H(MF,MAF))-ABS(HBIG)) 302,302,301
  301 HBIG=H(MF,MAF)
      MFCOL=MF
      MAFROW=MAF
  302 IF(MAF-10) 303,305,305
  303 MAF=MAF+1
      GO TO 300
  305 IF(MF-10) 306,310,310
  306 MF=MF+1
  307 MAF=1
      GO TO 300
  310 CONTINUE
      WRITE (NOUT,16)
   16 FORMAT(10X,7H COLUMN,10X,4H ROW,10X,6H VALUE)
      WRITE (NOUT,17) MFCOL,MAFROW, HBIG
   17 FORMAT(14X,I3,11X,I3,8X,F8.4)
C     SECTION TO LOCATE AND IDENTIFY THE CORRELATION COEFFICIENTS
C     DIAGONAL TO HBIG
      MFCEL=MFCOL
      MAFREW=MAFROW
      IF(MFCEL-1)360,360,370
  360 MFCEL=2
  370 CONTINUE
      IF(MAFREW-1)380,380,390
  380 MAFREW=2
  390 CONTINUE
      IF(MFCEL-10) 410,400,400
  400 MFCEL=9
  410 CONTINUE
      IF(MAFREW-10) 430,420,420
  420 MAFREW=9
  430 CONTINUE
      HNW=H(MFCEL-1,MAFREW-1)
      MFNW=MFCEL-1
      MAFNW=MAFREW-1
      HNE=H(MFCEL+1,MAFREW-1)
      MFNE=MFCEL+1
      MAFNE=MAFREW-1
      HSW=H(MFCEL-1,MAFREW+1)
      MFSW=MFCEL-1
      MAFSW=MAFREW+1
      HSE=H(MFCEL+1,MAFREW+1)
      MFSE=MFCEL+1
      MAFSE=MAFREW+1
  440 CONTINUE
      WRITE (NOUT,18)
   18 FORMAT(7X,5H MFNW,7X,6H MAFNW,10X,4H HNW,7X,4HMFNE,7X,6H MAFNE,
     1 10X,4H HNE)
      WRITE(NOUT,21) MFNW,MAFNW,HNW,MFNE,MAFNE,HNE
      WRITE(NOUT,20)
```

```
C     CORRELATION COEFFICIENT
C
      PP=MGCOL
      PQ=MGROW
      XX=SMAX
      YY=SMAY
      XXX=XX+((AAA-1.0)*VA)
      YYY=YY+((BBB-1.0)*VA)
      XXXX=XXX+((CCC-1.0)*HA)
      YYYY=YYY+((DDD-1.0)*HA)
      PX=XXXX+((PP-1.0)*GA)
      PY=YYYY+((PQ-1.0)*GA)
      WRITE(NOUT,30)
   30 FORMAT(20X,32H MAP CO-ORDS OF HIGH CORRELATION)
      WRITE (NOUT,31)
   31 FORMAT (17X,14H X CO-ORDINATE,5X,14H Y CO-ORDINATE)
      WRITE (NOUT,32) PX,PY
   32 FORMAT (10X,2F10.2)
C
C     SECTION TO CALCULATE THE RESIDUALS
C
      RA=N
      DA=0.0
  641 DO 642 I=1,N
  642 D(I)=SQRT(((X(I)-PX)**2)+((Y(I)-PY)**2))
C
C     RECIPROCAL TRANSFORMATIONS    KOT
C
      IF (KOT) 647,647,646
  646 DO 750 I=1,N
  750 D(I)=1.0/(D(I)+1.0)
  647 CONTINUE
C
C     LOG TRANSFORMATION          KET
C
      IF (KET) 701,701,702
  702 DO 703 I=1,N
  703 D(I)=ALOG(D(I))
  701 CONTINUE
      DO 643 I=1,N
  643 DA=DA+D(I)
      DA=DA/RA
      BARA=0.0
      SLOP=0.0
      DO 644 I=1,N
      BAR(I)=(D(I)-DA)*Z(I)
      BARA=BARA+BAR(I)
      SLOPE(I)=(D(I)-DA)*(D(I)-DA)
  644 SLOP=SLOP+SLOPE(I)
      SLOP=BARA/SLOP
      DO 645 I=1,N
  645 ZZ(I)=Z(I)-SLOP*(D(I)-DA)
      IF (KAT) 649,649,996
  996 WRITE (NOUT,995)
  994 FORMAT (40X,10H RESIDUALS)
      WRITE (NOUT,996) (ZZ(I), I=1,N)
  995 FORMAT (10X,8F10.0)
  649 CONTINUE
C
C     SECTION TO CALCULATE EXPLAINED VARIANCE
C
      ZAN=0.0
      DO 650 I=1,N
  650 ZAN=ZAN+ZZ(I)
      ZAN=ZAN/RA
      ZVAN=0.0
  651 DO 655 I=1,N
      ZZ(I)=ZZ(I)-ZAN
  655 ZVAN=ZVAN+(ZZ(I)**2)
      ZVAN=ZVAN/RA
  660 EXVAR=GRIG*GRIG
  661 CONTINUE
      WRITE (NOUT,26)
   26 FORMAT (1H ,5X,19H EXPLAINED VARIANCE)
      WRITE (NOUT,27) EXVAR
   27 FORMAT (8X,F10.5)
  662 CONTINUE
  670 IF(EXVAR-0.05) 700,700,680
  680 ZVA=SQRT(ZVAN)
      DO 681 I=1,N
  681 Z(I)=ZZ(I)
      GO TO 56
  700 CONTINUE
      STOP
      END
*DATA
    1    1    0
(2(1X,3A6,F8.0,F9.0,12X,F6.0)
 RICHMOND HILL              320        260          -1403    87
 GEORGETOWN                 285        288          -1375    71
 PORT CREDIT                310        295          -1342    44
 MARKHAM                    332        262          -1217     2
 PRESCOTT                   588        172          -1188    -6
 STONEY CREEK               303        325          -1201   -17
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AURORA	328	250	-1176	-18
STREETSVILLE	300	290	-1104	-62
CHIPPAWA	348	343	-1189	-67
MILTON	290	300	-1094	-71
GRIMSBY	312	328	-1053	-105
[illegible]	527	108	-975	-126
WOODBRIDGE	318	260	-1047	-161
ACTON	280	289	-902	-183
ALLISTON	292	236	-900	-203
FONT HILL	328	338	-966	-204
AYLMER	211	366	-805	-236
BANCROFT	428	248	-814	-269
STOUFFVILLE	335	253	-785	-271
STURGEON FALLS	287	22	-729	-276
GODERICH	163	273	-740	-277
GANANOQUE	545	213	-758	-278
ORANGEVILLE	275	258	-725	-296
ESSEX	080	390	-824	-296
DELHI	245	361	-799	-301
DUNNVILLE	309	355	-724	-304
BRADFORD	313	266	-809	-308
PETROLIA	130	354	-716	-326
ELMIRA	243	288	-690	-336
FERGUS	254	278	-705	-339
CLINTON	175	285	-672	-348
AMHERSTBURG	059	427	-692	-351
PICTON	400	155	-677	-355
TECUMSEH	075	455	-685	-355
ST. MARY,S	202	319	-678	-356
NIAGARA	344	324	-776	-363
BEAMSVILLE	319	329	-655	-367
CARDINAL	597	165	-778	-371
CRYSTAL BEACH	347	360	-734	-371
STRATHROY	167	348	-605	-391
HUNTSVILLE	335	122	-642	-406
HESPELER	260	304	-574	-419
EXETER	138	311	-516	-420
ALMONTE	540	128	-587	-422
NAPANEE	482	223	-560	-425
GRAVENHURST	326	167	-607	-426
CARLETON PLACE	543	135	-548	-441
PORT PERRY	350	240	-548	-446
LISTOWEL	215	275	-599	-448
BRACEBRIDGE	330	150	-587	-450
KINGSVILLE	085	435	-650	-451
UXBRIDGE	342	240	-556	-453
WATERDOWN	290	314	-607	-461
CALEDONIA	285	338	-584	-462
HAGERSVILLE	277	350	-548	-474
HANOVER	211	234	-560	-488
KEMPTVILLE	579	145	-517	-496
PORT DOVER	266	356	-468	-499
STIRLING	451	219	-509	-504
LAKEFIELD	401	208	-488	-515
WALKERTON	203	237	-188	-518
BRIGHTON	439	244	-476	-531
CAMPBELLFORD	434	219	-437	-547
CHESTERVILLE	606	135	-470	-561
MITCHELL	199	301	-460	-566
WINGHAM	192	259	-433	-576
MEAFORD	241	151	-399	-578
BRIDGEPORT	240	275	-455	-586
MARMORA	441	203	-478	-588
MORRISBURG	610	158	-486	-588
IROQUOIS	601	160	-478	-591
FRANKFORD	448	230	-410	-593
TWEED	467	207	-396	-598
PALMERSTON	223	266	-410	-599
HARROW	072	434	-410	-603
TILBURY	108	412	-380	-604
PENETANGUISHENE	286	176	-345	-607
WATERFORD	260	353	-337	-619
SUTTON	327	229	-456	-624
BELLE RIVER	88	410	-378	-632
NEW HAMBURG	232	310	-341	-638
ROCKLAND	598	93	-299	-649
MADOC	455	200	-355	-658
PORT ELGIN	188	207	-399	-664
MATTAWA	371	027	-303	-674
ALEXANDRIA	644	104	-277	-676
WINCHESTER	598	137	-343	-676
DESERONTO	486	228	-269	-684
DRESDEN	127	382	-278	-695
BLENHEIM	139	407	-318	-698
WIARTON	205	178	-253	-706
WHEATLEY	106	427	-307	-712
KINCARDINE	169	231	-286	-713
DURHAM	225	233	-275	-739
FOREST	140	340	-272	-741
BEAVERTON	341	208	-260	-747
RIDGETOWN	147	396	-208	-757
SOUTHAMPTON	189	201	-281	-758
STAYNER	230	251	-154	-759
SEAFORTH	184	292	-199	-761
NORWICH	239	348	-148	-772
PORT STANLEY	195	377	-268	-776
MOUNT FOREST	249	218	-193	-780
COLBORNE	479	247	-206	-781
LITTLE CURRENT	155	057	-240	-791
ELORA	276	210	-189	-792
WEST LORNE	163	385	-238	-792
VANKLEEK HILL	640	097	-141	-807
SHELBURNE	267	242	-251	-825
HARRISTON	281	259	-193	-826
EGANVILLE	477	100	-112	-846
FENELON FALLS	370	198	-136	-847
HAVELOCK	427	207	-110	-856
MARKDALE	236	220	-130	-876
WATFORD	153	350	-75	-879
CHESLEY	215	220	-132	-887
MILVERTON	218	291	-109	-893
BARRY,S BAY	439	105	-57	-894
ARTHUR	245	265	-106	-897
BOBCAYGEON	382	199	-57	-897
CASSELMAN	616	117	-28	-951