

## "Patterns are Morphological Laws"

Fred K. Schaefer

### PATTERNS OF LOCATION

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Schaefer's "morphological laws" are receiving increased attention by theoretical geographers. We explore the subject.

#### I. Introduction

This paper is intended to be a second supplement to the book, Theoretical Geography.<sup>1</sup> In the basic book, geography was related to science after Christaller's primary substantive example and Schaefer's primary methodology. The central problem of theoretical geography was tentatively identified as "the nearness problem," that is, "to located interacting objects as near to each other as possible." The first supplement to the basic book is an article entitled "Spatial Relations: The Subject of Theoretical Geography."<sup>2</sup> There the focus was sharpened by a hard examination of the methodological comments of select contemporary theoretical geographers and by a systematic review of some of the mathematics of space as applied to geography.

## II. Three Types of Location

Predictive or theoretical geography has important old roots in geography; certainly Davis and Köppen were predictive geographers and a close historical examination might reveal much earlier antecedents. As in prospecting for oil, so in history, we usually find traces of what we seek if we look hard enough. But the main historical upthrust of the American-Swedish school of predictive geography has been through Christaller. Schaefer crystallized the methodological implications of Christaller's and related substantive work. Schaefer described the essence of predictive geography as the discovery of predictive patterns. Schaefer's insistence on spatial patterns (or the earth's geometry, or spatial structure, or whatever term you prefer) seems to run head on into predictive geographers' interest in movements (or circulations or spatial process or whatever term you prefer). The history of this important methodological contribution is reported elsewhere.<sup>3</sup> Tentative conclusions are that most locations on the earth's surface are explained, which is logically equivalent to predicted, on the basis of optimal movements (or geodesics, or least effort paths or whatever term you favor). Earlier the author has identified this recurring theme as the nearness problem - the problem of locating interacting objects as near to each other as possible. The catholic claim made for this approach is that it cuts across all branches of traditional systematic geography. Also, it is admitted outright that the probabilistic aspects of predictive geography do not conform to optimal arrangements.<sup>4</sup> With this important admission behind us the remainder of this paper will contain no further comment on location theory based on probabilities.

The "Schaefer versus almost all others" argument in favor of patterns rather than movements as the subject of theoretical geography dissolves to nothing when it is realized that particular optimal movements result in particular patterns, that the geometry and movements are intertwined in spatial harmony. In this paper, we will approach our subject from the point of view of Schaefer's patterns rather than interactions or movements of Ullman and others.<sup>5</sup> In some ways Schaefer's pattern approach to this dialectic is the more appealing of the two since the patterns are so easy to "map." Of course, the "maps" are not true geographic maps since they do not apply to any particular locations on the earth's surface; rather, they are idealized "maps" such as Köppen's Hypothetical Continent.

Very recent substantive work, especially by Boyce and Clark, Dacey, Nordbeck, and Tobler, is sharply in the spirit of a Universal Systematic Geography.<sup>6</sup> Dacey especially has been engaged in work on patterns. But the

basic inspiration for this paper was received from Christaller and one of Tobler's favorite nongeographic teachers, Thompson.<sup>7</sup>

Christaller makes two isotropic assumptions; (1) that the density of his base object is uniform (even distribution of rural population) and (2) that his space is not twisted (uniform transportation in all directions). Thompson's work, dealing with the spatial but nongeographic aspects of life forms, contains most suggestive material. The very title of Thompson's book, On Growth and Form, can be paraphrased On Movement and Geometry without too much violence to his intended meaning.

Startling resemblances between geographic problems and those in biology might tempt readers to conclude, using our own argument of conservation of academic effort, that a really efficient science of location would include the spatial aspects of biology, physics, and so forth, as a study in General Systems. I think not. The very range of spatial scale between various sciences introduces worlds so weird that spatial experts in one have little to say to spatial experts in another. Subatomic physics is at a scale so different from the earth's surface as the home of man that the space is qualitatively almost totally foreign. Subatomic geometries have been superficially apposed to astronomical geometry by comparisons between the orbits of planets and electrons. In spite of early and prolonged attempts at such synthesis, efforts have been unsuccessful and the persistent experience of failure has been codified as "reductionism." However, while grandiose comparisons have been general failures some cross inspiration can be expected. For instance, Eigen values obtained from the characteristics of matrices have been used to explain ring jumps of electrons. In geography we might explore the characteristic of linear transformations as revealed in their matrices to see if Christaller's settlements occur in rings.<sup>8</sup> This exploration in the geometry of symmetry might shed light on Christaller's crucial "fixed k" assumption.

Electrical engineers, biologists, economists and others will learn much from us, and we from them, but this merely expresses the ultimate universality and interconnection of all knowledge. The logical conclusion to be drawn from successful borrowing of spatial notions in biology and elsewhere is not necessarily the horror of feeling compelled to claim all spatial problems are geography simply because all geography seems to be spatial. Throughout this paper references are made to nongeographic subjects where geographers can expect both to learn and to teach, but no wild territorial claims are made.

Returning to the main point of Thompson's insights, he noticed that one could accurately measure the length of a given specie of fish by measuring its weight. He assumed a constant density and shape to the fish. The outline of an object can be considered to be a property of density. Where the object ends its density suddenly drops to zero, a discontinuity to its density. Therefore, internal density and external density (shape) are in fact essentially the single element of density. To a modern geographer the resemblances and differences to Christaller are immediately apparent. Christaller avoided the problems that the irregular density of shape introduce, that is, he avoided the boundary problem, by the often used device of imagining an infinite plain. Both Thompson and Christaller assumed uniform internal density. Christaller also assumed a uniform transportation surface while Thompson simply ignored the internal circulation of his fish. Furthermore, one of the ways in which we have powerfully extended Christaller's original work is in the direction of understanding the basic "dimensional tension" involved in the problem of trying to spread points over an area.<sup>9</sup> So that while Christaller himself did not explore the dimensional problems implicit in his work, his intellectual descendents were compelled too. Thus, when Thompson emphasized the dimensional aspects of his fish, another correspondence to Christaller's work is established.

To translate biology into geography, consider the problem of predicting the volume of water at a river's mouth (weight of fish). If the rainfall is constant over the valley basins (fish are of uniform density) and the shapes of the basins (fish) the same, then the lengths of the valleys (fish) are proportional to the capacities (weights) of the rivers (fish). In addition, the pattern of the river system (veins and arteries) depends on the slope (internal transportability) of the terrain (fishes body). We now identify the spatial elements involved as dimension, morphology and density. The three elements all appear to be mathematical groups and therefore highly independent of one another.<sup>16</sup> At this speculative point in our understanding it also appears that there is an order to their fundamentality. Dimensions seem to be the most basic followed by morphology and lastly density. Christaller, Thunen and others exhibit a tendency to want to reduce the problems to elemental dimensional form. Geographers are fairly skilled in shifting dimensions. A dot map of elevation or a dot map of mean annual rainfall has every bit as much claim to legitimacy as hidebound isoline representations, but these are transformations within the set of dimensions. The transformations between dimensions and morphology and/or density are much more violent and more rare.

The truly elemental nature of our discussion strikes home when we notice that the three elements are actually three fundamentally different ways of



predicting location. Therefore, I believe, we are at the heart of theoretical geography.

#### A. Patterns of Dimensional Location

In this section of our discussion only dimensions will be allowed to vary. Morphology and density are assumed to be uniform.

To obtain more accuracy and cover certain areas in classical climatology and oceanography and new problems in political geography, we could deal with three dimensions (volumes) as well as zero (points), one (lines) and two (areas) but it would merely clutter the argument so we will not include three or higher space.

1. "Locate points on lines so that they are as near to the line as possible!" The dual and equivalent statement is, "Arrange points on lines so that they are as far from each other as possible."

The pattern is a uniform distribution of points along a line. (Figure 1.)



Figure 1. Points space uniformly along a line.

The scale of the spacing in this and subsequent examples depends on the ratios of the objects to each other and is trivial in terms of our discussion. Notice that if only one point is involved the median center is the solution; therefore, the problem can be thought of as one of "multiple medians." The dual statement suggests an analogue computer based on magnets.<sup>11</sup> If small bar magnets are skewered through corks and all the magnets turned the same way so as to mutually repel each other and then the corked magnets are allowed to float in a long narrow trough of water, they will form a uniform pattern along a line.

One method of obtaining a grasp of the power of the pattern is to stare at the unlabelled pattern and ask yourself "Of what is this a map?" Some possible answers include filling stations along a highway, major volcanic peaks along the Cascades and the distribution of ice cream vendors along a beach. Notice that these suggested applications to the earth's surface are more than shallow spatial coincidences. For instance, the total travel cost along a beach for the consumer of ice cream is minimized by such a pattern.<sup>12</sup> The volcanic pattern minimizes the movement of magma in the fissure, or put in another way, the uniform distribution marks points of the greatest internal pressure.

Since we are approaching our subject this time from Schaefer's point of view of patterns, in the remainder of the article we will not point out the rather

obvious examples of minimized movements which each pattern represents!

2. "Locate straight lines in an area so that the lines are as near to the area as possible." The dual; "Arrange straight lines in an area so that the lines are as far from each other as possible." The pattern is one of straight parallel lines evenly spaced. (Figure 2.)

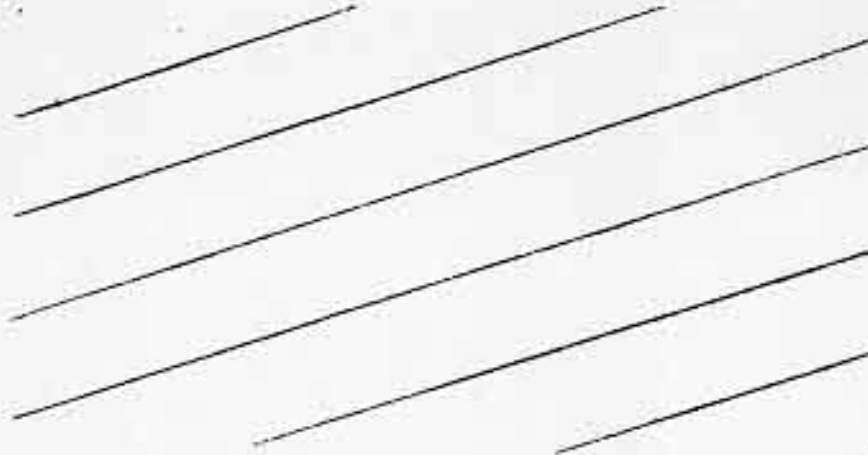


Figure 2. Parallel lines evenly spaced.

Straight lines are implied since we are allowing no morphology in our dimensional locations. Again the problem might be thought of in terms of self-repulsing floating magnets. This time the magnets are all tied together by strings to form self-repulsing lines. Notice that a single line "trying to get away from itself" is a straight line. The dual statement of placing objects as far from each other as possible is becoming rather tedious and will be dropped in most subsequent examples. It is well known that any extremum problem can be stated as a maximum or a minimum but still the reader might gain insight by conducting the exercise by himself.

Viewing the pattern as a map, such obvious translations as ridge and valley topography come to mind. I believe that the parallel line pattern is the most basic line-in-area pattern and that it is the fundamental river system, railroad pattern, etcetera.

3. "Locate points in an area as near to the area as possible." The pattern is the equilateral triangular distribution made so famous by Christaller. (Fig. 3a)



Figure 3a. Equilateral triangular distribution of points.

Possible maps include a range of objects such as classical settlement patterns, distributions of wild animals and efficient arrangements of oil wells.

So far we have drawn attention to the dialectic between the geometry and the movements but the dialectic can be even further expanded. Notice that the equilateral triangular distribution formally implies a hexagonal net. (Figure 3b.)

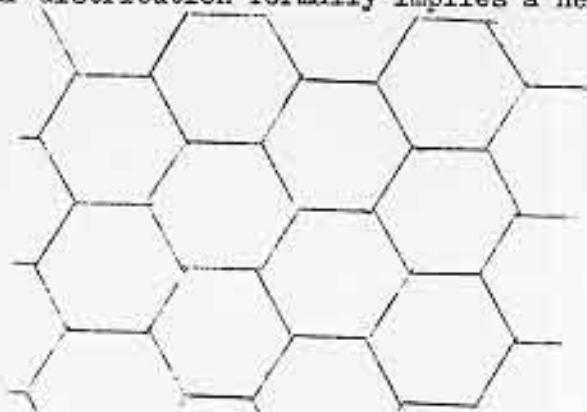


Figure 3b. Net of regular hexagons without points.

That is, we can establish the center of each cell in the net and thus generate the equilateral triangular distribution, or we can ask for the locus of all points halfway between the equilateral triangular distribution and immediately generate the net of hexagons. Stated in central place terms, the location of the central places determines the trade area and vice versa. It is really redundant to give both. These geometric duals appear often and, in combination with the implied movements, give a rich understanding from little. For instance, in Christaller's theory, with the single exogenous variable of  $k$ , say equals seven, consider the rich map we can draw. We can place settlements of various sizes, know their range of goods, their market boundaries, the movements to and from the centers to their hinterlands and even a great deal about the structure of their prices. The richness of Christaller's theoretical concepts have their foundations in the various formal mathematical dualities.

4. "Locate (straight) lines as near to points uniformly (equilateral triangularly) distributed in an area as possible." The pattern is, not too surprisingly, a set of parallel evenly spaced lines. (Figure 4.) The points must be



Figure 4. Parallel lines evenly spaced among an equilateral triangular distribution of points.

arranged in a uniform pattern or they will take on a variable density. Since one can project a mathematical surface into a density surface and conversely, obviously, only a uniform distribution of points will be "flat."

If we consider just one line notice how closely the problem resembles that of regression. In the first example and in the immediately previous example, the problems can be thought of as one of finding multiple medians. Here the problem is one of multiple regression in the sense of simultaneously fitting many lines to the "data."

This last example has introduced objects of three different dimensions at once--lines and points located in an area. There is no reason to stop here. We could have four different kinds of points, say representing farmsteads, villages, towns and cities all simultaneously located as near to each other as possible. What is evidently needed is some notation system for expressing the possible dimensional combinations. Often good notation systems are suggestive of mathematical relationships that further simplify and provide deeper insight, but a search for notation would place us ahead of ourselves. Instead, let us continue an informal exploration of the half seen theoretical landscape swirling out of the fog.

#### B. Patterns of Morphological Location

Points can not be shaped but both lines and areas can. Some mist still hangs over what we mean by shaped space. An area of disuniform transport cost can be thought of as stretched and puckered so that circles of equal cost would not draw on the area as circles of metric distance. These circles can be thought of as not just Tissot's circles which preserve circles in the small in conformal maps, but circles in the large as well so that both small and large angles, and thus all shapes, are preserved. A true "conformality" in the broadest application of the word. We refer to these space puckerings and stretchings as "internally shaped." Notice, again that lines as well as areas have the possibility of being internally shaped.

As an aside, we should be dissatisfied with our over reference to cost-miles and time-miles and even with Tobler's sophisticated utile-miles.<sup>13</sup> Perhaps our almost exclusive concern with such space-warper is due to the disproportionate influence of economic geography in current theoretical work. We need a grisly "death-miles" distance to explain human migration of a gross planetary sort. In climatology there exists the space twister of Coriolis acceleration. Coriolis-miles are just as legitimate as cost-miles.

All the previously mentioned patterns are seriously affected if we introduce the twisting of space. Obviously, if two points are at half the real miles as compared to the earth miles, they are located twice as close as an areal photo-



graph would indicate. The reader can readily imagine many examples but some are explicated here because they seem to shed genuine insight of a rather startling sort.

1. "Locate finite areas of different real-miles as near to a single point as possible." The pattern is one of concentric circles whose radii increase in unequal increments. (Figure 5) Treated as a map, Thünen rings of agriculture

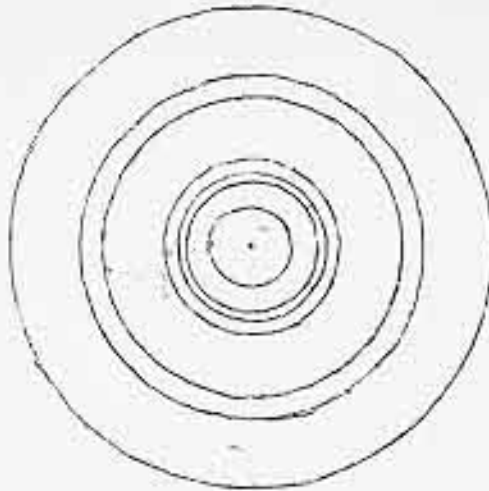


Figure 5. Concentric circles whose radii increase in unequal increments to produce irregular rings.

immediately come to mind. The rings suggest ecological circles of animals, from frogs to camels around a desert water hole. Rings of volcanic debris are caused by varying transportability of expelled material.

2. "Locate finite areas of different real miles as near to a straight line as possible." (Figure 6.) Thünen strips, with the central point replaced by a

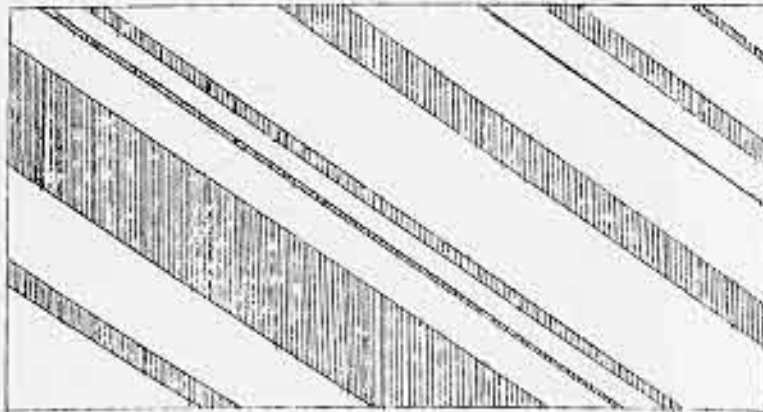


Figure 6. Strips of unequal width parallel to a base line.

central line, the most obvious extension imaginable but to my knowledge completely neglected, is apparent. Besides the strips of agricultural land use along

wilderness roads, perhaps littoral-using ocean animals are similarly arranged with a band of short flight birds toward the inside and salmon and seals in the farthest band.

We have accepted that much of the movement on the earth's surface can be explained by least-effort paths. Therefore, the shape of the space, the morphological pattern to the space, can yield infinite patterns of objects. If the space is symmetrically shaped, the objects often form beautiful patterns. Let us examine some of the patterns of flow due to fairly symmetric space twisting. The brilliant Beckmann has produced several "maps" showing the flow of economic goods with various twistings.<sup>14</sup> (Figure 7.) Prager, in the Beckmann tradition

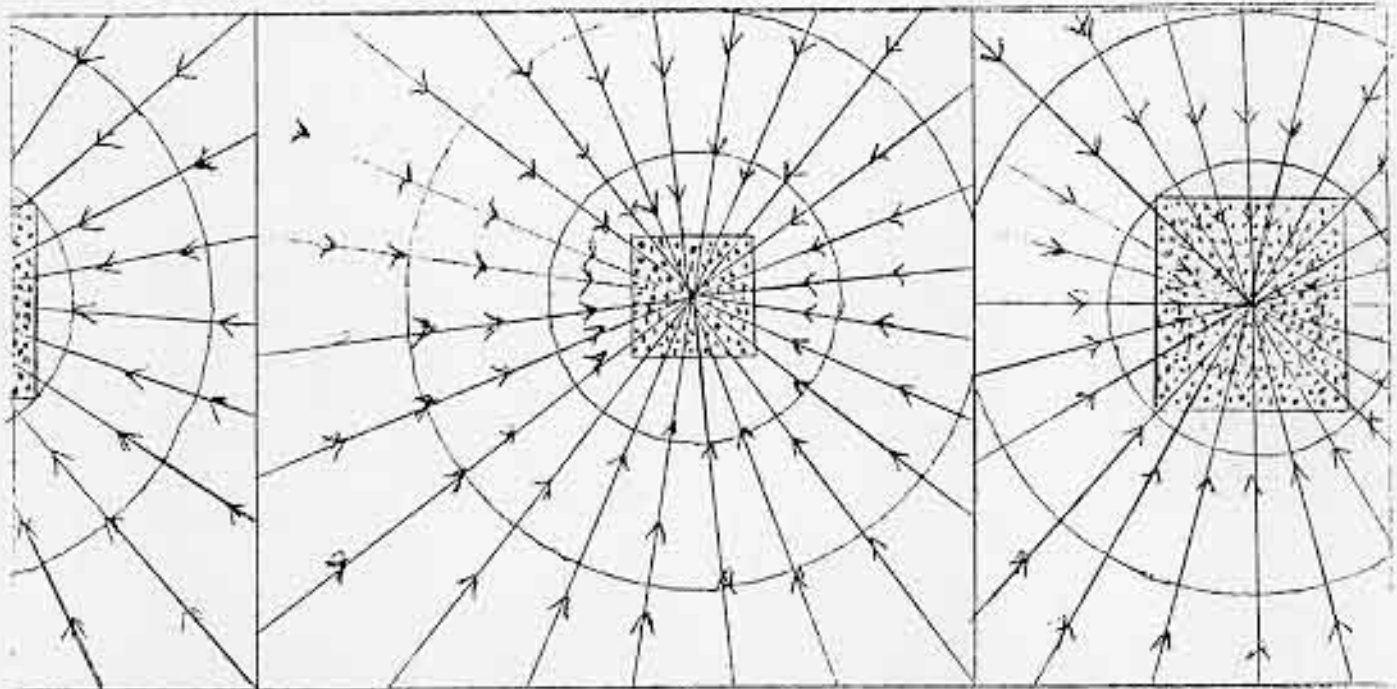


Figure 7. Beckmann patterns of flow. Stippled region indicates area of consumption.

has produced "maps" of telephone or any-other-type-of-network patterns.<sup>15</sup> (Fig. 8.)

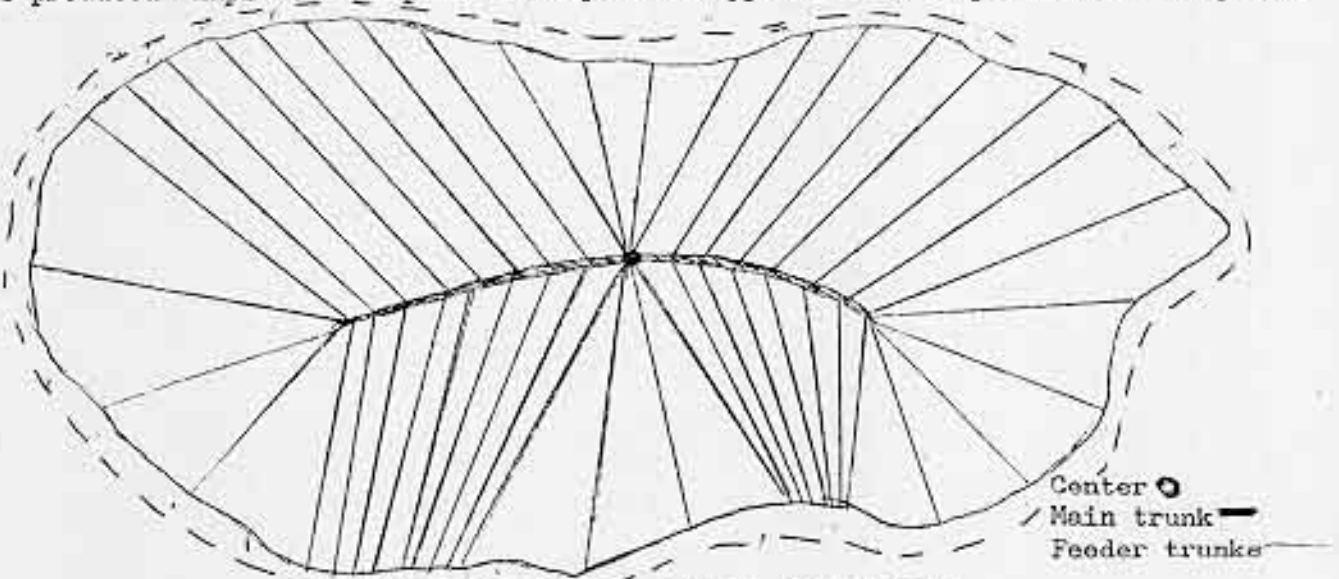


Figure 8. Prager pattern of networks.

There are several examples that are available to geography from our traditional area of interest in physical geography. . Figure 9 shows the pattern of wind flow

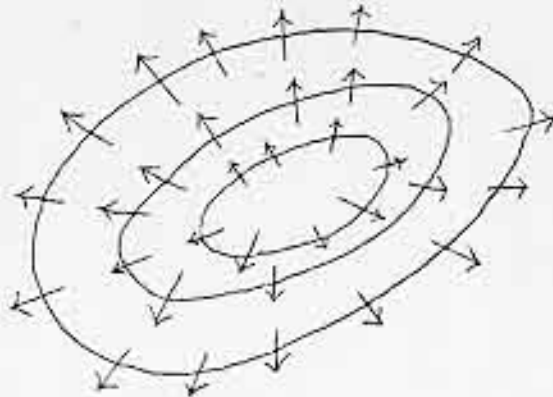
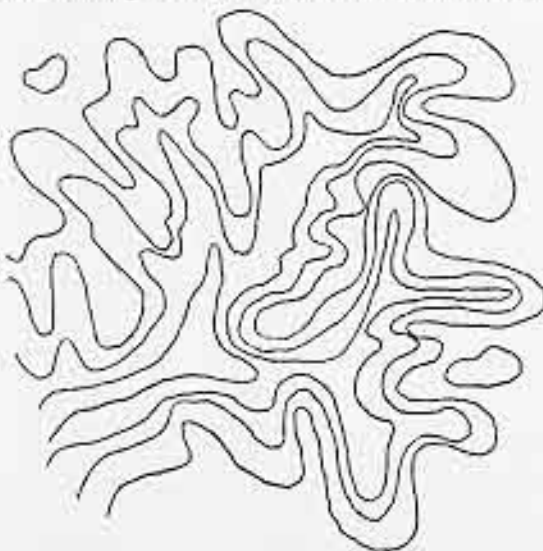


Figure 9. Wind pattern in a high, while discounting earth's rotation.  
discounting the effects of the earth's rotation. Figures 10a and 10b show a



10a Contours of Stream Valley Pattern    10b Velocity Vectors - Length of arrows  
proportional to slope.

typical stream pattern literally flowing down the potential surface. Maybe it occurred to traditional physical geographers that their slopes were merely space-twisters varying the transportability of water. Maybe it occurred to them that the flow of water over the terrain is a vector flow over the potential map. But after Lehman in 1874 presented the geographic world with the hachure map, why is it not generally known among geographers almost a hundred years later that his hachures were merely a crude vector representation of the potential (contour) dual? Flow diagrams such as streams, are density vectors as opposed to velocity vectors. Actually the density vectors can be computed from the velocity vectors but not the other way around. That is, Figure 10a is reproducible from Figure 10b but not from Figure 11.