Rules for Constructing Root Locus for $0 \leq K \leq \infty$:

![Block Diagram](image)

**Loop Transfer Function:**
\[ KGH = KG \frac{H_N}{G_D H_D} = K \frac{\prod_{i=1}^{n_p} (s - p_i)}{\prod_{i=1}^{n_z} (s - z_i)} \]

**Closed-loop Transfer Function:**
\[ \frac{KG}{1 + KGH} = \frac{\frac{K G_N}{G_D}}{1 + \frac{K G_N H_N}{G_D H_D}} = \frac{K G_N H_D}{G_D H_D + K G_N H_N} \]

**Characteristic Equation:**
\[ 1 + KGH = 0 \quad \text{OR} \quad G_D H_D + K G_N H_N = 0 \]

\[ \Rightarrow K = -\frac{1}{GH} = -\frac{G_D H_D}{G_N H_N} = \frac{\prod_{i=1}^{n_p} (s - p_i)}{\prod_{i=1}^{n_z} (s - z_i)} \]

\[ \Rightarrow \sum_{i=1}^{n_p} \text{angles}\{(s - p_i)\} - \sum_{i=1}^{n_z} \text{angles}\{(s - z_i)\} = \pm k(180^\circ), \quad k = 1, 3, 5, \ldots \quad \text{Angle Criterion} \]

and
\[ K = -\frac{\prod_{i=1}^{n_p} (s - p_i)}{\prod_{i=1}^{n_z} (s - z_i)} \quad \text{Magnitude Criterion} \]

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**Rule 1.** The root locus has $n = \max(n_p, n_z)$ branches where $n_p$ is the number of finite poles of the loop transfer function and $n_z$ is the number of finite zeros of the loop transfer function.

**Rule 2.** The root locus is symmetric about the real axis in the $s$-plane.

**Rule 3.** The root locus branches begin ($K = 0$) at the loop transfer function poles and end ($K = \infty$) on the loop transfer function zeros.

**Rule 4.** The root locus exists on the real axis if there is an odd number of poles and zeros (of the loop transfer function) to the right on the real axis.

**Rule 5.** The branches of the root locus which go off to the loop transfer function zeros at infinity approach asymptotically the straight lines with angles...
Angles of the asymptotes: \[ \theta = \frac{\pm k(180^\circ)}{n_p - n_z}, \quad k = 1, 3, 5, \ldots \]

These asymptotes intersect the real axis at

\[ \sigma = \sum_{i=1}^{n_p} p_i - \sum_{i=1}^{n_z} z_i \]

where \( p_i \) and \( z_i \) are respectively the finite poles and zeros of the loop transfer function. The number of asymptotes is equal to \( (n_p - n_z) \).

**Rule 6.** The root locus branches intersect the real axis at points where

\[ K = \frac{-1}{G(s)H(s)} \]

is at an extremum for real values of \( s \), i.e., for real values of \( s \) where

\[ \frac{d}{ds} \left( \frac{-1}{G(s)H(s)} \right) = 0. \]

**Rule 7.** \( j\omega \) – axis crossings may be determined by solving the characteristic equation with \( s = j\omega \).

**Rule 8 (a)** The root locus angle of departure from a complex pole \( p_c \) can be found by subtracting from \( 180^\circ \) the angle contribution at this pole of all other finite poles and adding the angle contribution of all finite zeros of the loop transfer function.

\[ \theta_{d|_{p_c}} = 180^\circ - \sum (\text{Angles of all other finite poles to } p_c) + \sum (\text{Angles of all finite zeros to } p_c) \]

**Rule 8 (b)** The root locus angle of arrival at a complex zero \( z_c \) can be found by adding to \( -180^\circ \) the angle contribution at this zero of all finite poles and subtracting the angle contribution of all other finite zeros of the loop transfer function.

\[ \theta_{a|_{z_c}} = -180^\circ + \sum (\text{Angles of all finite poles to } z_c) - \sum (\text{Angles of all other finite zeros to } z_c) \]

**Rule 9.** The value of \( K \) corresponding to a point \( s = s_r \) on the root locus is determined by using the magnitude criterion:

\[ K = \frac{\prod_{i=1}^{n_p} |s_r - p_i|}{\prod_{i=1}^{n_z} |s_r - z_i|} \]
In case of systems with positive feedback instead of negative feedback:

Closed-loop Transfer Function:

\[
\frac{KG}{1 - KGH} = \frac{\frac{KG_N}{G_D}}{1 - \frac{KG_N H_N}{G_D H_D}} = \frac{KG_N H_D}{G_D H_D - KG_N H_N}
\]

In Rule 4, replace odd by even.

In Rule 5, change \(\theta\) to:

\[
\theta = \frac{\pm k(360^\circ)}{n_p - n_z}, \quad k = 1, 2, 3, ...
\]

In Rule 8, change 180° to 360°.