Example 2.21

Transfer function—system with lossless gears

**Problem** Find the transfer function, \( \theta_2(s)/T_1(s) \), for the system of Figure 2.30(a).

**Figure 2.30**

a. Rotational mechanical system with gears;
b. system after reflection of torques and impedances to the output shaft;
c. block diagram

\[
\begin{align*}
T_1(t) & \quad \theta_1(t) \\
N_1 & \quad \theta_2(t) \\
J_1 & \\
D_1 & \\
J_2 & \\
K_2 & \\
(a) &
\end{align*}
\]

\[
\begin{align*}
T_1(t) \left( \frac{N_2}{N_1} \right) & \quad \theta_2(t) \\
D_e & = D_1 \left( \frac{N_2}{N_1} \right)^2 + D_2; \\
J_e & = J_1 \left( \frac{N_2}{N_1} \right)^2 + J_2; \\
K_e & = K_2 \\
(b) &
\end{align*}
\]

\[
\begin{align*}
T_1(s) & \quad \frac{N_2/N_1}{J_e s^2 + D_e s + K_e} & \quad \theta_2(s) \\
(c) &
\end{align*}
\]

**Solution** It may be tempting at this point to search for two simultaneous equations corresponding to each inertia. The inertias, however, do not undergo linearly independent motion, since they are tied together by the gears. Thus, there is only one degree of freedom and hence one equation of motion.

Let us first reflect the impedances \((J_1\text{ and } D_1)\) and torque \((T_1)\) on the input shaft to the output as shown in Figure 2.30(b), where the impedances are reflected by \((N_2/N_1)^2\) and the torque is reflected by \((N_2/N_1)\). The equation of motion can now be written as

\[
(J_e s^2 + D_e s + K_e) \theta_2(s) - T_1(s) \frac{N_2}{N_1} = 0 \tag{2.139}
\]

where

\[
J_e = J_1 \left( \frac{N_2}{N_1} \right)^2 + J_2; \quad D_e = D_1 \left( \frac{N_2}{N_1} \right)^2 + D_2; \quad K_e = K_2
\]

Solving for \(\theta_2(s)/T_1(s)\), the transfer function is found to be

\[
G(s) = \frac{\theta_2(s)}{T_1(s)} = \frac{N_2/N_1}{J_e s^2 + D_e s + K_e} \tag{2.140}
\]

as shown in Figure 2.30(c).