Name:

Honor Code: KEY – C

Instructions:

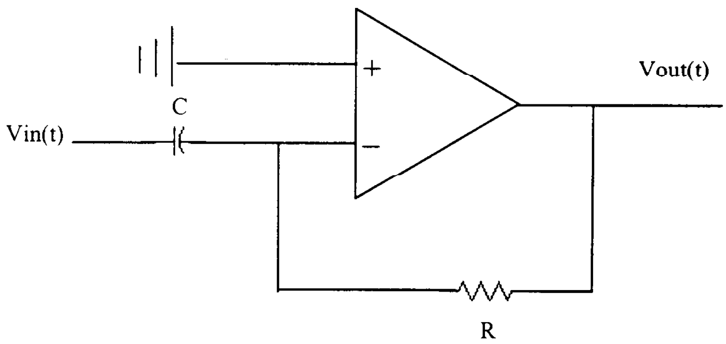
- Use the space on the accompanying pages to work the problems. Do not use a bluebook. Attach additional worksheets if necessary.
- If you wish to have partial credit awarded for any of your incorrect answers **you must write clearly and legibly.** Explain your work in words, if necessary.
- Read the instructions provided with each problem.
- Don't Panic.

1.

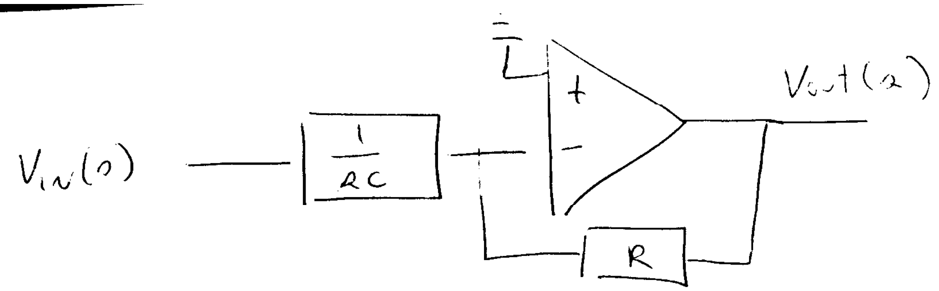
(a) [5 points] Find the Transfer Function Vout(s)/Vin(s) for the following system.

(b) [5 points] Determine and plot the step response.

(c) [5 points] Find Vout(t) when Vin(t)=cos 10πt.



a)

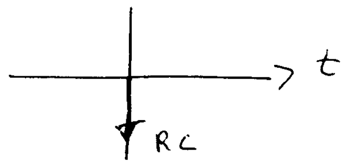


$$\frac{0 - V_{in}(\Omega)}{1/\Omega C} + \frac{0 - V_{out}(\Omega)}{R} = 0$$

$$\frac{V_{out}(\Omega)}{V_{in}(\Omega)} = \frac{-R}{1/\Omega C} = \boxed{-R\Omega C}$$

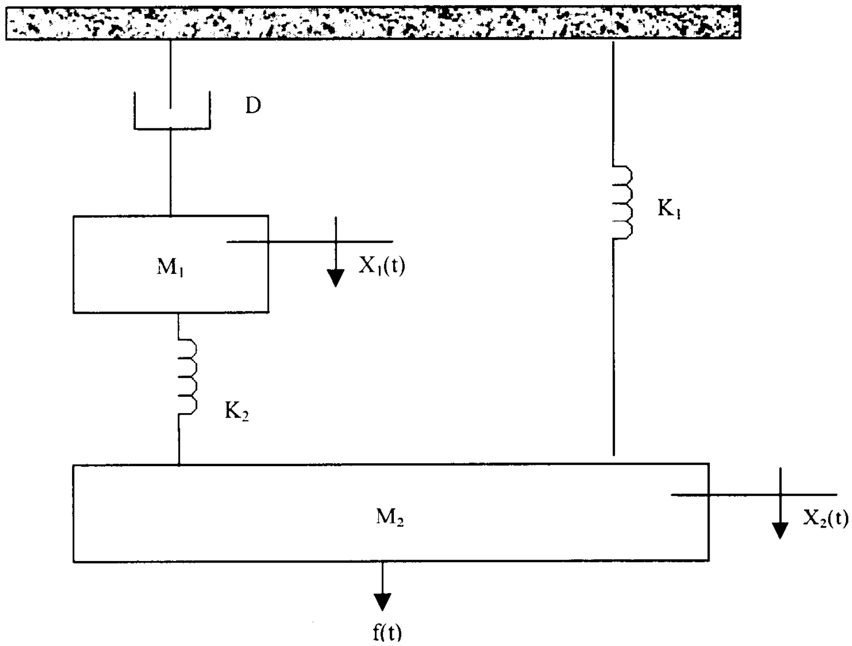
b) $\text{Output}(\Omega) = -R\Omega C \cdot \frac{1}{\Omega} = -RC$

$\text{output}(t) = -RC\,\delta(t)$



c) $\text{output}(\Omega) = -R\Omega C \cdot \frac{\Omega}{\Omega^2 + (10\pi)^2} = -RC\,\frac{\Omega^2}{\Omega^2 + (10\pi)^2}$

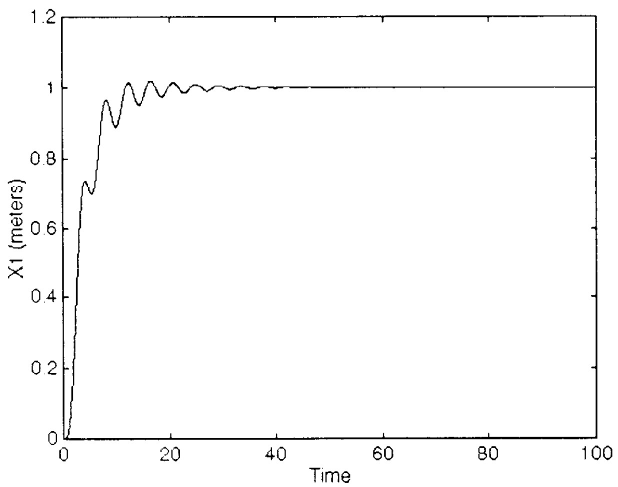
2.
(a) [15 points] Find the Transfer Function $X_1(s)/F(s)$



(b) [2 points] Let $M_1=M_2=1$, $K_1=K_2=1$, and $D=2$. Use the initial value theorem to show that when excited by a step input, $x_1(t)\rightarrow 0$ as $t\rightarrow 0$.

(c) [2 points] Use the final value theorem to show that $x_1(t)\rightarrow 1$ as $t\rightarrow\infty$ when $f(t)=u(t)$.

(d) [6 points] The following graph shows the step response of the system. Explain why it looks the way it does in two or three sentences.



(a)

FBD:

$\boxed{M_1}$ $\uparrow M_1 s^2 X_1(s)$ $\uparrow D s X_1(s)$ $\uparrow K_2(X_1(s) - X_2(s))$

(1) $$X_1(s)\left[s^2 M_1 + D s + K_2\right] + X_2(s)\left[-K_2\right] = 0$$

$F(s)\downarrow$ $\boxed{M_2}$ $\uparrow M_2 s^2 X_2(s)$ $\uparrow K_1 X_2(s)$ $\uparrow K_2(X_2(s) - X_1(s))$

(2) $$F(s) = X_1(s)\left[-K_2\right] + X_2(s)\left[M_2 s^2 + K_1 + K_2\right]$$

From (1) $$X_2(s) = X_1(s)\left[\frac{M_1 s^2 + D s + K_2}{K_2}\right]$$

$$\Rightarrow F(s) = X_1(s)\left[\left[-K_2\right] + \left[\frac{M_1 s^2 + D s + K_2}{K_2}\right]\left[M_2 s^2 + K_1 + K_2\right]\right]$$

$$\frac{X_1(s)}{F(s)} = \frac{1}{-K_2 + \frac{(M_1 s^2 + D s + K_2)}{K_2}(M_2 s^2 + K_1 + K_2)}$$

$$= \frac{K_2}{(M_1 s^2 + D s + K_2)(M_2 s^2 + K_1 + K_2) - K_2^2}$$

(c) $$\lim_{s\to 0} s\,G(s) = \frac{K_2}{(K_2)(K_1 + K_2) - K_2^2} = \frac{K_2}{K_1 K_2} = \underline{1}$$

(b) $$\lim_{s\to \infty} s\,G(s) = \underline{0}$$

d) The force is applied starting at $t=0$. The block responds by moving down quickly. The springs and dampers then pull back causing the mass to recoil. As time goes on, eventually the block settles at position $= 1$.

3. [20 points] Answer the following 10 questions True or False.

Answer true if and only if the system is stable for each of the closed loop denominators.

T i) Denominator(s)=s^2+3s+2

F ii) Denominator(s)=(s-1)(-s^3+4s^2-2s+1)

F iii) Denominator(s)=(s+1)(s+2)(-4s^2+4s+3)

F iv) Denominator(s)=(s+1)(s+2)(-2s^2-4s+3)

Answer the following second order systems questions true of false

T v) A CLTF with denominator s^2+3s+12 is underdamped

T vi) It is possible to choose K in to get 10% overshoot in a system with CLTF s^2+3s+K.

Answer the following partial fraction expansion questions true or false

F vii)

$$\frac{(s+1)}{s^2(s+2)} = \frac{1.25}{s} + \frac{.5}{s^2} + \frac{-.25}{s+2}$$

F viii)

$$\frac{(s+1)}{s(s+2)} = \frac{.25}{s} + \frac{-.25}{s+2}$$

F ix)

$$\frac{(s+1)}{s(s^2+2s+2)} = \frac{1}{s} + \frac{-1}{s+1}$$

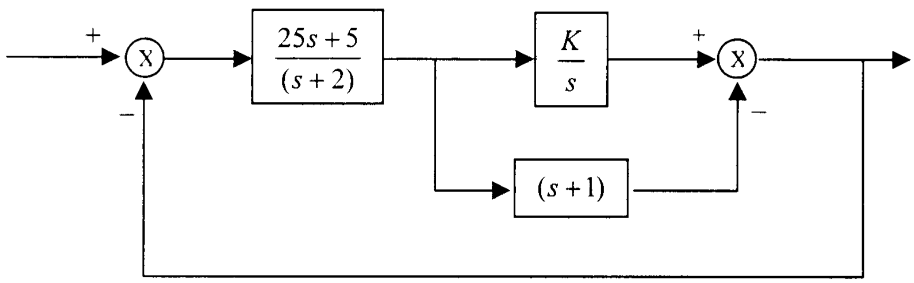
F x) The inverse Laplace Transform of

$$\frac{(s+1)}{s(s^2+2s-2)}$$

Includes an $e^{-at}\cos(\omega t)$ term for some ω and a.

4.

(a) [16 points] Find the range of K for stability in the following system
(b) [2 points] *Roughly* sketch the step response for K=-100 [use your results from (a) as a guide].
(c) [2 points] *Roughly* sketch the step response for K=+100 [use your results from (a) as a guide].



Parallel $\quad \frac{K}{s} - (s+1) = \frac{K - s(s+1)}{s} = \frac{K - s^2 - s}{s}$

Cascade $\quad \left(\frac{25s+5}{s+2}\right)\left(\frac{K - s^2 - s}{s}\right) = \frac{(25s+5)(K-s^2-s)}{s(s+2)}$

Feedback $\quad \frac{(25s+5)(K-s^2-s)}{s(s+2)+(25s+5)(K-s^2-s)}$

$$= \frac{(25s+5)(K-s^2-s)}{(s^2+2s)+(25Ks-25s^3-25s^2+5K-5s^2-5s)}$$

$$= \frac{(25s+5)(K-s^2-s)}{s^3\left[-25\right]+s^2\left[-29\right]+s\left[-3+25K\right]+5K}$$

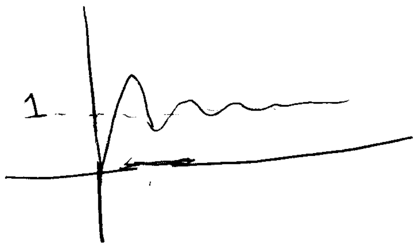
R-H table

s^3	-25	$-3+25K$
s^2	-29	$5K$
s^1	$\frac{600K}{29}-3$	
s^0	$5K$	

$\frac{600K}{29}-3<0$ $\qquad K<\frac{87}{600}$

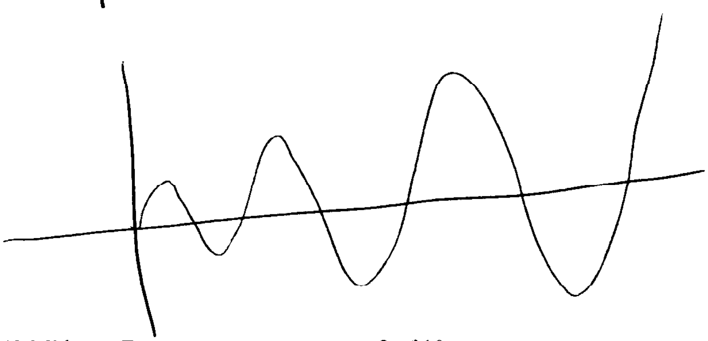
$5K<0$ $\qquad \boxed{K<0}$

b) Stable

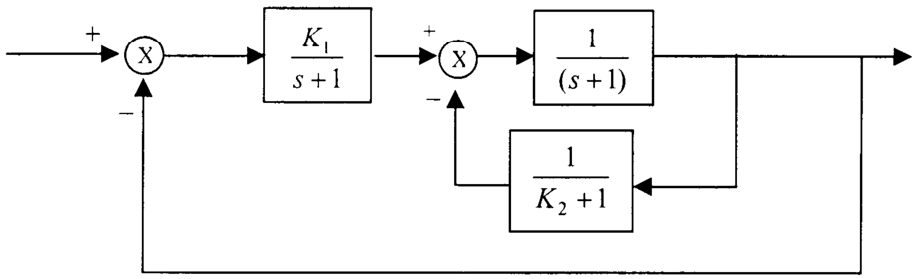


Final value = 1
via FV theorem

c) Unstable



5. Consider the following system



(a) [8 points] Write the Closed Loop Transfer Function.

(b) [6 points] Find all relevant second-order parameters of the system when $K_1=1$ and $K_2=2$ and sketch the output when the system is excited by a step input. Show that the system is stable.

(c) [6 points] Repeat (b) for $K_1=2$ and $K_2=1$. Show that the system is stable.

(a)

Feedback

$$\frac{\frac{1}{s+1}}{1+\left(\frac{1}{s+1}\right)\left(\frac{1}{K_2+1}\right)} \times \frac{(s+1)(K_2+1)}{(s+1)(K_2+1)}$$

$$= \frac{K_2+1}{(s+1)(K_2+1)+1} = \frac{K_2+1}{K_2 s + s + K_2 + 2}$$

Series

$$\left(\frac{K_1}{s+1}\right)\left(\frac{K_2+1}{s(K_2+1)+(K_2+2)}\right) = \frac{K_1(K_2+1)}{s^2[K_2+1]+s[2K_2+3]+[K_2+2]}$$

Feedback

$$\frac{K_1(K_2+1)}{s^2[K_2+1]+s[2K_2+3]+[K_1K_2+K_1+K_2+2]}$$

b) $\quad TF = \dfrac{3}{3s^2 + 7s + 7} = \dfrac{1}{s^2 + \frac{7}{3}s + \frac{7}{3}}$

$\omega_n^2 = \frac{7}{3}, \quad 2\zeta\omega_n = \frac{7}{3}$

$\Rightarrow \boxed{OS = 2.437\% \quad T_s = 3.430 \quad T_p = 3.19}$

stability

s^2	3	7
s^1	7	
s^0	7	

or

s^2	1	7/3
s^1	7/3	
s^0	7/3	

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c) $\quad TF = \dfrac{4}{2s^2 + 5s + 7} = \dfrac{2}{s^2 + \frac{5}{2}s + \frac{7}{2}}$

$\omega_n^2 = \frac{7}{2}, \quad 2\zeta\omega_n = \frac{5}{2}$

$\Rightarrow \boxed{OS = 5.45\% \quad T_s = 3.20 \quad T_p = 2.32}$

Stability

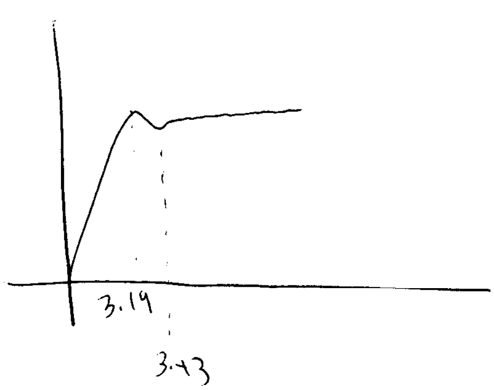
s^2	2	7
s^1	5	
s^0	7	

or

s^2	1	7/2
s^1	5/2	
s^0	7/2	

graphs

(a)



(b)